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Sergey N. Makarov
Worcester Polytechnic Institute, makarov@wpi.edu

S. Kulkarni

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Microwave radiation force on a parallel-plate resonator

S. Makarov and S. Kulkarni

Department of Electrical and Computer Engineering, Worcester Polytechnic Institute, 100 Institute Road, Worcester, Massachusetts 01609-2280

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A simulation method is proposed and tested in order to determine the radiation force on metal targets whose size is comparable to wavelength. The method is based on the method of moments solution of the electric-field integral equation, accurate calculation of the near field, and removal of the self-interaction terms responsible for the pinch effect. The method is used to determine the local force distribution for a parallel-plate metal resonator. It is observed that, at the resonance, the individual metal plates may experience large force densities, despite the fact that the net radiation force on the resonator still remains very small. A potential use of this observation is discussed, which is directed toward possible excitation of acoustic vibrations. © 2004 American Institute of Physics. [DOI: 10.1063/1.1650901]

The radiation forces and torques of a laser beam on small transparent dielectric objects were investigated long ago, both theoretically and experimentally. Recent (macro) applications of the optical radiation force include laser or optical tweezers and optically driven micro electro mechanical systems. The advantage of applying the laser beam to a dielectric is a relatively high intensity of the primary field as well as a relatively low absorption loss in the transparent dielectric.

The radiation forces on metal targets have received little attention. One portion of work was devoted to very small metal particles. On the other hand, one should perhaps mention limited research related to solar radiation pressure on satellites. In that research, the radiation pressure is usually estimated using the basic high-frequency formula

\[ f = \frac{1}{2} \text{Re}(J \times B^*) + \frac{1}{2} \text{Re} (\sigma_\text{S} E^*) \]

where \( E = E + E' \), \( H = H + H' \), and index \( i \) denotes the incident field. The scattered fields \( E', H' \) are found numerically using already known current and charge densities \( J_\text{S} \) and \( \sigma_\text{S} \), respectively, solutions of the MoM equations. The total Lorentz force density on a metal surface is given by

\[ F = \int f(r) ds \]
lengths along the plate dimension \(a\). Figure 1(b) shows three components of the total radiation force \(F\) on the plate as functions of the ratio \(a/\lambda\). The total force is normalized to its high-frequency limit \(F_0 = 2a^2J/c\), where \(J\) is the intensity of the incident signal. One can see that the axial force \(F_x\) asymptotically tends to \(2a^2J/c\) at higher frequencies. At lower frequencies and, especially close to the first resonance, the force values appear to be somewhat higher than the high-frequency limit. The components \(F_y, F_z\) are at least 100 times smaller than \(F_x\), as they should be for reasons of symmetry. In addition, the magnetic (Lorentz) component of the force in Eq. (1) clearly dominates as it follows from the solution analysis.

The scattering analysis of parallel-plate resonators has been among the first to be carried out by computer techniques about 40 years ago.\(^{18}\) Eigenmode charts are available for basic resonator shapes.\(^{19,20}\) However, the data on the radiation force or torque has not yet been reported.

The net radiation force on a parallel-plate resonator [Fig. 2(a)] is found in Fig. 2(b), in the vicinity of the first resonance (TM). The distance between plates \(d\) is 15% of the plate size \(a\). The behavior of the net force is relatively smooth close to the resonance. The opposite situation, however, occurs if we consider the absolute value (magnitude) of the force density and then integrate the result over the surface area. Figure 2(c) shows the result for

\[
F_x^{\text{abs}} = \int_S |f_x(r)| \, ds.
\]

One can see that \(F_x^{\text{abs}}\) at the resonance is nearly 325 times the high-frequency limit \(F_0\). The absolute force on a single plate is approximately one-half of this value.

To understand why \(F_x^{\text{abs}}\) is so different from the net force, we plotted the local force distribution. In Fig. 3(a), the surface shading indicates the surface value of the local force.
density \( f_s(r) \), for the same resonator with \( d/a = 0.15 \). The white color corresponds to positively directed force density (along the \( x \) axis), whereas the black color indicates the opposite direction. The local force density becomes very high at the resonance. Integrally, however, the forces cancel each other. This cancellation takes place not only between two plates but also for every individual plate. The net force \( F_x \) on a single plate was found to be about \( 25F_0 \) for the front plate and about \( 24F_0 \) for the second plate. Compared to the previous value of \( 325F_0 \), this indicates almost perfect cancellation for an individual plate. It was also found that it is the contribution of the electric component of the force (insignificant for single plate) that is responsible for such a cancellation.

Similar results were observed for different plate size/resonator thickness ratios equal to \( d/a = 0.04, 0.06, 0.08, 0.15, 0.20, \) and \( 0.40 \). Figure 3(b) shows the maximum of \( F_{abs}^x \) at the resonance as a function of plate spacing. One can see that \( F_{abs}^x \) may be as high as \( 2000F_0 \) and tends to increase as the distance between plates decreases (\( Q \) of the resonator increases).

In conclusion, we discuss a possibility to use the obtained force distribution in order to directly excite an acoustic mode in a parallel-plate resonator. The force density distribution shown in Fig. 3(a) creates a lateral surface loading \( p(y,z) = f_x(y,z) \) of a single plate. This loading is the source of plate bending, which is described by the lateral deflection \( w(y,z) \). When the incident signal is amplitude modulated, plate loading becomes a function of time: \( p(y,z,t) \). Alternatively, the incident signal may be frequency modulated. In that case, a deviation in frequency leads to detuning the parallel-plate resonator, which also implies a time-varying loading \( p(y,z,t) \).

The time-varying loading \( p(y,z,t) \) excites flexural vibrations of a single plate that are governed by the well-known thin-plate equation (see, for example, Ref. 21, p. 210). This equation is subject to boundary conditions that may be defined from a possible experimental arrangement. For a rough estimate, we will use the average work done by the lateral surface loading (see Ref. 22, p. 90):

\[
W^i = \frac{1}{2} \int_S p(y,z,t) \cdot w(y,z,t) \, ds; \quad i = 1, 2,
\]

where the overbar denotes time averaging (over the acoustic period) and \( i \) is the plate number. When the structure in Fig. 3(a) experiences an acoustic resonance, the local lateral deflection may be approximately assumed to be proportional to the local lateral pressure. The average work thus becomes proportional to the integral of the squared component of the local axial force. This could lead to an even more dramatic dependency of the delivered acoustic power on the \( Q \) of the resonator, compared to the dependency shown in Fig. 3(b).