April 2013

Dual-Axis Solar Tracker

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Dual-Axis Solar Tracker: Functional Model Realization and Full-Scale Simulations

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Degree of Bachelor of Science  

by  

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and  
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Date:  
April 25 2013  

Sponsored by  
French Development Enterprises  

Submitted to:  
Professor Alexander Emanuel, Advisor, WPI  
Professor Stephen S. Nestinger, Advisor, WPI  

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EXECUTIVE SUMMARY

Solar tracking is a technology aimed to improve the power output, especially for photovoltaic solar panels. The French Development Enterprises (FDE) has patented a Dual-Axis Solar Tracking System 444 (STS 444) capable of withstanding wind loads of 89.4 m/s (200 mph) and snow loads of 200 kg/m² (45 lb/ft²). This project analyzed the full-scale Dual-Axis Solar Tracker System 444 under critical weather conditions and realized a small-scale functional model for demonstration purposes. Force and stress analyses were carried out on the STS 444 analytically and simulated in ANSYS Fluent. The data were experimentally verified using a wind tunnel with a reduced scale prototype. The maximum drag occurred at 80° tilt and measured 1.95 MN. The maximum lift occurred at 50° tilt and measured 5.54 MN. Using structural steel for the solar tracker construction material, the maximum simulated stress occurred in the horizontal beams of the frame support structure and measured 880 MPa. The maximum torque required by the linear actuator for snow removal was obtained at different angular velocities of the panel rotation. The functional model was designed to be representative of the STS 444 with maximum polar and azimuthal angles of 80° and 360°, respectively. The functional model was made transportable with a size envelope of 1.60 m by 0.80 m by 0.27 m. Light, load and wind sensors were integrated to the functional model to detect simulated weather conditions. Mathematical models of the static and dynamic motion of the functional model were developed to make necessary design changes and to find the right motor and actuator to provide the desired tracking motions.
ABSTRACT

A Dual-Axis Solar Tracker with a 22.9m (75ft) by 8.23m (27ft) panel was analyzed under critical weather conditions. Analytical and simulated force and stress analyses were conducted for maximum wind loads of 89.4 m/s (200 mph) and maximum snow loads of 200 kg/m² (45lb/ft²). The results were verified by wind tunnel experiments using a reduced scale prototype yielding an expected full-scale drag and lift of 2.33 MN (0.524 × 10⁶ lbf) and 5.11 MN (1.15 × 10⁶ lbf) respectively. For demonstration purposes, a small scale functional model of the tracker was designed with a maximum polar and azimuthal angle of 80° and 360° respectively. Light, panel load and wind sensors were integrated into the functional model to detect simulated weather conditions. Static and dynamic mathematical models of the functional model were developed for component selection and to control the tracking motion. The final tracking resolution was as low as 0.3° for the rotation in polar axis.
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1. INTRODUCTION

Solar power is the fastest growing means of renewable energy production. Globally, the grid-connected solar capacity increased on average by 60% annually from 2004 to 2009 according to the National Center for Policy Analysis\textsuperscript{1}. Yet solar energy only contributed 0.18%\textsuperscript{2} of the total energy produced in the United States in 2011. Of many types of solar energy, the photovoltaic (PV) solar cell technology, has only improved by 0.10% of its total US energy contribution from 2010 to 2011\textsuperscript{3}. However, future of PV solar technologies is promising considering favorable location and continued federal tax subsidies\textsuperscript{1} as well as state renewable standard protocol\textsuperscript{4}. With the continued trend in decreasing cost of PV panels and government subsidies, PV solar energy might become cost competitive in the next 10 years (subsidy-free), for commercial installations while utility-scale installations might take longer\textsuperscript{1}. The August 2010 White House report, on the other hand, predicted that PV solar power will reach grid parity by 2015.

One way to increase the energy collected from the PV panels is by tracking the sun. Novel Dual-Axis solar trackers allow precise control of the polar and azimuth angle of the panel, allowing the panel to always be perpendicular to the sunlight. Tracking the sun can potentially double the energy output of a fixed PV solar system\textsuperscript{5}. Figure 1 compared the power output of four different kinds of tracker (dual-axis, North-South, vertical, East-West) to the power output of a fixed panel\textsuperscript{6}. Figure 1 indicates an increase of output power gain up to 43.87%, 37.53%, 34.43% and 15.69% for the dual-axes, east–west, vertical and north–south trackers, respectively, as compared with the fixed mounted panel. The dual-axis trackers produce the maximum power compare to other types of trackers.
The project sponsor, French Development Enterprises (FDE) is currently developing a patented STS 444 Dual-Axis Solar system that, in addition to tracking the sun in two axes, can also be rapidly deployed and transported. The tracker contains a wind speed sensor and a snow load sensor, which enable the tracker to go into a low drag resistance mode when there is high wind, and tilt the panel to dispose of accumulated snow on the panels. The design also includes a non-ground-penetrating concrete base, which enables the tracker to be deployed on landfill and brownfield sites.

The main purpose of this project was to carry out simulated analysis on the full-scale STS 444 tracker and design a small-scale functional model of the STS 444 for demonstration purposes. The first part of this report focuses on the static and dynamic analysis of the STS 444 under simulated critical wind and snow conditions. The second part of the report discusses the design and fabrication of a small-scale functional model based on the patented STS 444. The designated use of the small-scale functional model is for FDE demonstrations to potential clients. The features of this functional model include a maximum tilting angle of 80° with the horizontal axis to remove accumulated snow and a tracking resolution of 0.3° rotation along the polar axis. The functional model was made to be easily transported, assembled and disassembled.
2. BACKGROUND

The power output of a solar panel depends on the efficiency of the solar cell and the intensity of light falling on the cell. The efficiency depends on cell technology, which determines the price of the solar panel and the power output per square area.

The type of cell commercially in use today is the crystalline silicon cell, which is around 13%-20% efficient. Standard Test Conditions (STC) of 1,000 W/m² solar irradiance and 25°C PV module temperature are used for testing PV cells. So for a cell that is 20% efficient, it produces 200 W of usable power per meter square.

When a solar panel with a specific solar cell is chosen, the only other way to increase power output is by increasing the amount of light falling on the panel. The effectiveness of a PV panel in general, is directly correlated to the amount of sunlight that it is being exposed to. A PV panel is most effective when it is greeted by a light source at a perfectly perpendicular angle, i.e. angle of incidence is zero. Figure 2 below showed how the measured power output of the solar panel depends on the angle of incident of the light. In order to keep the angle of incident at zero in a real-world situation, the PV panel must move with the sun to maintain this perpendicular angle \[ \angle \]. This is why solar tracking is important to get the most power out of a solar panel.

Like most technology today, a large collection of solar tracking systems exist, ranging in price, effectiveness and reliability among others. If key aspects of the application needs were to be neglected, the solar tracker could actually under-perform a well-positioned stationary PV panel.

Although solar tracking could inherently provide more power output than a stationary PV panel, tracking is not always the ideal option. Due to the higher cost for solar tracking technology versus stationary PV panels, solar tracking is not always the best option for a given application. If a stationary PV panel is utilized, it is strategically placed facing the sun. The considerations to be taken regarding PV panel placement is that the panel must be placed in a spot where it will always have a clear line-of-sight (LOS) to the sun, and the panel must be positioned at an optimal angle facing the equator, depending on its latitude on earth \[ \theta \] (Figure 3).
Figure 2: Graph of measured power output of a panel against angle of incidence

Figure 3: Stationary PV panel orientation

Figure 3: Stationary PV panel orientation
Due to the fact that the earth is rotating on a tilted axis and takes an elliptical path around the sun as shown in Figure 4, a stationary PV panel’s output will drastically vary throughout the course of a day and even throughout the year.

![Figure 4: Earth's axis tilt/orbital path affecting solar angle](image)

Because of the sun's movement, a standard PV panel will only observe about 20-35% efficiency under ideal conditions, while solar tracking has been shown to potentially double that with 50% efficiency under ideal conditions.\(^5\)

### 2.1. Types of solar tracker systems

In general there are two main categories of solar tracker systems: single-axis trackers and dual-axis trackers.

#### 2.1.1. Single-axis Tracker

Single axis trackers follow the Sun’s East-West (or even North-South) movement and can either have a horizontal or a polar axis. The horizontal type, shown in Figure 5, is used in regions near the equator where the sun gets very high at noon, thus not having to adjust to vertical changes so much as horizontal changes.
The polar type single-axis trackers, shown in Figure 6, is used in high latitudes where the sun does not get very high, but where summer days can be longer, thus using the fact that vertical movement does not have to be compensated for as much as horizontal movement. Polar type single-axis trackers are good for regions near the north or south poles \cite{10}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.png}
\caption{A horizontal type single-axis tracker}
\end{figure}

\subsection{Dual-axis Tracker}

Two-axis or dual-axis trackers, shown in Figure 7, follow the sun’s movement irrespective of the axis of rotation. Dual-axis solar trackers have both a horizontal and a vertical axis and thus they can track the sun's apparent motion in the sky, no matter where it is positioned on earth. Having dual-axis motion maximizes the total power output by keeping the panels in direct sunlight longer than single-axis trackers or stationary panels \cite{9,10}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7.png}
\caption{A horizontal type single-axis tracker}
\end{figure}
2.2. Tracking system

Solar tracking system can be classified into two main types: passive tracking and active tracking.

2.2.1. Passive Tracking

Passive trackers use compressed gas to move the tracker. Depending on the angle of sunlight with the gas containers, a difference in gas pressure is created. This moves the tracker until it gets to an equilibrium position. The advantage of passive tracker is that the tracking system does not require a controller. But passive trackers are slow in response and are vulnerable to wind gusts.

2.2.2. Active Tracking

Active tracking uses an electromechanical system to position the solar tracker to keep the panel perpendicular to the sun. Trackers that use sensors to track the sun’s position input data into the controller unit, which drives the motors and actuators to position the tracker. There are trackers that use a solar map. Depending on the location, a solar map gives information on where the sun is at different times of day throughout the year. Trackers that use a solar map do not need sensors to track the sun. Some trackers use both sensors and a solar map. During sunny weather, the sensors would be used to track the sun. During cloudy conditions, the information from the solar map would be used. It is important to track the sun even in cloudy conditions since solar panels can still produce energy under those conditions.
2.3. Future prospects

The debate on solar energy production is a controversial one with concern about the efficiency, reliability and, above all, the commercial feasibility pertaining to the investment cost and grid parity of the system. Government subsidies have encouraged research and development of PV solar system as the alternative to natural gas which is the biggest competitor in the production of electricity. However, skeptics are still dubious about solar power potential. In the limit that solar power may take another century to be the sole provider of electricity for the whole world, there are great expectations that it will be able to provide for more than the 0.2% energy contribution².

2.3.1. Efficiency

The efficiency of a dual-axis PV solar system, under ideal conditions, is 20-35% higher than fixed PV solar systems. However, Mousazadeh et al. found that solar tracking could potentially double the efficiency under ideal conditions⁵. With sunlight variations due to the earth rotation and cloud cover, the average electrical power obtainable over a year would be about 20% of its peak wattage⁵. Another published article conducted in Jordan have reported increases in power gain of up to 43.87% in dual-axis tracking as compared to 15.69% for North-South tracking⁶.

Despite the fact that solar cells can last 20 to 25 years, output power is reported to decline by approximately 0.5% per year, even with periodic maintenance. This implies that by the end of 20 years they will only produce, at max, 80% of their rated capacity which will amount to only 16% of their peak wattage under ideal conditions¹.

2.3.2. Reliability

One of the biggest advantages of the conversion of solar energy to electricity via photovoltaic means is the autonomy of each solar panel collector¹¹. Most solar panels are connected in parallel so that in the event of a defective panel, it will not affect the other panels and can be easily replaced without affecting the whole connected system.

Despite the long-term benefits of solar energy, the biggest problems remain the low energy generation and the energy storage ability.
2.3.3. Investment Capital and Pay Back

Currently, subsidized solar energy costs between $0.22 per kilowatt-hour and $0.30 per kilowatt-hour, according to independent analyses. By contrast, the average cost of electricity nationwide is expected to remain roughly $0.11 per kilowatt-hour through 2015. The cost for at home installation is estimated at around $5,000 and it is difficult to determine the exact time for pay back considering the declining performance of the PV cells and other factors such as economic inflation.

2.3.4. Feasibility

The most important concern for investing in solar energy production is the time required for the subsidies to remain in place for these systems to be autonomously profitable and feasible. The fact that the price point of grid parity depends on a number of variables such as the amount of sunlight an area receives, the orientation of the solar array, whether the solar arrays are fixed or track the sun, construction costs and financing options, resulted in significance difference in breakeven costs by more than a factor of 10. Nevertheless, grid disparity was reached in Hawaii where the average price for electricity was $0.41. Arizona is another location which receives abundant sunlight and where grid parity would have been reached if not for its limited transmission access and low electricity prices.

Despite these economic factors as well as practical concerns, solar power could become the most important power source if the technology catches up to increase the efficiency of the photovoltaic collection and the means by which the energy is collected.
3. GOAL STATEMENT

This project had two main goals as specified by the sponsor:

(i) To conduct wind and snow load analyses on the STS 444 to determine the maximum force and stress acting on the system at different panel polar angles.

(ii) To design and build a small-scale functional prototype to demonstrate tracking abilities of the STS 444. The sponsor would be using this model to showcase to its future customers.
4. SOLAR TRACKER SYSTEM 444

The STS 444 is a second generation dual-axis solar tracker under development by FDE. The analyses completed in this report were accomplished using a model based on preliminary blueprints of the STS 444 provided by FDE with rough dimension estimates.

The following sections provide detailed discussion on the wind and snow load analyses using the full-scale STS 444 model. Data were compared via non-dimensional values for ease of application to similar systems.

4.1. Wind load analysis

Of all the loads that solar tracker systems are subjected to, wind load is the most important as it is responsible for the largest loading forces that vary in all direction and can cause mechanical damage due to resonance. In this report, only the most critical configurations of the STS 444 will be analyzed against a specified one-directional wind velocity of 89.4 ms\(^{-1}\) (200mph).

The forces acting on the STS 444 were calculated using three approaches: analytical, simulated and experimental methods (Figure 8). Estimate of forces were obtained from the full scale STS 444 and used to find the scaling factor for the experimental wind tunnel testing. The same conditions used in the wind tunnel were simulated to compare with the experimental data. Once validated, correction was made on the simulation for the full scale STS 444. Force data were collected at different tilt angle \(\theta\) as shown in Figure 9 of the PV collector and compared for the three different approaches. A good analysis is one in which the three approaches – analytical, simulated and experimental - yield relatively close values.
Figure 8: Flow chart of wind drag analysis comparison between three different methods namely analytical, simulated and experimental drag forces

### Forces

The drag and lift forces (Figure 9) were the horizontal and vertical forces acting on the STS 444 with a horizontal wind load. The analytical and simulated forces were calculated on the full scale STS 444. Preliminary blueprints provided by the FDE were used as rough model for the designing of the full scale STS 444. The dimensions of the STS 444 are given in Appendix A.

Both the drag and lift forces were used for comparison between the three approaches. Once verified, the absolute simulated forces on the STS 444 were then used to calculate the Von Mises stresses acting on the structure based on an assumed material.
ii. **Area**

The reference area was crucial in the determination of the force coefficients. The reference area is usually the surface area in direct contact with the wind and has the most influence in creating drag or lift. To be consistent with literature values, the reference area in this report was taken to be the fixed PV surface area, $A_p$ (viewed from the top)\textsuperscript{13,14}. The change in the projected area was accounted for later in the wind blockage correction on the velocity. The fixed reference area allowed for some standard comparison medium between the drag and lift forces.

iii. **Force coefficient**

Calculations for drag and lift coefficients, $C_D$ and $C_L$ respectively, were based on the drag and lift forces, $F_D$ and $F_L$ respectively, acting on the PV collector which is given by Equation 1 where $C$ is the force coefficient, $F$ is the force, $\rho$ is the fluid density, $A_p$ is the PV collector area and $V$ is the velocity of fluid.

\[
C = \frac{F}{\frac{1}{2}\rho A_p V^2} \quad (1)
\]

The following sections 4.1.1 – 4.1.3 described the steps taken in calculating $C_D$ and $C_L$ using the three approaches for tilt angle at 0 degrees. All other equivalent results for the range of angles from 0 degrees (horizontal) to 80 degrees (tilted) were given in the appropriate appendices.

4.1.1. **Analytical Approach**

The Reynolds number of the flow is given by Equation 2 where $V$ is the velocity of fluid, $d$ is the length of the panel and $v$ is the kinematic viscosity of fluid.
The Reynolds number given $V = 89.4 \text{ ms}^{-1}$, $d = 22.86 \text{ m}$ and $\nu = 1.57 \times 10^{-5} \text{ Pa.s}$ along the length of the PV collector was calculated to be $1.3 \times 10^8$. This indicated that the general flow was fully turbulent.

Table 1 shows the drag forces broken down by parts considered to contribute to total drag forces as calculated in Appendix B.

**Table 1: Typical Drag coefficients based on reference area and corresponding drag force for each part in the STS444 for angle at 0 degree**

<table>
<thead>
<tr>
<th>Part</th>
<th>$C_D$ based on</th>
<th>$C_D$</th>
<th>$F_D$ ($10^5 \text{ N}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV collector</td>
<td>Platform area</td>
<td>0.00215</td>
<td>0.0383</td>
</tr>
<tr>
<td>Horizontal beams x2</td>
<td>Frontal area</td>
<td>2.1</td>
<td>0.0066</td>
</tr>
<tr>
<td>Frame support x2</td>
<td>Frontal area</td>
<td>$\sim$ 1.8</td>
<td>0.1320</td>
</tr>
<tr>
<td>Rectangular turntable</td>
<td>Platform area</td>
<td>0.00744</td>
<td>0.0050</td>
</tr>
<tr>
<td>Gear</td>
<td>Circular cylinder</td>
<td>$\sim$ 2.2</td>
<td>0.1512</td>
</tr>
<tr>
<td>Base</td>
<td>Frontal area</td>
<td>$\sim$ 1.2</td>
<td>0.7707</td>
</tr>
<tr>
<td><strong>Total Analytical Force</strong></td>
<td></td>
<td></td>
<td><strong>1.104</strong></td>
</tr>
</tbody>
</table>

For air density $\rho$, PV collector area $A_p$ and velocity $V$ of air at $1.184 \text{ kg.m}^{-3}$, $188.1 \text{ m}^2$ and $89.4 \text{ ms}^{-1}$ respectively, analytical $C_D$ were determined to be $0.124$ at polar angle $\theta$ of $0$ degree. Calculated analytical drag could vary as much as $\pm 10000\text{ N}$ from estimation of individual $C_D$, so that final $C_D$ for the STS 444 could vary by $\pm 0.01$.

Lift forces, $F_L$, are more complicated to calculate and require solving the boundary layer differential equation. A simplified estimate of the lift was obtained using Equation 3 which is the vertical component of the drag.

$$F_L = \frac{F_D}{\tan \theta} \quad (3)$$

$C_D$ and $C_L$ were calculated for each increment polar angle $\theta$ of 10 degrees from 0 – 80 degrees tilt angle and listed in are listed in Table 2.
Table 2: Corresponding analytical drag and lift coefficients for different polar angle of the panel

<table>
<thead>
<tr>
<th>θ (deg)</th>
<th>C_D</th>
<th>C_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>0.42</td>
<td>2.40</td>
</tr>
<tr>
<td>20</td>
<td>0.89</td>
<td>2.46</td>
</tr>
<tr>
<td>30</td>
<td>1.45</td>
<td>2.51</td>
</tr>
<tr>
<td>40</td>
<td>1.72</td>
<td>2.05</td>
</tr>
<tr>
<td>50</td>
<td>1.94</td>
<td>1.62</td>
</tr>
<tr>
<td>60</td>
<td>2.11</td>
<td>1.22</td>
</tr>
<tr>
<td>70</td>
<td>2.20</td>
<td>0.80</td>
</tr>
<tr>
<td>80</td>
<td>2.34</td>
<td>0.41</td>
</tr>
</tbody>
</table>

4.1.2. Wind Tunnel Testing

Reproducing the wind drag on a full scale model at a high wind speed is very costly and time consuming to build the model. However, with the appropriate scaling, it is possible to simulate the forces on a full scale model in a wind tunnel. Scaling and experimental set-up are among the crucial steps in getting accurate force values in wind tunnel testing. An estimate of the magnitude of the forces was obtained from the analytical approach. Using the force ratio from Equation 1, the maximum forces could be used in designing the force probe fixture and choosing the right force sensors.

4.1.2.1. Scaling Factors

Scaling requires geometric, kinematic and dynamic similarities between the real-size model and the prototype to ensure accurate comparison. Geometric similarity should be ensured first before any kinematic and dynamic similarities can exist. If the force and pressure coefficients in the model and prototype are identical, then the Reynolds number should match for incompressible flow with no free surface\(^\text{15}\).

However, perfect dynamic similarity can never be achieved owing to discrepancies in air testing. Similarity in this project was limited by more than just discrepancies in the wind tunnel; as the dimensions decreased, according to Bernoulli relation, velocity increased by the same factor which could not be achieved in the wind tunnels available.
The Reynolds number along both the width and length of the PV collector was calculated to be completely turbulent as listed in Table 3.

**Table 3: Parameters and corresponding values for determining Reynolds number on the panel of the full scale STS 444**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_L$ (length) (m)</td>
<td>Length of PV collector</td>
<td>22.86</td>
</tr>
<tr>
<td>$d_W$ (width) (m)</td>
<td>Width of the PV collector</td>
<td>8.230</td>
</tr>
<tr>
<td>$V^*$ (ms$^{-1}$)</td>
<td>Maximum wind velocity</td>
<td>89.4</td>
</tr>
<tr>
<td>$\nu$ (m$^2$s$^{-1}$) (25 C)</td>
<td>Kinematic viscosity (x 10$^{-3}$)</td>
<td>1.57</td>
</tr>
<tr>
<td>$Re_L$</td>
<td>Reynolds number (along length) (x10$^8$)</td>
<td>1.30</td>
</tr>
<tr>
<td>$Re_W$</td>
<td>Reynolds number (along width) (x10$^7$)</td>
<td>4.70</td>
</tr>
</tbody>
</table>

*1 mph = 0.447 m/s

**4.1.2.2. Scaling Methods**

There are three methods to determine the scaling factor of a model in wind tunnel testing: Reynolds number, force ratio and blockage factor.

**i. Reynolds number:**

For low wind velocity ($\leq$ 200mph), compressibility effects are negligible. This means that viscosity effects which are equivalent to the Reynolds number become more relevant than the Mach number.

Since Reynolds number along both length and width are fully turbulent, trying to keep the Reynolds number for the scaled model consistent with the Reynolds number for the full scale model is very difficult at such high value. The resulting velocity for the reduced scale model would be too high to reproduce. It could only be ensured that the final flow for the reduced prototype is turbulent.

**ii. Force ratio:**

By using analytical and simulated analysis to get an estimate of the resulting drag and moment at the base of the model, $C_D$ and $C_L$ can be calculated using Equation 2.
The panel area $A_p$ is consistently used in all the calculations and the corresponding scaling factor is determined.

iii. Blockage factor:

By using analytical and simulated analysis, an estimate of the resulting drag and moment at the base of the model was found. The aspect ratio of the model surface area perpendicular to the wind flow and the cross-sectional area of the wind tunnel was calculated. Based on blockage correction a scaling factor was determined.

Among the many blockage correction methodologies available are those of Glauert, Maskell, Ashill & Keating, Hensel, Hackett-Wilsden and Sorensen & Mikkelsen.

A simpler wind correction was used as given by Equation 4 to obtain a good scaling factor.

$$V' = V \frac{A}{A - A_p \sin(90 - \theta)}$$  (4)

Since the ratio of the model surface area to the wind tunnel cross-sectional area was low, wind blockage correction was not significant and therefore scaling by force was used to scale down the STS 444. If time was not a constraint, the proper way to choose the best correction methodology would be to compare experimental and theoretical drag values for the different methodologies so that the one with the least disparity would be assumed the best for this system.

Another consideration for the scaling factor was the manufacturing process. The scaling factor of model was limited to the available materials especially for the beams and panel sheet thicknesses.

Taking into consideration the force ratio, manufacturing and wind tunnel set up, a scaling factor of 90 and a wind tunnel speed of 45 ms$^{-1}$ were found to be optimum given that the Reynolds number was still turbulent at $7.28 \times 10^5$ as shown in Appendix D. The experimental drag and lift were expected to reach a maximum of 65 N and 70 N, respectively, based on the analytical values for the drag and lift coefficients. Table 4 summarized the parameters used for the wind tunnel testing.
Table 4: Parameters for scaled model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaling factor</td>
<td>90</td>
</tr>
<tr>
<td>Wind tunnel datum Velocity (ms(^{-1}))</td>
<td>45</td>
</tr>
<tr>
<td>Reynolds number (PV collector) ((x \times 10^5))</td>
<td>7.28</td>
</tr>
<tr>
<td>Estimated max Drag at the base (N)</td>
<td>65</td>
</tr>
<tr>
<td>Estimated max Lift at the base (N)</td>
<td>70</td>
</tr>
</tbody>
</table>

4.1.2.3. Scale Model Design and Fabrication

i. Reduced scale model

The reduced scale prototype was based mostly on the design model used for the analytical analysis given in Appendix A.

The first iteration was that of a rigid rapid prototyped body (Figure 10) which had no degrees of freedom and with the panel at the default horizontal position. The support beams were removed and replaced by a flat sheet through which the panel could be screwed in while keeping the surface area hit by the wind consistent with the real scale model. A support was added at the bottom of the base through which the force probe would be secured (Figure 10).

![Figure 10: Side view of rapid prototyped reduced scale model at 1:90 scaling ratio](image)

The next iteration was fabricated completely out of a combination of Aluminum sheet for the panel, acrylic for frame support and rapid prototyping for more complex parts (Figure 11). This second prototype was made to be able to vary the polar angle of the panel and find the corresponding drag and lift exerted at the base. The prototype was assembled with screws and glue at appropriate places.
Figure 11: Second iteration for the prototype with modification made to the parts to allow for panel rotation and parts all screwed and glued together to reinforce the model

ii. Wind tunnel

The WPI Recirculating wind tunnel, with a cross-sectional dimensions of 0.61m x 0.61m (24 inch x 24 inch) and a maximum wind speed of 60 m/s, was used for testing. The frequency of the blades at 62.5 Hz corresponded to a wind speed of 55 m/s approximately and was assumed linear given in Appendix F.

iii. Force Probe System

A force probe system was designed to find drag and lift on the prototype simultaneously (Figure 12).
Figure 12: Fixture set-up for the prototype in the wind tunnel to measure drag and lift

The drag force on the prototype was determined using a system of moments measured by a force transducer \(^{16}\) (*Phidgets* compression load cell - RB-Phi-113) with 0V – 5V output for maximum force of 44.5N (10 lbf) (Figure 13). The original design (Figure 14) was iterated from a shorter arm to a longer arm about 0.4m since the maximum force was exceeded on the force transducer. The force transducer was connected to a National Instrument USB6008 and measurement type specified as Referenced Single Ended (RSE) at channel ai02. With a maximum force of 65 N, the force balance system would yield a maximum voltage value of 1.2V. A better sensitivity was desired however due to limitations in set-up materials, the range of voltage acquisition was left at 1/5 of total range.
The lift force on the prototype was measured directly using a dual-range (+10N - 50N) force transducer (Figure 15) and data was collected using a Vernier LabPro device. Analytical study at all polar angles yielded lift direction pointing downwards so that the measured force will be compressive if the sensor is placed beneath the force probe. The force sensor was therefore placed under the shaft to measure the corresponding lift as a compressive force. Since lift was measured by direct method and that analytical lift was estimated at approximately 70 N in Appendix D, the system was roughly calibrated with a 20 N weight which was extrapolated to ensure that the maximum recordable force was less than 50. The recordable force for an applied load of 50 N was only 32 N approximately.
4.1.2.4. Methodology

The prototype was fixed onto the lid and inserted into the wind tunnel (Figure 15) with the force probe firmly screwed to the base. The prototype was inserted such that the angle of attack of the wind was always directed onto the top of the panel which caused the force probe arm to contact with both force sensors (Figure 12).

![Figure 15: Schematic section of the prototype in the wind tunnel](image)

i. Calibration of the force probe:

Both force transducers were calibrated with a combination of 100g masses from 100g to 500g and a calibration curve was plotted in Appendix F. The force probe system was set up in the corresponding configuration for drag and lift measurements (Figure 16 - Figure 17).

The horizontal configuration (Figure 16) measured the drag by measuring the moment produced by the drag. The pulley was placed on a stable surface ensuring that the surface was perfectly parallel to the ground. An initial mass reading was recorded with no load. A string attached to the mass and the force probe was passed over a frictionless pulley (Figure 16) so that the mass was pulling on the force probe in a direction exactly perpendicular to the force probe stand to simulate drag. The voltage reading was recorded. The load was removed and the no-load voltage reading was recorded again to check for hysteresis error. An average of no-load voltage reading was carried out if the two no-load values did not match. A calibration curve was plotted in Appendix E.
The vertical configuration (Figure 17) allowed for lift measurement alone since drag produced a moment and lift produced a translational displacement assumed to pass directly through the pivot. Therefore by restricting the rotation of the arm and allowing only for translation, the lift component can be measured (in this case the rotation restriction was provided by the force transducer measuring drag simultaneously). Since the distances moved are minimal, the rotational motion could be neglected so that translational motion is possible.

To calibrate the lift measurement configuration, the masses will be pushing parallel to the force probe pivot in a downward direction (Figure 17). It was ensured that the force probe was initially in contact with the force transducer and that the pivot was found slightly in the middle of the slot to allow for displacement down (Figure 18). Therefore the initial no-load error reading on the force sensor for the lift will be that of the weight of the prototype.
ii. Drag and lift measurements:

Without disturbing the setup configuration after calibration, the polar angle of the panel was adjusted and an initial no-load reading was recorded for both force sensors. The wind tunnel was tuned at a frequency of 52.1 Hz corresponding to approximately 45 ms⁻¹ based on the wind tunnel calibration in Appendix E. The wind flow took about 1 minute to stabilize. The voltage and force readings were taken when the readings stabilized. A final no-load voltage reading was recorded to check for hysteresis. The actual drag and lift were found from the calibration curve. The polar angle θ was increased an increment of 10 degrees from 0 – 80 degrees (Figure 17) and corresponding drag and lift measured for each angle. A simplified version wind blockage correction given by Equation 4 was applied to the wind flow to find the drag and lift coefficients.

4.1.2.5. Results & Discussion

Typical experimental drag and lift values were listed in Appendix F. Experimental $C_D$ and $C_L$ were listed in Table 5.

Table 5: Corresponding experimental drag and lift coefficient for incremental polar angles

<table>
<thead>
<tr>
<th>θ (deg)</th>
<th>$C_D$</th>
<th>$C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.202</td>
<td>0.113</td>
</tr>
<tr>
<td>10</td>
<td>0.408</td>
<td>0.489</td>
</tr>
<tr>
<td>20</td>
<td>0.694</td>
<td>0.723</td>
</tr>
<tr>
<td>30</td>
<td>0.985</td>
<td>0.793</td>
</tr>
<tr>
<td>40</td>
<td>1.140</td>
<td>0.818</td>
</tr>
<tr>
<td>50</td>
<td>1.470</td>
<td>0.940</td>
</tr>
<tr>
<td>60</td>
<td>1.640</td>
<td>0.942</td>
</tr>
<tr>
<td>70</td>
<td>1.82</td>
<td>0.947</td>
</tr>
<tr>
<td>80</td>
<td>2.42</td>
<td>0.956</td>
</tr>
</tbody>
</table>
Experimental $C_D$ increased as $\theta$ increased. This relation was expected considering that the surface area in facing the wind increased which would increase the resultant force acting on the panel. $C_L$ however increased at a decreasing rate to reach almost a constant value.

The limitations of the set-up included a gap between the base and the bottom floor with spaces where the pivot probe passed through which also caused air to escape. Owing to the high forces bending occurred in some of the fixture components and screws which caused the whole prototype system to be slightly tilted. This introduced a systematic error for all the values of $\theta$. In addition, at higher values of $\theta$, slight tilting was causing the prototype to rotate around the vertical axis about the pivot which was very difficult to control at such high forces acting on the system.

4.1.3. Simulated Approach

There were two steps involved in the simulation:

(i) Conduct simulation based on the reduced prototype with exactly the same set-up and conditions as the wind tunnel experiment. The simulated data was validated with the experimental data.

(ii) Conduct simulation on the full scale STS 444 with real-life conditions and obtain more realistic values of the forces acting on the system.

The resultant force acting on the prototype was simulated using ANSYS workbench. The wind flow was modeled using Fluent with uni-directional wind speed in – Z-direction (Figure 19). The flow type was modeled using a simple standard $k-\omega$ turbulent model as described in Appendix C. In both cases, the wind pressure was imported onto the structure and a static structural analysis was carried out on the prototype or STS 444. Materials were appropriately assigned to the different parts of the system and reaction forces added at the fixed support. For time and computation considerations, all simulations were carried out on the model symmetry. Therefore drag and lift forces obtained were half of their actual value.

Estimation of simulated $C_D$ on prototype using Equation 1 was based on density of air $\rho$, frontal area $A$ and velocity $V$ of air at 1.184 kgm$^{-3}$, 0.0201 m$^2$ and 45 ms$^{-1}$ respectively. Calculated $C_D$ and $C_L$ for the prototype were listed in Table 6.
Figure 19: Streamline representation of the CFD analysis on the symmetry face of the boxed fluid for wind speed at 45 m/s in Z – directions.

Table 6: Corresponding simulated drag and lift coefficient for the reduced scale prototype based on similar set-up used in experimental wind tunnel testing

<table>
<thead>
<tr>
<th>θ (deg)</th>
<th>C_D</th>
<th>C_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.215</td>
<td>0.371</td>
</tr>
<tr>
<td>10</td>
<td>0.325</td>
<td>0.839</td>
</tr>
<tr>
<td>20</td>
<td>0.675</td>
<td>1.300</td>
</tr>
<tr>
<td>30</td>
<td>1.083</td>
<td>1.619</td>
</tr>
<tr>
<td>40</td>
<td>1.464</td>
<td>1.763</td>
</tr>
<tr>
<td>50</td>
<td>1.838</td>
<td>1.676</td>
</tr>
<tr>
<td>60</td>
<td>2.087</td>
<td>1.542</td>
</tr>
<tr>
<td>70</td>
<td>2.276</td>
<td>1.369</td>
</tr>
<tr>
<td>80</td>
<td>2.510</td>
<td>1.136</td>
</tr>
</tbody>
</table>
The corrected simulation was then carried out using a wind flow of 89.4 ms\(^{-1}\) (200 mph) with modified Turbulence intensity and Intensity scale length characteristics as described in Appendix C. Correction made between the prototype and the full scale STS 444 were based on location, type of grounds, weather conditions, materials selection and geometrical modifications as listed in Table 7. The turbulent intensity and intensity length were estimated from scaling ratio.

Table 7: Parameters for the reduced prototype and full scale model in the CFD modeling

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Reduced prototype</th>
<th>Full scale model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>Turbulent intensity</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>Intensity scale</td>
<td>0.028</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Table 8: Corrected Drag against lift force (ANSYS – real open space one without gap)

<table>
<thead>
<tr>
<th>Corrected Simulation</th>
<th>(\theta) /deg</th>
<th>(C_D)</th>
<th>(C_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.220</td>
<td>5.24</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.298</td>
<td>5.65</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.553</td>
<td>5.97</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.905</td>
<td>6.14</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1.29</td>
<td>6.19</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.77</td>
<td>6.23</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>2.04</td>
<td>6.10</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>2.10</td>
<td>5.92</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>2.19</td>
<td>5.79</td>
<td></td>
</tr>
</tbody>
</table>

4.1.4. Results

The graph of drag coefficient with polar angle was plotted (Figure 20) for the three approaches: analytical, experimental and simulated. Comparison between the three approaches showed almost linear increase with increase in polar angle. The end points for all three methods
coincided with some deviation occurring in between depending on the way they were approximated.

![CD against θ](image)

**Figure 20:** Graph of Drag coefficient against polar angle for analytical (blue), experimental (red) and simulated (green) showing CD increased almost linearly with polar angle. End points coincide because most of the approximation occurred in between the end points.

Graph of lift coefficient against polar angle was plotted (Figure 21) for the three approaches: analytical, experimental and simulated. Comparison between the three approaches showed all three methods followed different trend according to the ways in which they were approximated. Value tended to converge only at endpoint $\theta = 0$ while at $\theta = 80$ degrees, analytical diverge completely since it followed a sinusoidal curve.
Figure 21: Graph of lift coefficient against polar angle for analytical (blue), experimental (red) and simulated (green) showing all three approaches with different trend for CL as polar angle is increased. Values tend to converge at the endpoints but diverge in between because of the different ways in which they were approximated.

Graph of corrected drag and lift against polar angle was plotted (Figure 22). There was not too much discrepancy between the experimental drag and the corrected drag and the trend was fairly linear. The same similarity was observed between the simulated lift and the corrected simulated lift with a concave down shape, reaching peak at around 40 – 45 degrees.

Graph of ratio of $F_D$ over $F_L$ showed that the rate of increase of drag is greater than that of the increase in lift as $\theta$ is increased (Figure 23). The lift remained much higher than the drag by a factor of at least more than a factor of 2.
Figure 22: Graph of corrected drag and lift against polar angle for full scale STS444

Figure 23: Ratio of corrected drag and lift against polar angle showing an increase in the ratio as polar angle increases. Drag force increases at a faster rate than lift to reach about 38% of the total lift at θ = 80 degrees.
Maximum Von mises stresses (Figure 24) were applied on the simulated system with the appropriate material assigned to the different parts: concrete to the base, steel to the frame and aluminum to the panel. Maximum stresses were observed to occur on the steel beams at the horizontal and vertical beam junctions. Maximum stresses for different polar angles were listed in Table 9 and followed an increasing trend as polar angle was increase. For a tensile strength of 250 MPa for steel, all the maximum stresses exceed this value by far which indicated failure of the beams at those locations of maximum stresses.

Table 9: Maximum Von Mises stress based on different polar angles occurring mostly on the horizontal and vertical beams

<table>
<thead>
<tr>
<th>θ (deg)</th>
<th>Max stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>266</td>
</tr>
<tr>
<td>10</td>
<td>504</td>
</tr>
<tr>
<td>20</td>
<td>623</td>
</tr>
<tr>
<td>30</td>
<td>707</td>
</tr>
<tr>
<td>40</td>
<td>780</td>
</tr>
<tr>
<td>50</td>
<td>799</td>
</tr>
<tr>
<td>60</td>
<td>755</td>
</tr>
<tr>
<td>70</td>
<td>881</td>
</tr>
<tr>
<td>80</td>
<td>748</td>
</tr>
</tbody>
</table>
Figure 24: Von Mises stress contour graph on the symmetry of the full scale STS444 showing maximum stresses at 748 MPa occurring at the horizontal and vertical beam junctions

A contour plot of safety factor at θ = 80 degrees was plotted with a safety factor of more than 1.5 being acceptable. As expected, the safety factor would be zero at places (Figure 25) where the maximum stresses exceeded the tensile strength of the respective material. Based on this model, only the horizontal and vertical beams would be subject to failure at the junctions.
Figure 25: Safety factor contour graph on the frame part of the full scale STS444 showing that failure would occur at the horizontal and vertical beam junctions where safety factor is almost zero (red)
4.1.5. Discussion

Results showed that drag coefficient was directly proportional to polar angle. This was expected since as angle was increased, the surface in contact with the wind increased which increased resistance of flow opposing the wind load. From Equation 1, $C_D$ is directly proportional to drag for constant variables. This expected trend was confirmed by all three approaches with deviation observed in between the endpoints. At the endpoints, the surface areas in contact with the wind are clear-cut and it is easier to approximate without introducing too much error. At 80 degrees for example, the panel consisted of mostly two rectangular shapes: the base and the panel as seen from the frontal area which introduced minimum error in the analytical approach. Whereas in between the endpoints, the analytical approach for example assumed the flow to be blocked completely by the surfaces lying in the path of the horizontal wind flow. This caused an overestimation of the drag. In both the experimental and simulated however, the flow continued past the panel as would be expected to happen in reality. In addition the shapes of the parts contributed significantly in estimating the analytical drag coefficient for the system and these shapes have been in turn simplified to determine their corresponding individual drag coefficient from textbooks. This is why the analytical drag was much higher than the simulated and experimental drag in between the endpoints (Figure 20). The maximum deviation in the values between the three approaches was $\pm$ 40% approximately.

Consider the resultant force as the force perpendicular to the panel surface. The horizontal and vertical components would then be the drag and lift. When $\theta = 0$, the force perpendicular to the panel surface is at its minimum so that the lift which will be almost in the same direction as the resultant force would be at its minimum. As $\theta$ is increased, the resultant force acting on the panel would increase and so will the vertical component which is the lift. At $\theta = 45$ degrees, the resultant force is at its maximum. As $\theta$ is increase, the vertical component will decrease again but not to zero since the panel is never at 90 degrees. The parabolic trend for the lift therefore would be expected to be concave down.

Lift coefficients for all three approaches yielded very diverse trends and values. The general trend was an increase in $C_L$ as $\theta$ increased with inconclusive trend towards the end when $\theta$ was at a maximum. The analytical approach for example was bound to introduce significant discrepancies since it was calculated from the sinusoidal component of the drag. The analytical
lift was expected to follow an inverse tangent curve as expected with an undefined value at $\theta = 0$ (Figure 21). The trend is comparable to the experimental and simulated values with a concave down parabolic shape but with peaks occurring at different values of $\theta$.

The simulated lift values on the other hand were significantly affected by the turbulence model. To prove this, the same set-up conditions were simulated one with laminar and the other with turbulent flow at $\theta = 0$. The pressure distribution for laminar flow (Figure 26) resulted in greater pressure difference between the top and bottom pressure on the panel than for turbulent flow (Figure 27). It was suspected that in the experimental wind tunnel the flow might be more turbulent than the simulated model in ANSYS which might explain the overestimation of the simulated values over the experimental ones.

![Figure 26: Pressure distribution on STS 444 for laminar flow](image)

![Figure 27: Pressure distribution on STS 444 for turbulent flow](image)
Similarity between the three approaches was difficult to achieve in terms of geometry, surface roughness, turbulence model and measurement standards. The beams from the simulated model were modified to a flat plate for the prototype to prevent it from breaking. While the frontal surface area was kept identical, the shape which plays a significant part in the resulting drag and lift could cause comparable deviation.

Surface roughness affects the nature of flow significantly. In order to keep the data as generalized as possible, it was assumed that the roughness of the walls were negligible. In reality and in the wind tunnel however, roughness is present from the objects on the ground for example some minor trees and buildings nearby and in the wind tunnel the wall roughness itself.

According to Peterka et al. 17, standard $k-\omega$ turbulent model is not appropriate to model fully turbulent flow as obtained in the wind tunnel testing since this turbulent flow model does not fully represent the flow in reality. It was reported by Pfahl et al. 14, that a simulated turbulent flow generation as implemented commercially would be a vortex method which would ensure that the turbulence does not dissipate before reaching the body. It was also reported that using the common RaNS (Reynolds averaged Navier-Stokes) simulations averaging eliminated turbulent structures in the flow. The appropriate turbulent models to capture the largest turbulent structures would be the Large Eddy Simulation (LES) or the Detached Eddy Simulation (DES). In addition to the complexity of these flow models which require in-depth knowledge of the characteristics of the turbulence, the fine grid meshing requirement would involve incredible amount of computational time.

The experimental set up for the lift could be improved greatly since it was difficult to prevent the prototype from rotating in the horizontal plane sideways (yaw) which would make the contact probe lose contact with the sensors. The fixture did not restrict the prototype fully in the unwanted planes and direction which caused it to be slightly tilted during the experiment due to very high forces. This introduced a systematic error in all the angles especially for the maximum polar angle of 80. The experimental lift curve indicated that the maximum was reached but did not decrease as predicted by the simulation which confirmed that the tilt of the whole prototype resulted in an absolute panel tilt of less than 80 degrees. This explained why the curve was stuck at maximum at polar angle 80 degrees.
Other sources of error included bending in the fixture and prototype frame support, human error and other measurement inaccuracies and limitations. The fluctuations of the force sensor readings were significant at times due to resonance and values were approximated by taking the mean over a large range of values. The right way would have been to solve the equation of motion for the forced vibration problem and find the appropriate force however this was left for future work. Some hysteresis was observed in the measurements since the vibration caused the pivot arm to shift from its original place.

The correction carried out on the simulated STS 444 was modeled such that it was assumed the ground was a barren land with almost no obstacles considering that the possible location for the installations of the STS 444 might be in brownfield lands. Turbulent intensity for the equivalent simulation of the full-scale model in a full-scale wind tunnel was calculated at 1.46% in Appendix B. However, the turbulent intensity was reported to be 17% based on typical surroundings of locations of solar trackers (mostly barren land with scarce trees and buildings) for an effective surface roughness of 0.03m

Failure at the horizontal and vertical frame beams junctions are only based on the geometry and size of the beams as modeled. Since information was not disclosed as to the design of those beams, it can only be recommended that with a cross sectional area of about 0.36 m by 0.36 m for the horizontal beams and only 0.25 m by 0.13 m for the vertical beams, they would fail at the junctions. Cross support beams should solve this problem with modification as to the shape of the beams to reinforced metal I-beams.

4.1.6. Summary

Low reproducibility of wind conditions and profile make it difficult if not impossible to exactly compare the analytical, simulated and experimental values. The analytical give only a general value for drag based on generic force coefficient based on simple generalized shape in laminar flow while for the experimental values, the wind flow was set a certain speed with no knowledge of the kind of turbulence or other characteristic like vortices. The analytical approach provided an estimate of the forces in order to calculate the appropriate scaling ratio for the experimental approach.
The first part of the simulation was based entirely on the experimental set-up and was validated using the experimental values. Once validated, the final realistic forces could then be simulated with changes to the wind conditions encountered in reality. Therefore the most reliable results will be the simulated values whereby the wind flow will yield more predictive values that it was set to simulate. The surface roughness was neglected in the correction which included only wind blockage, material properties and geometric modifications.

The Von Mises stress analysis showed that the horizontal and vertical beams would fail at the junctions. Failure depends on geometry and materials assigned to the beams. Future work will focus on the design and material properties of the beams for a structurally stable system.

4.2. **Snow Load Analysis**

Part of the autonomy of the STS 444 in withstanding harsh weather conditions will be its ability to tow snow by rotating the panel. This will be driven by a linear actuator. The speed, mechanical capacity and electrical power of the linear actuator will depend on the maximum torque exerted by the snow load. Therefore deriving a snow torque profile to determine maximum torque exerted by the snow load while sliding off was crucial in selecting the right linear actuator.

4.2.1. **Background**

Although snow loads are not as significant as compared to the wind loads, they contribute to factors that affect the linear actuator kinetics. Modeling of a snow sliding profile was crucial in finding the time taken for the snow to be completely stowed and the maximum polar angle required. The snow sliding profile was also very important in finding the torque applied to the linear actuator shaft by the sliding snow.

It was thought that the sliding snow could be compared to a mini-avalanche effect whereby after a triggering effect, the snow would usually accelerate rapidly and the amount of snow sliding would increase exponentially. The equilibrium slope angle of a loose particulate dry snow on a slope \(^{18}\) can be approximated by Equation 5 where \(\alpha\) is the angle of inclination of the snow surface and \(\varphi\) is the internal friction angle.

\[
\alpha = \varphi \quad (5)
\]
In this problem however, the panel is in motion and the equilibrium profile provided in the literature will need to be calculated at each angle increment of the rotation to determine the amount of snow sliding off. This is a tedious undertaking that was not covered in this report.

Therefore, in order to obtain the snow sliding profile, the differential equation was solved analytically and the same conditions were fed into simulation software to complete the results of the snow sliding profile. The analytical solution was missing boundary conditions that were necessary to find all the constants and while the simulation package was powerful, it could not model a variable load with variable location. By combining both methods, the limitations that were met analytically were overcome by simulations and vice versa.

4.2.2. Snow Torque Profile

The problem was defined as a simple rotating panel and a snow load sliding along the surface of the panel (Figure 28). The complexity in deriving the snow torque profile stemmed from the fact that both the magnitude and the location of the load was varying non-linearly.

Figure 28: Schematic of the snow sliding off the rotating panel as a single rigid body with cross section area $A_v$ of snow that would fall off at each incremental $\theta$
The list of the variables used to derive the differential equation of motion of the system is given in Table 10.

**Table 10: Terms and definitions of the variables used to derive the differential equation of the snow sliding torque profile.**

<table>
<thead>
<tr>
<th>Terms</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>Width of the panel and snow part</td>
</tr>
<tr>
<td>( x )</td>
<td>Height of the snow part</td>
</tr>
<tr>
<td>( L )</td>
<td>Length of the panel</td>
</tr>
<tr>
<td>( A )</td>
<td>Cross-sectional area of snow ((A = wx))</td>
</tr>
<tr>
<td>( A_v )</td>
<td>Cross-sectional area of snow removed (hatched area)</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass of the snow</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Tilt angle of panel</td>
</tr>
<tr>
<td>( r(\theta) )</td>
<td>Separation distance between edge of snow part and panel edge (function of ( \theta ))</td>
</tr>
</tbody>
</table>

The first part of the derivation consisted of determining the differential equation that governed the sliding of the snow onto the panel. The differential equation defined by Equation 5 was solved for \( r(\theta) \) as derived in Appendix G where \( \mu_k \) is the dynamic coefficient of friction.

\[
\ddot{r} + 2\mu_k \dot{r}\dot{\theta} - r\dot{\theta}^2 = g\sin\theta - \mu_k g\cos\theta \quad (6)
\]

The solution to the differential equation was given by Equation 7.

\[
r = C_1 r_1 + C_2 r_2 + Ur_1 + V r_2 \quad (7)
\]

where

\[
C_1, C_2 = \text{constants}
\]

\[
r_1 = e^{-\omega(\mu_k + A)t}
\]

\[
r_2 = e^{-\omega(\mu_k - A)t}
\]

\[
U = \frac{-g(\mu_k + A)e^{(\mu_k + A)\theta}}{2\omega A[\omega^2(\mu_k + A)^2 + 1]} \left[ \sin\theta - \frac{\cos\theta}{\omega(\mu_k + A)} - \frac{\mu_k \cos\theta}{\omega(\mu_k + A)} - \frac{\mu_k \sin\theta}{\omega(\mu_k + A)} \right]
\]

\[
V = \frac{g(\mu_k - A)e^{(\mu_k - A)\theta}}{2\omega A[\omega^2(\mu_k - A)^2 + 1]} \left[ \sin\theta - \frac{\cos\theta}{\omega(\mu_k - A)} - \frac{\mu_k \cos\theta}{\omega(\mu_k - A)} - \frac{\mu_k \sin\theta}{\omega(\mu_k - A)} \right]
\]

\[
\theta = \omega t
\]
\[ A = \sqrt{(\mu_k^2 + 1)} \]

The initial condition was obtained by finding the angle at which the static snow was just about to slip which was given by Equation 8 where \( \mu_s \) is the static coefficient of friction. The relation was derived in Appendix G.

\[ g \sin \theta + \mu_s g \cos \theta = -r \dot{\theta} \quad (8) \]

For an arbitrary value of the angular velocity \( \omega \) at 0.1 rad/s, the angle of slip was just about 10.5 degrees for \( r(\theta) = 0 \). However another set of boundary conditions were needed to determine the values of the constants \( C_1 \) and \( C_2 \).

This limitation was overcome by simulating the sliding based on the initial conditions and the values of \( \mu_s \) and \( \mu_k \) to obtain a set of boundary conditions.

The second part of the derivation consisted of determining the magnitude of the load. It was assumed that incremental area \( A_v \) (Figure 28) would represent the decrease of load with \( \theta \). Other approximation of the snow leaving the panel would result in higher or lower estimation than what was used here. The variation in the magnitude of the load was given by Equation 9 as derived in Appendix G.

\[ W_s(\theta) = \rho g L \left[ xw - r(\theta)x + \frac{x}{2} \tan \theta \right] \quad (9) \]

Hence the torque exerted by the snow at the pivot point \( O \) was defined by Equation 10

\[ T_s = \frac{1}{2} W_s(\theta) r(\theta) \quad (10) \]

Assuming no power is lost, the force exerted axially on the linear actuator bearings would then be given by Equation 11. For small angles of \( \theta \), \( \omega \) can be approximated as constant.

\[ F(\theta) = \frac{T(\theta) \omega(\theta)}{v} \quad (11) \]

4.2.2.1. Model parts and Assembly

Most simulate package required prior knowledge of the snow sliding profile which needed to be fed to the simulation set-up which was what was sought after. The problem in this analysis being more of deductive problem than inductive required over-simplification of the snow sliding
profile. Therefore assumptions were needed for the snow unloading simulation with detailed simulation parameters given in Appendix G:

i. Snow was assumed to be a discrete load and was modeled as a part to capture both the change in load direction and load magnitude.

ii. Snow was taken as fresh new snow \(^{18}\) with a density of 50 – 70 kgm\(^{-3}\)

iii. Gravity was the only external force acting on the system

iv. Static and dynamic friction of snow on PV panel was approximated as 0.1 and 0.03 \(^{19}\) respectively

v. All other connections and contact surfaces were assumed to be frictionless

vi. Linear actuator was assigned a constant speed

The problem was having the location of the applied load change as well as the magnitude of the applied load. While the change in magnitude was easy enough to be set up, the change in direction of the force could only be simulated by modeling the snow as a part interacting with the mechanism in motion. The snow was modeled as a flat rectangular body to simplify the mechanism analysis set-up.

The solar tracker system subassembly consisted of 4 parts: panel, linear actuator motor housing, linear actuator shaft and frame support. The frame (Figure 29) was assembled with a rigid connection while the linear actuator motor housing (Figure 29) was constraint with a pin connection at the axis of rotation with the frame support. The linear actuator shaft (Figure 29) was constraint with both a slider connection and a bearing connection with the linear actuator motor housing and PV panel respectively. The snow part (Figure 29) was constraint with a *slider connection* between the snow surface and the panel surface.
4.2.3. **Mechanism Analysis**

After properly constraining the assembly, a defined linear servo motor was added to the linear actuator shaft slider connection at a constant speed to control the rotation of the panel and gravity was enabled as external force at 9.806 m s\(^{-2}\). Static and dynamic coefficients of friction were enabled at 0.1 and 0.03 respectively. The angular position, angular velocity, angular acceleration and torque measurements were defined at the pin connection (Figure 29) between the panel and the frame support while another torque measurement was defined on the linear actuator shaft (Figure 29).

Material for snow at maximum weight was applied to the snow part and aluminum applied to the panel and linear actuator parts.

The simulation was run for the servo motor acting downwards with constant velocity and panel at 0 degree. Gravity and friction were enabled. The only parameter needed for this simulation was the distance between the edge of the snow relative to the edge of the panel (Figure 30) to obtain the distance slid by the snow along the panel.
4.2.4. Results and Discussion

For a snow load of 200 kg/m$^2$ with a total force of 0.37 MN for panel size of 22.9 m by 8.23 m, the separation distance between the snow edge and the panel edge was plotted against polar angle (Figure 31). Remaining snow on top of panel was plotted with polar angle to show variation in magnitude of the load (Figure 32). Angular velocities of 0.05 rad/s, 0.1 rad/s and 0.5 rad/s were chosen to provide wider range of the effect of angular velocities on torque.

As angular velocity of the panel was increased, the polar angle of the panel at which the snow completely left the panel increased. This was partly because the panel was rotating at a faster rate than the snow was sliding.
Figure 31: Separation distance against polar angle showing an increase in angular velocities result in increase in angle at which the snow load completely left the panel.

Figure 32: Graph of snow load exerted on panel against polar angle for different angular velocities.
In order to verify that the simulation worked, the torque at pin joint O was plotted with and without snow load. The expected graph should show a curve that deviated from the curve without load only when the snow load is interacting with the panel. The result proved that the snow interacted with the panel only when the snow started sliding along the panel which occurred at around 10 degrees which proved that the snow did not interact with the panel in other unanticipated ways than what it was set to do. However graph (Figure 33) was not representative of the real physics since the sliding connection between the panel and the snow was to infinity. Therefore this simulation only served to obtain the location of the snow along the panel and to ensure that the sliding connection was appropriate enough.

![Graph of reaction torque at the pin joint O against polar angle with (black) and without (Red) snow load. Result showed that with snow load, there is a drastic increase in the torque to 14000 Nm at an angular velocity of 0.1 rad/s while the torque curve increased almost linearly without any snow load.](image)

Figure 33: Graph of reaction torque at the pin joint O against polar angle with (black) and without (Red) snow load. Result showed that with snow load, there is a drastic increase in the torque to 14000 Nm at an angular velocity of 0.1 rad/s while the torque curve increased almost linearly without any snow load.

To obtain the effect of the torque of the snow on the panel, the torque of snow applied on panel at each different angular velocities (Figure 34) were applied at the pin joint O. As was expected the larger the angular velocity of the panel rotation, the larger will be the polar angle and the lower the maximum torque exerted since when load is passing at the location where torque is maximum more of the snow has had time to leave the panel.
Figure 34: Graph of torque applied by snow sliding off the panel against polar angle at different angular velocities.

The graph of the resultant torque profile at pin joint O is the result of applying the snow torque to the pin joint O (Figure 35). The result as expected should have the same trend as the sliding snow torque profile (Figure 34).

Figure 35: Graph of resultant torque when snow torque was applied at pin joint O against polar angle for the different angular velocities
An important parameter to consider the speed and type of linear actuators will be the time taken to completely remove snow. For example, the amount of time it takes to tow the snow will result in decrease of energy from unavailability of sunlight collection and having to provide energy to tow the snow. Therefore the final selection will be a compromise between the energy lost in inability to capture the sunlight, the energy needed to power the linear actuator and the energy needed to power the linear actuator to return the panel to its respective orientation depending on the angle of tilt at which the snow was removed completely (Figure 36). Therefore maximum torque against time of removal and angle at which the snow is completely removed as listed in Table 11 are among the important parameters in considering the linear actuator speed and torque capacity.

Table 11: Maximum torque, angle at maximum torque, angle of slip and time for complete removal for the different angular velocities

<table>
<thead>
<tr>
<th>Angular speeds (rad/s)</th>
<th>$\omega = 0.05$</th>
<th>$\omega = 0.1$</th>
<th>$\omega = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum torque (kNm)</td>
<td>14.7</td>
<td>13.8</td>
<td>13.7</td>
</tr>
<tr>
<td>Angle at max torque (deg)</td>
<td>15</td>
<td>20</td>
<td>48</td>
</tr>
<tr>
<td>Angle of slip (deg)</td>
<td>6.7</td>
<td>7.3</td>
<td>9.9</td>
</tr>
<tr>
<td>Time taken for removal (s)</td>
<td>5.33</td>
<td>3.95</td>
<td>2.15</td>
</tr>
</tbody>
</table>

Figure 36: Graph of maximum torque against removal time for snow to completely be towed for different angular velocities
It is important to note that the simulated results were based on the specific conditions such as constant speed driving the linear actuator shaft, the mass of a discrete rigid body, coefficients of friction, initial position of panel and snow load and shape of the snow load.

4.2.5. Summary

The sliding snow torque profile was based on numerous assumptions that were made as general as possible. The maximum torque was found to be 14.7 kNm approximately for an angular velocity of 0.01 rad/s of the panel rotation and for snow load at 200 kg/m². The time for complete removal at that angular velocity was around 5 – 6 seconds based on a coefficient of kinetic and static friction of 0.03 and 0.1 respectively. It was also assumed all other joints were frictionless and that no other frictional sources were acting on the system for example wind drag.

Higher angular velocities of the panel were also analyzed which yielded lower maximum torque and lower removal time but higher angle of rotation. This introduced the concept of cost performance whereby the cost of the energy lost in rotating the panel a greater angle against higher torque needed for a smaller displacement requires optimization in order to select the right speed and capacity for the linear actuator. The economics aspect of the solar tracker is a subject that would be addressed in future work.

4.3. Conclusions and Recommendations

Results showed that the snow load was not as significant as compared to the wind load by at least a factor of 10. In addition the wind load can cause a great amount of mechanical damage in the sense that if in addition to the maximum wind load, the frequency of the wind equaled that of the mechanism system this will result in resonance.

Therefore the optimization of the beams supporting the panel and the base should be based on the wind force and stress analyses while the snow load analysis will be mostly geared towards selecting the right linear actuator. Based on the large amount of drag, the non-ground penetrating base should be strong enough not to be uprooted while part of the lift will be pushing vertically down on the solar tracker, this might help in anchoring the base to the ground during wind storms with wind at 89.4 m/s (200mph).

It was recommended based on the stress analysis of the solar tracker that the horizontal and vertical beams supporting the panel should be increased and reinforced further to prevent failure.
and the assumptions used in this report for the snow load analysis was that the solar tracker parts were negligible in weight as compared to the snow load. For the selection of the linear actuator, weights of the beams and panel parts need to be included for correctly predicting the maximum torque of the snow.

Both the wind and the snow load are critical to optimizing the design of the STS 444. This project only offered a rough preliminary estimate of the forces acting on the STS 444. Future work will focus on optimization of in-depth structural design and how parameters such as energy power to the driving actuators and gears affect the economics of the STS 444 depending on the speed of the cycles, the amount of tracking, type of tracking and default configurations in response to critical conditions.
5. FUNCTIONAL MODEL

The purpose of the functional model was to demonstrate to FDE’s customers how the large scale model (STS 444) duel-axis tracker would operate. The first step of realizing the functional model was to define the design specifications for the model.

5.1. Design Specifications

The design specifications for the functional model were based on the specifications set by the FDE and additional specifications generated by the team.

1. The functional model should be at most a three-piece assembly that could be disassembled into the panel, frame support and base

2. A panel with dimensions 1.58m × 0.81m at 16 kg was provided and all other parts should be designed correspondingly

3. The three disassembled parts should fit within a design envelop of 62 in × 32 in × 12 in (1.58m × 0.81m × 0.3 m) for storage and transportation

4. The function model should be able to assembled or disassembled within 2 minute and should be easily transported by one person

5. The total weight must be less than 36 kg (80 lbs.)

6. The panel should be able to rotate a full 360 degrees around the azimuth (vertical) axis

7. The panel should be able to tilt a maximum of 80 degrees with the polar (horizontal) axis

8. The model should be able to demonstrate snow removal simulation, solar tracking up to ±5 degree accuracy, and go into the low-drag mode in windy conditions to reduce drag forces

9. The model should be able to use power from the solar panel during outdoor use or the 110V AC socket during indoor use

10. The model should contain an internal battery when the two power sources are not available

11. The sensors should respond to external stimuli for wind, snow load and light simulation

12. Parts used should be easily obtained and compatible with all assembly parts used

13. The prototype must be easy to troubleshoot and easily assembled by a trained mechanic
5.2. **Design Description**

A system of linear actuator and slewing gear are used to rotate the PV panel in polar and altitude angles which are controlled by the linear actuator and the gear system respectively. Some of the mechanical and electrical components were both manufactured and purchased as described in the following section.

5.2.1. **System Kinematics**

The most commonly used type of linkage for rotation motion for solar panel is roughly that of a four-bar linkage with the linear actuator comprising the third linkage which is the slider (Figure 37). In this case the four-bar will be that of an infinitely long four-bar mechanism because of the slider. Link 4 and Link 3 are pivoted to ground and Link 3 is a slider assumed frictionless. Link 3 is assumed to act as the input link and is driven at an assigned linear constant speed \( \nu \). With this initial configuration (Figure 37), the effective length of Link 3 decreased in length and Link 4 is pivoted at ground in a clockwise direction through a maximum polar angle of 80 degrees.

![Figure 37: Schematic of infinitely long four-bar linkage equivalence of the linear actuator system connected to the PV collector and the frame support showing the Links 1 – 4 and transmission angle \( \mu \)](image)

5.2.2. **Linear actuator positioning**

The position of the linear actuator attachment points are defined by the size and stroke length of the linear actuator, the maximum angle of tilt of panel and the transmission angle. The exact location for the position of the linear actuator was found by graphical and verified by analytical methods considered with holonomic constraints only.
Ideally the linear actuator would be in compression when carrying the highest load, that is, when
the panel is being pushed up from a tilted position to the horizontal position. From a material
engineering point of view, the microvoids and shear bands tend to form in relatively brittle
materials in tension which limit the ability of the material to undergo strain. Although in such a
small scale system as the functional prototype sudden fracture is not likely to cause any damage,
for the much larger STS 444 system, creep may be one of the most critical material behaviors to
anticipate failure. This is why keeping the linear actuator in compression is so important.

Selection of linear actuator

The selection of the linear actuator was a compromise between what was available commercially
and the corresponding sizes, weights, costs and performances. Linear actuators from ServoCity\textsuperscript{20}
offered a range of different linear actuator that was within the allocated budget. The stroke
lengths for the available linear actuators were listed in Table 12. The cost and the performance
for the different stroke lengths of the chosen linear actuator type were comparable with
maximum velocity of 2 inch/sec (0.05m/s) and a maximum load capacity of 115lbs (512N
approximately).

The limiting conditions were the dimension and mass of the linear actuators: the linear actuator
should have a minimum length that fits into the functional model when fully retracted at least
and the longer the stroke length the heavier the linear actuator will weigh.

Dimensional limitation

The dimensional limitation was the most critical limitation when selecting the linear actuator
based on the dimensions of the solar panel as specified in the design specifications.

Parts were modeled based on available standard parts and materials are available and
dimensional as well as other limitations as will be described in following sections.
5.3. Design Iterations for Linear Actuator Positioning

5.3.1. Design Iteration 1

This linear actuator positioning iteration was assumed to have the bottom pivot passing through the center line of the pin connection between the panel and the frame support at point B and at point O (Figure 38). The horizontal and vertical dimensions to position the linear actuator was defined as \( x \) and \( y \) respectively.

![Figure 38: Side view of iteration 1 with x and y defined as horizontal and vertical distances respectively from points of attachments at A and B of the linear actuator to the point of origin O.](image)

The mathematical relation of \( xy \) (Figure 40) given in Equation 12 was derived in Appendix A.

\[
xy = \frac{L_1^2 - L_2^2}{2\cos\theta} \quad (12)
\]

Plotting the graphs of functions \( y (f(x), g(x), h(x)\ldots k(x)) \) against \( x \) distance from center O where \( (f(x), g(x), h(x)\ldots k(x)) \) are the \( y \) distances for linear actuator types 1 – 6 respectively (available from manufacturer) as listed in Table 12.

<table>
<thead>
<tr>
<th>Type</th>
<th>Stroke length/in</th>
<th>Extended ( L_1 ) (max)/in</th>
<th>Retracted ( L_2 ) (min)/in</th>
<th>( xy/\text{in}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.96</td>
<td>9.64</td>
<td>7.68</td>
<td>17.23</td>
</tr>
<tr>
<td>2</td>
<td>3.93</td>
<td>13.62</td>
<td>9.69</td>
<td>46.51</td>
</tr>
<tr>
<td>3</td>
<td>5.90</td>
<td>17.59</td>
<td>11.69</td>
<td>87.71</td>
</tr>
<tr>
<td>4</td>
<td>7.87</td>
<td>21.57</td>
<td>13.70</td>
<td>140.93</td>
</tr>
<tr>
<td>5</td>
<td>9.94</td>
<td>25.65</td>
<td>15.71</td>
<td>208.73</td>
</tr>
<tr>
<td>6</td>
<td>11.81</td>
<td>29.53</td>
<td>17.72</td>
<td>283.32</td>
</tr>
</tbody>
</table>

Table 12: Stroke lengths range and product \( xy \) for available linear actuator from ServoCity TM
The resulting curves (Figure 39) obtained were as expected with asymptotes at low values of x and with increasing values as stroke lengths increased. The selection for the linear actuator was based on the dimensional constraint of the PV panel where r is half the length of the panel width at 0.405 m (15.94 in): \( x \leq r \).

![Graph of y (f(x), g(x)…k(x)) against x distance from center O with functions (f(x), g(x)…k(x)) representing y distances for linear actuator types 1 – 6 respectively (available from manufacturer). Vertical line at x = 15.9 represented the maximum limit.](image)

**Figure 39:** Graph of y (f(x), g(x)…k(x)) against x distance from center O with functions (f(x), g(x)…k(x)) representing y distances for linear actuator types 1 – 6 respectively (available from manufacturer). Vertical line at x = 15.9 represented the maximum limit.

The values of x and stroke lengths are chosen based on the compromise that low values of y are desired as it reduces amount of material and size of the functional model while high values of x are desired for lower torque required and greater sensitivity in the panel rotation. However, high values of x will also mean that the linear actuator will be protruding from the sides which might be cumbersome. Lower stroke lengths were too short and would require high torque to rotate the panel while higher stroke length implied heavier linear actuators and use of only a portion of the total linear actuator stroke length. Therefore, linear actuator (type 3) with stroke length 5.90 inch
was chosen since it was the minimum stroke length to yield reasonable y values \((x = 10.69\text{in}, y = 13.96\text{in})\).

The limitations for this design iteration were as followed:

i. The dimension ranges were calculated only at the different extreme positions and dynamic motions were not taken into consideration

ii. Rotation point was taken at the center of mass of panel when in reality pivot point is found slightly offset along the vertical axis

iii. Attachment points of linear actuator on panel and beam were taken to be lying right onto the mid-axis. Optimum position of the attachment points might not be lying on the vertical and horizontal axes.

Although simple in concept, this iteration failed as it did not satisfy the “maximum polar angle of the panel at 80 degrees” design specification requirement.

5.3.2. **Design Iteration 2**

This design iteration addressed the maximum tilt angle of 80 degrees requirement by removing the constraint for point B (point of attachment between the bottom of the linear actuator and the frame). In the previous design iteration, B was constraint along center axis of frame passing through O. A new variable \(\delta\) (Figure 40) was introduced as the offset distance between O and O’.

![Figure 40: Side view of iteration 1 with x and y defined as horizontal and vertical distances respectively from points of attachments at A and B of the linear actuator to the point of origin O’ and \(\delta\) as offset distance between O and O’.]
The optimal value of $\delta$ was derived in Appendix I using both analytical and geometrical approach. The analytical approach produced two possible solutions for each specific length of linear actuator which were checked graphically and the transmission angles were subsequently compared for both solutions.

The 5.90 inch stroke length linear actuator was chosen for its compactness and minimum transmission angle at greater than 40 degrees.

With a 5.90 inch stroke length, x and y at 10.70 inches and 13.96 inches respectively, the minimum transmission angle was 52 degrees as compared to a transmission angle of 19.6 degrees at the second location (Figure 41). The other stroke length transmission angles were measured and listed in Table 27 in Appendix I.

![Figure 41: Graph of transmission angle against polar angle of the rotating panel for stroke length of 5.90 inch linear actuator.](image)

The closer the linear actuator center of gravity is to the center of gravity of the support frame, the more stable the assembly will be which will in turn require a smaller surface area for the base. The second solution for x and y would have resulted in a larger distance difference from the two centers of gravity which cause instability in addition to the angular momentum from the moving parts.
5.3.3. Design Iteration 3

Another alternative to the infinitely long four-bar mechanism was a six-bar mechanism based on an aircraft landing gear system. This iteration consisted of a Six-bar linkage which can be broken down into two Four-bar mechanisms connected with a ternary link (Figure 42).

For the determination of the lengths, the right hand side four-bar mechanism “PV_Fourbar” (Figure 42-blue line), was analyzed first through Linkages program by varying the amount of lengths while keeping in mind the transmission angle should be greater than 40 degrees minimum (Figure 43).
Transmission angle as calculated ranged from 44.4 degrees to 86.5 degrees which was acceptable since it was higher than the minimum 40 degrees and varied to within \( \pm 50 \) degrees (Figure 44).

The linear actuator length was limited by the retracted and extended lengths, which was used to optimize the system for a maximum polar angle of 80 degrees. As an acceptable range for linkage lengths for “PV_Four-bar” mechanism both transmission angles \( \mu_1 \) and \( \mu_2 \) should be at least 40 degrees. The optimized design for the “PV_Fourbar” was found in Linkages software.

The size of the mechanism was then scaled depending on the stroke length of the chosen linear actuator which this case would be 15 cm (5.90 inch) stroke length as provided by manufacturer. Ideally the height of the mechanism should be slightly more than half the width of the PV collector for complete tilting without any interference.
Once the model was scaled based on the optimization of the “PV_Fourbar” and the linear actuator length, the linkages were modeled in CAD (Figure 45). Motion was simulated with a constant speed of 0.01 m/s for servo motor added to the linear actuator. Measurement of transmission angle $\mu_1$ was then determined during the simulation which yielded 6.6 degrees and 55 degrees approximately at the two extreme positions (Figure 46).

Figure 45: Model of the six-bar mechanism showing the two extreme positions at $\theta = 0$ degrees (left) and $\theta = 80$ degrees (left).
Transmission angles for the six-bar $\mu_1$ and $\mu_2$ were 40 degrees and 6.6 degrees at 0 degree polar angle and 44.4 degrees and 55 degrees at 80 degrees polar angle respectively. The minimum transmission angle was significantly below 40 degrees which was undesirable.

5.4. Static Force Analysis

A static force analysis was carried out on the linear actuator mechanism iterations to determine whether the load acting on the linear actuator will be within the range of the maximum load capacity of the commercial linear actuator.

5.4.1. Design Iteration 1

From a preliminary measurement of the transmission angle for Design Iteration 1, since the transmission angle dropped lower than 40 degrees, all analyses thereon will be disregarded and not reported.
5.4.2. Design iteration 2

Static forces were derived analytically using FDBs as drawn in Appendix J. The parameters used in the derivation were listed in Table 28 in Appendix J. Maximum static force acting on the system would occur at different positions depending on the connections as derived in Appendix J. Reaction static forces at the pin joints where the linear actuator connected to the panel approximated a maximum of 2.64 N when polar angle was at 0 degrees. Reaction static forces at the bottom connection of the linear actuator where it connected with the frame support approximated 12 N at a polar angle of 80 degrees.

5.4.3. Design Iteration 3

A full static analysis of the six-bar mechanism was carried out by simulation. Results showed that at polar angle of 0 degree the static reaction force was at 2 N approximately while at polar angle of 80 degrees, the static reaction force was at approximately 157 N (Figure 47).

5.4.4. Selection of linear actuator mechanism

The factors considered in the selection of the mechanism for the linear actuator were transmission angle, dimensions, number of parts and torque as listed in Table 13.
Table 13: Parameters in the selection of the type of mechanism at a constant linear actuator speed of 0.01 m/s

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Transmission angle (Deg)</th>
<th>Size – height x width (cm)</th>
<th>No. of parts</th>
<th>Max Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 – Bar</td>
<td>μ = 48 – 90</td>
<td>55 x 30</td>
<td>2</td>
<td>0.145</td>
</tr>
<tr>
<td>6 – Bar</td>
<td>μ₁ = 6 – 55</td>
<td>68 x 48</td>
<td>4</td>
<td>0.175</td>
</tr>
</tbody>
</table>

The full cycle motion of the panel, which consisted of rotating the panel from 0 degrees to 80 degrees and back to 0 degrees using a servo motor with a constant linear velocity at 0.01 m/s, was simulated so that a full cycle will take about 30 seconds. The torque was measured at the pin joints where the linear actuator arm was connected to the panel. Similar conditions of servo motor and speed were applied to both four-bar and six-bar mechanisms and the maximum torque noted.

Based on the transmission angle, size of mechanism and number of parts and maximum torque the four-bar mechanism was at an advantage over the six-bar. The four-bar had a higher minimum transmission angle which was reflected in the six-bar having a higher torque at some locations. The four-bar required clearly less torque to operate than the six-bar which was desirable for less power needed. In addition, because of the design specifications that pertained to the transportability, compactness and easy assembly of the functional model, size and number of parts were crucial factors in selecting the four-bar as the right mechanism.

5.5. Dynamic Analysis

5.5.1. Velocity Analysis of the four-bar linkage

A velocity analysis was conducted on the four-bar linkage to get an analytical estimation of the velocity the panel will be rotating at a specific speed of the linear actuator. The velocity of linear actuator $V_{BA}$ (extending) was set at 0.01 m/s. The angular velocity of the panel $\omega_2$ (Figure 48) was determined by Equations 13 – 15.
Figure 48: FBD of the four-bar linkage used for the velocity analysis

\[ V_{OA} = x_1 \times \omega_1 \]

where \( \omega_1 = \frac{d\beta}{dt} = \dot{\beta} \)

\[ V_{OB} = (x_1 + x_2) \times \omega_1 \]

\[ V_{CB} = x_3 \times \omega_2 \]

where \( \omega_2 = \frac{d\alpha}{dt} = \dot{\alpha} \)

\[ V_{CB} = V_{OB} + V_{BA} \quad (13) \]

From geometry,

\[ \frac{\sin \beta}{x_3} = \frac{\sin \alpha}{x} \quad (14) \]

where \( x = x_1 + x_2 \) and \( \dot{x} = V_{BA} \)

Differentiating with time

\[ \omega_2 = \frac{x_1 \omega_1 \cos \beta + V_{BA} \sin \beta}{x_3 \cos \alpha} \quad (15) \]

For \( \theta = 70, \beta = 20, \alpha = 76, x_3 = 0.11 \) and for constant linear speed of 0.01 m/s, angular velocity of panel will be at 6 deg/s which presented a slight deviation as compared to the real value at around 5.3 deg/s.

The relation represented by Equation 15 was not practical since it required knowledge of angular velocity of the linear actuator \( \omega_1 \) about O (Figure 49), values for angles \( \alpha \) and \( \beta \) and linear actuator total length \( x \). Without simulation to look up the value of \( \omega_1 \), this relation derived by velocity analysis was not complete enough.
Figure 49: Graph of angular velocity $\omega_2$ for the rotation of the panel about point C against polar angle. Plot showed that the angular velocity of the panel reached a minimum at around 40 degrees polar angle.

Angular Displacement

The CAD functional model was used to obtain graphical data of the simulated motion (Figure 52). Angular displacement was measured at the pin connection between the frame and the panel with linear servo motor set at 0.01 ms$^{-1}$. Figure 51 shows the plot of the polar angle as the panel is rotated. The rotational displacement of the panel is nearly linear with time.
Figure 50: The functional model modeled in CAD and simulations were run to find the kinematics of the motion applied to it.

![Functional model](image)

Figure 51: Graph of polar angle against time showing an almost linear motion for polar angle 0 – 80 degrees by a constant linear actuator speed motor at 0.01 m/s. The half cycle lasted for about 16 seconds.

If the motion were to be considered linear, an R-squared error of 0.9995 would be obtained

\[ \theta = 4.68t + 1.19 \]

As compared to the polynomial relation with an R-squared error of 1

\[ \theta = 0.0056t^3 - 0.137t^2 + 5.58t - 0.0084 \]
A linear relationship is desirable such that correction for non-linearity of the torque sent to the linear actuator does not need to be modeled. Although there seemed to be no difference between the error squared of the linear and polynomial relations, these small differences become significant after differentiation for the angular velocity and the angular acceleration.

**Angular Velocity**

Angular velocity is almost zero at the start suggesting low transmission angles at the bearings. Maximum angular velocity reaches to about 47 degree/s (Figure 52).

![Angular velocity against polar angle](image)

**Figure 52:** Graph of Angular velocity against polar angle showing the relationship as a fourth order polynomial. Angular velocity dropped to a minimum at about 4.5 deg/s to increase again to an angular velocity of about 6.4 deg/s.

**Angular Acceleration**

The angular acceleration of the panel is related to the force transmitted to the linkage and therefore related to the transmission angle. It was clearly seen that at 40 degrees transmission angle was almost zero (Figure 53) which is where transmission is at 90 degrees.
Figure 53: Graph of Angular acceleration against polar angle showing acceleration reaching zero at around 40 degrees (transmission angle at 90 degrees) to a maximum of about 0.9 deg/s² at 80 degrees

Dynamic Torque

The torque is by far the most important parameter in selecting the right linear actuator. The maximum torque calculated from the simulation (Figure 54) was obtained at around 0.11 Nm while the rated maximum torque from the manufacturer was at a maximum value of 56 Nm.
Summary

The dynamic analysis showed that the angular displacement was almost linear while the panel slowed down at a polar angle of around 40 degrees. Acceleration profile presented no jerk motion. The slowing down of the panel at polar angle 40 degrees might not be as desirable as it would introduce sensitivity variation during tracking which would require control if a similar tracking system were used as the one in this report. However, for the purpose of a functional model it would not present any problem.

The maximum torque needed to rotate the panel fully was only at around 0.11 Nm which was way beyond the maximum torque capacity of the linear actuator from the manufacturer \textsuperscript{20}. A range of linear actuators was available but based on cost to performance, this specific linear actuator shown in Figure 76 was chosen as will be described further in section 5.6.5.
5.5.2. Power Analysis

Power: Linear actuator

The power required to rotate the panel was essential in estimating the feasibility of the linear actuator motor and the amount of power to send to it in order to follow the assigned kinematic motion.

A number of assumptions were made for the kinematic motion which was used as validation for the simulation. It was desirable that the azimuth rotation should be executed in less than 15 second (half cycle) and it was assumed that the model accelerated in 0.25 second only.

The definitions and values of the variables used were listed in Table 14 while mass moment of inertia was calculated for the whole functional model in Appendix K. Power was found using Equation 16. Power needed was found to be analytically 0.16 W however when simulated the linear actuator was assigned a constant velocity which resulted in a varying angular velocity from 4.5 rad/s to 6.4 rad/s. Simulated power increased to a maximum of about 0.7 W (Figure 55) owing to the larger angular velocity and deceleration of 0.9 rad/s².

Table 14: Definitions and values for the terms used in deriving the initial linear actuator power

<table>
<thead>
<tr>
<th>Terms</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>Time for half cycle (s)</td>
<td>15</td>
</tr>
<tr>
<td>( \omega_p )</td>
<td>Angular velocity of panel (rad/s)</td>
<td>0.093</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>Angular acceleration of panel (rad/s²)</td>
<td>0.37</td>
</tr>
<tr>
<td>( I_p )</td>
<td>Mass moment of Inertia of panel (kgm²)</td>
<td>4.67</td>
</tr>
<tr>
<td>( P_{la} )</td>
<td>Power required to rotate panel (W)</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Power needed for linear actuator to rotate the panel initially was given by Equation 16

\[
P_{la} \geq I \omega \alpha \quad (16)
\]
Figure 55: Graph of power required against polar angle showing maximum power of about 0.73 W needed at polar angle 80 degrees. Half a cycle lasted about 16 seconds.

**Power: Gear**

The tracking motion of the functional model about the azimuth axis could be achieved by using a motor and spur gears. Optimally, motor with low RPM and gears with high pitch are favorable since they could produce smoother motion. But the selections of motor and gears that can be used are limited by what is commercially available.

A set of spur gear made by Boston Gear was selected:

(i) NB96 spur gear with 14.5 Pressure Angle, 16 Pitch, 96 Teeth, and Pitch Diameter of 6”

(ii) NB20B-1/2 spur gear with 14.5 Pressure Angle, 16 Pitch, and 20 Teeth, and Pitch Diameter of 1.25”
Given that polar rotation should be executed in less than thirty second and that model accelerated in 1 second only, the power $P_g$ required to turn the tracker could be calculated given by Equation 16. Torque was consequently calculated from the power as given by Equation 17. Values for variables used in the calculation of power and torque was given in Table 15.

Torque needed to rotate the model:

$$\tau = \frac{P}{\omega} \quad (17)$$

**Table 15: Definitions and values of the variables used to derive gear power**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>Time to complete one cycle/s</td>
<td>$\leq 30$</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Angular velocity of the gear/ rad/s</td>
<td>$2\pi/30$</td>
</tr>
<tr>
<td>$a_p$</td>
<td>Angular acceleration of the gear initially/ rad/s$^2$</td>
<td>$2\pi/30$</td>
</tr>
<tr>
<td>$P_g$</td>
<td>Power needed to rotate gear/W</td>
<td>$\geq 0.219$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Torque required/ Nm</td>
<td>1.406</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Power of motor/ Nm</td>
<td>14</td>
</tr>
<tr>
<td>$R$</td>
<td>Gear Ratio</td>
<td>4.8</td>
</tr>
</tbody>
</table>

The motor selected for the application (Figure 77) had a rated torque of 14N.m at 12V as calculated by Equation 18. The motor would be connected to the smaller spur gear which rotates around the bigger spur gear. The torque exerted on the model by the motor can be calculated by:

$$\tau = P_m \cdot R \quad (18)$$

The torque output was found to be 67.2 Nm, which was a lot more than what the tracker needed, so a pulse-width modulated signal would be used to power the motor in order to reduce the torque.

5.5.3. **State equation for the system motion**

Determination of the Equation of motion of the system with both the linear actuator and the gear driving the motion was derived using the Lagrangian Method. Virtual work equations for each link in the functional model were derived in Appendix L:
The assumptions made for this analysis were:

1. All part bodies are rigid bodies so that only a change in CGs will alter the energy of the system.

2. Gravitational forces of the members in moderate to high speed machinery often tend to be dwarfed by the dynamic forces from the accelerating masses. In this project, the speed of the functional model was assumed to be low so that gravitational force and gravitational potential energy should be considered. However the CG of the panel is the only one that changed with height and the change in distance was assumed small enough such that the change in potential energy was neglected all along.

3. Pin jointed linkages with low-friction bearings at the pivots can have high efficiencies of 95% and above which is why it is reasonable to assume that the pin joints are frictionless. Therefore system was assumed a conservative system in an ideal environment.

The Equation of motion was given by Equation 19 and derived in Appendix L

\[ mb^2 (1 + \dot{\theta})\dot{\phi} - m (b^2 + l^2) N^2 \sin\phi \cos\phi = 0 \]  

(19)

However this was true only if panel was fixed at polar angle 0 degree and if no rotation was imposed by the linear actuator as explained further in Appendix L which will reduce to the simple basic equation of motion given as Equation 20.

\[ \sum M = I_{xx} \dot{\omega}_x \]  

(20)

5.6. Tracker Electronics

Figure 56 shows the overall electrical system of the functional model. The main power source of the tracker was the 12V onboard battery, which could be charged using power from the solar panel itself or from 110V AC socket. Both of them passed through a DC regulator and a charge controller to avoid overcharging the battery. The user could choose which power source to use via a switch on the tracker. The solar panel could be used during outdoor demonstration and the AC socket could be used as power source for indoor demonstration of the model.
The onboard battery supplied power to the microcontroller and the motor controller, which in turn controlled the DC motor and the linear actuator. An inverter was also included to demonstrate power output from the tracker.

![Figure 56: Overall tracker electrical system](image)

The microcontroller took input from the three sensors: light sensor, wind sensor, and the load sensor. Depending on the input signals, the microcontroller controls the motor and the linear actuator, which provided the tracking motion.

### 5.6.1. Panel

The solar panel used for the tracker, as shown in Figure 57 was provided by the FDE.
Figure 57: SST 175-72M solar panel

Information about the panel is listed in Table 15.

Table 16: Information about the SST 175-72M panel

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>CEEG Shanghai Solar Science and Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>SST 175-72M</td>
</tr>
<tr>
<td>Cell Type</td>
<td>Monocrystalline Cell</td>
</tr>
<tr>
<td>Cell Size</td>
<td>125 mm x 125 mm</td>
</tr>
<tr>
<td></td>
<td>(4.92 in x 4.92 in)</td>
</tr>
<tr>
<td>Cells</td>
<td>6 x 12</td>
</tr>
<tr>
<td>Dimensions</td>
<td>1580 mm x 808 mm x 50 mm</td>
</tr>
<tr>
<td></td>
<td>(62.2 in x 31.8 in x 1.9 in)</td>
</tr>
<tr>
<td>Weight</td>
<td>16 kg</td>
</tr>
<tr>
<td></td>
<td>(35.5 lbs)</td>
</tr>
<tr>
<td>STC Power Rating</td>
<td>175 W</td>
</tr>
<tr>
<td>PTC Power Rating</td>
<td>156.5 W</td>
</tr>
<tr>
<td>Open Circuit Voltage</td>
<td>44.5 V</td>
</tr>
<tr>
<td>Short Circuit Current</td>
<td>5.34 A</td>
</tr>
<tr>
<td>Voltage at Maximum Power</td>
<td>35.3 V</td>
</tr>
<tr>
<td>Current at Maximum Power</td>
<td>4.96 A</td>
</tr>
<tr>
<td>Panel Efficiency</td>
<td>13.7%</td>
</tr>
</tbody>
</table>
5.6.2. Light sensor

The light sensor in a solar tracker determines where the light source is. Two light sensors, which make up a pair, will be placed at an angle and the signal produced from the sensors would be inputted into a microcontroller. For a dual-axis tracker, two pairs of light sensors were used, one for each axis. Three different sensors that can detect light were deemed suitable: light dependent resistor (LDR) or photocells, light emitting diode (LED), and photovoltaic cells. An experiment was carried out to determine the performance of the sensors. Figure 58 shows the setup of the experiment.

Figure 58: Experiment setup to compare three types of light sensors

A light source was rotated from $0^\circ$ to $180^\circ$ from a constant distance, while the voltage output of each sensor was recorded. The LED and PV produces their own voltage when light shines on them, but LDR, on the other hand, is a resistance device; it does not produce its own power. So to use it as a light sensor, a voltage divider circuit (Figure 59) was used. The voltage output of each sensor from the experiment is shown in Figure 60.

Figure 59: Voltage divider circuit for the LDRs
The experiment showed that the LDR produced the highest signal, a maximum of 3.2V, for the same light source as compare to PV and LED, which produced a maximum of around 1V. Since the voltage outputs would be read by the microcontroller, strong signal strength is desired. Therefore, the LDR was chosen to be used as the light sensor.

A single LDR alone would not be able to track a light source. But by using two LDR positioned at an angle, tracking became possible. Figure 61 below showed the idea behind the process.
Two LDR were positioned opposite to each other, making an angle of $\beta$ with the horizontal plane. Voltages $V_1$ and $V_2$ were produced by the two LDRs. At this configuration, when the light source is at 80°, more light would fall on left LDR than the right LDR and $V_1$ would be higher than $V_2$. At 90°, both sensors would get the same amount of light and would produce the same voltages. At 100°, $V_2$ would be greater than $V_1$. The different between $V_1$ and $V_2$ could be found and it depends on the angle of the light source. The goal of tracking is to have the light source right above the sensor, where the difference between $V_1$ and $V_2$ would be zero. So whenever the difference is non-zero, there is an error.

Another experiment is carried out to find which angle $\beta$ should be used. This angle would determine the strength of the error signal, and thus the accuracy of the tracker. The same setup shown in Figure 61 was used, but $\beta$ was varied from 40° to 80° at a 10° increment, each time moving the light source from 80° to 100°. Figure 62 below showed the result of the experiment.

![Error depending on $\beta$ (LDR)](image)

**Figure 62: Error output depending on angle $\beta$**

The experiment showed that using $\beta$ of 80° gives the strongest error signal. Based on this result, a light sensor with an angle $\beta$ of 80° was developed (Figure 63).
Figure 64 shows how the light sensor works. When the light source is on the right side of the sensor, as shown in the figure, $V_2$ is greater than $V_1$ and the sensor produced a positive error. The goal of the tracker is to minimize the error or make it zero. But since it is impossible to get a complete zero error, a tolerance gap $T_\theta$ is specified. Whenever the light source is within the tolerance limit (-$T$ and +$T$), the tracker maintains its position. Only when the light source or the error is outside the tolerance limit, the tracker rotates either clockwise or counterclockwise until the error goes back into the tolerance limit.

The following showed the pseudo code the microcontroller uses to track the light source. The complete microcontroller code is given in Appendix M.

```plaintext
tolerance = user defined number
error = V_2 - V_1

if (error > tolerance)
    rotate panel clockwise
else if (error < -tolerance)
    rotate panel counter clockwise
else
    maintain panel position
```
Figure 64: Operation of the solar sensor

5.6.3. Wind sensor

The purpose of the wind sensor was to detect three guests of simulated wind blow, which would send the tracker into a low-drag mode. At this position, the panel is horizontal and the surface area perpendicular to the wind is kept at minimum.

The wind sensor implementation used a piezo vibration sensor to detect wind gusts (Figure 65). Wind blowing on the sensor caused it to vibrate and produced a signal. The ALD110800A MOSFET is used to limit the range of the signal given to the microcontroller since it is configured to read analog values from 0V-5V. In order for the user to be able to adjust the sensitivity of the sensor to the wind, a variable resistor is placed at the source of the transistor so the gate voltage necessary to signal the microcontroller at the source will vary with the resistance and the voltage at the source will be offset.
The vibration sensor produced voltages based on the degree to which its 28µm thick piezoelectric PVDF polymer film is bent. The plot showing the relation between the two is shown in Figure 66. As indicated the output voltage could reach 24V but with very low current.

The voltage produced from the vibration sensor goes to a MOSFET, which is highly sensitive to even the smallest increases in voltage. The vibration sensor is only moved about a millimeter or two when blown by a hair dryer, small fan, or by mouth. Therefore the ALD110800APCL MOSFET was used to increase that signal if it were too small, or on the other hand, keep the output voltage within the 0-5V range in case of large voltage spikes from the sensor due to large deflections. Figure 67 below showed the MOSFET’s characteristic.
The vibration sensor output was connected to the MOSFETs gate, which was implemented in the following schematic. The resistor R1 was used to keep the output low when the vibration sensor was disconnected using the switch connecting it to the MOSFET.

The variable resistor R3 was used to adjust the sensitivity of the MOSFET to the vibration sensor. It is a 0Ω to 250kΩ resistor, so when it is zero, the total resistance between the two resistors R2 and R3 is 100Ω. When a voltage greater than 0V was detected three times by the microcontroller, it triggers the wind safe mode for the panel. The wind safe mode for the track was initiated by having the solar tracking code overridden by code that drove the linear actuator toward the panel’s horizontal direction.
During initial testing, the wind sensor worked but the variable resistor had to be turned all the way down to make the wind sensor sensitive to wind blow from the mouth. The piezo sensor was made of a hard plastic material and it was hard to bend it using wind force. Another problem came up when the sensor was tested on the panel. During tracking, the tracker vibrates and the wind sensor, made of a vibration sensor, picked up the movement causing false readings.

After unsuccessful tryouts, the wind sensor was replaced with a thermal anemometer (Figure 69). The device works by heating a resistor to a constant temperature and the electrical power required to maintain temperature is measured. As the wind blew over the resistor, the temperature dropped and the resistor drew more power to keep the temperature constant.

![Figure 69: Thermal anemometer](image)

The anemometer already includes an onboard circuit which takes in 5V and outputs a voltage ranging from 0-5V depending on the wind speed. The output voltage could the inputted directly into the microcontroller. The anemometer was rated to sense wind speeds from 0 to 30mph.

5.6.4. Load sensor

The functional model also required a load sensor to detect the simulated snow weight that may accumulate on the solar panel. The load sensor was designed to measure the amount of weight placed on the sensor. The purpose is to tilt the solar panel to 80° when the weight measured by the sensor reached a certain threshold. The panel would be held at the 80° position until the weight slide off the panel.

The main challenge presented was determining how to design a load sensor. From research, it was found that the resistive property of a type of conductive foam could be used to implement a load sensor. The conductive foam used to package integrated circuits has an inverse relationship between compression and resistance. A piece of the foam has an initial resistance with no weight
on it. But once weight was applied to the foam, the foam compressed and the resistance across the foam started to decrease. To test this theory, a simple experiment was carried out that involved sandwiching the conductive foam of thickness 0.394” between two brass plates of size 5.9” by 4.75”. Figure 70 showed the setup of the load sensor.

Because the load sensor is a resistive device, a voltage divider has to be used to measure the output voltage. A 100kΩ variable resistor was connected in series with the load sensor, powered by a 5V supply. The output voltage, $V_1$ can be measured by the microcontroller. A series of different weight was put on the sensor while the sensor resistance and the output voltages were recorded. The data is shown in Figure 71 and Figure 72.
Figure 72: Load sensor output voltage

Looking at the voltage output, a threshold around 2.5V can be used. So whenever the microcontroller measured a voltage greater than 2.5V, the panel would be tilted to the 80° position.

The problem with the load sensor is that to register a voltage of 2.5V, a 6lb weight was needed to be applied on the sensor. The load sensor would be attached to a retractable plate on the side of the panel and putting a 6lb weight would buckle the retractable plate. The threshold voltage could be reduced to sense a smaller weight, but the output voltage is unstable at lower weights.

An alternative solution was found which utilized force sensing resistors. An FSR 406 43.69mm square force sensor from Interlink (Figure 73) was tested and found to be suitable for sensing weight. The sensitivity range of the sensor was rated from 0.1N to 10N (0.02lb to 2.25lb). Because it is also a resistive device like the previously developed load sensor, the force sensing resistor could be implement directly to the existing circuit without any changes to the design. The variable resistor could be adjusted to compensate for the difference in resistance between the two sensors.
5.6.5. Linear Actuator

A 12V 6” stroke linear actuator, shown in Figure 74, is used to provide motion to the panel about the polar or horizontal axis.

![ServoCity 6-12V DC 6” stroke linear actuator](image)

Information about the linear actuator is shown below in Table 16.

### Table 17: Information about the ServoCity linear actuator

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>ServoCity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Voltage</td>
<td>6.0 ~ 12.0 V DC</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>-20 ~ +65 °C</td>
</tr>
<tr>
<td></td>
<td>-14.8 ~ +149 °F</td>
</tr>
<tr>
<td>Operating Speed @ 12 V</td>
<td>.50”/second (No load)</td>
</tr>
<tr>
<td></td>
<td>.39”/second (Maximum load)</td>
</tr>
<tr>
<td>Dynamic Thrust</td>
<td>115 lbs</td>
</tr>
<tr>
<td>Static Load</td>
<td>500 lbs</td>
</tr>
<tr>
<td>Current Rating @ 12 V</td>
<td>700mA (No load)</td>
</tr>
<tr>
<td>Motor Type</td>
<td>3 Pole Ferrite</td>
</tr>
<tr>
<td>Potentiometer</td>
<td>10kΩ</td>
</tr>
<tr>
<td>Gear Ratio</td>
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</tr>
<tr>
<td>Duty Cycle</td>
<td>25%</td>
</tr>
</tbody>
</table>
5.6.6. DC Motor

The 12V DC motor (Figure 75) is used to provide rotational motion about the vertical or azimuth axis.

![AM Equipment 214 Series Gearhead Motor](image)

**Figure 75: AM Equipment 214 Series Gearhead Motor**

Information about the motor is shown below in Table 17

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>AM Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>12.0 V DC</td>
</tr>
<tr>
<td>Torque</td>
<td>14 Nm</td>
</tr>
<tr>
<td>Weight</td>
<td>1.6 lb</td>
</tr>
<tr>
<td>No load Speed (rpm)</td>
<td>75.9 ~ 62.1</td>
</tr>
<tr>
<td></td>
<td>3.3 ~ 2.7</td>
</tr>
<tr>
<td>Stall load Speed (rpm)</td>
<td>19.4 ~ 15.9</td>
</tr>
<tr>
<td></td>
<td>31.7 ~ 26</td>
</tr>
<tr>
<td>Peak power Speed (rpm)</td>
<td>36.5 ~ 30</td>
</tr>
<tr>
<td></td>
<td>10.5 ~ 8.6</td>
</tr>
</tbody>
</table>

Table 18: Information about the AM Equipment 214 Series Gearhead Motor
5.7. Design and Manufacturing

Most of the design features from the STS444 were included in the functional model; it contained the base with the slewing gears and motor, two vertical beams (or) dual towers, two vertical cross beams to support the panel, and a linear actuator in place of the screw jack. Figure 76 below showed the overview of the functional model. The blueprints of the major parts are shown in Appendix N.

Figure 76: Overview design of the functional model

5.7.1. Panel Assembly

The panel assembly is shown in Figure 77 below. To connect the solar panel to the tracker frame, two vertical cross beams made from 3/8” thick aluminum bars were used. They were bolted to four aluminum brackets (made from 1” x 2” aluminum tubing), which were then bolted to the panel’s side frame. Another aluminum beam, made out of 1” x 1” square tubing was also attached across the center of the panel. It would be attached to the linear actuator, which rotates the panel around the horizontal axis.
Figure 77: Two vertical cross beams and the center beam connected to the panel

Figure 80 showed how the two vertical cross beams were connected to the towers using a shoulder bolt and two flanged bearings on each side, so that the cross beams could rotate about the horizontal axis (See Figure 79 for the cross section view of the connection). The towers were made of 1” x 2” rectangular tubing. Another horizontal beam was attached to the side of the tower beams. A bracket was connected to it, which supports the lower end of linear actuator used to rotate the panel. The top end of the linear actuator was connected to the center square tubing using a ball-lock pin for easy removal.

Figure 78: Two tower beams connected to the cross beams
The assembly shown in Figure 78 made up the upper half of the tracker, the panel module. The panel module could be attached to the lower half of the tracker, called the base module. The base module provides stability to the tracker, contains a DC motor and gears to provide a rotation along the vertical axis, and contains an electronic box to house the control systems and a battery.

5.7.2. **Base Assembly**

The assembly of the base module, which is 40” long and 32” wide, started with the legs (Figure 80), welded together using 1” x 2” aluminum tubes.
As shown in Figure 81, an aluminum plate of diameter 11.6” and ¼” thick was bolted to the center of the base legs. A turntable was then bolted to the bottom aluminum plate. A spacer ring of ½” thickness is also attached on top of the turntable to give space for the gears inside. At the center of the base plate, a spur gear with a 6” pitch diameter is attached.

Two smaller spur gears, with a pitch diameter of 1.25”, were assembled. One of the gear was attached to the motor (Figure 75) and another gear was attached to the first gear by a bar mechanism. Moving the bar left or right disengages or engages the gears connection with the main spur gear. When the gears are disengaged, it enables the tracker to be rotated manually. Without this mechanism, the motor using a worm gear mechanism, the tracker would not be rotatable unless the motor turns. Another ¼” aluminum plate covers the gear mechanism inside.
The guide blocks would be used to attach the base module and the panel module together. The motor rotates the tracker around the vertical axis, as shown in Figure 84 below.

![Figure 82: Gears assembly inside the base](image)

To complete the base assembly, an enclosure box is attached on top of the base module to house the electrical components, as shown in Figure 83.

![Figure 83: Base module assembled](image)

5.8. **Assembly**

The panel module (Figure 78) and the base module (Figure 83) can be connected together by sliding the dual tower tubing onto the two guide blocks on the upper plate of the base module.
The two ball lock pins could then be inserted into the hole through the dual towers and the guide blocks to lock the two assemblies. Finally, the cable from the solar panel, sensors, and the linear actuator were attached to their specific ports on the electric box. The power cable can be connected to the wall socket to power the tracker and to charge the battery.

The panel module and the base module could also be connected to form a configuration for easy transportation, as show in Figure 85 below. By utilizing the casters and the handles, the tracker can either be pulled like a luggage or be pushed like a cart (Figure 86). Figure 89 showed the procedure on how to assemble the tracker from the transportation mode.
Figure 85: Transportation mode

Figure 86: Transporting the tracker

Figure 87: Assembling the tracker
5.9. Final design

The figures below show the actual photograph of the assembled functional model.

Figure 88: Functional model features

Figure 89: Functional model at low-drag position
5.10. Tracking Performance Analysis

As a last step, the functional model was tested to ensure that it met the design specifications, which required a tracking accuracy of ±5°. But as shown in Figure 2, the power output from a panel is greater than 90% as long as the light source is within ±25° with respect to the panel. An experiment, shown in Figure 90, was carried out to measure tracking resolution of the functional model on its polar axis. An LED flash light was moved slowly, simulating the movement of the sun. A potentiometer was fixed on the rotating axis to measure the panel’s rotation and the result is shown in Figure 91.

Figure 90: Tracking performance experiment setup

Figure 91: Tracking resolution
The data showed that the tracker could rotate at an increment of around 0.3° in the polar axis, which is the resolution of the tracker. The result was a lot better than the required resolution of ±5°.

5.11. Conclusion

The functional model manufactured met all the design specifications and exceeds the specification in some cases. The functional model was designed to have two parts, exceeding the design requirement. It could be assembled and disassembled by two people and could be transported by a single person with the help of the casters and handles attached to the panel. The model, in its transportation mode, also met the design envelope stated. It could be assembled from the transportation mode to a tracking mode in 4 simple steps. The weight of the final functional model was around 70lb. The three required sensor were successfully integrated into the function model. Finally, the tracking performance showed that the tracking resolution for the polar axis was 0.3°, exceeding the required resolution of ±5°.
APPENDIX A

The design model of the STS 444 was modeled based on the blueprints provided by the FDE. Most dimensions (Figure 92 - Figure 94) that were given were kept to ensure accuracy while those that were not given were estimated to the best of our abilities. Stabilizing pods and some extra reinforcing beams were omitted to simplify both the simulations and reduced scaled prototype fabrication. Since deformation analysis were not carried out on the beams with lack of material knowledge and proper dimensions, omitting beams would not affect the overall wind load analysis.

Figure 92: Top view of STS444 with dimensions in meters.

Figure 93: Side view of STS444 with dimensions in meters.
Figure 94: Front view of STS444 with dimensions in meters.
APPENDIX B

The drag was calculated on the simplified STS 444 with wind facing Side view (Figure 95)

Figure 95: Side View Contour lines of wind velocity coming from inlet (left – z-direction) perpendicular to the Side view of the model.

The Reynolds number given by Equation 1 along the PV collector (22.857m long)

\[
Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{89.4 \times 22.857}{1.57 \times 10^{-5}} = 1.3 \times 10^8
\]

Therefore the trailing-edge flow is fully turbulent. It is estimated that drag force will act on:

a. **PV collector (thin flat plate – platform area)**

   For fully turbulent smooth-wall flow, the drag coefficient on one side of the PV collector:

   \[
   C_D = \frac{0.031}{Re^{1/7}} = \frac{0.031}{(1.3 \times 10^8)^{1/7}} = 0.00215
   \]

   Then the drag on both sides of the PV collector is approximately (platform area)

   \[
   D = 2 \left( \frac{1}{2} C_D \rho V^2 A \right) = 0.00215(1.184)(89.4^2)(22.857)(8.230) = 3827.23N
   \]

b. **Horizontal beams x2 (square cylinder – frontal area)**

   Estimated \( C_D = 2.1 \)
\[ D = 2 \left( \frac{1}{2} C_D \rho V^2 A \right) = (2.1)(1.184)(89.4^2)(0.254)(0.13) = 656.20N \]

c. **Frame support x2 (flat nose section – frontal area)**

L/H ratio = 1.270/0.610 = 2.1

Estimated \( C_D \approx 1.8 \)

\[ D = 2 \left( \frac{1}{2} C_D \rho V^2 A \right) = 1.8(1.184)(89.4^2)(1.270)(0.610) = 13195.71N \]

d. **Rectangular turn-table (thin flat plate – platform area)**

Reynolds Number \( Re = \frac{89.4 \times 2.44}{1.57 \times 10^{-5}} = 1.4 \times 10^7 \) (Fully turbulent)

Estimated \( C_D = \frac{0.031}{Re^{1/7}} = \frac{0.031}{(14 \times 10^7)^{1/7}} = 0.00296 \)

\[ D = 2 \left( \frac{1}{2} C_D \rho V^2 A \right) = 0.00296(1.184)(89.4^2)(2.440)(7.315) = 500N \]

e. **Gear (circular cylinder – frontal area)**

Estimated \( C_D \approx 2.2 \) (worst case)

\[ D = \left( \frac{1}{2} C_D \rho V^2 A \right) = 0.5(2.2)(1.184)(89.4^2) \left( \frac{\pi \times 1.36^2}{4} \right) = 15121.23N \]

f. **Base (flat nose section – frontal area)**

L/H ratio = 1.016/13.360=0.076

Estimated \( C_D \approx 1.2 \)

\[ D = \left( \frac{1}{2} C_D \rho V^2 A \right) = 0.5(1.2)(1.184)(89.4^2)(1.016)(13.360) = 77068.72N \]

**Total drag in X-direction = 1.11E5N**

Determining \( C_D \) of panel at different angle based on projected frontal area only. All other components have a fixed \( C_D \) since the frontal area is not changing. However drag on other components will depend on how much of the component surface area is blocked by the panel.

For example when tilt angle is at 0 degrees, the only wetted area will be the PV panel and the base or when the tilt angle is at 30 degrees the wetted areas include those of the panel, base, cylinder, turn-table and half of the frame support towers surface area listed in Table 18.

At angle = 10 degrees
Drag due to PV collector:

L/H ratio = 0.38

Estimated $C_D \sim 2.29$

$$D = \left(\frac{1}{2} C_D \rho V^2 A\right) = 0.5(2.29)(1.184)(89.4^2)(22.857 \cos 10)(8.23) = 2.0073E6N$$

Drag due to base

L/H ratio = 0.076

Estimated $C_D \approx 1.2$

$$D = \left(\frac{1}{2} C_D \rho V^2 A\right) = 0.5(1.2)(1.184)(89.4^2)(1.016)(13.360) = 77068.72N$$

**Total drag in X-direction = 2.0054E6N**

Table 19: Sum of individual drag owing to the surfaces in direct contact with the wind flow and corresponding drag coefficient

<table>
<thead>
<tr>
<th>Tilt angle</th>
<th>$C_D$ of panel</th>
<th>Drag per component/10^6 N</th>
<th>$C_D$ Equivalence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV panel</td>
<td>Base</td>
<td>Cylinder</td>
</tr>
<tr>
<td>0</td>
<td>0.00215</td>
<td>0.003827</td>
<td>0.07707</td>
</tr>
<tr>
<td>10</td>
<td>1.744</td>
<td>0.2696</td>
<td>0.07707</td>
</tr>
<tr>
<td>20</td>
<td>2.276</td>
<td>0.6929</td>
<td>0.07707</td>
</tr>
<tr>
<td>30</td>
<td>2.676</td>
<td>1.1909</td>
<td>0.07707</td>
</tr>
<tr>
<td>40</td>
<td>2.510</td>
<td>1.4360</td>
<td>0.07707</td>
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<td>50</td>
<td>2.390</td>
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<td>60</td>
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<td>80</td>
<td>2.200</td>
<td>2.0073</td>
<td>0.07707</td>
</tr>
</tbody>
</table>
APPENDIX C

Using the STS 444 based on dimensions given in Appendix A, a turbulent flow model was set up in FLUENT first and imported to a static structural analysis tab onto the model (Figure 96). This example was based on the configuration where angle of tilt was zero degrees. The same procedures were followed for incremental angle of tilt from 0 – 80 degrees.

![Figure 96: Project Schematic for the CFD and static structural analysis in ANSYS of the STS444 in a boxed fluid](image)

1. **FLUENT**

1.1. Geometry

The CAD models of the STS 444 and the fluid around the model were assembled and converted to a STEP file. Since the system was perfectly symmetrical, only one half of the symmetry of the assembly was considered to reduce the number of elements for the meshing portion.

The parts were assigned appropriate materials. Since the base, frame and PV collector were all of different materials, they were modeled in single parts and assembled together so that materials could be assigned to each one of them.

The mates used in the assembly will appear as connections in ANSYS workbench. Define and delete unnecessary connections.

All the parts were suppressed except for the fluid box.
1.2. Meshing

Meshing was crucial to obtaining accurate result of the fluid flow. Sizing and inflation were used to improve meshing. Better meshing was achieved with the body of influence option however a limit of 512000 elements was imposed on the ANSYS solver available and the total meshing elements was capped at 491122 elements.

Since there were few curvatures, the advanced size function was set on proximity and inflation layers were set at a maximum of 5 layers with smooth transition as inflation option (Figure 97). Inflation was imposed on upstream surfaces to capture the boundary layer regions especially around the PV collector and bottom ground (Figure 98). Sizing was applied mostly at the wake of the flow.

<table>
<thead>
<tr>
<th>Defaults</th>
<th>Inflation</th>
<th>Assembly Meshing</th>
<th>Patch Conforming Options</th>
<th>Advanced</th>
<th>Statistics</th>
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<tr>
<td>Sizing</td>
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<td>Use Advanced Size Function</td>
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<td>Relevance Center</td>
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<td>Num Cells Across Gap</td>
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<td>Proximity Min Size</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Face Size</td>
<td>1000.0 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Size</td>
<td>1100.0 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth Rate</td>
<td>Default (1.20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Edge Length</td>
<td>60.6220 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use Automatic Inflation | All Faces in Chosen Named Selection |
| Named Selection | inner |
| Inflation Option | Smooth Transition |
| Transition Ratio | 0.272 |
| Maximum layers | 5 |
| Growth Rate | 1.2 |
| Inflation Algorithm | Pre |
| View Advanced Options | No |
| Method | None |
| Triangle Surface Mesher | Program Controlled |

Figure 97: Meshing set-up in ANSYS FLUENT for the fluid box around the STS444.
However the inflation created stairstep mesh at some location because of those body to body angles present so that by increasing the maximum angle the stair stepping can be eliminated.

Smooth transition option was used because it uses the local tetra element size to compute each local initial height.

1.3. Fluid flow Set-up

A standard $k-\omega$ turbulent model was chosen since it was estimated that the shape of the STS 444 would cause quite a significant flow separation. $k-\omega$ models are typically better in predicting adverse pressure gradient boundary layer flows and separation as compared to the other common model $k-\varepsilon$ model which are not commonly used in external aerodynamics. A better model would be that of the Shear-stress Transport $k-\omega$ model but that is beyond the scope of this report.
The default values for $α^*, α_e, α_0, β_∞, β_i, R_β, R_k, R_ω, ζ^*, M_{t0}$, TKE prandtl Number and SDR Prandtl number were 1, 0.52, 1/9, 0.09, 0.072, 8, 6, 2.95, 1.5, 0.25, 2 and 2 respectively (Figure 99).

Boundary conditions were only edited for the inlet and outlet values (Figure 100) while all others were left at default values. The turbulent intensity $I$ and turbulent length scale $l$ were estimated from Equations 21 – 22 where $L$ is the hydraulic diameter of the tunnel.

Reynolds number in this set-up:

$$Re = \frac{Vd}{v} = \frac{89.4 \times 37.5}{1.57 \times 10^{-5}} = 2.135 \times 10^8$$

Turbulent Intensity:

$$I = 0.16(Re_D)^{-1/8} = 1.46 \% \quad (21)$$

Turbulent length scale:

$$l = 0.07L = 0.49 \text{ m} \quad (22)$$
Figure 100: Velocity magnitude, turbulent intensity and turbulent length scale at the Inlet face of the fluid box. The Outlet values were assigned the exact same as the Inlet.

Note that for the reduced scale model a turbulent intensity of 2% and a turbulent scale length of 0.028 was used based on the wind tunnel set-up.

Solution was initialized at the inlet with Turbulent Kinetic energy $k$ and Turbulent Dissipation rate $\varepsilon$ with Equations 23 – 24 respectively. The average velocity $u_{avg}$ and the empirical constant specified in the turbulence model $C_\mu$ were approximately 89.4 ms$^{-1}$ and 0.09. $k$ and $\varepsilon$ were calculated to be 2.55 m$^2$s$^{-2}$ and 5.96s$^{-1}$ respectively (Figure 101).

\[
k = \frac{3}{2} (u_{avg} l)^2 \tag{23}
\]

\[
\varepsilon = C_\mu \frac{3}{l} \frac{k^\frac{3}{2}}{l} \tag{24}
\]

Convergence was reached after 30 iterations.
1.4 Solution and Results

In the results section the data was represented by applying contour plot on the symmetry (Figure 102).

Figure 101: The Solution Initialization was computed from the Inlet conditions and the velocity was set at 89.4 ms⁻¹. Turbulent Kinetic Energy and the Specific Dissipation rate were calculated automatically from the values of turbulent intensity and turbulent length scale.

Figure 102: Contour plots representation of the CFD analysis on the symmetry of the boxed fluid with the tilt angle at 70 degrees showing pressure ranging from 8.23E3 Pa (Red) to (Blue)
1.5. Static Structural Analysis

The second part of the analysis consisted of importing the geometry and the solution from ANSYS FLUENT to the static structural analysis tab (Figure 103).

This time in the geometry list, the box fluid was suppressed while the other three were unsuppressed. Material assignments were user-defined mass of 12 N, user-defined mass of 160 N, Aluminum 1061 and concrete for the PV collector, linear actuator, frame and base respectively.

Meshing was applied to model (Figure 104) since total number of elements from FLUENT meshing and static structural meshing of the structure should be below 512 000 elements.

<table>
<thead>
<tr>
<th>Defaults</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use Automatic Inflation</td>
</tr>
<tr>
<td></td>
<td>Inflation Option</td>
</tr>
<tr>
<td></td>
<td>Maximum Layers</td>
</tr>
<tr>
<td></td>
<td>Growth Rate</td>
</tr>
<tr>
<td></td>
<td>Inflation Algorithm</td>
</tr>
<tr>
<td></td>
<td>View Advanced Options</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Patch Conforming Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced</td>
</tr>
<tr>
<td>Deforming</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes 14994</td>
</tr>
<tr>
<td>Elements 1077</td>
</tr>
<tr>
<td>Mesh Metric None</td>
</tr>
</tbody>
</table>

Figure 103: Meshing sizing and inflation parameters used on the STS444 in Static Structural

Figure 104: View of the 105162 elements on the STS444 symmetry with no sizing and no inflation.
1.6. Set up

Fixed support constraint was added to the base and frictionless support added to the symmetry. The pressure is imported onto the corresponding surfaces on the model in contact with the fluid (Figure 105). If named selection had different meshing, the imported pressure will need to be done each separately and applied to the appropriate surfaces.

![Figure 105: View of the pressure imported from the CFD analysis onto the STS444 with wind flow in the direction from the left arrow (turquoise) to the right arrow (red).](image)

1.7. Solution

Solution for total deformation and a force probe at the fixed support constraint were added as measure to the analysis. Total deformation of the whole system and the resultant reaction force at the fixed support located at the base were solved (Figure 106). Note that since the symmetry of the model was used, the resultant forces would be twice the amount obtained in the results. Von Mises stress and safety factor contour plot were added to the results (Figure 107 - Figure 108).
Figure 106: Resultant reaction force acting on the base of STS444 at wind flow of 89.4 ms⁻¹

Figure 107: Von Mises stresses on the STS444 ranging from
Figure 108: Safety factor on the STS444 ranging from 0.284 (red) to 15 with safety factor below 1.5 considered as failure. Usually a safety factor of 1.5 is adequate but depending on the risk factor a safety factor of 5 is sometimes used instead.
Determination of the scaling factor in the wind tunnel testing involved numerous trial and errors to find the matching combinations. Constraints to take into consideration before estimating the value of the scaling factor:

i. For the scaled model to fit exactly in the side view position for the recirculating wind tunnel \(^{24}\), the model must be scaled down by a factor of at least 40.

ii. Maximum velocity in wind tunnel was 60 m\(^{-1}\) \(^{24}\).

iii. Depending on the manufacturing mode, the scaling should be matched. For the purpose of this project, the base of the model was rapid prototyped. The maximum build dimensions of the rapid prototyping machine are 10” x 10” x 12” (254 x 254 x 305 cm) while the minimum recommended wall thickness is 0.06” (1.5mm).

iv. The only available estimate of the drag and lift coefficients are analytically ones. The maximum drag or lift coefficient will be used in the force ratio to estimate maximum force.

**Trial 1**

**Table 20: Maximum \(C_L\), scaling factor and velocity of wind used for the parameters in the first trial.**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Trial values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical (C_L)</td>
<td>2.5</td>
</tr>
<tr>
<td>(A/m^2)</td>
<td>0.36</td>
</tr>
<tr>
<td>Full scale (A_p/m^2)</td>
<td>188</td>
</tr>
<tr>
<td>Scaling factor</td>
<td>40</td>
</tr>
<tr>
<td>(V/ms^{-1})</td>
<td>60</td>
</tr>
</tbody>
</table>

Using Equation 4 and values listed in Table 19, the wind blockage correction at a polar angle of 80 degrees resulted in an actual wind velocity of

\[
V' = V \frac{A}{A - A_p \sin(90 - \theta)} = \frac{60 \times 0.36}{0.36 - \left(\frac{188.11}{40^2}\right) \sin 10} = 63.6 \text{ m/s}
\]

From Equation 3,

\[
Drag = \frac{1}{2} \times 2.5 \times 1.184 \times \frac{188.11}{40^2} \times 63.6^2 = 704 \text{ N} \text{ (Too high for such a small model)}
\]
Trial 2
For $V = 60$ m/s and scaling factor 100, the calculated drag would be around 102 N (depends on sensitivity of force sensor). Therefore to increase the force slightly, velocity could be decreased to 45 m/s and scaling factor reduced to 90. This resulted in a maximum force of about 70 N which was reasonable to measure.

Check for compatibility

1. Equivalent Drag on reduced model at scaling ratio 90 at speed 45 m/s:

$$D = \frac{1}{2} C_D \rho A V^2 = \frac{1}{2} (2.5)(1.184) \left(\frac{188}{90^2}\right) (45^2) = 70 \, N$$

2. Reynolds number:

$$Re = \frac{40(22.857)}{90(1.57\times10^{-5})} = 6.47 \times 10^5 \, \text{(turbulent)}$$

3. Manufacturing constraints

Dimensions for rapid prototyping:

*The minimum recommended wall thickness for the rapid prototyping machine is 0.060” (1.5mm), and features thinner than 0.045” (1.1mm) will likely not print. The maximum build dimensions are 10” x 10” x 12 (height)” (254x254x305mm)*

The thinnest edge at scaling factor of 90 was 2.82 mm which is greater than 1.5mm and the longest length was about 137 mm which is less than 254mm. therefore the scaling factor of 90 is appropriate for the rapid prototype.
APPENDIX E

Drag and lift fixture (Figure 109) for the wind tunnel testing was designed in two configurations for drag and lift measurements. While Drag was measured by a system of moments, the lift was measured directly from the lift force on the force transducer.

The arm was about 7 cm and was later extended to about 40 cm to reduce the force at the end of the arm since the force sensor used for the drag could measure to only about 24N.

Figure 109: CAD model of the drag and lift fixture used in the wind tunnel testing
APPENDIX F

Wind tunnel calibration (calibrated on 9/25/2012) for the recirculating wind tunnel giving the relation between the wind tunnel turbine frequency and the actual velocity produced in the tunnel. The corresponding turbulence intensity was measured.

<table>
<thead>
<tr>
<th>Measurement Location</th>
<th>Averaged velocity data (ms$^{-1}$)</th>
<th>Turbulence Intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tunnel Frequency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.5 11 12</td>
<td>10.5 11 12</td>
</tr>
<tr>
<td>1</td>
<td>10.07614 10.47701 11.30686</td>
<td>1 0.33 0.34 0.39</td>
</tr>
<tr>
<td>2</td>
<td>10.03993 10.431 11.24964</td>
<td>2 0.35 0.38 0.40</td>
</tr>
<tr>
<td>3</td>
<td>10.04359 10.43982 11.2978</td>
<td>3 0.37 0.35 0.41</td>
</tr>
<tr>
<td>4</td>
<td>10.08865 10.49302 11.34405</td>
<td>4 0.37 0.37 0.38</td>
</tr>
</tbody>
</table>

For the small range tested, the relationship was almost linear and could be extrapolated to higher values.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Velocity (ms$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5.4682</td>
</tr>
<tr>
<td>6</td>
<td>6.3071</td>
</tr>
<tr>
<td>7</td>
<td>7.146</td>
</tr>
<tr>
<td>8</td>
<td>7.9849</td>
</tr>
<tr>
<td>9</td>
<td>8.8238</td>
</tr>
<tr>
<td>10</td>
<td>9.6627</td>
</tr>
<tr>
<td>11</td>
<td>10.5016</td>
</tr>
<tr>
<td>12</td>
<td>11.3405</td>
</tr>
<tr>
<td>13</td>
<td>12.1794</td>
</tr>
<tr>
<td>14</td>
<td>13.0183</td>
</tr>
<tr>
<td>15</td>
<td>13.8572</td>
</tr>
</tbody>
</table>

y = 0.8389x + 1.2737
R² = 0.9998
Experimental Drag

Calibration curve using a combination of 100 g masses from 100 – 500g was plotted showing a linear relationship between load (y) and measured voltage (x):

\[ y = 81.26x - 39.816 \]

![Load against Measured Voltage](image)

**Table 21:** Typical values obtained during experimental drag with calibration using 100 g masses

<table>
<thead>
<tr>
<th>Expt No.</th>
<th>θ /deg</th>
<th>Measured voltage/V</th>
<th>Drag/ N</th>
<th>V'</th>
<th>C_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.489</td>
<td>4.877</td>
<td>45.00</td>
<td>0.2022</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.489</td>
<td>9.996</td>
<td>45.49</td>
<td>0.4076</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0.489</td>
<td>17.39</td>
<td>45.98</td>
<td>0.6941</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.489</td>
<td>25.19</td>
<td>46.45</td>
<td>0.9852</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>0.490</td>
<td>29.66</td>
<td>46.88</td>
<td>1.139</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>0.489</td>
<td>38.84</td>
<td>47.26</td>
<td>1.467</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>0.489</td>
<td>43.88</td>
<td>47.57</td>
<td>1.636</td>
</tr>
<tr>
<td>8</td>
<td>70</td>
<td>0.489</td>
<td>49.16</td>
<td>47.81</td>
<td>1.815</td>
</tr>
<tr>
<td>9</td>
<td>80</td>
<td>0.489</td>
<td>57.70</td>
<td>47.95</td>
<td>2.416</td>
</tr>
</tbody>
</table>
Experimental Lift

**Table 22: Typical values obtained during experimental lift with calibration using 100 g and 200 g mass**

<table>
<thead>
<tr>
<th>Expt No.</th>
<th>Polar angle /deg</th>
<th>Cor Lift/ N</th>
<th>V'</th>
<th>C_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2.72</td>
<td>45.00</td>
<td>0.113</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>12.0</td>
<td>45.49</td>
<td>0.489</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>18.2</td>
<td>45.98</td>
<td>0.723</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>20.4</td>
<td>46.45</td>
<td>0.793</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>21.4</td>
<td>46.88</td>
<td>0.818</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>25.0</td>
<td>47.26</td>
<td>0.940</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>25.4</td>
<td>47.57</td>
<td>0.942</td>
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<tr>
<td>8</td>
<td>70</td>
<td>25.8</td>
<td>47.81</td>
<td>0.947</td>
</tr>
<tr>
<td>9</td>
<td>80</td>
<td>26.2</td>
<td>47.95</td>
<td>0.956</td>
</tr>
</tbody>
</table>

**Load against Measured Voltage - calibration curve**

\[ y = 0.9929x + 0.1605 \]

\[ R^2 = 0.9976 \]
APPENDIX G

When the snow was considered as a continuous mass, it was assumed that the snow was fresh and fluffy. This implied that the individual particles all behave like independent entities with minimum interaction with one another such as stickiness and agglomeration as seen in settled packed snow or wet firm snow. As the panel rotates to tow the snow, the particles theoretically slide over one another thus changing the shape and the center of mass of the snow pile. The change in center of mass will induce an increasing moment as the difference between the panel and pile center of masses increases. This problem involves both location and magnitude variation of the snow load. To counter for both variations, the variable has to be added as a variable torque with analytical calculations as to the amount of torque exerted by the snow on the panel at different angle of rotation.

The considerations are various for profiling the snow slide:

a) The amount of snow accumulation on top of the panel set initially at 220 kgm\(^{-2}\) (45lb/ft\(^2\))
b) The initial shape of the snow pile
c) Determining the static and dynamic friction of the snow particles to find at what angle the shape starts to change
d) Shape variation at each angle
e) The amount of snow that leave the panel at each angle

In order to simplify the amount of snow that fell off of the panel was cut-off at a vertical section and represented by cross-sectional area \(A_c\) (Figure 110).
Assumptions made for the snow removal analysis:

i. The snow was fresh snow with coefficient of static and dynamic friction at 0.1 and 0.03 respectively

ii. The snow did not accumulate at the bottom to hinder incoming snow sliding off the panel

iii. The snow shape was treated as a rigid rectangular shaped body

iv. In calculating the snow left on top of the panel at every tilt angle, it was assumed the volume of snow falling off was cut off along a vertical line with the edge represented by area $A_v$ (Figure 110).
Force equations based on the FBD:

\[
\begin{align*}
\sum F_r &= mgsin\theta + \mu_s N = m(\ddot{r} - r\dot{\theta}^2) \\
\sum F_r &= mgsin\theta - \mu_k N = m(\ddot{r} - r\dot{\theta}^2) \\
\sum F_\theta &= mcos\theta - N = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})
\end{align*}
\]

Assuming panel rotates with constant angular velocity \(\dot{\theta}\) so that \(\ddot{\theta} = 0\)

For static friction, when body is just about to slide, \(\dot{r} = 0, \ddot{r} = 0\), angle of slip is calculated by Equations 29 – 30

\[
N = mgcos\theta
\]

\[
\frac{gsin\theta + \mu_s gcos\theta}{g} = -r\dot{\theta} \quad \rightarrow \quad \theta = \sin^{-1}\left(\frac{-r\dot{\theta}}{g} - \mu_s cos\theta\right)
\]

For \(\omega = 0.1 \frac{rad}{s}\), \(\theta = 10.5\ degrees\)

The sliding snow was simulated and the curve showed that the snow started sliding at around 10 degrees (Figure 111) which validated the simulation.

![Separation against Polar angle](image)

**Figure 111:** Separation r(\(\theta\)) against polar angle of the panel showing that snow load start to slip at around 10 degrees validating the analytical angle of slip
For dynamic friction given by Equations 31 - 32,

\[ \sum F_\theta = mg \cos \theta - N = 2m \dot{r} \dot{\theta} \]

\[ \sum F_r = g \sin \theta - \mu_k (g \cos \theta - 2 \dot{r} \dot{\theta}) = \dot{r} - r \dot{\theta}^2 \]

Equation of motion given by Equation 33,

\[ \ddot{r} + 2 \mu_k \dot{r} \dot{\theta} - r \dot{\theta}^2 = g \sin \theta - \mu_k g \cos \theta \]

Let \( A = \sqrt{\mu_k^2 + 1} \)

The second order non-homogenous differential equation was solved to obtain solution in the form given by Equation

\[ r = C_1 r_1 + C_2 r_2 + U r_1 + V r_2 \]

where

\[ r_h = C_1 r_1 + C_2 r_2 \]
\[ r_p = U r_1 + V r_2 \]
\[ \theta = \omega t \]

\[ r_1 = e^{-\omega (\mu_k + A) t} \]
\[ r_2 = e^{-\omega (\mu_k - A) t} \]

\[ U = \frac{-g (\mu_k + A) e^{(\mu_k + A) \theta}}{2 \omega A [\omega^2 (\mu_k + A)^2 + 1]} \left[ \sin \theta - \frac{\cos \theta}{\omega (\mu_k + A)} - \frac{\mu_k \cos \theta - \mu_k \sin \theta}{\omega (\mu_k + A)} \right] \]

\[ V = \frac{g (\mu_k - A) e^{(\mu_k - A) \theta}}{2 \omega A [\omega^2 (\mu_k - A)^2 + 1]} \left[ \sin \theta - \frac{\cos \theta}{\omega (\mu_k - A)} - \frac{\mu_k \cos \theta - \mu_k \sin \theta}{\omega (\mu_k - A)} \right] \]

Initial conditions for \( \mu_s = 0.1 \) and \( \mu_k = 0.03 \):

\[ \theta = 10.5 \ degrees; \ r = 0 \]
\[ C_1 + 1.443 C_2 = 18.607 \]

From validated simulation, the boundary condition was found: \( \theta = 24.7 \ degrees; \ r = 8.23 \)

\[ r = 0.91C_1 + 1.092C_2 + 528.719(0.91) - 446.656(1.092) = 0 \]
\[ C_1 + 2.367C_2 = 68.216 \]

Therefore for \( \omega = 0.1 rad/s \)

\[ C_1 = -58.60 \ and \ C_2 = 53.70 \]
For \( \omega \) at 0.005 rad/s, 0.01 rad/s and 0.05 rad/s, the polar angle reached maximum before the snow left the panel surface. Therefore since the panel would be tilted at 80 degrees, the snow would fall mostly under gravity. The time taken for the snow to be completely removed would be therefore a little more than the time taken for the panel to reach maximum tilt since acceleration due to gravity is much higher at a tilt angle of 80 degrees. The only reasonable values for angular velocities considered were those of 0.05 rad/s, 0.1 rad/s and 0.5 rad/s as listed in Table 23.

<table>
<thead>
<tr>
<th>Time to reach 80 degrees</th>
<th>( \omega ) (rad/s)</th>
<th>If ( \theta &gt; 80 ) deg with constant ( \omega )</th>
<th>If ( \theta \leq 80 ) deg with constant ( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta ) (deg)</td>
<td>Time to slide</td>
<td>( \theta ) (deg)</td>
</tr>
<tr>
<td>4.65 min</td>
<td>0.005</td>
<td>160</td>
<td>9.31min</td>
</tr>
<tr>
<td>2.33 min</td>
<td>0.01</td>
<td>147</td>
<td>4.28 min</td>
</tr>
<tr>
<td>27.9 sec</td>
<td>0.05</td>
<td>89.0</td>
<td>31.1 sec</td>
</tr>
<tr>
<td>14.0 sec</td>
<td>0.1</td>
<td>39.7</td>
<td>6.93 sec</td>
</tr>
<tr>
<td>2.79 sec</td>
<td>0.5</td>
<td>7.38</td>
<td>0.258 sec</td>
</tr>
</tbody>
</table>

Based on the location of the center of mass of the rigid snow, the cross-sectional area \( A_v \) of the snow sticking outside of the panel is shaved off and a new instantaneous center of mass for the remaining snow is calculated. The torque is then calculated based on the location and magnitude of the remaining snow on top of the panel and a torque profile was made for different angular velocities. The torque profile was simulated and the resultant torque was calculated on the linear actuator shaft.
A compromise had to be made between the time taken for the snow to be completely removed and the amount of tilt angle from the panel. At a low angular velocity of 0.01 rad/s, the snow slipped completely after about 13 seconds at a maximum tilt angle of 9 degrees approximately as compared to an angular velocity of 0.5 rad/s whereby the snow was completely removed after about 2 seconds for a maximum tilt angle of 64 degrees (Figure 113). During windy days where the lift will be pressing down on the panel in addition to having a snow load, a low angular velocity might be more appropriate for minimum tilt angle. During sun tracking, time is a very important factor and therefore a higher angular velocity is desired to minimize time lost during snow towing.

The following calculations for determining the variation of the magnitude of the snow load was derived below for maximum weight per area at 200 kg.m⁻² with variables listed in Table 24.

**Table 24: Term and corresponding values used in the determination of magnitude of load**

<table>
<thead>
<tr>
<th>Terms</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>x/m</td>
<td>1</td>
</tr>
<tr>
<td>L/m</td>
<td>8.23</td>
</tr>
<tr>
<td>A/m²</td>
<td>188</td>
</tr>
<tr>
<td>V/m³</td>
<td>A</td>
</tr>
<tr>
<td>ρ/ kg.m⁻³</td>
<td>200</td>
</tr>
</tbody>
</table>
At angle $\theta$:

Volume of snow left on panel $V_s = L[xw - A_v(\theta)]$ where

$$A_v(\theta) = r(\theta)x + \frac{x}{2} \tan \theta$$

Total snow load on top of panel $W_s = V_s \rho g$

$$W_s(\theta) = \rho g L \left[ xw - r(\theta)x + \frac{x}{2} \tan \theta \right]$$

Distance $|OP| = \frac{r(\theta)}{2}$

Torque exerted by snow which is calculated based on the offset distance from panel pivot to center of mass of newly formed snow pile:

$$T_s = W_s |OP| = \frac{1}{2} W_s(\theta) r(\theta)$$

Conservation of power around O

$$P = Fv = T\omega$$

Axial force along the linear actuator

$$F(\theta) = \frac{T(\theta) \omega(\theta)}{v}$$

For small angle $\theta$, angular velocity $\omega$ can be assumed constant so that the profile of the force will look somewhat similar to that of the torque profile around pin joint O multiplied by a factor $v$. 
APPENDIX H

The position of the linear actuator is calculated for the design iteration 1 by using two static cases, one when the linear actuator is fully extended and the other when it is fully retracted. A correlation between the dimensional variables as listed in Table 25 is derived to compare the different linear actuator available and find the optimum one.

Table 25: Terms and definition used to find optimum position of the linear actuator

<table>
<thead>
<tr>
<th>Terms</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>Horizontal distance between point of linear actuator and panel to the axis of rotation of panel</td>
</tr>
<tr>
<td>y</td>
<td>Vertical distance between point of linear actuator and frame to the axis of rotation of panel</td>
</tr>
<tr>
<td>r</td>
<td>Half the length of panel width</td>
</tr>
<tr>
<td>H</td>
<td>Total height of frame</td>
</tr>
<tr>
<td>θ</td>
<td>Angle between panel and frame</td>
</tr>
<tr>
<td>a</td>
<td>Total length of linear actuator when fully extended</td>
</tr>
<tr>
<td>b</td>
<td>Total length of linear actuator when fully retracted</td>
</tr>
</tbody>
</table>

Case 1: Extended (θ = 90 deg)

Using Pythagoras law,

\[ a = \sqrt{x^2 + y^2} \]

Constraints: \[ x \leq r \text{ where } r = 0.405 \text{ m} \text{ and } y \leq H \]
Case 2: Retracted ($\theta = 10$ deg)

Using laws of cosine,

\[ b = \sqrt{x^2 + y^2 - 2xycos\theta} \]

Equating (1) and (2),

\[ xy = \frac{a^2 - b^2}{2cos\theta} \]
APPENDIX I

Analytical Approach

The following conditions applied are the maximum tilt of the collector from the horizontal at 80 degrees, extended length \( a \), retracted length \( b \) of the linear actuator and the stroke length \( l \) as per the manufacturer’s specifications \(^{20}\) and as defined by Table 26.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Extended length of linear actuator</td>
</tr>
<tr>
<td>( b )</td>
<td>Retracted length of linear actuator</td>
</tr>
<tr>
<td>( l )</td>
<td>Stroke length of linear actuator (( b-a ))</td>
</tr>
<tr>
<td>( x )</td>
<td>horizontal distance from bottom to upper attachment points of the linear actuator when completely extended</td>
</tr>
<tr>
<td>( y )</td>
<td>vertical distance from bottom to upper attachment points of the linear actuator when completely extended</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Offset distance of linear actuator attachment point on the collector to the pivot of the collector on the frame support.</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Angle between panel and vertical axis</td>
</tr>
</tbody>
</table>
The first step is to find the offset distance \((x - \delta)\) (Figure 114) depending on the stroke length \(l\) as calculated by Equation 24 using Sine Rule

\[
(x - \delta) = \frac{l}{2 \sin 40°}
\]  

Equation 24

From Figure 114, two solutions \(x_1/y_1\) and \(x_2/y_2\) are obtained from the two simultaneous Equations 25 and 26 by eliminating \(\delta\):

\[
x^2 + y^2 = a^2
\]

Equation 25

\[
[\delta + (x - \delta)\cos 10°]^2 + [y - (x - \delta)\sin 10°]^2 = b^2
\]

Equation 26
Linear actuator stroke length = 5.90 inch

Calculating horizontal distance x using simple iteration

Optimum offset (x-δ) defined as c
\[ c = \frac{5.90}{2 \sin \left(\frac{2 \pi}{9}\right)} \]

Variables  
\[ a = 17.39 \quad b = 11.69 \quad c = 4.389 \]

Eliminating y from equation and writing function in terms of x only to get two functions

\[ f_1(x) = \frac{b^2 - a^2 - 2\sqrt{\left(\frac{a^2 - x^2}{2}\right)c \cdot \sin(10) - c^2 \cdot (\cos(10) - 1)^2 - (c \cdot \sin(10))^2}}{2 \cdot c \cdot (\cos(10) - 1)} \]

\[ f_2(x) = \frac{b^2 - a^2 + 2\sqrt{\left(\frac{a^2 - x^2}{2}\right)c \cdot \sin(10) - c^2 \cdot (\cos(10) - 1)^2 - (c \cdot \sin(10))^2}}{2 \cdot c \cdot (\cos(10) - 1)} \]

Finding solution where \( F(x) = x \)

\[ h(x) = x \quad x = 0, 1, 20 \]

Solution yields two sets of values for x and y. Transmission angles for both sets of values in the position of the linear actuator will determine the optimum values for x and y for stroke length at 5.90 inch

\[ f_1(10.691) = 10.691 \quad f_2(16.569) = 16.57 \]

\[ y_1 = \sqrt{\left(\frac{a^2 - 10.691^2}{2}\right)} \quad y_2 = \sqrt{\left(\frac{a^2 - 16.569^2}{2}\right)} \]

\[ y_1 = 13.968 \quad y_2 = 5.903 \]

![Horizontal distance f(x) against x](image)

Figure 114: Calculation sheet and plot of horizontal distance f(x) against x carried out in Mathcad. Equation was iterated with f(x) = x at intersection with line h(x) = x.
Geometrical Approach

The two analytical solutions obtained for the 5.90 inch stroke length linear actuator were drawn graphically and the corresponding transmission angles $\mu$ were measured at each location. A radius of 4.59 inch was drawn on the horizontal panel extended through 80 degrees to the panel tilting at that angle. Resulting values for $x$ and $y$ were 10.88 in and 13.76 in respectively with 1.7% and 1.5% deviation from the analytical approach. This confirmed that the two solution obtained for $x$ and $y$ (Figure 115)
For all the available linear actuator stroke lengths from the manufacturer, transmission angles were determined for both solutions for each stroke length as listed in Table 27. Of the types of linear actuator available, Type 3 with stroke length 5.90 inches with at least 46 degrees as the transmission angle was chosen considering the small size against transmission angle.
APPENDIX J
FBD of panel:
\[ \sum F_x = R_{Ax} - R_{Ox} = 0 \quad \Rightarrow \quad R_{Ox} = R_{Ax} \]
\[ \sum F_y = R_{Ay} - R_{Oy} - W = 0 \quad \Rightarrow \quad R_{Oy} = W - R_{Ay} \]
\[ \sum M_O = R_{Ay}(x - \delta)\sin\theta - R_{Ax}(x - \delta)\cos\theta = 0 \quad \Rightarrow \quad \frac{R_{Ax}}{R_{Ay}} = \tan\theta \]

FBD of linear actuator:
\[ \sum F_x = R_{Ax} + R_{Bx} = 0 \quad \Rightarrow \quad R_{Ax} = -R_{Bx} \]
\[ \sum F_y = R_{Ay} - R_{By} + w_{la} = 0 \quad \Rightarrow \quad R_{Ay} = R_{By} - w_{la} \]
\[ \sum M_A = (x - a)w_{la} + yR_{Bx} - xR_{By} = 0 \quad \Rightarrow \quad R_{By} = \frac{(x-a)w_{la} + yR_{Bx}}{x} \quad \Rightarrow \quad R_{By} = \frac{(x-a)w_{la} + yw_{la}\tan\theta}{x + y\tan\theta} \]

FBD of one vertical beam support:
All reaction forces that are in contact with other objects are halved since there are two vertical beams. FBD showed forces on an equivalent single beam so that all forces obtained at contact places O and C will be halved on each beam
\[ \sum F_x = R_{Ox} + R_{Cx} + R_{Bx} = 0 \quad \Rightarrow \quad R_{Cx} = -R_{Ox} - R_{Bx} \]
\[ \sum F_y = R_{Oy} + R_{By} + w_b - R_{Cy} = 0 \quad \Rightarrow \quad R_{Cy} = R_{Oy} + R_{By} + w_b \]
\[ \sum M_O = -L_bR_{Cx} - yR_{Bx} + \delta R_{By} = 0 \quad \Rightarrow \quad R_{Cx} = \frac{\delta R_{By} - yR_{Bx}}{L_b} \]
FBD of turn-table plate:

Assumptions:

1. All forces acting collinearly in a single plane
2. Cross section thickness chosen is so small that the slight curvature at the ends are almost straight lines when angle is small \((\tan \beta \approx \beta)\)

Considering cross section of circular turn-table in the y-z plane, since system is static, the force that motor shaft is exerting on the turn table is zero and therefore is not included in the plane of analysis. The turn table will be more prone to bending depending on the variables \(z_b\) and \(D\). Distance \(z_b\) will in turn affect stability of the functional model and the angular momentum for dynamic motion.

\[
\sum F_z = R_{Cz} - R_{Dz} + R_{Dz} - R_{Dz} = 0
\]

\[
\sum F_y = R_{Cy} - 2R_{Dy} + w_{tt} = 0 \quad \Rightarrow \quad R_{Dy}
\]

\[
\sum M_C = \frac{z_b}{2} w_{tt} + \frac{z_b}{2} R_{Cy} + \frac{(D - z_b)}{2} R_{Dy} - (z_b + \frac{D - z_b}{2}) R_{Dy} = 0 \quad \Rightarrow \quad R_{Dy}
\]

\[
\sum F_x = R_{Ex} - R_{Cx} + R_{D1x} - R_{D2x} = 0 \quad \Rightarrow \quad R_{Ex}
\]

\[
= R_{Cx} - R_{D1x} + R_{D2x}
\]
\[ \sum F_y = R_{Cy} - 2R_{Dy} + w_{tt} + R_{Ey} = 0 \implies R_{Ey} = 0 \]
\[ = 2R_{Dy} - w_{tt} - R_{Cy} = 0 \]
\[ \sum M_{D_1} = dR_{Ey} + \frac{D}{2} w_{tt} + \frac{D}{2} R_{Cy} - DR_{Dy} + tR_{Ex} - tR_{Cx} = 0 \implies R_{D1x} = R_{D2x} \]
\[ \implies R_{Ex} = R_{Cx} \]

FBD of pinion and motor:
The gear set used is that of an internal gear set and the difference between external and internal gear set is that the former reverses direction of rotation between the cylinders whereas the latter will have the same direction of rotation on input and output shafts.

\[ \sum F_x = R_{Fx} - R_{Ex} = 0 \implies R_{Fx} = R_{Ex} \]
\[ \sum F_y = R_{Fy} + R_{Ey} - w_p = 0 \implies R_{Fy} = w_p - R_{Ey} \]
\[ \implies R_{Fy} = w_p \]

FBD of base and ball bearing:

\[ \sum F_x = R_{Dx} - R_{Dx} - R_{Fx} = 0 \]
\[ \sum F_y = 2R_{Dy} + w_B - R_G + R_{Fy} = 0 \implies R_G = 2R_{Dy} + w_B + R_{Fy} \]
\[ \implies R_G = W + w_{tt} + w_{la} + w_p + w_b + \]
\[ w_B \]
Assumptions for static analysis:

i. Mass center of linear actuator is approximated to about 1/3 way from the more bulky end.

ii. All connections are assumed to be point contact disregarding force distribution on objects in large surface contacts such as bolts and screws

iii. All bodies are assumed to be rigid bodies

iv. Materials have been approximately assigned to the bodies as either aluminum or steel to estimate the weights

v. CG of linear actuator is assumed to be the same whether extended or retracted based on the assumption that the pushing rod has negligible weight compared to the bulk part of the linear actuator itself

Table 28: Definitions and values for the parameters used in the derivation of the reaction forces

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>Weight of the PV collector</td>
<td>160</td>
</tr>
<tr>
<td>$w_{la}$</td>
<td>Weight of the linear actuator</td>
<td>12.0</td>
</tr>
<tr>
<td>$w_b$</td>
<td>Weight of the beam (bearing??)</td>
<td>1.00</td>
</tr>
<tr>
<td>$w_{tt}$</td>
<td>Weight of the turn-table</td>
<td>4.20</td>
</tr>
<tr>
<td>$w_p$</td>
<td>Weight of the pinion (this includes the motor weight)</td>
<td>3.00</td>
</tr>
<tr>
<td>$w_B$</td>
<td>Weight of the base</td>
<td>108</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{ext}$</td>
<td>Extended length of linear actuator</td>
<td>0.447</td>
</tr>
<tr>
<td>$l_{ret}$</td>
<td>Retracted length of linear actuator</td>
<td>0.297</td>
</tr>
<tr>
<td>$l$</td>
<td>Stroke length of linear actuator (b-a)</td>
<td>0.150</td>
</tr>
<tr>
<td>$a$</td>
<td>Horizontal distance of CG of linear actuator from bottom end</td>
<td>0.0389</td>
</tr>
<tr>
<td>$b$</td>
<td>Vertical distance of CG of linear actuator from bottom end</td>
<td>0.118</td>
</tr>
</tbody>
</table>
x  horizontal distance from bottom to upper attachment points of the linear actuator when completely extended 0.117
y  vertical distance from bottom to upper attachment points of the linear actuator when completely extended 0.355
δ  Offset distance of linear actuator attachment point on the collector to the pivot of the collector on the frame support. 0.155
zb  Horizontal distance between the centers of the two vertical frame support beam
D  Diameter of outer slewing gear

<table>
<thead>
<tr>
<th>Force</th>
<th>Analytical Equation</th>
<th>Relation</th>
<th>θ = 0</th>
<th>θ = 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROx</td>
<td>( w_{la} \tan(90 - \theta) \left( \frac{a}{x + y \tan(90 - \theta)} \right) )</td>
<td>( R_{Ax} )</td>
<td>0.460</td>
<td>0</td>
</tr>
<tr>
<td>ROy</td>
<td>( W + w_{la} \left( \frac{a}{x + y \tan(90 - \theta)} \right) )</td>
<td>( W - R_{Ay} )</td>
<td>157.4</td>
<td>160</td>
</tr>
<tr>
<td>RAx</td>
<td>( w_{la} \tan(90 - \theta) \left( \frac{a}{x + y \tan(90 - \theta)} \right) )</td>
<td>( R_{Ay} \tan(90 - \theta) )</td>
<td>0.460</td>
<td>0</td>
</tr>
<tr>
<td>RAy</td>
<td>( -w_{la} \left( \frac{a}{x + y \tan(90 - \theta)} \right) )</td>
<td>( R_{By} - w_{la} )</td>
<td>2.600</td>
<td>0</td>
</tr>
<tr>
<td>RBx</td>
<td>( -w_{la} \tan\theta \left( \frac{a}{x + y \tan(90 - \theta)} \right) )</td>
<td>( -R_{Ax} )</td>
<td>0.460</td>
<td>0</td>
</tr>
<tr>
<td>RBy</td>
<td>( w_{la}(1 - \frac{a}{x + y \tan(90 - \theta)}) )</td>
<td>( R_{By} )</td>
<td>9.400</td>
<td>12.00</td>
</tr>
<tr>
<td>RCx</td>
<td>( 0, \frac{R_{By}}{R_{Bx}} = \frac{y}{\delta} )  ( -R_{Bx} - R_{Ox}, )  ( \frac{\delta R_{By} - y R_{Bx}}{L_b} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RCy</td>
<td>( W + w_{la} + w_b )</td>
<td>( R_{Oy} + R_{By} + w_b )</td>
<td>173.0</td>
<td>173</td>
</tr>
<tr>
<td>RCz</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 29: Reaction forces, relation to one another and corresponding mathematical equation derived with absolute values given at the extreme polar angle values.
<table>
<thead>
<tr>
<th></th>
<th>Expression</th>
<th>R&lt;sub&gt;Dx&lt;/sub&gt;</th>
<th>R&lt;sub&gt;Dy&lt;/sub&gt;</th>
<th>R&lt;sub&gt;Ey&lt;/sub&gt;</th>
<th>R&lt;sub&gt;Ex&lt;/sub&gt;</th>
<th>R&lt;sub&gt;Fy&lt;/sub&gt;</th>
<th>R&lt;sub&gt;Fx&lt;/sub&gt;</th>
<th>R&lt;sub&gt;G&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\frac{W + w_{la} + w_b + w_{tt}}{2})</td>
<td>(\frac{R_{Cy} + w_{tt}}{2})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R&lt;sub&gt;Cx&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R&lt;sub&gt;Cx&lt;/sub&gt;, R&lt;sub&gt;Ex&lt;/sub&gt;</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>w&lt;sub&gt;p&lt;/sub&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>w&lt;sub&gt;p&lt;/sub&gt;</td>
<td>3.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total weight</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>288</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>288</td>
</tr>
</tbody>
</table>
APPENDIX K

Equations used for the individual parts to find the mass moment of Inertia listed in Table 30 and analytical and simulated values listed in Table 31.

Table 30: Equation to find mass moment of inertia for individual parts of the functional model

<table>
<thead>
<tr>
<th>Part</th>
<th>Mass m/kg</th>
<th>x/m</th>
<th>y/m</th>
<th>d/m</th>
<th>Mass moment of Inertia Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Panel</td>
<td>16.05</td>
<td>1.580</td>
<td>0.808</td>
<td>n/a</td>
<td>$I_{zz} = \frac{1}{12}m(x^2 + y^2)$</td>
</tr>
<tr>
<td>2. (Vertical beam support) × 2</td>
<td>0.610</td>
<td>0.038</td>
<td>0.038</td>
<td>0.176</td>
<td>$I_{zz} = I_o + md^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$I_{zz} = \frac{1}{12}m(x^2 + y^2) + md^2$</td>
</tr>
<tr>
<td>3. (Horizontal beam support) × 2</td>
<td>0.970</td>
<td>0.025</td>
<td>0.817</td>
<td>0.218</td>
<td>$I_{zz} = I_o + md^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$I_{zz} = \frac{1}{12}m(x^2 + y^2) + md^2$</td>
</tr>
<tr>
<td>4. Horizontal Frame support</td>
<td>0.293</td>
<td>0.005</td>
<td>0.390</td>
<td>0.050</td>
<td>$I_{zz} = I_o + md^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$I_{zz} = \frac{1}{12}m(x^2 + y^2) + md^2$</td>
</tr>
<tr>
<td>5. Linear actuator</td>
<td>1.20</td>
<td>0.050</td>
<td>0.200</td>
<td>0.100</td>
<td>$I_{zz} = I_o + md^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$I_o = \frac{1}{2} \left( \frac{1}{4}mx^2 + \frac{1}{12}my^2 \right)$</td>
</tr>
<tr>
<td>6. Cross beams</td>
<td>0.823</td>
<td>0.051</td>
<td>0.508</td>
<td>n/a</td>
<td>$I_{zz} = \frac{1}{12}m(x^2 + y^2)$</td>
</tr>
<tr>
<td></td>
<td>0.152</td>
<td>0.051</td>
<td>0.260</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Circular base</td>
<td>0.490</td>
<td>0.150</td>
<td>n/a</td>
<td>n/a</td>
<td>$I_{zz} = \frac{1}{2}mx^2$</td>
</tr>
</tbody>
</table>
Table 31: Mass moment of Inertia for individual parts in the functional model obtained analytically and by simulation

<table>
<thead>
<tr>
<th>Part</th>
<th>Mass Moment of Inertia $(I_{zz}/kgm^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculations</td>
</tr>
<tr>
<td>Panel</td>
<td>4.2121</td>
</tr>
<tr>
<td>(Vertical beam support) x2</td>
<td>0.0381</td>
</tr>
<tr>
<td>(Horizontal beam support) x2</td>
<td>0.2002</td>
</tr>
<tr>
<td>Horizontal Frame support</td>
<td>0.0044</td>
</tr>
<tr>
<td>Linear actuator</td>
<td>0.0031</td>
</tr>
<tr>
<td>Cross beams</td>
<td>0.0188</td>
</tr>
<tr>
<td>Circular base</td>
<td>0.0055</td>
</tr>
<tr>
<td>Total</td>
<td>4.4822</td>
</tr>
</tbody>
</table>

Note:

1. In calculations of mass moment of inertia for PV collector, supporting welded beams were not taken into account unlike in the Solidworks simulations.

2. The vertical beams and horizontal beams calculations were an overestimate since in reality they were hollow while in the calculations they were treated as filled.

3. Center of gravity for linear actuator in calculations was placed one third of total length in all directions from the base.

4. Mass moment of inertia for linear actuator in Solidworks was taken when linear actuator was completely vertical with center of gravity at one third distance of total length in all directions from the base.

5. Both results for mass moment of inertia by calculation and Solidworks simulations do not include miscellaneous items such as the electric box, bolts, screws and other items that will add slightly to the mass moment of inertia.
APPENDIX L

Deriving the equation of motion for each link using the D’Alembert method assuming all forces are conservative. By conservation of energy \((i = \text{number of links})\):

\[
\sum_{k=i}^{n} \vec{F}_k \cdot \vec{v}_k + \sum_{k=i}^{n} \vec{\tau}_k \cdot \vec{\omega}_k = \sum_{k=i}^{n} m_k \vec{a}_k \cdot \vec{v}_k + \sum_{k=i}^{n} I_k(\theta) \vec{\omega}_k \cdot \vec{\omega}_k
\]

Number of links identified in the system: \(k = 4\)

**Link 1:**

\[\tau_1 \omega_1 = \omega_1^2 (J_1 r_1 + J_2 r_2)\]

**Link 2:**

\[\tau_1 \omega_1 = J_3 \omega_1^2 r_3\]

**Link 3:**

\[F v_{LA} + \tau_1 \omega_1 = m_3 v_{OB} a_{OB} + J_4 \omega_{OB}^2 r_{OB}\]
Deriving the equation of motion for the system driven by both motors which exert a rotational angular velocity

For the purpose of simplification $\dot{\theta} = 0$

Resolving angular velocity vector $\vec{N}$ along the $x-y-z$ coordinates to obtain the resultant angular velocity vector $\vec{\omega}$

$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k} = -N \sin \theta \vec{i} - N \cos \theta \vec{j} + \dot{\phi} \vec{k}$$

Angular momentum,

$$\vec{H} = I_{xx} \omega_x \vec{i} - I_{zz} \omega_z \vec{k}$$

where

$$I_{xx} = \frac{1}{12} m (b^2 + l^2)$$
\[ l_{zz} = \frac{1}{12} mb^2 \]

Kinetic energy,
\[ T = \frac{1}{2} \ddot{\omega} \cdot \ddot{H} \]

Assuming potential energy negligible,
\[ V = 0 \]

Lagrangian Equation,
\[ L = T - V \]
\[ \left( \frac{d}{dt} \right) \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i^{nc} \text{ where } i = 1, 2, \ldots n \]

Assuming system is frictionless so that forces are conservative,
\[ \left( \frac{d}{dt} \right) \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 \]

Partial derivatives,
\[ \left( \frac{d}{dt} \right) \left( \frac{\partial L}{\partial \dot{\phi}} \right) = \frac{1}{12} mb^2 \ddot{\phi} \]
\[ \frac{\partial L}{\partial \phi} = \frac{1}{12} m(b^2 + l^2)N^2 \sin \phi \cos \phi \]

Therefore Equation of motion,
\[ mb^2 \ddot{\phi} - m(b^2 + l^2)N^2 \sin \phi \cos \phi = 0 \]

It turns out that equation above would only work if:

i. It can be seen that if a rotation N is imposed, since \( l_{zz} \) is changing the total energy will not be conserved since there will be coriolis effects and change in inertia if in addition angular velocity \( \dot{\theta} \) was imposed.

ii. The change in inertia can be solved by introducing a rotation matrix that would transform the axes of rotation to change from one plane to another plane. However coriolis effect will still need to be taken into consideration with imposed rotation.

iii. Therefore the above equation would only work in the stall position where \( \phi = 90 \)
iv. In the simplified version of $\varphi = 90$

The full analysis of the motion was a little more complex and was beyond the scope of this project. Therefore in the simplified version, the equation of motion would be based on the Euler Equations:

*Euler’s Equations*

\[
\sum M_x = I_{xx} \dot{\omega}_x - (I_{yy} - I_{zz}) \omega_y \omega_z \\
\sum M_y = I_{yy} \dot{\omega}_y - (I_{zz} - I_{xx}) \omega_z \omega_x \\
\sum M_z = I_{zz} \dot{\omega}_z - (I_{xx} - I_{yy}) \omega_x \omega_y
\]

Where if rotation is considered in x–axis only with constant angular velocity and at polar angle of 0 degree, equation of motion will simply be:

\[
\sum M = I_{xx} \dot{\omega}_x
\]
APPENDIX M

//Arduino code for solar tracker
//Two light sensors, wind sensor, load sensor
#define sensitivity 100   //sensitivity of the tracker (lower value -> higher sensitivity)
#define LA_PWM 100             //PWM output for linear actuator (255 -> full power)
#define BM_PWM 100                 //PWM output for base motor (255 -> full power)
#define LA_extended_value 840      //Feedback value from LA when extended (panel at 0 degree)
#define LA_retracted_value 180     //Feedback value from LA when retracted (panel at 80 degree)
#define wind_sensor_limit 500
#define load_sensor_limit 400
#define time_limit 10000           //Time limit to reset wind_gust_count to 0

//Pin assignment & initialization
int LA_out_A = 3;     //Linear actuator output A
int LA_out_B = 2;     //Linear actuator output B
int BM_out_A = 4;   //Base motor output A
int BM_out_B = 5;   //Base motor output B
int wind_state = 0;
int wind_gust_count = 0;
int last_wind_state = 0;
unsigned long previousMillis = 0;
unsigned long currentMillis = 0;

void setup() {
  pinMode(LA_out_A, OUTPUT);
  pinMode(LA_out_B, OUTPUT);
  pinMode(BM_out_A, OUTPUT);
  pinMode(BM_out_B, OUTPUT);
  Serial.begin(9600);
}

void loop() {

  //Tracking/////////////////////////////////////////////////////////////////////////
  int LA_error = (analogRead(A0) - (analogRead(A1)));
  int BM_error = (analogRead(A2) - (analogRead(A3)));

  if ((LA_error > sensitivity) && (analogRead(A6) > LA_retracted_value)) {
    analogWrite(LA_out_A, 0);
    analogWrite(LA_out_B, LA_PWM);
  } else if ((LA_error < -(sensitivity)) && (analogRead(A6) < LA_extended_value)) {
    analogWrite(LA_out_A, LA_PWM);
    analogWrite(LA_out_B, 0);
  } else {
    analogWrite(LA_out_A, 0);
    analogWrite(LA_out_B, 0);
  }
}
if (BM_error > sensitivity) {
  analogWrite(BM_out_A, 0);
  analogWrite(BM_out_B, BM_PWM);
}
else if (BM_error < -(sensitivity)) {
  analogWrite(BM_out_A, BM_PWM);
  analogWrite(BM_out_B, 0);
}
else {
  analogWrite(BM_out_A, 0);
  analogWrite(BM_out_B, 0);
}

//Wind Sensor

//After 3 gusts of wind, rotate the panel to 0 degree.
unsigned long currentMillis = millis();

if (analogRead(A4) > wind_sensor_limit) {
  while(analogRead(A4) > (wind_sensor_limit - 50)){
    wind_state = 1;
  }
} else
  wind_state = 0;

if (wind_state != last_wind_state) {
  if (wind_state == 1) {
    wind_gust_count++;
    Serial.println("Wind");
    Serial.println(wind_gust_count);
    previousMillis = currentMillis;
  }
}

last_wind_state = wind_state;

if (((wind_gust_count == 1) || (wind_gust_count == 2)) && (currentMillis - previousMillis > time_limit))
  wind_gust_count = 0;

if (wind_gust_count >= 3) {
  while (analogRead(A6) < LA_extended_value) {
    analogWrite(LA_out_A, 255);
    analogWrite(LA_out_B, 0);
  }
  wind_gust_count = 0;
}

//Force sensor

//If the force sensor is pressed, rotate the panel to 80 degree.
if (analogRead(A5) > load_sensor_limit) {
  while(analogRead(A6) > LA_retracted_value) {
    analogWrite(LA_out_A, 0);
    analogWrite(LA_out_B, 255);
  }
}
}
REFERENCES


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13 Wind loads on an azimuthal photovoltaic platform. Experimental study by Velicu et al.

14 Wind loads on heliostats and photovoltaic trackers of various aspect ratios by Pfahl et al.

17 Wind load design methods for ground based heliostats and parabolic dish collectors by Peterka and Derickson.


[24] Prof. Evans website: gtrl.wpi.edu