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Determination of sensitivity vectors in hologram interferometry from two known rotations of the object

Ryszard J. Pryputniewicz and Karl A. Stetson

The fringes generated in holographic interferograms by independent rotations of an object are used to determine the sensitivity vectors of a hologram recording setup. This is done by identifying the fringes observed on the surface of a 3-D object with a fringe vector, which, in turn, equals the vector product of the sensitivity vector with object rotation. These vector relationships are formulated in terms of the projection matrices, and a least square error solution is derived that extracts the sensitivity vectors from the fringe data from two or more known rotations.

1. Introduction

In the holographic analysis of body deformations, both the illumination \( K_1 \) and the observation \( K_2 \) directions must be known before the sensitivity vector, \( K = K_2 - K_1 \), can be computed (Fig. 1). The process of determining these quantities is both inaccurate and time-consuming when done with standard measuring instruments, particularly when several of the holograms (with the same recording and observation geometries) are to be analyzed. Further complications arise (1) for objects of complicated 3-D shape, (2) for wide-angle or multiple illumination directions, (3) when mirrors are used for back or side views of the tested objects [often needed to satisfy condition (1)], etc.

If an object is illuminated from a stationary point source of light, two (or more) double-exposure (calibration) holograms can be made, each recording a different but known rotation of the object. Then the sensitivity vectors may be determined directly from these holograms, ensuring that each recorded hologram is analyzed from a number of the same directions of observation. These sensitivity vectors can, in turn, be used to analyze (data) holograms recording unknown motions of an object, providing the illumination and observation geometries remain unchanged for all the (calibration and data) holograms.

It was shown previously\(^1\) that three independent rotations are necessary to provide pertinent information to calculate sensitivity vectors. However, this study indicates that the vector relationships between the fringes observed on the surface of a 3-D object, the sensitivity vectors, and object rotation may be reformulated in terms of the projection of the sensitivity vector onto a plane perpendicular to the object rotation. In this form it becomes apparent that even two object rotations provide more data than are necessary to solve for the sensitivity vector.

II. Theory

If an object undergoes a rotation about any axis, the fringes generated appear to be the lines resulting from the intersection of equidistant laminae with the object's surface; for example, Fig. 2 shows the object that rotated about a vertical axis located 15 cm away from its center. The laminae may be described by fringe vector\(^2\) \( K_f \), whose magnitude is inversely proportional to the normal distance between these laminae and whose direction coincides with the direction of this normal and points toward a fringe lamina of higher order. Fringe vector \( K_f \) may also be expressed as a vector product of the sensitivity and rotation vectors\(^1,2\) \( K \) and \( \theta \), respectively, that is,

\[
K_f = -\theta \times K. \tag{1}
\]

The rotation vector \( \theta \) appearing in Eq. (1) can be written as a product of the rotation magnitude \( |\theta| \), which is scalar, with the rotation unit vector \( \hat{\theta} \), as follows:

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\[ \mathbf{K} = \begin{pmatrix} K_x \\ K_y \\ K_z \end{pmatrix} \]
sensitivity vectors one at a time is rather cumbersome, and, therefore, it would be advantageous to develop an equation that allows calculation of all the necessary sensitivity vectors at the same time. This can be done in an elegant way by noting that observation of a holographic interferogram (recording rotation \( \theta_i \)) from any direction \( m = 1, 2, \ldots, r \), where \( r \) is the total number of observations, defined by a sensitivity vector \( \mathbf{K}^m \), as shown in Fig. 3, is accompanied by a corresponding change in the observed fringe pattern; this fringe pattern is uniquely defined by the \( m \)th fringe vector \( \mathbf{K}'^m \) corresponding to the \( i \)th rotation. Therefore, for each of the \( m \) observations we can write an equation of the type of Eq. (9). Since rotation vectors \( \theta_i \) and projection matrices \( \mathbf{P}_\theta \) are common to all these equations, we can write them as

\[
(\hat{\theta}_i \times \mathbf{K}^m)/|\theta_i| = \mathbf{P}_\theta \mathbf{K}^m, \quad i = 1, 2, \ldots, n\]
\( m = 1, 2, \ldots, r \) \hspace{1cm} (10)

The term \( \mathbf{P}_\theta \mathbf{K}^m \) on the right-hand side of Eq. (10) indicates the projection of the \( m \)th sensitivity vector \( \mathbf{K}^m \) onto a plane perpendicular to the \( i \)th object rotation \( \theta_i \). Equation (10) can be expanded first with respect to the number of rotations, that is, \( i = 1, 2, \ldots, n \), yielding

\[
\begin{bmatrix}
\hat{\theta}_1 \times \mathbf{K}^1 \\
\hat{\theta}_2 \times \mathbf{K}^2 \\
\vdots \\
\hat{\theta}_n \times \mathbf{K}^n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{P}_{\theta_1} \\
\mathbf{P}_{\theta_2} \\
\vdots \\
\mathbf{P}_{\theta_n}
\end{bmatrix} \mathbf{K}^n, \quad m = 1, 2, \ldots, r
\]

then, with respect to the number of observations, that is, \( m = 1, 2, \ldots, r \), which results in

\[
\begin{bmatrix}
\hat{\theta}_1 \times \mathbf{K}_1 \\
\hat{\theta}_2 \times \mathbf{K}_2 \\
\vdots \\
\hat{\theta}_n \times \mathbf{K}_n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{P}_{\theta_1} \\
\mathbf{P}_{\theta_2} \\
\vdots \\
\mathbf{P}_{\theta_n}
\end{bmatrix} \begin{bmatrix}
\mathbf{K}^1 \\
\mathbf{K}^2 \\
\vdots \\
\mathbf{K}^r
\end{bmatrix}. \hspace{1cm} (11)
\]

Equation (11), in turn, may be written in a more condensed form as

\[
\Gamma = \mathbf{PK}, \hspace{1cm} (12)
\]

where we have defined

\[
\Gamma = \begin{bmatrix}
\hat{\theta}_1 \times \mathbf{K}_1 \\
\hat{\theta}_2 \times \mathbf{K}_2 \\
\vdots \\
\hat{\theta}_n \times \mathbf{K}_n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{P}_{\theta_1} \\
\mathbf{P}_{\theta_2} \\
\vdots \\
\mathbf{P}_{\theta_n}
\end{bmatrix} \begin{bmatrix}
\mathbf{K}^1 \\
\mathbf{K}^2 \\
\vdots \\
\mathbf{K}^r
\end{bmatrix}. \hspace{1cm} (13)
\]

Premultiplying both sides of Eq. (12) by a transpose of rectangular matrix \( \mathbf{P} \) we obtain

\[
\mathbf{P}^T \Gamma = \mathbf{P}^T \mathbf{PK}. \hspace{1cm} (16)
\]

This procedure reduces the rectangular \( 3n \times 3 \) matrix \( \mathbf{P} \) to a square \( 3 \times 3 \) matrix \( \mathbf{P}^T \mathbf{P} \). The resulting Eq. (16) may then be solved for matrix \( \mathbf{K} \) of the sensitivity vectors, which has the least square error

\[
\mathbf{K} = [\mathbf{P}^T \mathbf{P}]^{-1} \mathbf{P}^T \mathbf{PK}. \hspace{1cm} (17)
\]

Studying the above vector relationships, formulated in terms of the projections of the sensitivity vector onto a plane perpendicular to the object rotation, it becomes apparent that even two object rotations provide more data than are necessary to solve for the sensitivity vectors. In the case of only two object rotations Eq. (11) becomes

\[
\begin{bmatrix}
\hat{\theta}_1 \times \mathbf{K}_1 \\
\hat{\theta}_2 \times \mathbf{K}_2 \\
\hat{\theta}_n \times \mathbf{K}_n
\end{bmatrix}
= \begin{bmatrix}
\mathbf{P}_{\theta_1} \\
\mathbf{P}_{\theta_2} \\
\vdots \\
\mathbf{P}_{\theta_n}
\end{bmatrix} \begin{bmatrix}
\mathbf{K}^1 \\
\mathbf{K}^2 \\
\vdots \\
\mathbf{K}^r
\end{bmatrix}. \hspace{1cm} (18)
\]

Equation (18) can readily be solved to obtain the sensi-
sitivity vectors $K_1, K_2, \ldots, K_r$ by the least squares procedure shown in Eq. (17).

III. Procedure, Results, and Discussion

To implement the above procedure for determination of the sensitivity vectors, a computer program was developed to solve Eq. (17), and a sample calculation was made. In this calculation, two independent rotations

$$\theta_1 = 46^\circ,$$

$$\theta_2 = 8^\circ + 32^\circ,$$

where coefficients are rotations in microradians, were recorded on two separate double-exposure holograms, respectively. For both of these recordings, the object was illuminated from a point source defined by a space vector

$$R_1 = 499.51 + 440.25k,$$

where coefficients are coordinates of a point source of illumination given in millimeters with respect to the origin of a coordinate system (Fig. 1). After processing, each reconstructed image was observed from five different points defined by the space vectors

$$R_1 = 3.51 + 27^\circ + 177^\circ,$$
$$R_2 = -32.51 + 28^\circ + 177^\circ,$$
$$R_3 = 31^\circ + 24.5^\circ + 177^\circ,$$
$$R_4 = -28.51 - 27^\circ + 177^\circ,$$
$$R_5 = -38.51 - 27^\circ + 177^\circ.$$

Corresponding to these observations, there were two sets of five fringe vectors (in mm$^{-1}$), one for each of the two independent rotations:

$$K_1 = -0.759i + 0.352j + 0.262k,$$
$$K_2 = -0.746i + 0.241j + 0.401k,$$
$$K_3 = -0.744i + 0.246k,$$
$$K_4 = -0.528i + 0.132j + 0.244k,$$
$$K_5 = -0.519i + 0.130j + 0.170k,$$
$$K_6 = -0.520i + 0.130j + 0.282k,$$
$$K_7 = -0.522i + 0.130j + 0.291k,$$
$$K_8 = -0.518i + 0.129j + 0.182k.$$

The fringe vectors given in Eqs. (23) and (24) were calculated using procedures developed in Refs. 4 and 5. The necessary parameters were obtained by digitization of photographs of holographically reconstructed images taken along the directions of observation through points (on a hologram) specified in Eq. (22).

The sensitivity vectors $K_0$ (in mm$^{-1}$) computed from Eq. (17), using rotations given in Eqs. (19) and (20) and the corresponding fringe vectors given in Eqs. (23) and (24), respectively, were

$$K_0 = 7652i + 109j + 16,500k,$$
$$K_1 = 5696i + 1533j + 16,219k,$$
$$K_2 = 9152i + 1359j + 16,255k,$$
$$K_3 = 8717i - 1505j + 16,507k,$$
$$K_4 = 5348i - 1359j + 16,176k.$$

On the other hand, using the space vectors given by Eqs. (21) and (22), sensitivity vectors $K_R$ were computed to be

$$K_0 = 7645i + 112j + 16,492k,$$
$$K_1 = 5703i + 1527j + 16,220k,$$
$$K_2 = 9146i + 1341j + 16,256k,$$
$$K_3 = 8714i - 1485j + 16,301k,$$
$$K_4 = 5358i - 1358j + 16,176k.$$

To compare the resulting sensitivity vectors shown in Eq. (25) with those given in Eq. (26), let us define error $E$ between $K_0$ and $K_R$ as the difference between their magnitudes $|K_0|$ and $|K_R|$, respectively, divided by the magnitude $|K_0|$, that is,

$$E_m = \left( |K_0| - |K_R| \right) / |K_0|.$$

Then, using Eq. (27) we obtain a set of the following five numbers:

$$E^1 = 0.056\%,$$
$$E^2 = 0.016\%,$$
$$E^3 = 0.018\%,$$
$$E^4 = 0.045\%,$$
$$E^5 = 0.001\%.$$

giving the percentage errors between $K_0$ and $K_R$. From Eqs. (28) it is obvious that the largest error between sensitivity vectors $K_0$ (computed from rotations) and sensitivity vectors $K_R$ (based on the actual space vectors) is <0.06%.

IV. Summary

The method presented for determination of the sensitivity vectors directly from holograms is very accurate and, no doubt, will prove very useful in holographic analysis. For example, if mirrors are employed to obtain side or back views of the object, the very cumbersome and inaccurate usual procedures for determining illumination and observation directions can be eliminated by the procedure presented in this paper.

The main problem remaining, however, is to choose the technique for performing the observations through the hologram. This can vary from the simple observation through the hologram with the naked eye to the sophisticated approach using the four-degrees-of-freedom positioners carrying the photographic or TV cameras. The required positioners would need adjustments in the vertical and horizontal directions (e.g., parallel to the plane of the hologram) and must be free to rotate with respect to these axes. The photographs

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or TV images obtained in such a way would be digitized, and the corresponding sensitivity vectors are obtained using a relatively straightforward computer program. These results in turn can be integrated with the interactive computer graphics procedure to solve for unknown rigid-body motions and deformations of the object.

Portions of this paper were presented at the Annual Meeting of the Optical Society of America at Rochester, New York, Oct. 1979.

References

Meetings Calendar continued from page 2195

1981
April
6-9 ISA/81 Industry Oriented Conf. & Exhibit, St. Louis ISA/81 St. Louis, Instrument Soc. of America, 67 Alexander Dr., P.O. Box 12277, Research Triangle Park, NC 27709
6-10 6th Inter. Symp.: Noise in Physical Systems, NBS, Gaithersburg, R. Mountain, N.M. Chairman, Los Alamos Conf. on Optics '81, Los Alamos Scientific Lab., E-10, MS 430, P.O. Box 1663, Los Alamos, N.M. 87545
7-10 Integrated Optics and Optical Fiber Communication, 3rd Int. Conf., San Francisco OSA, 1816 Jefferson Pl. N.W., Washington, D.C. 20036

June
7-12 8th Canadian Congress of Applied Mechanics, Moncton, Canada N. Srivastava, Faculty of Sci. & Eng., Université de Moncton, Moncton, N.B., E1A 3E9, Canada
8-12 Int. Conf. on Fourier Transform Infrared Spectroscopy, S. Carolina U J. Lehhardt, Philip Morris R&D, P.O. Box 26583, Richmond, Va. 23261
8-12 2nd Inter. Conf. on Precision Measurement and Fundamental Constants, NBS, Gaithersburg B. Taylor, B-298, Metrology Bldg., NBS, Wash., D.C. 20234
15-19 Int. Joint Conf. on Thermophysical Properties, NBS, Gaithersburg A. Cezairiyan, Rm. 124, Hazards Bldg., NBS, Wash., D.C. 20234

July
1-7 Int. Conf. on Luminescence, West Berlin AIP, 335 E. 45 St., New York, N.Y. 10017

August
31-5 Sept. ICO-12, Graz, Austria S. S. Ballard, Phys. Dept., U. Fla., Gainesville, Fla. 32611

September
8-11 7th European Conf. on Optical Communication, Copenhagen M. Danielsen, Tech. U. of Denmark, Electromag. Inst., Bldg. 348, DK-2800, Lyngby, Denmark
21-25 5th Int. Thin Films Congress, Israel Y. Shapira, School of Eng., Tel-Aviv U., Ramat Aviv, Israel

October

November
2-6 APS Div. of Plasma Physics, Washington, D.C. W. W. Havens, Jr., 335 E. 45 St., New York, N.Y. 10017
23-25 APS Div. of Fluid Dynamics, Monterey, Calif. W. W. Havens, Jr., 335 E. 45 St., New York, N.Y. 10017

1982
March

April

September

October