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Uniqueness of Complex Permittivity Reconstruction in a Parallel-Plane Waveguide

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Uniqueness of complex permittivity reconstruction in a parallel-plane waveguide

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[1] The paper presents a statement and a proof of uniqueness of solution to the inverse problem of determination of permittivity of a lossy dielectric inclusion in a parallel-plane waveguide from the reflection and transmission characteristics. The approach is based on the analysis of asymptotic representations of a solution to the direct problem of diffraction of a transverse electric wave and employs a generalization of the notion of partial far-field patterns applied for a guide.

1. Introduction

[2] With the recent remarkable progress of computational resources, computer-aided design has become a valuable component in developing systems of microwave power engineering. Knowledge of complex permittivity ($\varepsilon = \varepsilon' - i\varepsilon''$) of materials involved in an application is critical for creating an adequate model and thus for successful system design. However, the dielectric constant ε' and the loss factor ε'' are not always available. The lack of data regarding realistic materials motivates further development of robust and practical techniques of determining complex permittivity.

[3] Since ε' and ε'' are not directly measured, but calculated given the data on some measurable characteristics, a related numerical simulator may be made involved in determination of material parameters through a numerical solution of a corresponding inverse problem. This approach has been taken in a number of techniques using the finite element method [*Coccioli et al.*, 1999; *Thakur and Holmes*, 2001; *Olmi et al.*, 2002; *Santra and Limaye*, 2005], the finite-difference time-domain (FDTD) method [*Wäppling-Raaholt and Risman*, 2003] and the finite integration technique [*Requena-Pérez et al.*, 2006] for modeling of the entire experimental fixtures. Further exploring this trend, *Eves et al.* [2004] and *Yakovlev et al.* [2005] have recently developed the novel neural-network-based FDTD-backed technique capable of efficiently determining the dielectric constant and the loss factor of materials placed in a transmission-line-type

cavity. In this case, the experimental part is reduced to measuring the reflection and transmission coefficients of the systems. The technique is demonstrated to be versatile, robust, frequency- and cavity-independent, and applicable to the samples and fixtures of arbitrary configuration. However, while it has been shown by *Eves et al.* [2007] that the reconstructed ε' and ε'' can be easily validated and are proved to be accurate, uniqueness of this reconstruction remains to be an assumption. The latter circumstance may become an issue when using this or other modeling-based technique for determining effective complex permittivity of such increasingly important materials as nano-composites and metal powder, typically characterized by very high values of ε'. This paper presents the first results of the original study aiming to show that determination of complex permittivity of a body in a waveguide is unique when ε' and ε'' are reconstructed from the related reflection and transmission coefficients.

[4] More specifically, a goal of our study is to develop solution techniques elaborated by *Shestopalov and Sirenko* [1989] and *Shestopalov and Shestopalov* [1996] for the direct and inverse boundary value problems (BVPs) for Maxwell's and Helmholtz equations associated with the wave propagation in the waveguides with dielectric inclusions. Such problems arise also in mathematical models of the wave propagation and diffraction in inhomogeneous media [*Colton and Kress*, 1998]. In our approach, the BVPs and eigenvalue problems are formulated in unbounded domains and with partial radiation conditions at infinity that contain the spectral parameter; the method of solution employ integral equations (IEs) constructed using Green's function of the domain occupied by a regular guide. The methods of reconstructing the shape of the scatterer or its permittivity

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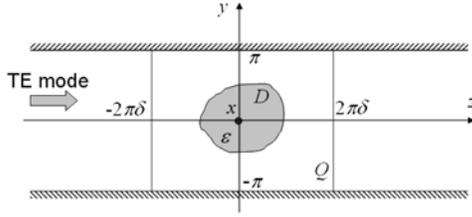
Figure 1. TE-mode diffraction by a dielectric inclusion in a parallel-plane waveguide.

are developed by *Colton and Kress* [1998] mainly for the cases when the obstacles are supposed to be perfectly conducting or dielectric bodies in two- or three-dimensional space. The recent paper by *Shestopalov and Lozhechko* [2003] suggests the technique for cylindrical scatterers whose (two-dimensional) cross sections are formed by domains with infinite noncompact boundaries. The uniqueness for such problems stated in the whole space or in the half-space is proved when the data in the inverse problem of finding the shape of the scatterer or permittivity of the inclusion (1) consist of the far-field patterns of the scattered field given for the plane wave irradiating the obstacle from all directions, and (2) are available for all frequencies varying in a certain interval.

[5] However, when a dielectric body is situated in a waveguide, similar results concerning the unique solvability and efficient solution techniques for the inverse scattering problems of reconstructing permittivity of the scatterer are not available. This fact becomes a driving force of our effort in developing a new approach to the solution of both direct and inverse scattering problems in waveguides. The present paper is devoted to the proof of uniqueness for a parallel-plane waveguide.

2. Diffraction Problem

[6] We consider a parallel-plane waveguide $S = \{(y, z): -\pi < y < \pi, z < \infty\}$ containing a nonmagnetic, isotropic, and inhomogeneous dielectric inclusion having the cross section $D \subset Q = \{(y, z): -\pi < y < \pi, -2\pi\delta < z < 2\pi\delta\}$ bounded by a piecewise smooth closed contour ∂D (Figure 1), where $Q \subset S$ denotes the so-called transition domain. The permittivity function $\varepsilon = \varepsilon (y, z)$ is assumed to be continuously differentiable and such that supp $m(y, z) \subset Q$, where $m(y, z) = 1 - \varepsilon (y, z)$. We also assume that the permittivity $\varepsilon (y, z)$ is a complex-valued function of two real arguments y, z continuously differentiable and bounded in S and denote

$$\varepsilon(y,z) = \varepsilon_1(y,z) \exp[i\varepsilon_2(y,z)] = g_1(y,z) + i\, g_2(y,z). \tag{1}$$

In accordance with physical assumptions of the model, the real and imaginary parts of equation (1) are positive, continuously differentiable, and bounded satisfying $g_1 (y, z) \geq 1$, so that the modulus ε_1 and argument ε_2 of the $\varepsilon (y, z)$ are also positive functions that are continuous and bounded on the line and satisfy $0 \leq \varepsilon_2 (y, z) < \pi/2$ and $\varepsilon_1 (y, z) \geq 1$.

[7] We further introduce the complex magnitude of the stationary electric and magnetic field, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$, respectively, where $\mathbf{r} = (x, y, z)$, and consider the problem of diffraction of the TE mode (assumed to be linearly polarized)

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \exp(-i\omega t), \quad \mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) \exp(-i\omega t), \tag{2}$$

$$\mathbf{E}(\mathbf{r}) = (E_x, 0, 0),$$

$$\mathbf{H}(\mathbf{r}) = \left(0, \frac{1}{i\omega\mu_0} \frac{\partial E_x}{\partial z}, -\frac{1}{i\omega\mu_0} \frac{\partial E_x}{\partial y} \right), \tag{3}$$

by a dielectric inclusion D.

[8] The total field $u(y, z) = E_x (y, z) = E_x^{inc} (y, z) + E_x^{scat} (y, z) = u^i (y, z) + u^s (y, z)$ of the diffraction by the D of the unit-magnitude TE wave with the only nonzero component is the solution to the BVP [*Shestopalov and Sirenko*, 1989]

$$\left[\Delta + \kappa^2\varepsilon(y,z)\right]u(y,z) = 0 \text{ in } S, \qquad u(\pm\pi,z) = 0, \tag{4}$$

$$u(y,z) = u^i(y,z) + u^s(y,z),$$

$$u^s(y,z) = \sum_{n=1}^{\infty} a_n^{\pm} \exp(i\Gamma_n z) \sin(ny), \tag{5}$$

where $\Delta = \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the Laplace operator, superscripts $^+$ and $^-$ correspond to the domains $z > 2\pi\delta$ and $z < -2\pi\delta$, $\omega = \kappa c$ is the dimensionless circular frequency, $\kappa = \omega/c = 2\pi/\lambda$ is the dimensionless frequency parameter (λ is the free-space wavelength), $c = (\varepsilon_0\mu_0)^{-1/2}$ is the speed of light in vacuum, and $\Gamma_n = (\kappa^2 - n^2)^{1/2}$ is the transverse wavenumber satisfying the conditions

$$Im\Gamma_n \geq 0, \quad \Gamma_n = i|\Gamma_n|, \quad |\Gamma_n| = Im\Gamma_n = \left(n^2 - \kappa^2\right)^{1/2}, \quad n > \kappa. \tag{6}$$

It is also assumed that the series in (5) converges absolutely and uniformly and allows for double differentiation with respect to y and z.

[9] Note that $u^i (y, z)$ satisfies (4) in S, the boundary condition, and radiation condition (5) only in the positive direction, so that the electromagnetic field with the

x-component u^i (y, z) may be interpreted as a mode coming from the domain $z < -2\pi\delta$.

[10] In the next section, we show that problem (4)–(6) can be reduced to a volume integral equation (IE). Then we show that if only one TE mode propagates in the waveguide without the inclusion, then the solution to (4)–(6) can be represented as a superposition of the reflected (in the domain $z < -2\pi\delta$) and transmitted (in the domain $z > 2\pi\delta$) fields and exponentially decreasing terms. This form of the solution is crucial for developing a method of solution to both direct and inverse scattering problems.

3. Integral Equation

[11] We solve the (direct) problem (4)–(6) in the transition domain Q by reducing it to a Lippmann-Schwinger volume IE [*Colton and Kress*, 1998]. To this end, we use Green's function G_0 of problem (4)–(6) [*Morse and Feshbach*, 1953] defined at $\varepsilon \equiv 1$ in the domain S from the expression

$$G_0(y,z;y_0,z_0) = \frac{1}{\pi} \sum_{n=1}^{\infty} \exp[i\Gamma_n|z-z_0|] \sin(ny)$$
$$\cdot \sin(ny_0)\Gamma_n^{-1}. \qquad (7)$$

The IE with respect to the sought for scattered field u (y, z) has the form

$$u(y,z) = -\int_{-2\pi\delta}^{2\pi\delta} \int_{-\pi}^{\pi} G_0(y,z;y_0,z_0)[1-\varepsilon(y_0,z_0)]$$
$$\cdot u(y_0,z_0)dy_0dz_0 + u^i(y,z), \quad (y,z) \in Q. \qquad (8)$$

The existence and uniqueness of the solution to (8) and its equivalence to BVP (4)–(6) can be proved on the basis of the approaches developed by *Shestopalov and Sirenko* [1989] and *Colton and Kress* [1998].

[12] Separating the singularity of Green's function (7) in the form

$$G_0(y,z;y_0,z_0) = -\frac{1}{2\pi} \ln\left[(y-y_0)^2+(z-z_0)^2\right]$$
$$+ N_0(y,z;y_0,z_0), \qquad (9)$$

where N_0 $(y, z; y_0, z_0)$ is differentiable for all (y, z), (y_0, z_0)

$\in Q$ one can show that the following result is valid.

[13] For Theorem 1, suppose that $m(y, z) = 0$ for $\sqrt{y^2 + z^2} \geq a$ with some $a > 0$ (so that the inclusion D is contained in a circle B_a^O of radius a centered at the origin) and $B_a^O \subset S$ and let $\kappa^2 < 2/Ma^2$, where $M = \max_{B_a^O} |m(y, z)|$. Then there exists a unique solution to the IE (8).

[14] In fact, following *Shestopalov and Sirenko* [1989], one can prove a more general result based on the fact that the integral operator of the second kind associated with IE (8) is a Fredholm and meromorphic operator-valued function of κ: IE (8) is uniquely solvable for all $\kappa > 0$ and, what is more, for all complex κ except for a countable set of isolated points.

[15] Lippmann-Schwinger IE (8) can be written in the operator form

$$(I - K)u = f, \qquad (10)$$

where I denotes the identity operator and the integral operator K and the right-hand side f are defined in (8). Under the conditions of Theorem 1, the (unique) solution to IE (8) can be written as the Neumann series

$$u = \sum_{n=0}^{\infty} K^n f = f + Kf + K^2 f + \dots. \qquad (11)$$

This representation can be used for the analytical solution to the inverse problem of determination of permittivity of a lossy dielectric inclusion inside a parallel-plane waveguide formulated in section 5.

[16] Let C (Q) denote the space of continuous functions in the transition domain Q with the norm $\|f\| = \|f\|_{C(Q)} = \max_{(y,z) \in Q} |f(y, z)|$. We can estimate the norm of the integral operator K in (8), which is bounded and continuous in C (Q) according to representation (9), as

$$\|K\| \leq M_0(D)|\varepsilon - 1|, \quad M_0(D) = \tilde{M}_0 \cdot \text{mes } D, \qquad (12)$$

where $\tilde{M}_0$ is a constant governed by the properties of the regular part N_0 of Green's function specified in (9). A sufficient condition for the convergence of the Neumann series (11) which implies the unique solvability of (8) can be written as

$$\tilde{M}_0 \cdot \text{mes } D|\varepsilon - 1| < 1. \qquad (13)$$

This implies that if electric dimensions of the scatterer are sufficiently small, the (direct) diffraction problem (4)–(6) is uniquely solvable.

4. Asymptotic Representation of the Field

[17] The goal of this section is to prove that, under the condition $1 < \kappa < 2$, the asymptotic representation of the total field has the form

$$u(y,z) = \begin{cases} \left[e^{i\Gamma_1 z} + R(\kappa)e^{-i\Gamma_1 z} \right] \sin(y) + O\!\left(e^{|\Gamma_2|z}\right) = u^i(y,z) + R(\kappa)u^i(y,-z) + O\!\left(e^{|\Gamma_2|z}\right), & z < -2\pi\delta, \\ T(\kappa)\sin(y)e^{i\Gamma_1 z} + O\!\left(e^{-|\Gamma_2|z}\right) = T(\kappa)u^i(y,z) + O\!\left(e^{-|\Gamma_2|z}\right), & z > 2\pi\delta, \end{cases} \quad (4)$$

where $R(\kappa)$ and $T(\kappa)$ are, respectively, the reflection and transmission coefficients. To this end, we use (8) and express the scattered field

$$u^s(y,z) = -\int_{-2\pi\delta}^{2\pi\delta} \int_{-\pi}^{\pi} G_0(y,z;y_0,z_0)m(y_0,z_0)u(y_0,z_0)$$
$$\cdot dy_0 dz_0, \quad m(y_0,z_0) = 1 - \varepsilon(y_0,z_0), \quad (15)$$

by substituting the expression (7) for Green's function for $z > 2\pi\delta$ into (15) to obtain

$$u^s(y,z) = -\sin(y)\exp(i\Gamma_1 z)u^{(1)}_{\infty,+}(\kappa)$$
$$-\sum_{n=2}^{\infty} \frac{i}{\pi} \frac{\exp(-|\Gamma_n|z)}{|\Gamma_n|} u^{(n)}_{\infty,+}(y;\kappa). \quad (16)$$

Here $\Gamma_1 = (\kappa^2 - 1)^{1/2}$, $\kappa \in (1,2)$, $|\Gamma_n| = (n^2 - \kappa^2)^{1/2}$ $(n > 1)$, and the partial far-field patterns

$$u^{(n)}_{\infty,+}(y;\kappa) = \sin(ny)u^{(n)}_{\infty,+}(\kappa), \qquad n = 2,3,\ldots, \quad (17)$$

where

$$u^{(n)}_{\infty,+}(\kappa) = \int\!\!\int_Q \exp(|\Gamma_n|z_0)\sin(ny_0)m(y_0,z_0)u(y_0,z_0)$$
$$\cdot dy_0 dz_0.$$

[18] For $z < -2\pi\delta$ we obtain the scattered field

$$u^s(y,z) = -\sin(y)\exp(-i\Gamma_1 z)u^{(1)}_{\infty,-}(\kappa)$$
$$-\sum_{n=2}^{\infty} \frac{i}{\pi} \frac{\exp(|\Gamma_n|z)}{|\Gamma_n|} u^{(n)}_{\infty,-}(y;\kappa) \quad (18)$$

and the partial far-field patterns

$$u^{(n)}_{\infty,-}(y;\kappa) = \sin(ny)u^{(n)}_{\infty,-}(\kappa), \qquad n = 2,3,\ldots,$$

where

$$u^{(n)}_{\infty,-}(\kappa) = \int\!\!\int_Q \exp(-|\Gamma_n|z_0)\sin(ny_0)m(y_0,z_0)$$
$$\cdot u(y_0,z_0)dy_0 dz_0.$$

[19] To sum up, we write the sought for asymptotic expressions

$$u^s(y,z) = -\sin(y)\exp(i\Gamma_1 z)u^{(1)}_{\infty,+}(\kappa) + O\!\left(e^{-|\Gamma_2|z}\right)$$
$$= -u^i(y,z)u^{(1)}_{\infty,+}(\kappa) + O\!\left(e^{-|\Gamma_2|z}\right), \quad z > 2\pi\delta,$$
$$(19)$$

$$u^s(y,z) = -\sin(y)\exp(-i\Gamma_1 z)u^{(1)}_{\infty,-}(\kappa) + O\!\left(e^{|\Gamma_2|z}\right)$$
$$= -u^i(y,-z)u^{(1)}_{\infty,-}(\kappa) + O\!\left(e^{|\Gamma_2|z}\right), \quad z < -2\pi\delta,$$
$$(20)$$

which proves representations (14).

[20] Expressions (19) and (20) prove also that $u = u^i + u^s$ is a twice continuously differentiable solution to (4)–(6) because (16) and (18) satisfy (4) and (5) and series in (16) and (18) converge absolutely and uniformly in every closed subdomain of S and admit termwise differentiation arbitrary number of times. Also representation (15) holds in the whole S.

5. Uniqueness of Solution to the Inverse Problem of Permittivity Reconstruction

[21] Assume that the domain D occupied by the dielectric inclusion in the waveguide is fixed and there are given sets of the partial far-field patterns (17) and (18)

$$U_{\infty,\pm}(y;\kappa) = U_{\infty,\pm}(y;\kappa;\varepsilon;u) = \left\{ u^{(n)}_{\infty,\pm}(y;\kappa) \right\}_{n=1}^{\infty}$$
$$= \left\{ \sin(ny)u^{(n)}_{\infty,\pm}(\kappa) \right\}_{n=1}^{\infty}, \quad (21)$$

for all $y \in [-\pi, \pi]$ and (i) for one frequency or (ii) for all values in an interval $\kappa \in K = (\kappa_1, \kappa_2) \subset (1, 2)$. It is necessary to determine the permittivity $\varepsilon_1 (y, z)$ of the inclusion, which is assumed to be a continuous function in the closed domain $\bar{D}$.

[22] Variable y in (21) plays the role of the observation angle of the far-field pattern. However, the angle of incidence of the plane wave which enters the far-field pattern representation in the case of scattering by an obstacle in the space *has no counterpart* in this statement. Therefore it is not possible to prove the uniqueness of finding the permittivity of the inclusion directly using the methods developed by *Colton and Kress* [1998].

[23] In order to prove the uniqueness of finding permittivity of the inclusion, assume that $\varepsilon_1 (y, z)$ and $\varepsilon_2(y, z)$ are two different permittivity functions (continuous in S) such that the corresponding (nontrivial) partial far-field pattern vectors (21)

$$U^j_{\infty,\pm}(y; \kappa) = U_{\infty,\pm}(y; \kappa; \varepsilon_j; u_j), \quad j = 1, 2,$$

where

$$U^j_{\infty,\pm}(y; \kappa) = \left\{ u^{(n),j}_{\infty,\pm}(y; \kappa) \right\}^\infty_{n=1}, \quad j = 1, 2,$$

coincide,

$$U^1_{\infty,\pm}(y; \kappa) - U^2_{\infty,\pm}(y; \kappa) \equiv 0, \quad y \in [-\pi, \pi]. \quad (22)$$

[24] It follows that

$$\int_Q \int e^{\pm i|\Gamma_1|z_0} \sin(y_0) r(y_0, z_0) dy_0 dz_0 = 0,$$

$$\int_Q \int e^{\pm |\Gamma_n|z_0} \sin(ny_0) r(y_0, z_0) dy_0 dz_0 = 0,$$

$$n = 2, 3, \ldots,$$

$$r(y_0, z_0) = m_1(y_0, z_0)u_1(y_0, z_0) - m_2(y_0, z_0)u_2(y_0, z_0).$$

[25] Using formulas (16)–(18) we obtain the scattered (transmitted and reflected) fields generated by $U^j_{\infty,\pm}(y; \kappa)$

$$u^s_j(y, z) = -\sin(y) \exp(i\Gamma_1 z) u^{(1),j}_{\infty,+}(y; \kappa)$$

$$- \sum_{n=2}^\infty \frac{i}{\pi} \frac{\exp(-|\Gamma_n|z)}{|\Gamma_n|} u^{(n),j}_{\infty,+}(y; \kappa), \quad j = 1, 2,$$

$$(23)$$

for $z > 2\pi\delta$ and

$$u^s_j(y, z) = -\sin(y) \exp(-i\Gamma_1 z) u^{(1),j}_{\infty,-}(y, \kappa)$$

$$- \sum_{n=2}^\infty \frac{i}{\pi} \frac{\exp(|\Gamma_n|z)}{|\Gamma_n|} u^{(n),j}_{\infty,-}(y, \kappa), \quad j = 1, 2,$$

$$(24)$$

for $z < -2\pi\delta$.

[26] Conditions (22) imply that the (twice continuously differentiable) functions specifying the scattered fields coincide in the whole domain S, $u^s_1 (y, z) \equiv u^s_2 (y, z) \equiv u^s (y, z)$, $(y, z) \in S$.

[27] Consequently, the total field $u = u^i + u^s$ is uniquely determined by the partial transmitted and reflected far-field pattern vector $U_{\infty,\pm} (y; \kappa)$ in the domains $z > 2\pi\delta$ and $z < -2\pi\delta$, respectively, and thus in S. We have that the total field $u(y, z)$ satisfies equation (1) with two different permittivity functions $\varepsilon_1 (y, z)$ and $\varepsilon_2(y, z)$: $[\Delta + \kappa^2\varepsilon_1 (y, z)]u(y, z) = 0$ and $[\Delta + \kappa^2\varepsilon_2 (y, z)]u(y, z) = 0$, $(y, z) \in S$. This yields $\varepsilon_2 (y, z) \equiv \varepsilon_1 (y, z)$ because $\kappa^2[\varepsilon_2 (y, z) - \varepsilon_1 (y, z)]u(y, z) = 0$ and $u(y, z) \neq 0$.

6. Conclusion and Discussion

[28] We have proved the uniqueness of the solution to the inverse problem of finding permittivity of a lossy arbitrarily shaped body inside a parallel-plane waveguide on the basis of knowledge of the partial far-field patterns of the transmitted and reflected fields. The analysis complements the inverse scattering theory and could be generalized to the general case when the domain containing the scatterer is a waveguide. The obtained asymptotic representation of the scattered field justifies the way of choosing the data for reconstructing complex permittivity in the form of the reflection and transmission coefficients.

[29] The presented results have a potential for diverse further development. For instance, the suggested proof can be extended to a more complex 3D case involving a regular waveguide. This will become a valuable supportive argument for additional enhancement of the available practical techniques of complex permittivity reconstruction. An important aspect of these applied methods is associated with the fact the corresponding measurements can be noise contaminated. In this situation, the uniqueness of permittivity reconstruction in a parallel-plane waveguide can be also proved, subject to certain restrictions, by the proposed method. If the problem is considered in the frequency band in which the irradiation is applied (i.e., when only one dominant mode propagates in the waveguide), then the scattered field can be calculated from the asymptotically exact expressions of the form (14) with exponentially small error. Thus if the partial far-field patterns (equation (21)) are given with a

noise component which does not violate the convergence of the series equations (23) and (24) the additional error of finding the scattered field subject to noise will be again exponentially small. One can then reconstruct the complex permittivity with an error that has the same order of magnitude as in the noiseless problem plus an exponentially small addend. In this study we consider only a theoretical proof of uniqueness of permittivity reconstruction rather than a mathematical method or algorithm of its practical determination. Therefore we leave a detailed analysis of noise-contaminated problems for a separate consideration.

[30] In conclusion, it seems to be important to reiterate the fact that the eventual goal of the presented analysis, being rather theoretical at its present stage, is to directly serve in support of a group of existing practical techniques of determination of material parameters in many industrial and engineering fields. Once our proof does not apply any restriction on the values of real and imaginary parts of complex permittivity, the corresponding applied technology will be given an appropriate level of confidence when working with new materials whose parameters cannot be found with the use of alternative methods.

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