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Evaluation of angle-dependent spectral distortion for infinite, planar elastic media via angular spectrum decomposition

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The angle-dependent spectral distortion for an infinite planar interface (IP-ASD) is examined for the case of fluid–fluid and fluid–solid (elastic) interfaces. The results are based on computation of the angular spectrum (plane wave) decomposition of the acoustic pressure field as radiated by an equivalent image source transducer, for each frequency of interest. The exact incident-angle-dependent reflection coefficient of the interface may thus be used in the computation, versus approximate methods previously required [D. P. Orofino and P. C. Pedersen, J. Acoust. Soc. Am. 92, 2883–2899 (1992)]. Comparisons of the previous approximate IP-ASD results to the current exact results are made. IP-ASD results for elastic parallel plates are also presented for a variety of plate orientations and geometries. For the special case of normal incidence, a comparison of the IP-ASD result to a previously published result is made. The IP-ASD is additionally examined for a reduced subset of plane wave components that approximate the radiated pressure field. The error in the IP-ASD versus the number of plane waves in the subset is discussed.

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LIST OF SYMBOLS

- \( a \): radius of source transducer
- \( A \): complex amplitude of \( q \)th plane wave component of angular spectrum decomposition of source pressure field
- \( b_2 \): shear (transverse) wave propagation velocity in elastic medium 2
- \( c_i \): longitudinal (compressional) wave propagation velocity in medium \( i \), \( i = 1, 2 \)
- \( k, k' \): wave number of shear and longitudinal waves, respectively
- \( k_\perp, k_y, k_z \): Cartesian components of wave vector \( k \)
- \( \text{OP} \): vector extending from the origin of the \((x,y,z)\) coordinate system to the origin of the \((u,v,w)\) coordinate system.
- \( \tilde{p}(r) \): acoustic pressure
- \( \tilde{P}_i, \tilde{P}_r, \tilde{P}_t \): incident, reflected, and transmitted (refracted) plane waves
- \( Q \): vector extending from the origin of the \((u,v,w)\) coordinate system, \( P \), to locations on the receiver surface.
- \( \tilde{R}_L \): complex longitudinal acoustic reflection coefficient of interface
- \( r_i \): acoustic impedance of medium \( i \), \( r_i = \rho_i c_i \), \( i = 1, 2 \)
- \( \hat{s} \): outward unit normal vector to the planar interface
- \( \tilde{\nu}_q(\omega) \): unit vectors in receiver (physical) transducer Cartesian coordinate system
- \( \tilde{\nu}_q(\omega) \): unit vectors in image transducer Cartesian coordinate system
- \( (x_\perp, y_\perp, z_\perp) \): unit vectors in receiver (physical) transducer Cartesian coordinate system
- \( (x, y, z) \): unit vectors in image transducer Cartesian coordinate system
- \( \omega_0 \): temporal frequency of acoustic pressure field
- \( n_x, n_y, n_z \): direction cosines of propagating plane wave density of medium \( i \), \( i = 1, 2 \)
- \( \rho_1 \): density of medium \( i \), \( i = 1, 2 \)
- \( \theta_i \): angles of incident, reflected, or refracted plane wave components of the field, \( i = 1, 2, 3 \)

INTRODUCTION

The angle-dependent spectral distortion (ASD) refers to the spectral content of the electrical output signal produced by a transducer with given geometry and infinite bandwidth, operating in the pulse-echo mode, when sonifying a planar reflecting interface with a given angular orientation. All other problem parameters (such as distance to interface, acoustic parameters of interface layers, etc.) are assumed to be known. A complete description of the radiation and reception characteristics of the transducer must be numerically obtainable, as must be the effects of the material interface on the reflected field. When the interface is assumed to be of infinite extent and planar, the problem can be further categorized as IP-ASD. Referring to Fig. 1, the planar interface formed between two dissimilar acoustic media...
Dia is misaligned by a small angle $\alpha$ relative to the plane of the source transducer.

The IP-ASD is appropriate not only for the direct problem of calculating the transducer output signal, but has been shown to be applicable to inverse problems, e.g., extracting the interface misalignment angle. Once the IP-ASD spectrum has been determined for a particular measurement situation, the spectral distortion can be removed from the measurement (using an appropriate deconvolution), leading to more accurate quantitative ultrasonic measurements of the material properties of interest than otherwise possible. This application requires the IP-ASD to be accurately obtainable in both magnitude and phase, and allows minimal approximation to be employed in its solution.

A detailed discussion of IP-ASD has previously been published, and was developed for the specific case of a fluid–fluid planar interface of infinite extent. In this prior investigation, the determination of the IP-ASD was based on integration of the reflected pressure field from its calculation at discrete points on the surface of the receiving transducer. The complex pressure field was calculated by an efficient numerical technique. This approach did not, however, permit incorporation of the exact, angle-dependent interface reflection coefficient. Instead, a constant was used to approximate the reflection coefficient. Instead, a constant was used to approximate the reflection coefficient, and was found to be an acceptable approximation of the fluid–fluid interface for two principal reasons: (1) the reflection coefficient for planar fluid–fluid interfaces is relatively constant over moderate angular deviations from normal plane wave incidence, and (2) nearly all of the acoustic power radiated from a given axisymmetric source transducer is carried by plane waves whose propagation directions are bounded by a cone of small solid angle centered about the acoustic axis.

This argument will be validated in Sec. V B of the present paper.

For the case of fluid–elastic planar interfaces, and in particular for planar elastic plates, however, the reflection (and transmission) coefficient may change rapidly with angle even for small deviations from normal incidence, and the reflection coefficient cannot be considered angle-independent, as was done in previous work. The behavior of the reflection coefficients from such reflecting structures will be numerically illustrated in Sec. I. An accurate determination of the IP-ASD for the cases of elastic interfaces and plates thus requires the incorporation of the exact angle- (and possibly frequency-) dependent reflection coefficient.

In this paper, a numerical strategy for calculating the IP-ASD is presented that incorporates the exact reflection coefficients of the above-mentioned planar interface between fluid and elastic media. For a single frequency of interest, the strategy can be divided into four principal steps: (1) Calculate the angular spectrum decomposition of the pressure field radiated by an equivalent image source, (2) modify each plane wave component by an angle-dependent (and possibly frequency-dependent) reflection coefficient, (3) propagate the plane wave components to the receiver surface, and (4) calculate and sum together the output electrical signal from the receiver for each plane wave component of the pressure field. These steps should be repeated for all temporal frequencies of interest; however, it will be shown that in many circumstances step (1) does not need to be repeated for each new frequency, requiring only a periodic update depending on the range of frequencies, transducer size, etc., used. In certain circumstances, the decomposition need be performed only once, and, by simple reinterpretation, the results for a pressure field decomposition at one frequency can be used for all frequencies of interest with little loss in accuracy.

Each step of the algorithm will be presented in a separate section of the paper. Reflections of a plane wave from plane boundaries will be discussed in Sec. I, including the case of fluid–elastic half-spaces and plates. A brief review of the angular spectrum decomposition will be presented in Sec. II, along with a discussion of several important numerical implementation considerations. Development of the equivalent image source will be given in Sec. III, and the calculation of the output electrical signal from the receiver will be discussed in Sec. IV. Simulation results for arbitrary alignment of the interface will be presented in Sec. V for the case of half-space interfaces, and for planar piston and non-diffracting transducer geometries. Results from perfectly reflecting, fluid–fluid and fluid–elastic half-spaces are discussed, along with several comparisons to a previous investigation. Next, results from an integral formulation for the special case of a planar circular piston aligned to normal incidence with an elastic interface will be presented in Sec. VI and compared to results obtained using the angular spectrum technique. This is followed by IP-ASD results for elastic plates with various misalignment angles and thicknesses. Finally, an approximation to the angular spectrum technique via a reduced set of plane waves is given in Sec. VII, along with an examination of the accompanying loss in accuracy.
I. REFLECTIONS AT A PLANE BOUNDARY

A. Homogeneous half-spaces

The half-space boundaries (interfaces) examined in this paper fall into the two general categories of fluid–fluid and fluid–elastic types, and it is convenient to examine each type separately. The reflection coefficients for both types of interfaces are frequency independent, but require a complete description of the acoustic properties of the two homogeneous half-spaces and of the incident plane wave. An exact determination of the reflection coefficient can be obtained only for a plane wave field; thus, a plane wave (angular spectrum) decomposition of the source field is required, and the total field result may be found from superposition of the individual plane wave components.

We will denote an arbitrary incident plane wave as

\[ \tilde{P}_i(r) = \tilde{A}_i \exp(\imath(\omega t - k \cdot r)), \]

where \( \tilde{A}_i \) is the amplitude (tilde denotes a complex value), \( k \) is the propagation vector, \( r \) is the observation position vector, and \( j = \sqrt{-1} \). The amplitude of a given reflected plane wave \( \tilde{A}_r \), is obtained from the amplitude of the incident plane wave simply as \( \tilde{A}_r = \tilde{A}_i \tilde{R}(\theta) \), where \( \tilde{R}(\theta) \) is the reflection coefficient. Assuming that the acoustic parameters of the two homogeneous planar half-spaces are specified, then the reflection coefficient is solely a function of incident plane wave angle \( \theta \) as measured relative to the interface normal.

For fluid–fluid interfaces, such as those formed between water and biological soft tissues the reflection coefficient takes a particularly simple form. Figure 1 (a) illustrates the general case and Fig. 2(a) illustrates a given plane wave, propagating in fluid medium 1, incident on the interface of fluid medium 2, at an angle \( \theta_1 \) with respect to the interface normal. The reflected longitudinal (compressional) plane wave also propagates with an angle \( \theta_1 \), and the transmitted longitudinal plane wave propagates through medium 2 with an angle \( \theta_2 \), all measured with respect to the interface normal. The longitudinal reflection coefficient, \( \tilde{R}_L \), for a plane wave obliquely incident on an infinite planar fluid–fluid interface is well known, and is restated here for convenience:

\[ \tilde{R}_L = \frac{(r_2/r_1) - (\cos \theta_2/\cos \theta_1)}{(r_2/r_1) + (\cos \theta_2/\cos \theta_1)}. \]

Medium 1 has a characteristic acoustic impedance \( r_1 = \rho_1 c_1 \), and medium 2 has a characteristic acoustic impedance \( r_2 = \rho_2 c_2 \), where \( \rho \) is the density and \( c \) is the propagation velocity of the appropriate half-space. Note that from Snell’s law, \( \theta_1 \) and \( \theta_2 \) are related as follows:

\[ \sin \theta_1 / c_1 = \sin \theta_2 / c_2. \]

Here, \( \tilde{R}_L \) can be complex since \( \theta_2 \) can take on complex values.

The wave interaction at a fluid–elastic interface is more difficult to determine than for the fluid–fluid case, because shear waves are supported in the elastic medium. A given incident longitudinal plane wave is denoted by \( P_i \), the corresponding reflected longitudinal plane wave by \( P_r \), the refracted longitudinal plane wave \( P_L \), and the refracted shear wave by \( P_S \).

The reflection coefficient for a plane wave obliquely incident on a fluid–elastic infinite planar half-space interface has been derived in detail elsewhere. Utilizing the notation of Brekhovskikh, the following equations summarize the interaction:

\[ \tilde{R}_L = \frac{Z_2 \cos^2 \gamma_2 + Z_{2s} \sin^2 \gamma_2 - Z_1}{Z_2 \cos^2 \gamma_2 + Z_{2s} \sin^2 \gamma_2 + Z_1}, \]

where

\[ \frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2} = \frac{\sin \gamma_2}{b_2}, \]

and

\[ Z_1 = \frac{\rho_1 c_1}{\cos \theta_1}, \quad Z_2 = \frac{\rho_2 c_2}{\cos \theta_2}, \quad Z_{2s} = \frac{\rho_2 b_2}{\cos \gamma_2}. \]

Note that \( c_1 \) is the (longitudinal) propagation velocity in fluid medium 1, and \( c_2 \) and \( b_2 \) are the longitudinal and shear propagation velocities in elastic medium 2, respectively. The relevant parameters are defined in Fig. 2 (b). Numerically, it is convenient to employ the admittance terms \( Y = 1/Z \) to remove the potential singularities from the calculation of \( \tilde{R}_L \) in (4).

For a water–aluminum interface, the reflection coefficient is calculated and plotted in Fig. 3. Because an elastic reflector supports mode conversion of the incident longitudinal wave, three different curves must be given in general: the reflected longitudinal, refracted shear, and refracted longitudinal wave components. If the solid is infinitely thick (or, more practically, is a layer thick enough that back wall and higher-order reflections can be effectively time-gated out of the received signal), these transmitted waves never reflect and strike the fluid–elastic boundary again; therefore, no secondary mode-converted longitudinal waves are developed in medium 1 and only the longitudinal wave reflection coefficient in (4) is considered for this case.

Note that the reflection coefficients for half-space interactions are frequency independent, assuming no frequency dependence of the medium parameters. Once determined, the coefficients can be used to scale the amplitude of each reflected plane wave component of the pressure field, ac-
According to their angle of incidence for all temporal acoustic frequencies of interest.

B. Elastic plate in a homogeneous fluid

In this section, the reflection coefficient for a simple planar layer is presented. In general, the reflection coefficient for a discretely layered medium can be difficult to obtain, especially when the layers are not aligned parallel to one another. If only two interfaces are present, and are constrained to be planar and parallel to each other (e.g., a plate) as shown in Fig. 1(b), then a closed-form expression for the reflection coefficient can be obtained; the reflection coefficient for such a layered medium is dependent upon both incident angle and frequency of the acoustic wave. We will confine the analysis to an elastic plate bounded by identical fluid half-spaces.

We denote the elastic plate as medium 2, and the surrounding fluid on either side as medium 1; this geometry is illustrated in Fig. 1(b). The fluid has propagation velocity $c_1$ and density $\rho_1$. The reflection of a monochromatic plane wave by an elastic plate, with longitudinal and shear sound speeds $c_2$ and $b_2$, respectively, density $\rho_2$, and thickness $h$, is given by the following:

$$ R_L = \frac{j(V^2 - U^2 + 1)}{2V + j(V^2 - U^2 - 1)}, \quad (6) $$

where

$$ U = \frac{Z_2 \cos^2 2\gamma_2}{Z_1 \sin P} + \frac{Z_2 \sin^2 2\gamma_2}{Z_1 \sin Q}, $$

$$ V = \frac{Z_2 \cos^2 2\gamma_2 \cot P + Z_2 \sin^2 2\gamma_2 \cot Q}{Z_1}, \quad (7) $$

and

$$ P = k_2 h \cos \theta_2, \quad Q = k_2 h \cos \gamma_2. \quad (8) $$

Description of the impedance terms and the relation between the various reflection and transmission angles can be found from (5). In (8), the wave numbers are $k_2 = \omega/c_2$ and $k_2 = \omega/b_2$, where $\omega$ is the radian temporal frequency. For the more general situation of two dissimilar fluid media surrounding the elastic plate, (6) becomes slightly more complicated.

Example calculations of the power reflection coefficient versus incident plane wave angle $\theta_1$ and frequency in MHz is shown in Fig. 4. In this example, the surrounding fluid medium is water ($c_1 = 1500 \text{ m/s}, \rho_1 = 1000 \text{ kg/m}^3$) and the elastic plate is aluminum ($c_2 = 6420 \text{ m/s}, b_2 = 3040 \text{ m/s}, \rho_2 = 2700 \text{ kg/m}^3$) with thickness $h = 0.945 \text{ cm}$. This value of $h$ is chosen for later comparison of IP-ASD results to similar results published in the literature, and is discussed later in Sec. V.

It is clear, upon examination of Fig. 4, that the reflection coefficient for the elastic plate cannot be reasonably approximated by a constant coefficient, with respect to neither angle nor frequency. Despite the fact that most of the power of the pressure field is carried by plane waves whose propagation vectors make only relatively small angles with respect to the acoustic axis, the reflection coefficient generally contains rapid fluctuations for small angular variations of $\theta_1$ from normal layer incidence. Behavior of this nature is the princi-

FIG. 3. Longitudinal (power) reflection coefficient $|R_L|^2$ and longitudinal and shear (power) transmission coefficients $|T_L|^2$ and $|T_S|^2$ of a plane wave incident from water on a planar aluminum half-space interface.

FIG. 4. Power reflection coefficients from a planar aluminum plate of infinite extent ($c_2 = 6420 \text{ m/s}, b_2 = 3040 \text{ m/s}, \rho_2 = 2700 \text{ kg/m}^3$) of thickness $h = 0.945 \text{ cm}$, immersed in water ($c_1 = 1500 \text{ m/s}, \rho_1 = 1000 \text{ kg/m}^3$), versus frequency and angle of plane wave incidence relative to the interface normal.
pal motivation for incorporating the exact reflection coefficient in the calculation of the IP-ASD for elastic interfaces.

II. ANGULAR SPECTRUM DECOMPOSITION

In order to make use of the reflection coefficients as presented in the previous section, a description of the incident pressure field in terms of plane waves is required. The angular spectrum is employed to decompose a monochromatic field specified over a plane into component plane waves. This plane, designated the source decomposition plane (SDP) in this paper, does not necessarily correspond to the plane of the source transducer, and thus allows the angular spectrum to be computed for a wide variety of arbitrary transducer geometries. Assuming the SDP to be located at \( z = 0 \) by a suitable choice of axes, then the angular spectrum decomposition can be expressed as:

\[
\tilde{A}(k_x, k_y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{g}(x, y, \omega) \exp\left(-j(k_x x + k_y y)\right) \, dx \, dy,
\]

where \( \tilde{g}(x, y, \omega) \) is the field in the SDP, and \( k_z = 2\pi f_z \), etc.

Each plane wave component has a complex amplitude \( \tilde{A}(k_x, k_y, \omega) \) and direction of propagation \( k \), where

\[
k = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}.
\]

Since \( |k| = k = 2\pi/\lambda \), where \( \lambda \) is the temporal wavelength of the monochromatic field, then \( k_z \) is completely determined by \( k_x \) and \( k_y \), and may be found from

\[
k_z = \sqrt{k^2 - k_x^2 - k_y^2}.
\]

When a single plane wave component is analyzed, chosen from the continuum of plane waves in the decomposition, we will denote its propagation direction as \( \hat{n} \) and its amplitude as \( \tilde{A}(\omega) \). Note that \( |k| = k \) for all plane waves in the decomposition.

It is convenient to express the propagation vector in terms of the direction angles \( \theta_x, \theta_y, \theta_z \) that \( \hat{n} \) makes with respect to the \( x, y, \) and \( z \) coordinate axes; these angles are illustrated in Fig. 5. The direction cosines \( n_x, n_y, \) and \( n_z \) can be used advantageously, as they are simply related to the propagation vector components in a straightforward fashion;

\[
k = \hat{n} h = n_x \hat{x} + n_y \hat{y} + n_z \hat{z},
\]

where \( \hat{n} \) is a unit vector in the direction of \( k \), and

\[
n_x = \cos \theta_x = k_x / k = \lambda f_x, \\
n_y = \cos \theta_y = k_y / k = \lambda f_y, \\
n_z = \cos \theta_z = \sqrt{1 - n_x^2 - n_y^2}.
\]

Here, \( n_z \) is uniquely specified upon determination of \( n_x \) and \( n_y \), since \( n_x^2 + n_y^2 + n_z^2 = 1 \). The generation of evanescent waves is anticipated in (13) and (11), where \( n_z \) will become complex when \( n_x^2 + n_y^2 > 1 \), or equivalently, when \( (f_x^2 + f_y^2) > \lambda^{-2} \).

Note that all plane waves in the decomposition (9) have the origin of the coordinate system as their common phase reference, and for a given observation point \( R \) in space, the distance that the \( q \)th plane wave propagates in arriving at point \( R \) is \( d^q = h^q r \), where \( r \) is the observation position vector extending from the origin, \( O \), to point \( R \). The phase change for a given plane wave is \( \exp(jkd^q) \) and the complex amplitude of a given plane wave component at point \( R \) is

\[
\tilde{A}(\omega, r) = \tilde{A}(\omega, O) \exp[jk(n_x r_x + n_y r_y + n_z r_z)],
\]

where the argument \( O \) refers to the evaluation of \( \tilde{A}(\omega) \) at the origin of the \( (x, y, z) \) coordinate system. This result will be of importance later when the individual plane waves must be propagated to the receiver. Note that \( \tilde{g} \) in (9) can either describe the complex pressure, \( \hat{p} \), in the SDP, or the normal velocity, \( \hat{u}_n \). In the latter case, (14) is modified to include an additional factor \( 1/jkn \) (Ref. 11), which causes the decomposition to represent velocity (and not normal velocity) amplitudes, which are directly related to pressure via the specific acoustic impedance.

A. Computation of the angular spectrum

The angular spectrum may be efficiently computed from (9) by making use of the two-dimensional fast Fourier transform (2D-FFT), with the discrete transformation frequencies interpreted in terms of direction cosines as given in (13). The field in the SDP must be evaluated over a set of discrete sampling points, where it is assumed that the discretization region is square with \( N \times N \) samples equispaced by a distance \( \delta \) in both the \( x \) and \( y \) directions. The \( N \times N \) FFT array is partitioned into a square region of \( M \times M \) samples, containing all source field points of significant amplitude, and beyond which the source field is negligible.

In general, discretization of the SDP is obtained from calculations of the complex pressure field as radiated by the given source transducer. Note that no limitations on the source transducer exist in this method (i.e., the transducer...
can be nonaxisymmetric, nonplanar, etc.) as long as the required pressure field computations over the chosen plane can be made. In some circumstances, such as for a planar transducer geometry, it is convenient to place the SDP directly in the plane of the transducer surface itself; the transducer surface velocity may then be sampled directly (if it is known) or a uniform excitation can be assumed. Thus, discretization of the SDP may require little computation, if any. For the situation of the SDP in the plane of a planar circular piston transducer of radius $a$, the sample spacing is generally adjusted such that the transducer maximally fills the $M \times M$ array partition, i.e., $2a/M = \delta$.

There are many important practical considerations\textsuperscript{12,13} that one should be aware of in computing (9) via the 2D-FFT; however, such detailed discussion falls beyond the scope of the present paper. Briefly, the primary concerns in angular spectrum computations can be characterized by three parameters: source discretization error, angular span, and angular resolution of the plane wave components in the decomposition. A cursory discussion of these parameters follows. The particular angular spectrum decomposition techniques employed here are given in detail in Refs. 11 and 14, and the interested reader should consult the references for further information.

The angular span of the decomposition defines the total angular range of plane wave propagation vectors that are represented in the 2D-FFT result and is influenced by temporal wavelength $\lambda$ and the sampling partition size $M$. The angular span is chosen to represent all but a negligible fraction of the power in the forward propagating field. Minimum bounds on the angular span may be obtained for a given transducer geometry.\textsuperscript{4} The angular resolution of the plane wave components is an important consideration when the interface reflection coefficient exhibits a strong angular dependence. The resolution must be sufficiently good to allow an accurate integration of the decomposed field over the receiver surface, since determination of the total electrical output signal from the receiver is based on the discrete integration of these plane wave components. A useful indicator of the integration accuracy is the ratio of the angular span to the angular resolution. This ratio is at most equal to $N$, the FFT size in one dimension. The ratio is adversely affected by the appearance of evanescent wave components in the 2D-FFT results, since these components are generally omitted from the integration, and thus care must be taken to minimize their presence in the array. Evanescent waves are present to the greatest extent at longer temporal wavelengths.

For a given transducer and a given temporal frequency, one may determine an optimal choice of decomposition parameters $N$ and $M$ with respect to the source discretization error, angular span, and resolution of the component plane waves. Since we are interested here in simulating the IP-ASD over a large span of frequencies, it would appear that the angular spectrum decomposition must be determined for each discrete temporal frequency of interest; although computationally intensive, this approach leads to the most exact results. However, the optimal decomposition parameter values for any two adjacent temporal frequencies in the simulation may not change, simply due to the discrete nature of the variables; in fact, the parameter values may not change significantly over a fairly large range of frequencies (on the order of several kHz to several MHz), depending on the transducer and interface parameters. It is possible to use just a single 2D-FFT decomposition for an entire IP-ASD calculation, which for example has been done for the situations of perfect reflectors and half-space interfaces in Sec. V, where the results obtained are accurate over a range of frequencies from dc to 15 MHz. Other IP-ASD simulations, notably those in Sec. VI for an elastic plate, require new values of $M$ and $N$ to be computed periodically; this indicates that decomposition parameter selection is highly situation dependent.

### III. EQUIVALENT IMAGE SOURCE

The propagation directions of the plane wave components of the reflected field may be found directly by “ray tracing” the plane wave propagation vectors $k^i$, i.e., by applying (3) or (5) as appropriate to redirect the propagation vector of each incident plane wave component, found from the angular spectrum decomposition, as it strikes the reflecting interface. The propagation vectors of several plane wave components of the incident pressure field are represented in Fig. 6(a) as $k^i$, along with their associated reflected propagation vectors $k^r$. In this figure, the actual transducer is shown located at the origin of the $(u,v,w)$ coordinate system. The front surface of the interface (shown here as a plate) is located a distance $w_0$ axially from the source, and rotated by an angle $\alpha$ with respect to the $v$ axis; the rotation is confined to the $v$-$w$ plane for simplicity, and is composed without loss of generality for an axisymmetric source transducer. For a nonaxisymmetric source, incorporation of a three-dimensional rotation in the $(u,v,w)$ coordinate system can be made. Once reflected, the amplitudes of the plane waves can then be scaled by the appropriate reflection coefficients describing the given interface. This procedure is tedious, but can be used to exactly determine the reflected pressure field.

(Note that in determining the reflected pressure field, the term “interface” is used to represent the boundary between the homogeneous half-space containing the actual transducer, referred to here as the “coupling medium,” and the first of possibly several planar layers whose properties generally differ from the coupling medium. The reflection coefficient for the interface is defined to take into account all wave interactions occurring in the layered medium, and thus no explicit calculation of internal reflections in the layered medium are required when determining the reflected field.)

A preferred alternative to “ray tracing” is the use of an equivalent image source\textsuperscript{5,15} which, in the pulse-echo case, takes the place of the actual transducer for acoustic pulse transmission; the actual transducer is used in the analysis for reception only. By definition, this equivalent image source must reproduce the reflected pressure field exactly at all points in the coupling medium. The development of the equivalent image source is shown for a plate in Fig. 6, but the arguments are equally valid for a planar interface. If we backpropagate the reflected plane wave components of Fig. 6(a) and assume that the coupling medium is extended to

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homogeneously fill all space, then it is seen that the propagation vectors intersect at a common origin lying to the right of the interface plane, as shown in Fig. 6(b). Assuming initially a perfect (unity) reflection coefficient of the interface and a planar (but not necessarily axisymmetric) transducer, then the plane waves converge to form an equivalent image source that is a mirror image of the actual transducer with respect to the interface plane (or more correctly, a mirror image of the original source only for the case of a perfect (unity) reflection coefficient.

For the particular choice of interface misalignment angle $\alpha$ illustrated in Figs. 1 and 6(b), the equivalent image source has its center located at the origin of the $(x,y,z)$ coordinate system, in which the interface is located a distance $z_0 = w_0$ from the image source. The figure is similar to 6(b), except that it has been rotated such that the equivalent image source is now drawn on the left, and the actual transducer operating as receiver is rotated in the $y$-$z$ plane by an angle $2\alpha$ relative to the $z$ axis. The outward unit normal vector of the planar interface, denoted $\hat{\mathbf{F}}$ in Fig. 6(c), is easily seen to be

$$\hat{\mathbf{F}} = \cos \alpha \hat{\mathbf{z}} - \sin \alpha \hat{\mathbf{y}} \quad (15)$$

with respect to the image source $(x,y,z)$ coordinate system. The output electrical signal from the receiver (actual transducer) is based exclusively on the formulation in Fig. 6(c).

### IV. ACOUSTIC PRESSURE FIELD RECEPTION

It is assumed that the output electrical signal from the receiver is proportional to the integral of the complex pressure field over the phase-sensitive transducer surface. In general, a numerical integration of the pressure field over the receiver surface is required, and in the prior investigation a technique based on discretization of the receiver surface into a mesh of planar elements was used. This technique is applicable to the general case of a nonplanar, nonaxisymmetric transducer. Since the pressure field here is decomposed into plane waves, then each plane wave component can be integrated over the receiver surface individually and the result summed over all plane waves in the decomposition to yield the receiver output signal. Finally, the calculation is repeated for all frequencies of interest.

For certain simple transducer geometries, however, an analytical solution to the integration of an arbitrary plane wave over the surface is possible, and is derived here for the specific case of a planar circular piston. Use of such solutions greatly increases the computational efficiency of the angular spectrum method, and hence is the reason for exclusively investigating the planar circular piston geometry throughout this paper. By application of superposition, the results for the planar circular piston can be used in formulating an analytic solution to the output electrical signal produced by a planar annular array transducer as well.
An analytical solution to the integration of a single arbitrary plane wave, illuminating a circular planar piston receiver surface, now follows. Recall that the plane wave components incident on the receiver surface are determined by angular spectrum decomposition of the source field in the \( z = 0 \) plane. The receiver-based coordinate system \( (u,v,w) \), as shown in Fig. 6(c), is chosen to simplify derivation of the receiver output function; thus, the plane wave components found from (9) must be expressed in the \( (u,v,w) \) coordinate system, in terms of the propagation direction unit vectors \( \hat{n}^q \) and the observation position vector \( r \), by suitable choice of rotation matrix. It will later be shown that only the angle between \( \hat{n}^q \) and the receiver normal is required to compute the receiver output signal, and so this rotation is merely a mathematical formality. Before doing so, however, it is convenient to break up the observation position vector \( r \), extending from the origin \( O \) to locations on the surface of the receiver, into the sum of two position vectors, \( r = OP + Q \), where \( OP = P - O \). Here, \( P \) is the position vector to the origin of the \( (u,v,w) \) coordinate system, and \( Q \) points from \( P \) to locations on the receiver surface. The propagation of a given plane wave from the plane of constant phase containing \( O \), to a parallel plane containing \( P \), is accomplished by simple application of (14), where

\[
\hat{A}^q(\omega, P) = \hat{A}^q(\omega, O) \exp[jk(\hat{n}^q OP)], \tag{16}
\]

after which the plane wave may be rotated into the \( (u,v,w) \) coordinate system. Note that \( P \) extends to the origin of the \( (u,v,w) \) coordinate system. After propagation to \( P \), the complex amplitude of the plane wave, \( \hat{A}^q(\omega, Q) \), is integrated over the receiver surface, where \( P \) now represents the new phase origin of the plane wave.

Let the components of \( Q \) be expressed as \( (Q_u, Q_v, Q_w) \) and those of \( \hat{n}^q \) as \( (n_u^q, n_v^q, n_w^q) \), in the \( (u,v,w) \) coordinate system. The components of both vectors are stated in cylindrical coordinates as

\[
(n_u^q, n_v^q, n_w^q) = (n_u^q \cos \theta_w^q, n_v^q \sin \theta_w^q, n_u^q),
\]

\[
(Q_u, Q_v, Q_w) = (Q_u \cos \theta_w, Q_v \sin \theta_w, 0),
\]

where \( (n_u, n_v, n_w) \) and \( (Q_u, Q_v, Q_w) \) are the cylindrical vector components, and \( \theta_w \) and \( \theta_w \) are angles measured with respect to the \( u \) axis. (See Fig. 7 for a description of the cylindrical vector components of \( \hat{n}^q \).) Note that \( Q_w = 0 \) for all points on the planar receiver surface. Propagation of the \( q \)th plane wave component of the radiated field from \( P \) to an arbitrary location on the receiver surface proceeds similar to (16), and may now be restated with the aid of (17) as

\[
\tilde{A}^q(\omega, Q) = \tilde{A}^q(\omega, P) \exp[jk \hat{n}^q Q]
= \tilde{A}^q(\omega, P) \exp[jk Q_u n_u^q \cos(\theta_w - n_w^q)], \tag{18}
\]

where simplification of trigonometric expressions has been introduced.

The \( q \)th plane wave component of the field is now integrated over the receiver, yielding an analytical solution to the output signal \( \tilde{V}^q(\omega) \),

\[
\tilde{V}^q(\omega) = \int_S u^q(\omega, Q) dQ.
\]

where \( S \) is the receiver surface, \( \theta_w^q \) is the angle of the \( q \)th plane wave propagation vector relative to the \( w \) axis (see Fig. 7), and \( \rho_1, c_1 \) is the acoustic impedance of the medium. The term \( \cos \theta_w^q / \rho_1, c_1 \) is known as the acoustic reciprocity factor. Expressing (19) as an integration in cylindrical coordinates, then,

\[
\tilde{V}^q(\omega) = \int_0^a 2\pi \tilde{A}^q(\omega, Q) \cos \theta_w^q \frac{Q_u}{\rho_1, c_1} dQ_u \cos \theta_w^q dQ_v dQ_w
= \tilde{A}^q(\omega, P) \cos \theta_w^q \int_0^a Q_u dQ_u \frac{Q_v}{\rho_1, c_1}
\times \int_0^{2\pi} \exp[jk Q_u n_u^q \cos(\theta_w - n_w^q)] d\theta_w,
\]

where the receiver is assumed to be of radius \( a \). The last integral on the right-hand side of (20) can be identified as \( 2\pi j_0(k Q_u n_u^q) \) (Ref. 17), where \( j_0 \) is the zeroth-order Bessel function of the first kind. Equation (20) may now be written as

\[
\tilde{V}^q(\omega) = \tilde{A}^q(\omega, P) \cos \theta_w^q \frac{2\pi}{\rho_1, c_1} \int_0^a Q_u j_0(k Q_u n_u^q) dQ_u.
\]

To further evaluate (21), a useful theorem states

\[
\int_0^\infty \xi J_0(\xi) d\xi = \eta J_1(\eta),
\]

such that

\[
\tilde{V} = \tilde{A}^q(\omega, P) \frac{2\pi a}{\rho_1, c_1} j_1(k n_u^q a).
\]

Now, referring to Fig. 7, \( n_u^q \) can be shown from simple geometrical considerations to be

\[
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\]

FIG. 7. Geometry used to determine the angle \( \theta_w^q \) as given in Sec. IV. Here, \( \hat{n}^q \) is the unit propagation vector describing the \( q \)th plane wave of the radiated pressure field, and \( n_u, n_v, \) and \( n_w \) are its components in cylindrical coordinates.
where $|\hat{n}| = 1$ and $\theta_w^q$ has been previously given; the corresponding angle in the $(x,y,z)$ coordinate system is $\theta_w^q$ from comparison with Fig. 5. After some rearrangement of terms, the receiver electrical output signal may be expressed in its final form as

$$\tilde{V}(\omega) = \sum_{q=1}^{N^2} \tilde{V}(\omega, P, D) \cos \theta_w^q \left( \frac{2J_1(ka \sin \theta_w^q)}{ka \sin \theta_w^q} \right),$$

(25)

where it is assumed that all plane waves have previously been propagated along $OP$ as given in (16). Note that (25) only requires the angle $\theta_w^q$ to be determined in the $(u,v,w)$ coordinate system, which may just as easily be computed as $\theta_w^q$ in the $(x,y,z)$ system from the scalar product of the receiver normal vector and $\hat{n}$; the angular spectrum of the source field could then be left in the original $(x,y,z)$ coordinate system for computational convenience. To determine the receiver output signal generated in response to the incident pressure field, a sum over all plane wave components in the angular spectrum is performed, yielding

$$\tilde{V}(\omega) = \sum_{q=1}^{N^2} \tilde{V}(\omega, P, D) \cos \theta_w^q \left( \frac{2J_1(ka \sin \theta_w^q)}{ka \sin \theta_w^q} \right),$$

(26)

The upper limit of the sum in (26) may be somewhat less than $N^2$ due to the possible presence of evanescent wave components in the decomposition (as discussed in Sec. II). The receiver output signal for the case of a temporal acoustic pulse is readily synthesized by repeated use of (26) for all temporal frequencies $\omega$ of interest.

V. IP-ASD RESULTS: HALF-SPACES

A. Perfect reflecting half-spaces

Results for the case of a half-space with constant unity reflection coefficient were obtained using the angular spectrum algorithm, for a planar piston transducer with radius $a = 1.27$ cm, positioned a distance $Z_0 = 5$ cm from the interface plane, and rotated by an angle $0.6^\circ$ with respect to the transducer axis. The propagation medium is water (sound speed $c = 1500$ m/s). The magnitude IP-ASD results were obtained using angular spectrum decomposition for two sets of simulation parameters, identified by method 1 and method 2 in Fig. 8. In method 1, the results are calculated using $N = 128$ and $M = 80$; this implies a sample spacing of 6 = 2a/M0.3 mm. Method 2 uses $N = 32$ and $M = 20$, which implies $\delta = 1.3$ mm. The results obtained using method 2 are slightly less accurate than the results using method 1, especially at higher frequencies, but are computed much faster (10 s versus 135 s on a DECStation 5000 RISC workstation running FORTRAN-77). Additionally, results obtained from a previous investigation (Ref. 1, Sec. II), employing discrete pressure field computations over a meshed receiver surface and a numerical sum to calculate the receiver output electrical signal, are presented in the figure as well. This previous method will hereafter be called the Multirate-DSP method, referring to the specific technique used in calculating the pressure field. The correspondence between the results is very good, and clearly indicates the numerical accuracy obtainable with the angular spectrum technique.

The IP-ASD was next computed for the case of a circular annular array transducer, configured as a nondiffracting transducer. The simulation is performed for a 10-annulus source transducer, with total radius $a = 2.54$ cm, and annuli widths as described in Ref. 19, Sec. II B. The receiver, however, is a single-element planar piston of radius $a = 2.54$ cm, which in practice is the same annular array used for transmission, but with equal weighting applied to all annuli in the receive mode. The transducer is placed a distance $Z_0 = 5$ cm from a perfectly reflecting half-space interface, and rotated by an angle $\alpha = 1.25^\circ$. This situation exactly corresponds to the result given in Ref. 1, Fig. 9, to which we will presently compare the results obtained using the angular spectrum method.

The most straightforward method of calculating the angular spectrum decomposition of the annular array is to treat the array as a uniformly driven planar piston source; discretization of the SDP is obtained by sampling the excitation weights of the array as a function of radial distance from the transducer center. This approach requires a fairly large number of discretization points in order to reproduce the annuli boundaries in the array as accurately as possible; $N = 128$ and $M = 80$ have been used with good results. In Fig. 9, the magnitude of the IP-ASD results using this method are plotted, along with the IP-ASD as obtained using the multirate-DSP method. Errors in the IP-ASD are greatest at low frequencies, where the resolution of the angular spectrum is at its lowest due to the presence of evanescent waves in the FFT results.

B. Fluid-fluid and fluid-elastic half-spaces

The IP-ASD was calculated for a fluid-fluid half-space boundary, formed at the planar interface between water
(c₁ = 1500 m/s, ρ₁ = 1000 kg/m³) and fatty tissue (c₂ = 1450 m/s, ρ₂ = 950 kg/m³). The reflection coefficient $\tilde{R}_L$, as given in (2), is used here to determine the amplitudes of the reflected plane wave components. Calculation of $\tilde{R}_L$ requires knowledge of $\theta_i$, the angle between the incident plane wave propagation vector and the interface normal vector, and $\theta_r$, which is found from $\theta_i$ using (3). Letting $\theta_\parallel$ be the angle between the unit vector in the direction of the qth plane wave, $\hat{r}_q$, and the outward interface normal vector, $\hat{z}$, then $\theta_\parallel$ may be found from

$$\theta_\parallel = \cos^{-1}(\hat{r}_q \cdot \hat{z}).$$

Results were obtained for a planar circular piston source transducer of radius $a = 1.27$ cm, with $z_{0} = 5$ cm, and oriented at an angle $\alpha = 0.6^\circ$ relative to the interface normal. The magnitude of the IP-ASD result was found to be identical to the results obtained for a perfectly reflecting planar interface, as given in Fig. 8, to within a frequency-independent scale factor. Furthermore, the IP-ASD was investigated for several fluid–elastic half-space boundaries, formed at angles $\alpha = 0.6^\circ$, relative to the interface normal.

VI. IP-ASD RESULTS: ELASTIC PLATES

In contrast to the half-space interfaces previously examined, the IP-ASD for an elastic planar plate exhibits significant changes relative to the IP-ASD for a perfect reflector. This is due to the far greater incident-angle dependence of the reflection coefficient for elastic plates as compared to perfectly reflecting interfaces. To begin, we note that the IP-ASD for an elastic plate aligned precisely parallel to the transducer surface can be described in terms of an integral equation, and is developed in the following section. The solution to this equation, obtained using numerical integration, will be used as a reference for comparison of the angular spectrum IP-ASD results.

A. Integral formulation for normal incidence

A solution to the IP-ASD in terms of an integral equation can be obtained for the specific case of a planar circular piston aligned parallel ($\alpha = 0$) to the elastic plate, and has been previously presented by Johnson et al. It will now be shown that the individual steps of the angular spectrum method given here, for the specific conditions as noted, lead directly to the integrand of that formulation, specifically [Ref. 9, Eq. (2)]. The steps proceed as follows: (1) determine angular spectrum decomposition of source pressure field, (2) propagate plane wave components to interface plane, (3) reflect plane waves from interface, (4) propagate reflected plane waves to receiver, and (5) determine output electrical signal from receiver.

The propagation steps (2) and (4) taken together may be found by application of Eq. (16). Recall that the receiver term in (25) contains the sum of the (complex) amplitudes of the given plane wave over the receiver surface, i.e., over the points defined by the position vector $\mathbf{Q}$. Thus we need only account for the phase shift of the plane wave along $\mathbf{OP}$; since $\alpha = 0$, the components of $\mathbf{OP}$ in the $(u,v,w)$ system are simply $(0,0,2w_{0})$. Additionally, since $\theta_i$ is now exactly the angle between the incident wave and the interface normal vector, it is easily seen that $\hat{r}_i = |r| \cos \theta_i$, simplifying the propagation term of steps (2) and (4) to

$$\exp[2jkw_{0} \cos \theta_i].$$

The plane wave reflection coefficient in step (3) was previously covered in Sec. I, and is here denoted $\tilde{R}_L(\theta_i, \alpha)$. We have shown that the receiver term in step (5) has the analytical solution as given in (25). Step (1), the analytical determination of the angular spectrum decomposition of the...
pressure field due to a planar circular piston transducer, has not been formally derived here, but is identical to (25) (to within a reciprocity factor) if we assume a reciprocal source with uniform surface excitation. The derivation of this result proceeds in a similar fashion to the solution of reception of an arbitrary plane wave by a planar circular piston, developed in Sec. III. The results of these five steps are multiplied together to form the integrand \( I(\theta_{z}, \omega) \), which is then integrated over all plane wave components of the radiated field to yield the IP-ASD for normal incidence.

Only major steps of the integration are given below. In order to simplify the integration, a new parameter \( K = k \sin \theta_{z} \) is introduced, which is the projection of the propagation vector onto the plane of the reflecting interface; it is clear that \( K \) varies from 0 to \( k \) for propagation directions spanning the forward hemisphere if evanescent waves are neglected. Integrating the contributions of the previous steps over all projections of \( K \) in the plane, we see that

\[
\bar{V}(\omega) = \frac{\cos \theta_{z}}{\rho_{1} c_{1}} \int_{0}^{2\pi} \int_{0}^{k} I(K, \omega) K \, d\phi \, dK,
\]

where \( \psi \) is the polar angle of the projection of \( K \) in the plane, and the reciprocity factor is explicitly introduced. The integral thus obtained is independent of \( \psi \), allowing (28) to be reduced to a single integral. Expanding the integrand in terms of \( \theta_{z} \), then \( dK = k \cos \theta_{z} \, d\theta_{z} \), and

\[
\bar{V}(\omega) = 2\pi \int_{0}^{\pi/2} \tilde{R}_{L}(\theta_{z}, \omega) k \sin \theta_{z} \times \frac{\cos \theta_{z}}{\rho_{1} c_{1}} \left( \frac{2J_{1}(ka \sin \theta_{z})}{ka \sin \theta_{z}} \right)^{2}
\times \exp(2jkw_{0} \cos \theta_{z}) k \cos \theta_{z} \, d\theta_{z},
\]

where the limits of integration have changed accordingly. Aside from notational differences, Eq. (29) is exactly the integrand solution as given in Ref. 9, Eq. (5). Thus, the numerical implementation of the two methods are for all purposes identical, as long as the angular spectrum decomposition is assumed to yield the theoretical solution to the source radiation.

A plot of the IP-ASD for a planar circular piston of radius 1.88 cm located a distance \( z_{0} = 7.5 \) cm from an aluminum plate, with thickness \( h = 0.945 \) cm, is shown in Fig. 10, under the constraint of normal incidence. Results were obtained using the angular spectrum method, with \( M = 10 \), and \( N \) decreasing with increasing excitation frequency, limited to the range \( 128 > N > 64 \). Additionally, a result obtained by numerical integration of (29) for the same interface parameters is plotted in Fig. 10 for comparison. (A simple trapezoidal numerical integration was used over \( \theta_{z} \) to evaluate this integral.) Note that the transducer is assumed to have a constant frequency response (infinite bandwidth) for the results given here. It is not surprising, in light of the previous correspondence between the angular spectrum and numerical integration methods, that the two curves are practically identical.

A comparison between the above numerical integration and experimental results was given in Ref. 9, Fig. 5 for the same aluminum plate where an experimentally determined transducer frequency response was multiplied together with the numerical integration results. In the present paper, numerical results are assumed to be of infinite bandwidth, hence the need for recalculation of the numerical integration results as shown in Fig. 10.

B. Misaligned elastic plates

The IP-ASD for an aluminum plate was examined above for the case of normal incidence \( (\alpha = 0) \); the same plate is now reexamined for misalignment angles \( 0 < \alpha < 5^\circ \). The magnitude of IP-ASD results are presented in Fig. 11 for the frequency range dc to 1 MHz, with angular spectrum

![FIG. 10. IP-ASD for aluminum plate aligned to normal incidence, thickness \( h = 0.945 \) cm, as calculated using numerical integration (250 frequencies) and angular spectrum decomposition (250 frequencies, \( 64 < N < 512 \), \( M = 10 \)). Transducer is circular planar piston, \( a = 1.88 \) cm, distance to interface \( z_{0} = 7.5 \) cm.](image1)

![FIG. 11. IP-ASD for same situation as Fig. 10, but with plate set to various misalignment angles \( \alpha \) as indicated. All methods computed using angular spectrum technique, \( 64 < N < 512 \), \( M = 10 \).](image2)
decomposition parameters \( M = 10 \) and \( 128 < N < 64 \). The transducer is again a planar circular piston of radius 1.88 cm, located at \( z_0 = 7.5 \) cm from the interface. The strong effect of interface misalignment on the receiver output electrical signal is clearly evident.

Next, the IP-ASD for an aluminum plate, set to normal incidence, was examined for several values of plate thickness \( h \). In Fig. 12, the magnitude of IP-ASD is shown for an aluminum plate with \( h = 0.5, 0.75, \) and 1.0 cm, for the same transducer and distance as in Fig. 11. Note that as the plate becomes thinner, the spectral features in the magnitude IP-ASD move higher in frequency, inversely proportional to \( h \). The spectral distortion versus plate thickness may be interpreted as a stretching or compression over frequency of the IP-ASD function at a given plate thickness, with little change to the spectral shape.

VII. REDUCED PLANE WAVE APPROXIMATION

It is of interest to examine the accuracy of the IP-ASD calculation when fewer than the total number of plane waves available in the decomposition are used in the receiver sum term in (26). It has been observed (for the case of the elastic plate) that the resolution of plane waves is quite critical when the reflection coefficient of the interface changes quickly with respect to angle, and a large number of plane waves are required to synthesize results to a suitable accuracy. However, it might be conjectured that situations in which the reflection coefficient is nearly constant and does not significantly influence the IP-ASD, other than by a simple attenuative factor (such as the case of half-space interfaces), do not require the contributions of many plane wave components for an accurate IP-ASD result.

In particular, assuming a planar circular piston source, and constraining discussion to half-space interfaces, it was desired to know to what extent the contribution made by a single plane wave, propagating in a direction normal to the transducer surface, was sufficient to estimate the IP-ASD function. This plane wave has the largest amplitude in the source decomposition at a given frequency, and, given a small angle of misalignment \( \alpha \), contributes the largest amount of power to the receiver electrical output signal. Thus it was reasoned that this plane wave, out of the set of all decomposed plane waves, would singularly make the best approximation to the IP-ASD function.

Simulations were conducted in which only this single plane wave was considered in the receiver output signal at each frequency of interest. The IP-ASD was calculated earlier for a planar circular piston, radius \( a = 1.27 \) cm, positioned at \( z_0 = 5 \) cm from a perfectly reflecting interface aligned to \( \alpha = 0.6^\circ \). In Fig. 13, this exact result is superimposed on the IP-ASD result calculated by considering only the single central plane wave. The single plane wave IP-ASD has been scaled to cause the greatest coincidence between the two curves, in order to better facilitate comparison. (Note that for this simulation, the parameters \( N = 128 \) and \( M = 50 \) are used, but such choices only affect the amplitude of the single plane wave, and not the form of the resulting approximate IP-ASD.) The single plane wave IP-ASD is found to be quite similar in form to the exact IP-ASD calculation, if one disregards the strictly attenuative factor intrinsic to the approximation. The most important differences between the curves are that the single plane wave IP-ASD contains true nulls in the spectrum [as should be expected...]

**FIG. 12.** IP-ASD for same situation as Fig. 10 (\( \alpha = 0 \)), except plate thickness \( h \) is varied as indicated.

**FIG. 13.** Comparison between single plane wave IP-ASD and exact IP-ASD calculation, for same situation as in Fig. 8.
when evaluating (25) for a single plane wave over frequency, and that the null locations are slightly shifted in frequency. This approximate solution is therefore usable to indicate the spectral shape of the IP-ASD function, but is not applicable in determining an appropriate inverse function for use with spectral compensation techniques.

One should not draw general conclusions from the agreement between the single plane wave IP-ASD and the exact IP-ASD, as shown in Fig. 13. The result does not imply that such approximations can be used for IP-ASD estimation in general. For instance, the same approximation applied to the case of an elastic plate yields a completely erroneous IP-ASD result. Even when the interface is a perfect reflector, the approximation leads to highly inaccurate IP-ASD results when applied to the case of a nondiffracting transducer. Furthermore, the single plane wave IP-ASD for the same situation as given in Fig. 13 yields increasing errors as the transducer radius becomes smaller.

An alternate approximation to the IP-ASD can be obtained by utilizing a reduced number of plane waves in the receiver output signal sum, given in (26). One straightforward way of reducing the total number of plane waves is by letting \( N \) become small; appropriate changes to \( M \) must also be made. In Fig. 14, three simulations of the IP-ASD are made for successively smaller values of \( N \). The calculations are performed for the same parameters as given in Fig. 8. One simulation is calculated for \( N = 16, M = 7 \), another for \( N = 8, M = 4 \), and the last for \( N = 4, M = 2 \). All 2D-FFT results are scaled by a factor of \( 1/N^2 \) for comparative purposes. The IP-ASD solutions thus obtained are much closer to the actual IP-ASD solution (as calculated for \( N = 128, M = 80 \), and plotted in Fig. 14 for convenience), than was the single plane wave IP-ASD.

VIII. CONCLUSIONS

The angular spectrum technique has been employed to calculate the spectral distortion of the receiver electrical output signal, for an infinite planar reflecting interface (IP-ASD). Incorporation of the exact reflection coefficient creates a robust algorithm equally applicable to both fluid and elastic half-spaces and plates. The solutions provided by this algorithm agree well with previous integral formulations to the normal incidence problem, and to previous numerical studies of the IP-ASD as well.

Use of the angular spectrum technique in computing the IP-ASD is required when investigating materials whose reflection coefficients exhibit rapid changes over small angles of incidence, such as elastic plates. These rapid variations significantly affect the reflected pressure field, influencing the receiver output signal and hence the IP-ASD. Examination of the planar piston IP-ASD for fluid–elastic half-spaces has shown that the reflection coefficients from such interfaces do not significantly influence the result, when compared to similar situations involving a perfectly reflecting interface. The reflection coefficients in this case are generally constant over small angles of incidence. In such situations, the multirate-DSP method can be employed with good accuracy.

For investigations involving the spherically focused piston geometry, it was shown in Ref. 1 that the associated IP-ASD does not attenuate as rapidly for large misalignment angles, as does the planar piston IP-ASD. Hence, ultrasonic experiments involving focused transducers may well be performed at misalignments large enough to shift the plane wave propagation angles into regions where the reflection coefficient begins to change rapidly for small deviations in angle. In such situations, the angular spectrum technique must be employed for accurate predictions of the IP-ASD.

The IP-ASD has been shown to offer quantitative information regarding the structure of elastic plates, and it has been demonstrated that the plate thickness and misalignment angle exhibit significant and calculable effects on the receiver output electrical signal. Such results may be important for the ultrasonic examination of industrial coatings and bonding layers, with applications to process control and nondestructive evaluation.

4 See Ref. 1, Appendix A.
7 L. M. Brekhovskikh, Waves in Layered Media (Academic, New York, 1980), Sec. 1.5-1.10.
8 See, for example, Ref. 6, Eqs. 10.3–10.5.