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Rapid Generation of Isomorphic Tutoring

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Rapid Generation of Isomorphic Tutoring Content

Acknowledgements

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This IQP report includes work that has been previously published in [2], [14], [18], [19]. While the overall work was done with several collaborators, the portion of the project described in the IQP report was undertaken by me, and overseen by Shane Almeida, graduate student at WPI. Following my work, Shane worked on the template feature to make it a full-fledged feature of the ASSISTment system. Therefore, while I claim credit for this contribution, I would like to acknowledge a healthy contribution from all the authors, in particular Shane Almeida.
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ABSTRACT

Traditional intelligent tutoring systems have been successful at fostering learning, but very few systems have been built due to their high cost of creation. It has been reported that it takes between 100 to 1000 hours to produce a single hour of tutoring content and authors require significant background in artificial intelligence rule-based programming. Our previous research reported a reduction in the cost of authoring content through the use of pseudo-tutors, constructs that mimic cognitive tutors but are limited in scope to a single problem. This reduction in complexity made the system accessible to a wider audience, and allowed users to build an hour of tutoring content in only 10 to 30 hours. However, building isomorphic content, sets of problems that are nearly identical except for surface features, was costly because each such problem required an independent effort. In this work, we present novel features of the ASSISTment system that allow rapid creation of isomorphic problems via variabilized templates. Authors can employ templates to create a range of isomorphic problems varying in contextual and numerical content. Based on a usability experiment and a case study involving a college-level statistics course, we report evidence that this feature reduces the time required to author isomorphic problems, and we suggest that the feature is accessible to novice users of the system. We also describe experiments carried out and planned in order to evaluate the effectiveness of templates. Finally, we discuss the template feature as a simple prototype for personalizing tutoring content, and extending existing ITS authoring tools to be culturally aware.
1 Introduction

Although model-tracing, rule-based tutors provide effective learning environments in a variety of domains [9], research has shown that developing a single hour of usable student content can take between 100 and 1000 hours [3], [9]. Creating cognitive tutors also requires considerable knowledge in the domains of cognitive psychology and computer science, particularly artificial intelligence rule-based programming [16]. The high ratio of creation time to usable content, and the high level of expertise required have resulted in few systems being built and consequently adopted by educators.

Authoring tools for intelligent tutoring systems is a vibrant area of research as indicated by the recent book by Murray et al that reviews over a dozen authoring tools [12]. Particularly relevant to our work is the CTAT system which also supports the generation of isomorphic content [1]. Aleven et al. have added support for parameterized templates in the CTAT authoring framework to enable mass production of example-tracing tutors, and to reduce the burden of producing a large number of solutions. Rather than require tedious problem-specific authoring for each solution, CTAT allows authors to create a template that uses variables for problem elements that can vary. In the CTAT implementation, Excel spreadsheets containing problem-specific information are used to populate the variables in the template.

In this work, we take a similar approach to support the creation of isomorphic content. However, our implementation of variabilized templates in the ASSISTment system is broad and supports a range of functionality. Variabilized templates in our system provide greater flexibility by allowing authors to define and use arbitrary mathematical and logical functions involving variables. Further, although our implementation allows users to populate variables using third-party applications, we have no dependency on external tools; the same authoring environment is
used to build variabilized templates and define variables. This consistency in the authoring interface has made our template framework easily accessible to content creators.

### 1.1 The ASSISTment Project

The ASSISTment project is a joint research effort by Worcester Polytechnic Institute and Carnegie Mellon University, and is funded by grants from the Department of Education, the National Science Foundation, and the Office of Naval Research. The mission of the ASSISTment project to provide cognitive-based assessment of students is supported by providing tutoring content to students, comprehensive and up-to-date reports to teachers, and tools to teachers for creating their own tutoring content. ASSISTment assists while it assesses. The system assists students in acquiring different skills through the extensive framework of scaffolding questions, hints, and incorrect messages [13]. Assessment of student performance is provided to teachers through real-time reports based on statistical analysis. The system includes builder tools that allow teachers to create content tailored to their classes. Our primary users are middle- and high-school teachers throughout Massachusetts who are teaching the curriculum of the Massachusetts Comprehensive Assessment System (MCAS). Presently, we have 3000 students and about fifty teachers using our system in conjunction with classroom instruction and traditional evaluations (e.g., exams and homework).

### 1.2 Pseudo-Tutors

The Office of Naval Research funded the ASSISTment project in part to explore ways of reducing the cost of making intelligent tutoring systems. Our goal was to facilitate rapid content
creation by users with little or no background in computer science and cognitive psychology. Rather than implement a rule-based tutor that generalized over a particular domain, our project focused on developing a framework and supporting tools for the creation of “pseudo-tutors” [17].

Pseudo-tutors, a concept introduced by Koedinger et al., are constructs that mimic cognitive tutors but are limited in scope to a single problem [10]. In our system, we have implemented a simplified version of pseudo-tutors that supports only a linear progression through a problem. The pseudo-tutors in our system (i.e., a problem with any scaffolding) are called “assistments.” This simplification makes content creation easier and more accessible to a general audience.

Previous research has shown that our pseudo-tutor-based system can reduce the time required to build a single hour of content from 100 to 1000 hours to 10 to 30 hours [17]. At the most basic level, an assistment consists of a single main problem. For any given problem, assistance to students is available through either a series of hints or scaffolding problems. Hints are messages that provide insights and suggestions for solving a specific problem. Scaffolding problems are designed to address specific skills needed to solve the original problem. Additionally, instructive messages called “buggy messages” are provided to students if certain incorrect answers are selected. For problems without scaffolding, a student will remain in a problem until the problem is answered correctly. If scaffolding is available, the student will be programmatically advanced to the scaffolding problems in the event of an incorrect answer. Hints, scaffolding, and buggy messages together help create assistments that are structurally simple but can address complex student behavior. The structure and the supporting interface used to build assistments are simple enough so that users with little or no computer science and cognitive psychology background can use it easily.
1.3 Pseudo-Tutors for Isomorphic Content

Although they exhibit similar behavior to rule-based tutors, pseudo-tutors lack the ability to generalize over similar problems [17]. Separate pseudo-tutors are required for each individual problem regardless of similarities in tutoring content. Indeed, a common task among content creators in our system is “morphing:” modifying existing assistments in subtle ways (e.g., changing quantities in the problem) to create new content. Before the template feature was introduced in ASSISTment, the system contained about 140 morphs (out of about 3500 assistments). Without the template feature, morphing was tedious and time consuming. In addition, ensuring coherency across all components of the assistment was challenging.

However, it has been observed that a large body of isomorphic content is pedagogically useful. Pavlik et al. report that learners, particularly beginners, need practice at closely spaced intervals [4], while McCandliss and others claim that beginners benefit from practice on closely related problems [11]. Applying these results to a tutoring system requires generating a significant body of content addressing the same skill sets. In addition, teachers using the ASSISTment system used to assign the same problem set to students multiple times for provide more practice. However, this approach was not very effective because students had already solved the given problems before and the repetition didn’t reinforce the learning.

Therefore, in this project, we built a template framework to allow isomorphic content to be created in a short amount of time. Our goal with the research was to not only extend our content-building tools, but also to ensure that the additional complexity did not affect the system usage adversely.
2 Variabilized Templates

One of the main goals of the ASSISTment system was to create a tool that instructors with no experience in cognitive psychology or computer science could use to build content with ease. In order to achieve this goal, a specialized system was built around the concept of simplified pseudo-tutors. However, this simplicity reduced the flexibility of the system with regard to isomorphic content. Through the use of templates, our research seeks to facilitate the reuse of tutoring content across multiple instantiations of an item. Our aim is to increase flexibility and reduce the time required to build isomorphic content while retaining the basic ease of use associated with the ASSISTment system.

2.1 What are templates?

Isomorphic content is tutoring content that addresses the same concepts through a set of problems that differ only in their surface features. Thus, isomorphic problems are identical except for features such as the numerical values used, contextual elements (e.g. names) etc. We decided to capture this similarity in isomorphic content through a template model. In this model, a template assistment consists of two parts, a static part and a dynamic part. The static part contains the content that doesn’t change across these problems while the dynamic part contains surface features like numeric and contextual values. Like blueprints, once these templates were created, they can be used to create any number of isomorphic problems. The figure below shows an example of a template that will be explained in the following sections.
2.2 Implementation of Templates

While the concept of templates is conceptually simple, a new framework had to be introduced in the ASSISTment system to support the creation and use of templates. The static parts of templates could be implemented using the existing architecture, but to implement the dynamic parts of templates, we had to introduce the concept of *variables* to the system.

Variables, as in math or computer science, refer to quantities that change. In a template, variables are placeholders for quantities and contextual elements that vary across isomorphic problems. Since these variables need to be used across different parts of an assistment such as the main problem, scaffolding problems, answers, hints, and buggy messages, our implementation associates variables with individual assistments. As a result, variables can be used consistently throughout the pseudo-tutor enabling tutoring elements to address specific details of the problem. The variables can relate to any contextual or numerical value in the problem. Depending on the degree of flexibility required, entire problem statements can be put into variables too.

Each variable has a name and either a set of possible values it can take or a value-generating function. Depending on the type of variable (numeric or contextual), our system provides different ways to generate variable values. Once a variable has been created, it can be used in
any part of the assistment, including complex math formulas. After all variables have been
created and incorporated into assistments, a template is ready for use and can generate any
number of isomorphic problems.

To support the concepts of variables and templates in the ASSISTment system, we extended
the existing database schema. By linking assistments to individual sets of variables, we created a
framework by which an assistment could find the variables assigned to it and use them as
required. For incorporating variables into actual assistments, we built a run-time template
evaluator as well as a template instantiator. The former allows teachers to assign students a
template assistment that is evaluated right before students view the assistment. The latter allows
a teacher to create morphs or “instantiations” of a template beforehand, and assign them to
students separately.

Throughout the implementation of templates, one of our main goals was to preserve the
simplicity of the system. Therefore, our template framework hid all the implementation details
from the user, and instead presented only a simple interface to create templates. The following
section demonstrates this interface through an example of how templates can be created from an
existing problem.

2.3 Creating a Template

The assistment shown in Figure 1 addresses the Pythagorean Theorem and is an assistment
commonly encountered by students using our system. It has thirteen hints and eight buggy
messages distributed between the main problem and four scaffolding problems. Variabilized
templates make it possible to take such an assistment and create multiple versions of the
assistment that differ in details such as the length of the fence (i.e., 17, 8), the name of the person (i.e., Manuel), the type of enclosure being built (i.e., flower garden) and so on.

The process of creating a template involves several steps as we shall now describe:

1. **Identifying Variables**: The first step in creating a template assistment is to determine candidate variables in the problem. Candidate variables are the parts of the problem that can be changed without changing the basic concept addressed by it. The remaining content of the problem is static, and is same across all morph or instances of the problem. In the figure, the possible candidates for variables are the numerical values (17, 8), and certain non-essential contextual elements like the name (Manuel) and type of enclosure (flower garden). Depending on the complexity of the problem and desired flexibility for the templates, more parts of the problem statement can be used as variables. For instance, one could imagine a generalized problem where the shape of the enclosure ranges from a Pythagorean triangle to an isosceles triangle or an equilateral triangle. Figure 2 identifies variables in the Pythagorean problem with circles.
2. **Defining Variables:** Once variables in a problem statement have been identified, they must be created and assigned values. As described previously, a variable in our system has a unique name and one or more values associated with it. So the second step in creating templates is to define these variables by giving them unique names and assigning them realistic values.

There are various ways to assign values to variables. The simplest way is to specify the **exact values** that a variable may take. Another way to specify variable values is to use mathematical functions. For example, the function “rand(x)” can be used to generate integers from 0 to x-1. Similarly, we can construct complicated functions using logarithms, squares, roots and other math functions. The third way to create variables is to use values of **previously defined variables**. For example, if variables “a” and “b” have been defined previously, then we can create a variable “c” which is any function of “a” and “b” such as “a+b” or “a^2 + 2*a*b + b^2). When the last two methods of creating variables are combined, we can get a large variety of variables.

Figure 3 shows the variables defined for this Pythagorean Theorem problem. Due to the nature of the problem, we chose to directly assign values to the variables. As we can see, some variables like name and structure have string values while others have numeric values. The variables of particular interest to us here are the variables corresponding to the three sides of the right triangle, namely “a”, “b”, and “c.” As we know, not all triples of numbers can correspond to a right triangle. Our system provides a way to ensure that the right sets of numbers are picked together. An important feature associated with variables is the provision to bind together specific values of multiple variables as a **set**. For example, in this Pythagorean Theorem problem, the set feature
allows common Pythagorean triples like (3, 4, 5), (5, 12, 13) and (7, 24, 25) to be grouped together. This is accomplished by using the “Variable in a set” option while creating the variables.

<table>
<thead>
<tr>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>name = {Manuel; Cody; Zack; Jozsef}</td>
</tr>
<tr>
<td>structure = {a flower garden; a shed; a vegetable garden; a parking lot}</td>
</tr>
<tr>
<td>a = {3; 5; 7; 8; 9; 11; 12; 13; 16; 20}</td>
</tr>
<tr>
<td>b = {4; 12; 24; 15; 40; 60; 84; 63; 21}</td>
</tr>
<tr>
<td>c = {5; 13; 25; 17; 41; 51; 37; 85; 65; 29}</td>
</tr>
</tbody>
</table>

Figure 3 Variables created for the Pythagorean Theorem problem

3. **Incorporating Variables in Problem:** Once variables have been defined, the next step is to incorporate them in the actual assistment. The architecture of templates ensures that once a variable has been defined, it can be used in all parts of the assistment including problem statements, answers, hints, buggy messages and scaffolding questions. Figure 4 shows the Pythagorean assistment with variables introduced in the places identified earlier. A special syntax in the form of \( %v\{variable-name\} \) is used to reference variables throughout the builder environment. A similar syntax can be used to include variables in other parts of the assistment as well. For example, a possible incorrect answer for the Pythagorean problem may be identified as \( %v\{c-a\} \).

Moreover, the same syntax can be used to define variable functions to be used in problems. For example, \( %v\{sqrt(c^2 - a^2)\} \) could be used to calculate the length of the second leg b from sides a and c. This ability is important in problems where the values of
answers of intermediate results cannot be known beforehand. As we discuss in later sections, this ability to use mathematical and logical functions in templates also allows content creators to interface with various third-party utilities.

![A sample variabilized Assistment on the Pythagorean Theorem. As shown in red, variables have been introduced for various parts of the present problem including numerical values and parts of the problem statement.](image)

Generation of variables in the system is simple and uses the same style of widgets for creating answers and hints. Maintaining consistency with other elements of the build tools minimizes the learning time for content creators. The system also allows users to preview a template with variables replaced with actual values. This feature assists authors during the editing cycle.

4. **Creating Instances:** Once variables have been defined and introduced into problems, scaffolding problems, answers, hints, and buggy messages, the template is ready. We now move to step four where we actually create multiple instantiations of the template.

An instantiation of a template is a variant of the template where each of the variables takes on a specific value. Once an instance has been created, this instance is static and cannot be used to create other instances. With a single click of a button we can generate as many variants of the original problem as we want. Here lies the main advantage of templates. Once a variabilized template is created, new assistments including their scaffolding problems, answers, hints, and buggy messages can be generated instantly.
Figure 5 shows the main problem in three such instantiated assistments for the Pythagorean problem. As we can see from the figure, the names of the persons, the enclosure to be built and the lengths of the sides of the triangle change with every instantiation of the assistment.

Thus, in summary, a short process of four steps is required to rapidly generate a large number of problems. While the Pythagorean Theorem problem demonstrates the use of variables in small parts of the problem statement and in the numerical values, variables can be used to completely change the context of the problem and non-essential contextual elements differ and provide variety.

2.4 Design and Creation of Content via Templates

Following the implementation of templates in the ASSISTment system, we built a large body of content using this feature. Shown in this section are some variabilized assistments. When
creating content for the ASSISTment system, our goals are threefold: to create problems that are easy to understand for the target grade level and precisely address a particular skill, to provide stepwise tutoring for helping students to solve the problem, and to create a diverse set of problem that address a given skill comprehensively. Consider for instance, the tutoring content we created for the concept of average or mean. The mean (average) content was formulated to specifically address the concept in diverse ways. The problems were designed so that they were easy for eight graders to understand, and the contexts for the problems (e.g. scores in math tests, sales at a store etc) were chosen so that the students could relate to them. Tutoring for these problems was provided by means of hints as well as scaffolding questions. Both strategies of tutoring followed the general sequence of first explaining the concept of mean and then taking the students through the process of actually calculating the mean. While designing the tutoring, our aim was again to create hints and scaffolds that were easy to follow, and that helped correct common misconceptions about the target concept.

We accomplished our goal of comprehensively addressing the concept of mean by designing problems that required the students to employ mean in various contexts. To start, we created problems that required the student to find the mean of a list of numbers (integers and decimals) with varying numbers of values. Next, we created problems that required the student to find the missing value in a list of numbers given the mean of the list. We also created problems where data was presented to students in a tabular form and they were required to interpret this data to calculate the mean. Finally, we designed problems that introduced contexts in all the above types of problems. For example, in one problem, the students were required to find the average of the sales at a hardware store. Another problem asked the student to find the score that a student, say Mary, would have to obtain to get the given score as the mean. These contexts were aimed at
providing real life examples to students and showing them how these concepts applied outside of math. A comprehensive list of the content we created is available in the appendices.

While all the considerations discussed above are important for content created in the ASSISTment system, the use of templates introduces new challenges in this process. The problems need to be designed so that it is easy to create and use variables throughout the problem, and problem contexts need to be created so that the contexts themselves can be variabilized. To illustrate, we should be able to use the same variabilized template to create problems about calculating the mean of math scores as well as the mean of the sales at a hardware store. Moreover, variabilized templates also need careful thought in determining the values of variables. This is particularly important when the template variables need to bear specific relationships to one another. For example, while designing content for median problems, we need to create variables for which we know the ordering of all their possible values beforehand. In this case, the variables must be assigned values so that precise ordering relationships between them hold irrespective of the actual values of the variables.

In summary, we found that while creating variabilized content in the ASSISTment system, we need to invest more time in planning the problem and constructing appropriate variables for use in the problem. However, once this initial step has been completed, we can rapidly generate a large number of instances with just a click of a button. We now list a few of the templates created including their problem statements (with variables highlighted), answers, hints, and examples of instances of the template.
Example 1: Variabilized template for an assistance on finding the mean of a set of numbers.

1. Problem statement

2. Variables:
3. Answers

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔️ %v(ans1)</td>
</tr>
<tr>
<td><img src="image" alt="New Answer" /></td>
</tr>
</tbody>
</table>

4. Hints

<table>
<thead>
<tr>
<th>Hints</th>
</tr>
</thead>
<tbody>
<tr>
<td>The total sales equal the sum of all the monthly sales in 1997.</td>
</tr>
<tr>
<td>Now we have to add all the numbers in this list up.</td>
</tr>
<tr>
<td>%v(x1)</td>
</tr>
<tr>
<td>%v(x2)</td>
</tr>
<tr>
<td>%v(x3)</td>
</tr>
<tr>
<td>%v(x4)</td>
</tr>
<tr>
<td>%v(x5)</td>
</tr>
<tr>
<td>%v(x6)</td>
</tr>
<tr>
<td>%v(x7)</td>
</tr>
<tr>
<td>%v(x8)</td>
</tr>
<tr>
<td>%v(x9)</td>
</tr>
<tr>
<td>%v(x10)</td>
</tr>
<tr>
<td>%v(x11)</td>
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<tr>
<td>%v(x12)</td>
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<td>%v(x13)</td>
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<td>%v(x15)</td>
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<td>%v(x16)</td>
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<td>%v(x17)</td>
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<td>%v(x18)</td>
</tr>
<tr>
<td>%v(x19)</td>
</tr>
<tr>
<td>%v(x20)</td>
</tr>
<tr>
<td>%v(x21)</td>
</tr>
<tr>
<td>%v(x22)</td>
</tr>
<tr>
<td>Total: %v(sum)</td>
</tr>
</tbody>
</table>

Remember, the sum of all the numbers is %v(sum).
Now find the number of numbers and divide.

There are 12 different numbers in the table.

The mean is \%v(sum) / 12 = %v(sum/12)

Rounding the mean to the hundredths gives %v(ans1), type in %v(ans1).

5. Example of an Instance

<table>
<thead>
<tr>
<th>Ashley runs a hardware store, and listed below are the store sales for the year 1997. What were the average monthly sales in 1997?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>January</td>
</tr>
<tr>
<td>February</td>
</tr>
<tr>
<td>March</td>
</tr>
<tr>
<td>April</td>
</tr>
<tr>
<td>May</td>
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<td>June</td>
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<td>July</td>
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<tr>
<td>August</td>
</tr>
<tr>
<td>September</td>
</tr>
<tr>
<td>October</td>
</tr>
<tr>
<td>November</td>
</tr>
<tr>
<td>December</td>
</tr>
</tbody>
</table>

(round to hundredths place)
Example 2: Finding the median of a list of numbers

1. Problem statement

During a medical study, doctors recorded the blood pressure of all their volunteers. Some of the blood pressures (in mmHg) are provided here. What is the median blood pressure of the volunteers as listed below?

\( \text{v5, v6, v4, v2, v7, v9, v8, v1, v6, v11, v2} \)

2. Variables

<table>
<thead>
<tr>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>v10 = [0:120:150]</td>
</tr>
<tr>
<td>Variable is in the set.</td>
</tr>
<tr>
<td>v1 = 20 + rand(2) + v10</td>
</tr>
<tr>
<td>v2 = 20 + rand(2) + v10</td>
</tr>
<tr>
<td>v3 = 23 + rand(3) + v10</td>
</tr>
<tr>
<td>v4 = 26 + rand(2) + v10</td>
</tr>
<tr>
<td>v5 = 31 + rand(2) + v10</td>
</tr>
<tr>
<td>v6 = 35 + rand(3) + v10</td>
</tr>
<tr>
<td>v7 = 39 + rand(5) + v10</td>
</tr>
<tr>
<td>v8 = 43 + rand(5) + v10</td>
</tr>
<tr>
<td>v9 = 50 + rand(3) + v10</td>
</tr>
<tr>
<td>v11 = 55 + rand(10) + v10</td>
</tr>
</tbody>
</table>

param = [age, weight, height]
Variable is in the set.
Variable has string values.

unit = [years, lbs, centimeters]
Variable is in the set.
Variable has string values.

3. Answer

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔️ ( \text{\frac{(v5 + v6)}{2}} )</td>
</tr>
</tbody>
</table>

New Answer
4. Hints

Hints
Writing the numbers in increasing order, we get:
\[ 8v(v1), 8v(v2), 8v(v3), 8v(v4), 8v(v5), 8v(v6), 8v(v7), 8v(v8), 8v(v9), 8v(v11) \]

Since there are 10 numbers in the list, the median is \(\frac{5^{th} \text{ number} + 6^{th} \text{ number}}{2}\).

The 5^{th} number is \(8v(v5)\), and the 6^{th} number is \(8v(v6)\). So the median is:
\[
\frac{8v(v5) + 8v(v6)}{2} = \frac{8v(v5\cdot v6)}{2}
\]
Type \(\frac{8v(v5\cdot v6)}{2}\).

5. Example of an instance

**During a medical study, doctors recorded the weights of all their volunteers. Some of weights (in lbs) are provided here. What is the median weight of the volunteers as listed below?**

152, 155, 148, 145, 159, 171, 140, 166, 11, 140

**Break this problem into steps**

Type your answer below (mathematical expression):

**Submit Answer**
Example 3: Template for calculating the range of a list of numbers

1. Problem Statement

   The team coach needs to pick one of two people for the (what) team. Points obtained by (name1) and (name2) are given below.

   What is the range of points obtained by (name1)?

2. Variables

<table>
<thead>
<tr>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1 = rand(5) + 11</td>
</tr>
<tr>
<td>v2 = rand(5) + 17</td>
</tr>
<tr>
<td>v3 = rand(5) + 25</td>
</tr>
<tr>
<td>v4 = rand(5) + 15</td>
</tr>
<tr>
<td>v5 = rand(5) + 19</td>
</tr>
<tr>
<td>v6 = v3 + 3 * rand(3)</td>
</tr>
<tr>
<td>ans = v6 - v1</td>
</tr>
<tr>
<td>v7 = v3 - 5</td>
</tr>
<tr>
<td>name1 = [Beth; Amy; Rachel; Joe; Gary]</td>
</tr>
<tr>
<td>what = [basketball; All-USA Math; Computer Programming; All-USA Physics]</td>
</tr>
<tr>
<td>name2 = [Jenny; Ross; Don; Fleur; Jess]</td>
</tr>
</tbody>
</table>

   Variable has string values.

   | v11 = rand(6) + 5 |
   | v12 = rand(6) + 11 |
   | v13 = rand(6) + 14 |
   | v14 = rand(6) + 17 |
   | v15 = rand(6) + 18 |
   | v16 = rand(6) + 13 |
   | v17 = rand(6) - 23 |
3. **Answer**

```
<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
</tr>
<tr>
<td>![New Answer]</td>
</tr>
</tbody>
</table>
```

4. **Hints**

```
Remember, the **range** is the difference between the **largest value** and the **smallest value** of the set of numbers.

The list of %v[name1]'s scores is: %v[v7], %v[v3], %v[v2], %v[v1], %v[v5], %v[v4], %v[v6]

The **largest value** is %v[v6], and the **smallest value** is %v[v1].

The range is the difference between %v[v6] and %v[v1].

%v[v6] - %v[v1] - %v[v6] - %v[v1]. Type %v(ans)
```

5. **Example of an instance**

```
The All-USA Physics team coach needs to pick one of two people for the All-USA Physics team. Points obtained by Beth and Dan are given below.

What is the range of points obtained by Beth?

Beth: 21, 26, 17, 14, 19, 19, 30
Dan: 15, 8, 13, 20, 25, 11, 22
```

Following the creation of content, we ran several experiments to measure the effectiveness of isomorphic content and our template framework. These studies are discussed in the evaluation section of the report. Next, we discuss some of the extensions we made to the template system to increase their flexibility and use.
3. Extensions of Templates

The template framework of the ASSISTment system is not only a powerful means to generate a large body of tutoring content, but it is also a way to dynamically alter problem content to suit various audiences. Moreover, it can also be used to implement the feature of Mastery Learning. We now discuss extensions of the template framework that allow personalization and cultural awareness in tutors along with the implementation of Mastery Learning.

3.1 Personalization of Tutors

In their notable paper on intrinsic motivation through personalization and contextualization, Cordova and Lepper showed that personalizing teaching content and adding familiar as well as interesting contexts to math problems motivated kids to learn math [7]. While this study was conducted with elementary school children, it is reasonable to expect that the same principle holds for older students too. Therefore, building on the template framework defined above, we create a way in which student data could be incorporated into assiments. This feature allowed information like the student’s name, favorite restaurant, hobby etc. to be included in problem. These details were specific for each student and were inserted into the problems dynamically. The goal of this feature was to add personal details to the problem without changing any conceptual details. Figures 6 and 7 show an example of such a personalized problem.
As shown in the figure 6, we used special variables like “user_first_name” and “user_favorite_restaurant” to indicate that this data had to be pulled from the user information. Since this data is already available in the system, separate variables do not need to be defined for this purpose. The system has a built-in framework that will populate these variables before the student gets the personalized problem.

Personalized templates differ from the variabilized templates in that it is not possible to create instances of this template. The template is adapted just before the student views it and no static problems are created. As a result, all the students see the exact same problem, but their personal details have been added to make the problem interesting. Our case study described in the evaluation section describes the use of personalized tutors at a Worcester middle school.
3.2 Culturally Aware Tutors

Although intelligent tutoring systems are being used for distance learning, these systems have not been designed to be culturally sensitive despite evidence that culture has implications on how people learn and interpret information [8]. Extending existing ITS authoring tools to be culturally aware is a challenge due to the complexity of these tools, the cost for extending them, and the expected increase in content creation time. As part of this IQP, we explored the use of the template feature of ASSISTment as a simple prototype for extending existing ITS authoring tools to be culturally aware. While this feature has similarities to the authoring tool described by Blanchard et al. [5], it is important to notice that enabling the production of culturally-aware content is one of its several uses, and that since this feature is an extension of an existing authoring tool, it is limited in its scope in targeting cultural contexts.

3.2.1 Culturally Aware Templates

Similar to the personalized templates discussed in the previous section, our framework can be used to construct culturally aware templates, i.e. templates that are adapted according to culture. Figure 8 shows the basic functioning of culturally aware templates. The three parts involved in this system are a repository of student specific data obtained from user profiles of students in the system, the culturally aware templates created using the Assistment Builder, and a run-time system that retrieves student data and adapts the content based on culturally specific information.
3.2.2 Building Culturally Aware Templates

Variabilized templates in the ASSISTment system described above are supported by the concept of variables which are placeholders for elements that vary across otherwise similar assistments. In the case of culturally aware templates, variables relate to contextual data that varies with culture. As before, each variable in a template has a name and a value. To associate a cultural variable with user data, its name has to take the form “x depends on y” which implies that the variable depends on the existing data called y. For example, “sport depends on country” implies that this variable depends on the country the student belongs to. This framework allows authors to either use traditional culture-bearing units or to define and use new culture-bearing units [6]. For instance, if the author wanted to extend the definition of cultures to “micro-cultures” like schools or states, variables like “activity depends on school” or “team depends on state” could be constructed and used.
Values of variables are used to adapt the contextual elements of a problem based on culture. For instance, if the system is used in three countries say the United States, India, and Canada, the variable “currency_depends_on_country” can be defined to take the values of American dollars, rupees, and Canadian dollars respectively. This is defined using the syntax “default: dollars; United States: American dollars; India: rupees; Canada: Canadian dollars” where each country is followed by the appropriate currency and default signifies the default value for the currency. Figure 9 shows the construction of four such variables depending on the country of the student. Once defined, these variables can be used throughout the assistment using a special syntax of the form %v{variable-name}. Figure 10 shows a culturally aware template in which the name of the person, currency, currency symbol, and sport is decided by the country of the student.

Figure 9 Variables defined for use in the culturally aware template. Variables can depend on various user specific data; in this example, the variables depend on country.

Once a template has been created with the appropriate culturally dependent variables, it can be assigned to students. The run-time system retrieves student specific data relating to culture-bearing units and looks for variable values corresponding to this data. These values then adapt the template based on cultural information. For instance, if a student in India is using the
system, the previous assistment would have “cricket” substituted for the variable “sport depends on country,” “Rahul” for “name depends on country,” “Rs.” For “symbol depends on country,” and “rupees” for “currency depends on country.” Figure 11 shows how the template assistment from Figure 10 is adapted for users in the United States, India, and Canada.

Figure 10: A culturally aware template where the currency, sport, symbol, and name are defined based on the country the student belongs to.

Figure 11: The main problems of three instantiations of the variabilized template

While Figure 10 shows a simple example involving the introduction of culturally specific contexts, an author can associate complete cover stories with culture-bearing units. Further, since variables are associated with individual assistments, the author can vary culturally specific information with different assistments.
3.3 Mastery Learning

Mastery Learning is the instructional model where a student is provided as much instruction as necessary to “master” a topic and the student is allowed to proceed to the next topic only after he or she has mastered the current one. While Mastery Learning is an important instructional model, a large body of content on any given topic is necessary for implementing it. Hence, it was not possible to incorporate mastery learning in the ASSISTment system prior to the introduction of variabilized templates.

However, once the template infrastructure was implemented in the system, a Mastery Learning module was built by other collaborators. With a large number of isomorphic problems, students could practice problems on any topic till they mastered them. This provision of Templates + Mastery Learning has subsequently been used to run various studies in the ASSISTment system. Several of these studies have been part of other IQPs at WPI.
4 Evaluation

In this section we will describe the experiments that have been carried out as well as planned in order to evaluate the template feature of the ASSISTment system. We conducted one experiment and one user-study to examine the effect of templates on the ease of use of the system, two experiments to study the pedagogical significance of templates, and one case-study to investigate the use of personalized tutors. An experimented to study the effect of mixing content vs. not mixing content is planned.

4.1 Efficiency of Templates

Our first experiment was designed to measure how the introduction of templates affected the content creation time for isomorphic problems and how it affected the ease of use of the system.

4.1.1 Experimental Design

Our study on the efficiency of templates focused on the Pythagorean Theorem problem discussed above since it is representative of a problem that content creators would wish to morph. In addition, it has four scaffold questions and 13 hints, 13 answers, and eight buggy messages, which is also representative of a typical assistment. The template for this assistment is presented below for convenience.
We asked experienced content creators to construct morphs of the Pythagorean assistment. The possible values for numeric quantities and contextual elements were provided in advance. The participants were free to use traditional morphing techniques (e.g., copy and paste). We recorded the creation time per assistment for each individual. After the participants created their morphed assistments, we introduced the concept of variabilization with less than one hour of instruction and demonstration of the new tools. As part of the training, the participants were encouraged to experiment with variables as well as create variabilized versions of other existing assistments. At the conclusion of the training, the subjects were asked to create a single variabilized version of the Pythagorean Theorem assistment that would produce the five morphs they created previously. Again, we recorded the creation time of the variabilized assistment for each individual. After a variabilized assistment was built to construct five morphs, the number of possible values for the variables was increased to create more morphs. Although the system supports more interesting variabilization, the techniques used in this study were simple and, more importantly, representative of how teachers currently morph assistments.
4.1.2 Results

Data was obtained for the development of 23 morphs of assistments created by eight individuals, and 30 instances of variabilized assistments made by three participants. Table 1 lists creation times required for each of the individuals to create a morphed assistment. Our data indicates that the time to make morphs decreases with the construction of each subsequent morph, leveling off at about 12 minutes. Table 2 shows the time required to variabilize an assistment that creates the five instances that were previously created by morphing. Although the time required to create variabilized assistments decreases with each iteration, the reduction is not appreciable. After constructing a variabilized assistment that creates five morphs, it takes an average of five additional minutes to extend the assistment to generate five more.

Table 1: Average Morph Creation Time without Templates

<table>
<thead>
<tr>
<th>Participant</th>
<th>Number of Morphs</th>
<th>Average time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>15.7</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>14.8</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>23.8</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>14.25</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>20.18</td>
</tr>
</tbody>
</table>

Table 2: Average time required to create a variabilized assistment

<table>
<thead>
<tr>
<th>Participant</th>
<th>Number of Variabilizations</th>
<th>Resulting number of Assistments</th>
<th>Average time (min)</th>
<th>Average time per Assitment (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>10</td>
<td>36</td>
<td>7.2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>10</td>
<td>38</td>
<td>7.6</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>10</td>
<td>41.5</td>
<td>8.3</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>30</td>
<td>38.5</td>
<td>7.7</td>
</tr>
</tbody>
</table>
4.1.3 Analysis

Our data shows that variabilizing content reduces the average time required to generate content from 20.18 minutes to 7.76 minutes. This indicates a speedup by a factor of 2.6. It is important to note that this speedup is appreciable only if more than two assistments are to be created since creating one variabilized assistment requires 38.8 minutes on average as opposed to 20.18 minutes for a morphed assistment. However, the variabilized assistment can be used to produce several instances of the assistment while the morph is essentially a single assistment. Our studies on extending a variabilized assistment indicate that creating additional instances of an assistment can be done at the rate of one instance per minute.
4.2 **Student Acceptance of Templates**

In this experiment, we measured how accepting students were of isomorphic content. Our objective was to examine whether students observed any differences between using morphed assistments versus the same assistment, and whether they preferred isomorphic content.

### 4.2.1 Experimental Design

The five instances of the variabilized Pythagorean assistments created in the above study were assigned to an eighth grade class consisting of 25 students. All of the students were familiar with the ASSISTment system and had completed a number of assignments using it in the past. The students were asked to complete an assignment consisting of the five morphed assistments but were not told the purpose of the study. At the conclusion of the assignment, each student was given a survey asking if they noticed anything about the assignment that was similar or dissimilar to other assignments they had completed in their past use of the ASSISTment system. If the students did notice differences, they were asked if they liked or disliked the changes. Once the initial surveys were collected, all students were told about the experiment and given a second survey. They were asked about their thoughts in general as well as whether they preferred solving similar problems as opposed to those with more variety.

Finally, the classroom instructor was asked her opinion of students completing assignments with several morphed assistments. This instructor had previously expressed interest in using morphed assistments for practice.

### 4.2.2 Results
When asked about the experimental assignment, 13 out 25 students did not notice the difference between this assignment and a typical assignment. After being told the details of the experiment, 12 students reported that they liked the similarity in the assistments; seven said that they preferred more variety in the assistments, and six indicated no clear preference. While some students commented that they appreciated the opportunity for repeated exposure to similar content, others indicated that they would have preferred having a variety of questions. Some students specifically asked for content that taught different approaches to the same problem.

4.2.3 Analysis

Tests on student acceptance indicate that approximately half the students prefer seeing similar problems as this gives them an opportunity to practice. We believe that student acceptance is closely tied with the number of similar assistments they are assigned and the degree of variability in the assistments. Considering the number of students who did not notice the content similarity, our preliminary results suggest that five to ten morphs is probably a reasonable limit for a single assignment. We expect that the relationship between number of morphs and student acceptance is roughly represented by the curve in Figure 13.
Figure 13: Predicted Relationship between student acceptability and number of morphs
4.3 User-study of Templates

The objective of this user-study was to examine the use of templates by novice content creators in the ASSISTment system. For the purpose of our study, templates were also used to interface with “R” a third-party software for statistical computing.

4.3.1 Experimental Design

The template feature was used in an undergraduate statistics course at Worcester Polytechnic Institute. Three members of the course staff conducted a study evaluating the effectiveness of two pedagogical strategies. Templates were extensively used to create content for the study. Some of these templates interacted with R, an environment for statistical computing, to create problems involving complex statistical concepts. Prior to their study, the staff had no experience creating content in our system. In addition, only one of the three had any experience in computer science. The only training provided to these users was in the form of a sample template assistment containing variables.

4.3.2 Results

The course staff built 23 pairs of templates for the study. These templates were then used to generate roughly 500 instantiations, 460 of which were ultimately used. Subsets of these were then assigned to each of the 49 students in the course over a period of two weeks. On average, they spent 1.8 hours planning and creating a template. Thus, an average of 0.08 hours were required to create each instance of the assistment.

4.3.3 Analysis
Although these novice users required nearly twice as much time to create templates, the ability to generate ten times the content significantly reduced per problem development time. Previous studies have shown that experienced content creators spend an average of one hour creating and planning an assistment [17]. Compared to this data, we find that creating one assistment with templates requires 1/10\textsuperscript{th} of the time required without templates (refer Figure 14).

The fact that these novice users created a significant body of content in a short time is indicative of the ease of use of variables and the system in general. Moreover, because each of the templates built for the study was used to generate ten instantiations, and templates were created in pairs, there was a reasonable expectation that a problem with specific values would only be seen by approximately five students rather than the entire class. Unlike traditional pencil-and-paper homework or even computer-based homework where all students complete the same problems, the variety offered by templates reduces some of the opportunity for unwanted collaboration between students.

![Figure 14: Time required to build an assistment with and without templates](image)
The course staff used R to incorporate Student's t-tests and random numbers from specific probability distributions into their problems. The mathematical and logical functions supported natively by our variabilized templates were incapable of providing the power and flexibility needed by the course staff. Without the ability to interface with R, ASSISTment could not have been used in the course. This demonstrates the importance of augmenting our ability with external tools.
4.4 Learning with Isomorphic Content

A study on the Pythagorean theorem problem described previously showed that the ability to generate isomorphic content reduced the content creation time by a factor of 2 [18]. In addition, the same study showed that students were generally accepting of isomorphic content. However, the study did not measure student learning with isomorphic content. Therefore, we conducted an experiment to measure whether students learned more when given an opportunity to repeat an assignment with isomorphic content as opposed to repeating the original assignment itself. Since teachers in the ASSISTment system routinely reassigned same assignments to students, we wanted to evaluate if isomorphic problems improved learning.

4.4.1 Experimental Design

The skill addressed by our experiment was the Pythagorean Theorem. Thirty seven eight grade students were each given a pre-test consisting of six items. All students then completed an assignment with five items. Seventeen of these students were given the same assignment twice. The remaining twenty students were assigned isomorphic content. All students were then given a post-test consisting of the same six items from the pre-test. The pre- and post-tests contained three questions with isomorphic content and three that addressed the same skill in other contexts.

4.4.2 Results

For the control group, the average pre-test score was 4.59 out of 6 (standard deviation 1.54). The average post-test score was 5.18 out of 6 (standard deviation 1.13). The control group gained an average of 0.53 points (standard deviation 1.18). For the experimental group, the
average pre-test score was 4.25 out of 6 (standard deviation 1.33). The average post-test score was 4.95 out of 6 (standard deviation 1.36). The experimental group gained an average of 0.70 points (standard deviation 1.56). These findings are summarized in Table 3.

Table 3: Learning Gains with Isomorphic Content

<table>
<thead>
<tr>
<th>Condition</th>
<th>Average Pre-test (max = 6)</th>
<th>Average Post-test (max = 6)</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control group</td>
<td>4.59</td>
<td>5.18</td>
<td>0.53</td>
</tr>
<tr>
<td>Experimental group</td>
<td>4.25</td>
<td>4.95</td>
<td>0.70</td>
</tr>
</tbody>
</table>

4.4.3 Analysis

We expected learning to be greater with students who practiced with isomorphic content. Unfortunately, although the experimental group generally demonstrated larger gains, there was no statistically reliable difference in the means (p = 0.71). A possible explanation is the reduced opportunity for learning gains due to prior knowledge of the skill tested (as indicated by the high pre-test scores). In addition, the sample size as well as the number of test and practice items was rather small.
4.5  Case-Study with Personalized Content

Cordova and Lepper have shown that personalizing tutoring content motivates grade schools students to learn better [7]. In this short case-study, we examined whether personalized content improved learning in middle school students too.

4.5.1  Experimental Design

To create personalized content, we first presented twenty middle school students a survey. In this survey, the students filled out information about themselves like birth date, names of three friends, favorite restaurant, favorite afterschool activity, favorite music band etc. Once this information was collected, we built personalized content using the information provided by the students. Figure 15 and 16 show examples of personalized assistments built for this study. By providing personal references in the problems, we wanted to examine if the students could relate to the problems, and hence be motivated to work on the problems.

"%v{user_first_name}'s favorite restaurant is %v{user_favorite_restaurant}. On the last day of school, %v{user_first_name} went to %v{user_favorite_restaurant} with five friends. If the total bill for their meal came to $136 and they decided to split the bill evenly, how much did each person have to pay?"

Figure 15: Example of a Personalized Template
Figure 16: Example of a personalized template

Personalized templates used in this study utilize system data about individual students to populate variables in the personalized template. In the template shown in Figure 15, for example, the system would get information about the student’s first name and favorite restaurant to insert into the assistance. Thus, although all students got the same problem, the personal details were different in each problem. Once the templates were created, they were assigned to the students in the study. Half the students got generic, non-personalized problems whereas the remaining students got personalized problems.

4.5.2 Analysis and Results

In our experiment we did not find any differences in learning between the students with personalized content and those with non-personalized content. A possible reason for this result is that the students were already familiar with the material used in the experiment. However, the students who got personalized content enjoyed the experience a lot and were very keen on getting more personalized tutoring. This indicates that while personalization may not lead to better learning directly, it can help increase student motivation, and hence improve learning. We hope to undertake additional experiments to further examine the effect of personalization.
4.6 Mixing Experiment

We now discuss an experiment that has been set up, but could not be run due to the time constraints of the IQP. This experiment will compare whether teaching students closely-related math concepts separately i.e., “not-mixing”, is more beneficial than teaching these concepts together i.e., “mixing.” In [15], Rohrer argues that mixed review (presenting problems addressing different math concepts individually) actually leads to better learning than teaching the same concepts separately. While this seems correct intuitively, in this experiment we aim to find out if this is indeed true with respect to data analysis skills of mean, median, mode and range.

4.6.1 Experimental Design

In order to run this experiment, we created content addressing each of the skills of mean, median, mode and range. As before, for each skill we created a set of templates with enough variety to address the skill completely. In particular, each skill had the following types of problems: computing the specific measure, finding a data value given the particular measure, computing the specific measure in a particular context, and interpreting data from a table to compute the specific measure. For mean, we generated 35 templates; for median, we created 20 templates; for mode, we created 11 templates; and for range, we created 12 templates. The number of templates created was based on the amount of variety we expected to have for each skill. Once the templates had been created, we generated ten instances of each template to use in the study.

The actual experiment is set up as follows: we created a single problem set that enveloped the entire experiment. This main problem set consisted of three main parts. The first was a pre-test
designed to measure the students’ knowledge of mean, median, mode and range. The second part was the actual experiment and the third was a post-test designed to measure how much the students learned through the tutoring. The pre-test had 4 problems while the post-test had 8 problems. 4 problems of the 8 post-test problems were the same as the pre-test problems while the remaining 4 were different from the pre-test problems. The actual experiment had two conditions, a control condition and an experimental condition, to which students were to be randomly assigned. The experimental condition contained a single “Mastery Learning” problem set with problems addressing all the four data analysis skills. A student assigned to the experimental condition had to do the pre-test, get eight questions correct in a row to finish the experimental data analysis section and then proceed to the post-test. In contrast, the control condition contained four separate “Mastery Learning” sections for mean, median, mode and range. In this condition, a student had to take the pre-test, then complete each of the four Mastery Learning sections by getting 4 problems correct in a row, and then proceed to the post-test. Thus, all students get the same pre-test and post-test, and only the tutoring in between will vary. This will enable us to isolate the effect of mixing content versus not mixing content. The difference between the number of items required to “master” a skill stems from the fact that problems in the experimental section come from all four skills and hence we must ensure that the students see at least one problem addressing each skill. Figure 17 shows the problem set to be used for this experiment and its structure. Notice that the main problem set is of type “Linear” with the pre- and post-tests of type randomized. The actual experiment resides in a set of type “ChooseCondition” so that students are randomly assigned to the control and experimental condition. Within the experimental condition, there is a single mastery learning problem set. In
contrast, the control condition contains four separate mastery learning sections for each skill.

Instances of the templates generated for this study are provided in the appendices A to D.

![Figure 17: Problem Set for Mixing Experiment](image)

![Figure 18: Experimental problem set showing both conditions](image)

### 4.6.2 Results and Analysis

Following the experiment, the data will be analyzed for learning and also for effects of prior knowledge on the benefit of the learning strategy. Based on the pre-test and post-test scores for both these groups, we can calculate how much learning has taken place in each of the groups and hence compare the two approaches to learning.
5 Conclusions

One goal of the Assistment project was to provide a system that allowed ordinary users to create and edit tutoring content. Through extreme simplification of traditional intelligent tutoring concepts, the ASSISTment system has made content creation available to users outside of the cognitive psychology and computer science communities. Our previous research has demonstrated that our pseudo-tutor-based system and supporting build tools allow content to be created in a relatively short amount of time by ordinary users. By extending our authoring tools to support isomorphic content generation, we have increased flexibility and reduced authoring time while maintaining ease of use of our system. These tools will allow teachers to more rapidly develop tutoring content while giving students an opportunity to practice similar problems in weaker skill areas. Moreover, our studies indicate that templates significantly reduce content creation time and that students are accepting of isomorphic content. As demonstrated by various Mastery Learning studies conducted by collaborators, isomorphic content can now play a bigger role in tutoring students using our system. In this IQP, we have also presented the template feature of the ASSISTment system as a simple prototype for building personalized and culturally aware tutoring systems.
6 Future Work

To build upon our work on isomorphic content creation, we would like to investigate the extent to which isomorphic content can be utilized in our system. In particular, we are interested in determining the effectiveness of learning with isomorphic content versus unique content. We expect that unique content will provide greater learning, but knowing the relative worth of isomorphic content compared to unique content will allow us to make pedagogical decisions in curriculum creation. We are also interested in extending the capabilities of our implementation of variabilized templates. Although we already support the use of a number of arbitrary mathematical and logical functions within variables, we wish to explore the benefits of interfacing with external applications. The use of R in the statistics course demonstrated the usefulness and need for powerful variabilized templates in certain domains. Supporting a range of third-party applications would increase the applicability of ASSISTment to domains beyond basic middle- and high-school mathematics.
7 References


Appendix A:

Variabilized Templates for Mean problems
Assistment

You are previewing content.

Calculate the **mean** of the following numbers:

115,  70,  73,  61,  26,  110

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

*Type your answer below (mathematical expression):*

- 

Submit Answer
Let's move on and figure out this problem.

How do we find the **mean**?

Comment on this question
The **mean** of a set of numbers is the **average** of the numbers.

Comment on this hint
This means that the **mean** is the sum of the numbers divided by the number of numbers.

Comment on this hint
*Select one:*

- The number seen most often in this set of numbers.
- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The middle number in this set of numbers.

Submit Answer
Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

\[ 115, \ 70, \ 73, \ 61, \ 26, \ 110 \]

Comment on this question
In order to find the sum, we have to add all the numbers together.
Comment on this hint
\[ 115 + 70 + 73 + 61 + 26 + 110 = 455 \]
The sum is equal to 455, so type in 455
Comment on this hint
Type your answer below (mathematical expression):
• 455

Submit Answer
Correct!

Now that we know the sum of the numbers is 455, try the original problem again.

Calculate the mean of the following numbers:

\[ 115, \ 70, \ 73, \ 61, \ 26, \ 110 \]

(round to the nearest hundredths place)

Comment on this question
The mean of a set of numbers is the average of the numbers.
Comment on this hint
To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have six numbers.
Comment on this hint
\[ 455 \div 6 = 75.83 \] (answer is rounded to the nearest hundredths place)

The mean is 75.83, so type in 75.83.
Comment on this hint
Type your answer below (mathematical expression):
• 75.83

Submit Answer
Correct!
Assistment

You are previewing content.

Calculate the mean of the following numbers:

6, 10, 2, 8, 3, 8

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer
Let's move on and figure out this problem.

How do we find the mean?

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
This means that the mean is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

- The sum of the numbers divided by the number of numbers.
- The middle number in this set of numbers.
- The number seen most often in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.

Submit Answer
Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

6, 10, 2, 8, 3, 8

Comment on this question
In order to find the sum, we have to add all the numbers together.

Comment on this hint
6 + 10 + 2 + 8 + 3 + 8 = 37

The sum is equal to 37, so type in 37

Comment on this hint
Type your answer below (mathematical expression):

• 37

Submit Answer
Correct!

Now that we know the sum of the numbers is 37, try the original problem again.

Calculate the mean of the following numbers:

6, 10, 2, 8, 3, 8

(round to the nearest hundredths place)

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
To find the mean, we have to divide the sum by the amount of numbers that we have.
For this problem, we have six numbers.

Comment on this hint
37 ÷ 6 = 6.17 (answer is rounded to the nearest hundredths place)

The mean is 6.17, so type in 6.17.

Comment on this hint
Type your answer below (mathematical expression):

• 6.17

Submit Answer
Correct!
What number should be added to the list below to get a mean of 20.33?

12, 20, 15, 28, 23

(Mean is rounded to the nearest hundredths place)

Type your answer below (mathematical expression):

Submit Answer

Let's move on and figure out this problem.

The mean is the average of a set of number. We can get the mean by dividing the sum of the numbers by the number of numbers. So, let's start by finding the total.

What should the total be to get a mean of 20.33?

(Round to the ones place)

Remember, the list should have six numbers when we add the new number.

Since we know that

Total ÷ Number of Numbers = Mean,

then

Total = Mean x Number of Numbers
(Round to the ones place)

Comment on this hint
Total = 20.33 x 6 (Round to the ones place)
Total = 122.

So, the sum of this set of numbers is 122.

Type in 122.

Comment on this hint
Type your answer below (mathematical expression):

• 122

Submit Answer
Correct!

Now that we know the total is 122, try the original problem again.

What number should be added to the list below to get a mean of 20.33?

12, 20, 15, 28, 23

(Mean is rounded to the nearest hundredths place)

Comment on this question
Remember the sum of the set of numbers is 122.

Comment on this hint
In order to find the value of the new number, all we have to do is subtract the five current numbers from the total.

Comment on this hint
122 - 12 - 20 - 15 - 28 - 23 = 24

So, in order to get a mean of 20.33 we have to add 24 to the set of numbers.

Type in 24.

Comment on this hint
Type your answer below (mathematical expression):

• 24

Submit Answer
Correct!
The coach for the All-Star Basketball Game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jose</td>
<td>9, 3, 15, 16, 26, 18, 8, 1, 28, 10</td>
</tr>
<tr>
<td>Beth</td>
<td>18, 21, 3, 8, 13, 9, 17, 28, 17, 16</td>
</tr>
</tbody>
</table>

What is the **mean** (average) number of points scored by Jose?

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

- 

Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the **mean**?

**Comment on this question**

The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

This means that the **mean** is the sum of the numbers divided by the number of numbers.
Comment on this hint
Select one:

- The number seen most often in a set of numbers.
- The middle number in a set of ordered numbers.
- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value in a set of numbers.

Submit Answer
Correct!

Since part of finding the **mean** is finding the sum of all of the numbers
what is the sum of scores that Jose scored?

Jose 9,3,15,16,26,18,8,1,28,10

Comment on this question

First, write out all of the numbers in this plot on a piece of paper.

Comment on this hint

Your list of numbers should look something like this:

9,3,15,16,26,18,8,1,28,10

Comment on this hint

Now we have to add all the numbers in this list up.

\[
\begin{align*}
9 \\
3 \\
15 \\
16 \\
26 \\
18 \\
8 \\
1 \\
28 \\
+ 10 \\
= 134
\end{align*}
\]

Type your answer below (mathematical expression):

- 134

Submit Answer
Correct!
Now that we know the definition of the **mean**, let's try the original problem again.

The coach for the All-Star Basketball Game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jose</td>
<td>9,3,15,16,26,18,8,1,28,10</td>
</tr>
<tr>
<td>Beth</td>
<td>18,21,3,8,13,9,17,28,17,16</td>
</tr>
</tbody>
</table>

What is the **mean** (average) number of points scored by Jose?

*Comment on this question*

Remember, the sum of all the numbers is 134.

Now find the number of numbers and divide.

*Comment on this hint*

There are 10 different scores on the chart.

The mean is \(134 \div 10 = 13.4\)

Type 13.4.

*Comment on this hint*

The mean equals to \(134/10 = 13.4\). The correct answer is 13.4.

*Comment on this hint*

*Type your answer below (mathematical expression):*

- 13.4

Submit Answer

Correct!
Steve has scored the following points in his last five basketball games: 5, 7, 13, 7, 9. How many points must he score in the next game to average 14 points per game?

There are three steps to solve this problem. Here is a similar problem with steps to solve.

Jim has scored the following points in his last five basketball games: 3, 8, 9, 11, 5. How many points must he score in the next game to average 9 points per game?

**Step 1**
Add up the five scores.

You would normally add up all the scores but since one is missing this is all you can do for now.

\[ 3 + 8 + 9 + 11 + 5 = 36 \]

**Step 2**
Jim is about to play his 6th game so you multiply the average he wants, which is 9, by 6

This is so you know what the sum would be to get an average of 9 after 6 games.

\[ 9 \times 6 = 54 \]
Step 3
Subtract the two answers

If the sum should be 54 from all 6 games and so far you only have 36 this will tell you what the last score should be.

54 - 36 = 18 ← the answer to the example. Now you try to solve the given problem.

Comment on this hint
The first step to solve is to add up all 5 scores.

5 + 7 + 13 + 7 + 9 = 41

Comment on this hint
The second step is to multiply the mean you want 14 by 6.

14 * 6 = 84

Comment on this hint
The last step is to subtract.

84 - 41 = 43

Type in 43

Type your answer below (mathematical expression):

• 43

Submit Answer
Correct!
You are previewing content.

Calculate the mean of the following numbers:

6, 10, 20, 19, 13, 20

(round to the nearest tenths place)

Comment on this question

This is how to solve a problem similar to your problem.

Find the mean of the following group of numbers: 3, 8, 4, 9

3 + 8 + 4 + 9 = 24
24 ÷ 4 = 6

Comment on this hint

The first step is to add up the values.

6 + 10 + 20 + 19 + 13 + 20 = 88

Comment on this hint

The last step is to divide 88 by the number of values you have, which is 6.

88 ÷ 6 = 14.6666666666667
14.6666666666667 = 14.7 (rounded)

Type in 14.7
Comment on this hint

Type your answer below (mathematical expression):

- 14.7

Submit Answer

Correct!
Calculate the **mean** of the following numbers:

1.63, 1.16, 0.62, 1.58, 2.38, 1.91

(round to the nearest hundredths place)

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

• 

Submit Answer

Let's move on and figure out this problem.

How do we find the **mean**?

**Comment on this question**

The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

This means that the **mean** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

*Select one:*

- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The middle number in this set of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!

*Before we can find the mean, we must first find the sum of the numbers.*
Calculate the sum of the following numbers:

1.63, 1.16, 0.62, 1.58, 2.38, 1.91

In order to find the sum, we have to add all the numbers together.

1.63 + 1.16 + 0.62 + 1.58 + 2.38 + 1.91 = 9.28

The sum is equal to 9.28, so type in 9.28

Now that we know the sum of the numbers is 9.28, try the original problem again.

Calculate the mean of the following numbers:

1.63, 1.16, 0.62, 1.58, 2.38, 1.91

(round to the nearest hundredths place)

The mean of a set of numbers is the average of the numbers.

To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have six numbers.

9.28 ÷ 6 = 1.54666666666667

Rounding the answer to the hundredths place, we get 1.55, so type in 1.55.
Correct!
Assistment

You are previewing content.

Calculate the mean of the following numbers:

1.22, 1, 5, 0.75, 7, 2.86, 5

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How do we find the mean?

Comment on this question

The mean of a set of numbers is the average of the numbers.

Comment on this hint

This means that the mean is the sum of the numbers divided by the number of numbers.

Comment on this hint

Select one:

○ The sum of numbers divided by the number of numbers.
○ The difference between the largest value and the smallest value of the set of numbers.
○ The middle number in this set of numbers.
○ The number seen most often in this set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

\[1.22, \ 1, \ 5, \ 0.75, \ 7, \ 2.86, \ 5\]

Comment on this question
In order to find the sum, we have to add all the numbers together.

Comment on this hint
1.22 + 1 + 5 + 0.75 + 7 + 2.86 + 5 = 22.83

The sum is equal to 22.83, so type in 22.83

Comment on this hint
Type your answer below (mathematical expression):

• 22.83

Submit Answer
Correct!

Now that we know the sum of the numbers is 22.83, try the original problem again.

Calculate the mean of the following numbers:

\[1.22, \ 1, \ 5, \ 0.75, \ 7, \ 2.86, \ 5\]

(round to the nearest hundredths place)

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have seven numbers.

Comment on this hint
\[22.83 \div 7 = 3.26142857142857\]

Rounding the answer to the hundredths place, we get 3.26, so type in 3.26.
Type your answer below (mathematical expression):

- 3.26

Submit Answer

Correct!
You are previewing content.

Calculate the **mean** of the following numbers:

165, 146, 128, 78, 93

(round to the nearest hundredths place)

**Comment on this question**
Break this problem into steps

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

How do we find the **mean**?

**Comment on this question**
The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

This means that the **mean** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

*Select one:*

- The middle number in this set of numbers.
- The number seen most often in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The sum of the numbers divided by the number of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

165, 146, 128, 78, 93

In order to find the sum, we have to add all the numbers together.

165 + 146 + 128 + 78 + 93 = 610

The sum is equal to 610, so type in 610

To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have five numbers.

610 ÷ 5 = 122
Rounding the answer to the hundredths place, we get 122, so type in 122.

Comment on this hint

Type your answer below (mathematical expression):

• 122

Submit Answer

Correct!
Hannah obtained the following scores in 5 math tests. Calculate the mean of Hannah's math scores:

171, 145, 74, 47, 26

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

Submit Answer

Let's move on and figure out this problem.

How do we find the mean?

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
This means that the mean is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

- The sum of the numbers divided by the number of numbers.
- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the scores.

Calculate the **sum** of the following numbers:

171, 145, 74, 47, 26

Comment on this question

In order to find the **sum**, we have to **add** all the scores together.

Comment on this hint

171 + 145 + 74 + 47 + 26 = **463**

The **sum** is equal to **463**, so type in 463

Comment on this hint

Type your answer below (mathematical expression):

- 463

Submit Answer

Correct!

Now that we know the sum of the scores is **463**, try the original problem again.

Hannah obtained the following scores in 5 math tests. Calculate the **mean** of Hannah's math scores:

171, 145, 74, 47, 26

(round to the nearest hundredths place)

Comment on this question

The **mean** of the set of scores is the **average** of the scores.

Comment on this hint

To find the **mean**, we have to divide the **sum** of the scores by the number of scores that we have. For this problem, we have **five** scores.

Comment on this hint

**463 ÷ 5 = 92.6**
Rounding the answer to the hundredths place, we get 92.6, so type in 92.6.

Comment on this hint

Type your answer below (mathematical expression):

- 92.6

Submit Answer

Correct!
During a medical study, doctors recorded the weights in pounds of all their volunteers. Some of the weights are given here. What is the average weight of the volunteers listed below?

137, 154, 131, 119, 117, 133, 127, 150, 162, 103

(round to the nearest hundredths place)

**Comment on this question**

**Break this problem into steps**

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

How do we find the **average**?

**Comment on this question**

**Average** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

**Select one:**

- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The sum of the numbers divided by the number of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!

Before we can find the average, we must first find the sum of the numbers.
Calculate the sum of the following numbers:

137, 154, 131, 119, 117, 133, 127, 150, 162, 103

Comment on this question
In order to find the sum, we have to add all the numbers together.

Comment on this hint
137 + 154 + 131 + 119 + 117 + 133 + 127 + 150 + 162 + 103 = 1333

The sum is equal to 1333, so type in 1333

Comment on this hint
Type your answer below (mathematical expression):

• 1333

Submit Answer

Correct!

Now that we know the sum of the numbers is 1333, try the original problem again.

During a medical study, doctors recorded the weights in pounds of all their volunteers. Some of the weights are given here. What is the average weight of the volunteers listed below?

137, 154, 131, 119, 117, 133, 127, 150, 162, 103

(round to the nearest hundredths place)

Comment on this question
To find the average, we have to divide the sum by the amount of numbers that we have. For this problem, we have ten numbers.

Comment on this hint
1333 ÷ 10 = 133.3 (answer is rounded to the nearest hundredths place)

The mean is 133.3, so type in 133.3.

Comment on this hint
Type your answer below (mathematical expression):

• 133.3

Submit Answer
Correct!
You are previewing content.

Mary works at the local shoe store and has to process all the sales at the end of the day. The list below gives the dollar amounts of all the sales made on a particular day.

What is the **average** amount of these sales?

\[ 13.67, \ 16, \ 13, \ 14.57, \ 3, \ 4, \ 9 \]

(round to the nearest hundredths place)

*Comment on this question*

**Break this problem into steps**

*Type your answer below (mathematical expression):*

- \[
\]

Submit Answer

Let's move on and figure out this problem.

How do we find the **average (mean)**?

*Comment on this question*

**Average** is the sum of the numbers divided by the number of numbers.

*Comment on this hint*

**Select one:**

- The difference between the largest value and the smallest value of the set of numbers.
- The sum of the numbers divided by the number of numbers.
- The middle number in this set of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!
Before we can find the average, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

\[ 13.67, \ 16, \ 13, \ 14.57, \ 3, \ 4, \ 9 \]

**Comment on this question**
In order to find the sum, we have to add all the numbers together.

**Comment on this hint**

\[ 13.67 + 16 + 13 + 14.57 + 3 + 4 + 9 = 73.24 \]

The sum is equal to 73.24, so type in 73.24

**Comment on this hint**

Type your answer below (mathematical expression):

- 73.24

Submit Answer

Correct!

Now that we know the sum of the numbers is 73.24, try the original problem again.

Mary works at the local shoe store and has to process all the sales at the end of the day. The list below gives the dollar amounts of all the sales made on a particular day.

What is the average amount of these sales?

\[ 13.67, \ 16, \ 13, \ 14.57, \ 3, \ 4, \ 9 \]

(round to the nearest hundredths place)

**Comment on this question**

To find the average, we have to divide the sum by the amount of numbers that we have. For this problem, we have seven numbers.

**Comment on this hint**

\[ 73.24 \div 7 = 10.4628571428571 \]
Rounding the answer to the hundredths place, we get **10.46**, so type in 10.46.

*Comment on this hint*

*Type your answer below (mathematical expression):*

- 10.46

Submit Answer

Correct!
What number should be added to the list below to get a mean of 21.415?

14, 50, 12, 29, 6.29

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

Submit Answer
Let's move on and figure out this problem.

How is the mean of a set of numbers defined?

Comment on this question
Mean is the same as average. Average is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

- The number that is seen most often
- The middle number when the numbers are arranged in increasing order

Difference between the largest and smallest number.

Submit Answer
Correct!

We can get the mean by dividing the sum (that we do not know right now) of the numbers by the number of numbers (in this problem it is 6 numbers).

So, let's start by finding the expected sum of the numbers.

What should the sum be to get a mean of 21.415 with 6 numbers?
Remember, the list should have six numbers when we add the new number.

Since we know that

\[
\text{Total} \div \text{Number of Numbers} = \text{Mean},
\]

then

\[
\text{Total} = \text{Mean} \times \text{Number of Numbers}
\]

\[
\text{Total} = 21.415 \times 6
\]

\[
\text{Total} = 128.49.
\]

So, the sum of this set of numbers is 128.49.

Type in 128.49.

Now that we know the total is 128.49, try the original problem again.

What number should be added to the list below to get a mean of 21.415?

14, 50, 12, 29, 6.29
Remember the sum of the set of numbers is **128.49**.

In order to find the value of the new number, all we have to do is subtract the five current numbers from the **total**.

128.49 - 14 - 50 - 12 - 29 - 6.29 = 17.2

So, in order to get a **mean** of **21.415** we have to add 17.2 to the set of numbers.

Type in 17.2.

Type your answer below (mathematical expression):

- **17.2**

Submit Answer

Correct!
The coach for the School Computer Programming team needs to pick one of two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jimmy</td>
<td>9, 6, 8, 12, 24, 18, 5, 22, 14</td>
</tr>
<tr>
<td>Nathalie</td>
<td>22, 26, 7, 16, 10, 2, 19, 27, 18, 9</td>
</tr>
</tbody>
</table>

What is the mean (average) number of points scored by Jimmy?

Let's move on and figure out this problem. Let's start with the definition. Which of the following is the right definition for the mean?

The mean of a set of numbers is the average of the numbers.

This means that the mean is the sum of the numbers divided by the number of numbers.
Select one:

- The sum of the numbers divided by the number of numbers.
- The middle number in a set of ordered numbers.
- The difference between the largest value and the smallest value in a set of numbers.
- The number seen most often in a set of numbers.

Submit Answer

Correct!

Since part of finding the mean is finding the sum of all of the numbers
what is the sum of scores that Jimmy scored?

Jimmy 9,6,8,12,24,18,9,5,22,14

Comment on this question

First, write out all of the numbers in this plot on a piece of paper.

Comment on this hint

Your list of numbers should look something like this:

9,6,8,12,24,18,9,5,22,14

Comment on this hint

Now we have to add all the numbers in this list up.

9
6
8
12
24
18
9
5
22
+ 14

127

Comment on this hint

Type your answer below (mathematical expression):

- 127

Submit Answer

Correct!
Now that we know the definition of the **mean**, let's try the original problem again.

The coach for the All-Star Basketball Game needs to pick one of the two players for the team.

The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jimmy</td>
<td>9,6,8,12,24,18,9,5,22,14</td>
</tr>
<tr>
<td>Nathalie</td>
<td>22,26,7,16,10,2,19,27,18,9</td>
</tr>
</tbody>
</table>

What is the **mean** (average) number of points scored by Jimmy?

**Comment on this question**

Remember, the sum of all the numbers is 127.

Now find the number of numbers and divide.

**Comment on this hint**

There are 10 different scores on the chart.

The mean is $127 \div 10 = 12.7$

Type in 12.7.

**Comment on this hint**

*Type your answer below (mathematical expression):*

- 12.7

Submit Answer

Correct!
The coach for the All-USA Physics team needs to pick one of two students for the team. The table below shows the number of points each of the students obtained in their last 10 tests.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>13,5,12,21,20,23,14,5,29,10</td>
</tr>
<tr>
<td>Cristina</td>
<td>17,26,4,12,8,4,21,28,17,16</td>
</tr>
</tbody>
</table>

What is the **mean** (average) number of points obtained by Cristina?

Type your answer below (*mathematical expression)*:

- Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the **mean**?

The **mean** of a set of numbers is the **average** of the numbers.

This means that the **mean** is the sum of the numbers divided by the number of numbers.
The sum of the numbers divided by the number of numbers.
The number seen most often in a set of numbers.
The difference between the largest value and the smallest value in a set of numbers.
The middle number in a set of ordered numbers.

Submit Answer
Correct!

Since part of finding the mean is finding the sum of all of the numbers

what is the sum of scores that Cristina scored?

Cristina: 17,26,4,12,8,4,21,28,17,16

First, write out all of the numbers in this plot on a piece of paper.

Your list of numbers should look something like this:

17,26,4,12,8,4,21,28,17,16

Now we have to add all the numbers in this list up.

\[
\begin{align*}
17 \\
26 \\
4 \\
12 \\
8 \\
4 \\
21 \\
28 \\
17 \\
+16
\end{align*}
\]

153

Now that we know the definition of the mean, lets try the original problem again.
The coach for the All-USA Physics team needs to pick one of two students for the team. The table below shows the number of points each of the students obtained in their last 10 tests.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>13,5,12,21,20,23,14,5,29,10</td>
</tr>
<tr>
<td>Cristina</td>
<td>17,26,4,12,8,4,21,28,17,16</td>
</tr>
</tbody>
</table>

What is the mean (average) number of points obtained by Cristina?

Remember, the sum of all the numbers is 153.

Now find the number of numbers and divide.

There are 10 different scores on the chart.

The mean is $153 \div 10 = 15.3$

Type in 15.3.

The mean equals to $153/10 = 15.3$. The correct answer is 15.3.

Type your answer below (mathematical expression):

- $15.3$

Submit Answer

Correct!
Alex runs a shoe store, and listed below are the store sales for the year 1997. What were the average monthly sales in 1997?

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1001</td>
</tr>
<tr>
<td>February</td>
<td>1050</td>
</tr>
<tr>
<td>March</td>
<td>2505</td>
</tr>
<tr>
<td>April</td>
<td>1125</td>
</tr>
<tr>
<td>May</td>
<td>1501</td>
</tr>
<tr>
<td>June</td>
<td>606</td>
</tr>
<tr>
<td>July</td>
<td>1006</td>
</tr>
<tr>
<td>August</td>
<td>2201</td>
</tr>
<tr>
<td>September</td>
<td>1011</td>
</tr>
<tr>
<td>October</td>
<td>1637</td>
</tr>
<tr>
<td>November</td>
<td>1921</td>
</tr>
<tr>
<td>December</td>
<td>2058</td>
</tr>
</tbody>
</table>

(round to hundredths place)

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the **mean**?

**Comment on this question**

The **mean** of a set of numbers is the **average** of the numbers.
This means that the **mean** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

**Select one:**

- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value in a set of numbers.
- The middle number in a set of ordered numbers.
- The number seen most often in a set of numbers.

**Submit Answer**

**Correct!**

Since part of finding the **mean** is finding the sum of all of the numbers.

What were the total sales in 1997?

**Comment on this question**

The total sales equal the sum of all the monthly sales in 1997.

**Comment on this hint**

Now we have to add all the numbers in this list up.

<table>
<thead>
<tr>
<th>1001</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050</td>
</tr>
<tr>
<td>2505</td>
</tr>
<tr>
<td>1125</td>
</tr>
<tr>
<td>1501</td>
</tr>
<tr>
<td>606</td>
</tr>
<tr>
<td>1006</td>
</tr>
<tr>
<td>2201</td>
</tr>
<tr>
<td>1011</td>
</tr>
<tr>
<td>1637</td>
</tr>
<tr>
<td>1921</td>
</tr>
<tr>
<td>2058</td>
</tr>
</tbody>
</table>

**Total:** 17622

**Comment on this hint**

**Type your answer below (mathematical expression):**

- 17622

**Submit Answer**

**Correct!**
Now that we know the definition of the **mean**, lets try the original problem again.

Alex runs a shoe store, and listed below are the store sales for the year 1997. What were the average monthly sales in 1997?

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1001</td>
</tr>
<tr>
<td>February</td>
<td>1050</td>
</tr>
<tr>
<td>March</td>
<td>2505</td>
</tr>
<tr>
<td>April</td>
<td>1125</td>
</tr>
<tr>
<td>May</td>
<td>1501</td>
</tr>
<tr>
<td>June</td>
<td>606</td>
</tr>
<tr>
<td>July</td>
<td>1006</td>
</tr>
<tr>
<td>August</td>
<td>2201</td>
</tr>
<tr>
<td>September</td>
<td>1011</td>
</tr>
<tr>
<td>October</td>
<td>1637</td>
</tr>
<tr>
<td>November</td>
<td>1921</td>
</tr>
<tr>
<td>December</td>
<td>2058</td>
</tr>
</tbody>
</table>

(round to hundredths place)

Remember, the sum of all the numbers is 17622.

Now find the number of numbers and divide.

There are 12 different numbers in the table.

The mean is \( 17622 \div 12 = 1468.5 \)

Rounding the mean to the hundredths gives **1468.5**, type in 1468.5.

Type your answer below (mathematical expression):

- 1468.5

Submit Answer

Correct!
Calculate the mean of the following numbers:

1.65, 1.16, 2.29, 1.42, 1.15, 1.43, 1.16, 0.92, 9, 2.71, 7.33

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How do we find the mean?

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
This means that the mean is the sum of the numbers divided by the number of numbers.

Select one:

○ The sum of the numbers divided by the number of numbers.
○ The number seen most often in this set of numbers.
○ The middle number in this set of numbers.
○ The difference between the largest value and the smallest value of the set of numbers.

Submit Answer

Correct!

Before we can find the mean, we must first find the sum of the numbers.
Calculate the **sum** of the following numbers:

\[1.65, \ 1.16, \ 2.29, \ 1.42, \ 1.15, \ 1.43, \ 1.16, \ 0.92, \ 1.95, \ 30, \ 7.33\]

*Comment on this question*

In order to find the **sum**, we have to **add** all the numbers together.

*Comment on this hint*

\[1.65 + 1.16 + 2.29 + 1.42 + 1.15 + 1.43 + 1.16 + 0.92 + 1.95 + 2.71 + 7.33 = 23.17\]

The **sum** is equal to 23.17, so type in 23.17

*Comment on this hint*

*Type your answer below (mathematical expression):*

- 23.17

Submit Answer

Correct!

Now that we know the sum of the numbers is **23.17**, try the original problem again.

Calculate the **mean** of the following numbers:

\[1.65, \ 1.16, \ 2.29, \ 1.42, \ 1.15, \ 1.43, \ 1.16, \ 0.92, \ 1.95, \ 2.71, \ 7.33\]

*(round to the nearest hundredths place)*

*Comment on this question*

The **mean** of a set of numbers is the **average** of the numbers.

*Comment on this hint*

To find the **mean**, we have to divide the **sum** by the amount of numbers that we have.

For this problem, we have **eleven** numbers.

*Comment on this hint*

\[23.17 \div 11 = 2.106363636364\]

Rounding the answer to the hundredths place, we get **2.11**, so type in 2.11.

*Comment on this hint*
Type your answer below *(mathematical expression)*:

- 2.11

Submit Answer

Correct!
Assistment

You are previewing content.

Calculate the **mean** of the following numbers:

0.98, 0.37, 2.38, 0.84, 1.68, 1.46, 0.59, 3.1

*(round to the nearest hundredths place)*

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

•

Submit Answer

Let's move on and figure out this problem.

How do we find the **mean**?

**Comment on this question**

The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

This means that the **mean** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

**Select one:**

- The sum of the numbers divided by the number of numbers.
- The number seen most often in this set of numbers.
- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.

Submit Answer

Correct!

**Before we can find the mean, we must first find the sum of the numbers.**
Calculate the **sum** of the following numbers:

- 0.98, 0.37, 2.38, 0.84, 1.68, 1.46, 0.59, 3.1

Comment on this question
In order to find the **sum**, we have to **add** all the numbers together.

Comment on this hint

0.98 + 0.37 + 2.38 + 0.84 + 1.68 + 1.46 + 0.59 + 3.1 = **11.4**

The **sum** is equal to **11.4**, so type in 11.4

Comment on this hint

Type your answer below *(mathematical expression)*:

- **11.4**

Submit Answer

Correct!

Now that we know the sum of the numbers is **11.4**, try the original problem again.

Calculate the **mean** of the following numbers:

- 0.98, 0.37, 2.38, 0.84, 1.68, 1.46, 0.59, 3.1

*(round to the nearest hundredths place)*

Comment on this question
The **mean** of a set of numbers is the **average** of the numbers.

Comment on this hint

To find the **mean**, we have to divide the **sum** by the amount of numbers that we have. For this problem, we have **eight** numbers.

Comment on this hint

**11.4 ÷ 8 = 1.425**

Rounding the answer to the hundredths place, we get **1.43**, so type in 1.43.

Comment on this hint

Type your answer below *(mathematical expression)*:
Submit Answer

Correct!

- 1.43
Assistment

You are previewing content.

Calculate the **mean** of the following numbers:

0.67, 1, 1, 0.99, 3, 0.37, 7, 1.68, 11

(round to the nearest hundredths place)

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How do we find the **mean**?

The **mean** of a set of numbers is the **average** of the numbers.

This means that the **mean** is the sum of the numbers divided by the number of numbers.

Select one:

- The number seen most often in this set of numbers.
- The sum of numbers divided by the number of numbers.
- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

\[0.67, 1, 1, 0.99, 3, 0.37, 7, 1.68, 11\]

Comment on this question
In order to find the sum, we have to add all the numbers together.

Comment on this hint
\[0.67 + 1 + 1 + 0.99 + 3 + 0.37 + 7 + 1.68 + 11 = 26.71\]

The sum is equal to 26.71, so type in 26.71

Comment on this hint
Type your answer below (mathematical expression):

- 26.71

Submit Answer
Correct!

Now that we know the sum of the numbers is 26.71, try the original problem again.

Calculate the mean of the following numbers:

\[0.67, 1, 1, 0.99, 3, 0.37, 7, 1.68, 11\]

(round to the nearest hundredths place)

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint

To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have nine numbers.

Comment on this hint

\[26.71 \div 9 = 2.96777777777778\]

Rounding the answer to the hundredths place, we get 2.97, so type in 2.97.
Comment on this hint

Type your answer below (mathematical expression):

- 2.97

Submit Answer

Correct!
Assistment

You are previewing content.

Calculate the mean of the following numbers:

1.35, 3, 5, 1.55, 5, 1.29

(round to the nearest hundredths place)

Comment on this question

Break this problem into steps

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

How do we find the mean?

Comment on this question

The mean of a set of numbers is the average of the numbers.

Comment on this hint

This means that the mean is the sum of the numbers divided by the number of numbers.

Comment on this hint

*Select one:*

- The number seen most often in this set of numbers.
- The sum of numbers divided by the number of numbers.
- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

1.35, 3, 5, 1.55, 5, 1.29

In order to find the sum, we have to add all the numbers together.

1.35 + 3 + 5 + 1.55 + 5 + 1.29 = 17.19

The sum is equal to 17.19, so type in 17.19

Now that we know the sum of the numbers is 17.19, try the original problem again.

Calculate the mean of the following numbers:

1.35, 3, 5, 1.55, 5, 1.29

(round to the nearest hundredths place)

The mean of a set of numbers is the average of the numbers.

To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have six numbers.

17.19 ÷ 6 = 2.865

Rounding the answer to the hundredths place, we get 2.87, so type in 2.87.
Comment on this hint

Type your answer below (mathematical expression):

- 2.87

Submit Answer

Correct!
Assistment

You are previewing content.

Calculate the mean of the following numbers:

138, 164, 15, 92, 34, 26, 10

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How do we find the mean?

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
This means that the mean is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

○ The sum of the numbers divided by the number of numbers.
○ The middle number in this set of numbers.
○ The number seen most often in this set of numbers.
○ The difference between the largest value and the smallest value of the set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the **sum** of the following numbers:

\[ 138, \ 164, \ 15, \ 92, \ 34, \ 26, \ 10 \]

**Comment on this question**
In order to find the **sum**, we have to **add** all the numbers together.

**Comment on this hint**

\[ 138 + 164 + 15 + 92 + 34 + 26 + 10 = 479 \]

The **sum** is equal to **479**, so type in 479

**Comment on this hint**

*Type your answer below (mathematical expression):*

- 479

Submit Answer

Correct!

Now that we know the sum of the numbers is **479**, try the original problem again.

Calculate the **mean** of the following numbers:

\[ 138, \ 164, \ 15, \ 92, \ 34, \ 26, \ 10 \]

*(round to the nearest hundredths place)*

**Comment on this question**

The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

To find the **mean**, we have to divide the **sum** by the amount of numbers that we have. For this problem, we have **seven** numbers.

**Comment on this hint**

\[ 479 \div 7 = 68.4285714285714 \]

Rounding the answer to the hundredths place, we get **68.43**, so type in 68.43.
Type your answer below (mathematical expression):

- 68.43

Submit Answer

Correct!
Calculate the **mean** of the following numbers:

52, 16, 31, 132, 102, 145, 86, 48

(round to the nearest hundredths place)

**Comment on this question**
Break this problem into steps

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

How do we find the **mean**?

**Comment on this question**
The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

This means that the **mean** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

*Select one:*

- The sum of the numbers divided by the number of numbers.
- The number seen most often in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The middle number in this set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

\[ 52, \quad 16, \quad 31, \quad 132, \quad 102, \quad 145, \quad 86, \quad 48 \]

Comment on this question
In order to find the sum, we have to add all the numbers together.

\[ 52 + 16 + 31 + 132 + 102 + 145 + 86 + 48 = 612 \]

The sum is equal to 612, so type in 612

Comment on this hint
Type your answer below (mathematical expression):

- 612

Submit Answer
Correct!

Now that we know the sum of the numbers is 612, try the original problem again.

Calculate the mean of the following numbers:

\[ 52, \quad 16, \quad 31, \quad 132, \quad 102, \quad 145, \quad 86, \quad 48 \]

(round to the nearest hundredths place)

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
To find the mean, we have to divide the sum by the amount of numbers that we have. For this problem, we have eight numbers.

\[ 612 \div 8 = 76.5 \]

Rounding the answer to the hundredths place, we get 76.5, so type in 76.5.
Comment on this hint

Type your answer below (mathematical expression):

• 76.5

Submit Answer

Correct!
Assistment

You are previewing content.

Rose obtained the following scores in 7 math tests. Calculate the mean of Rose's math scores:

85, 23, 79, 97, 52, 89, 63

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How do we find the mean?

Comment on this question
The mean of a set of numbers is the average of the numbers.

Comment on this hint
This means that the mean is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The sum of the numbers divided by the number of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!
Before we can find the mean, we must first find the sum of the scores.

Calculate the sum of the following numbers:

85, 23, 79, 97, 52, 89, 63

In order to find the sum, we have to add all the scores together.

85 + 23 + 79 + 97 + 52 + 89 + 63 = 488

The sum is equal to 488, so type in 488

Now that we know the sum of the scores is 488, try the original problem again.

Rose obtained the following scores in 7 math tests. Calculate the mean of Rose's math scores:

85, 23, 79, 97, 52, 89, 63

(round to the nearest hundredths place)

The mean of the set of scores is the average of the scores.

To find the mean, we have to divide the sum of the scores by the number of scores that we have. For this problem, we have seven scores.

488 ÷ 7 = 69.7142857142857
Rounding the answer to the hundredths place, we get 69.71, so type in 69.71.

Comment on this hint
Type your answer below (mathematical expression):

• 69.71

Submit Answer

Correct!
Hannah obtained the following scores in 9 math tests. Calculate the **mean** of Hannah's math scores:

91, 68, 25, 91, 12, 80, 64, 19, 67

*(round to the nearest hundredths place)*

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

- 

Submit Answer

Let's move on and figure out this problem.

How do we find the **mean**?

**Comment on this question**

The **mean** of a set of numbers is the **average** of the numbers.

**Comment on this hint**

This means that the **mean** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

*Select one:*

- The sum of the numbers divided by the number of numbers.
- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!

**Before we can find the mean, we must first find the sum of the scores.**
Calculate the **sum** of the following numbers:

91, 68, 25, 91, 12, 80, 64, 19, 67

**Comment on this question**

In order to find the **sum**, we have to **add** all the scores together.

**Comment on this hint**

$$91 + 68 + 25 + 91 + 12 + 80 + 64 + 19 + 67 = 517$$

The **sum** is equal to **517**, so type in 517

**Comment on this hint**

*Type your answer below (mathematical expression):*

- **517**

Submit Answer

Correct!

**Now that we know the sum of the scores is 517, try the original problem again.**

Hannah obtained the following scores in 5 math tests. Calculate the **mean** of Hannah's math scores:

91, 68, 25, 91, 12, 80, 64, 19, 67

(Without rounding to the nearest hundredths place)

**Comment on this question**

The **mean** of the set of scores is the **average** of the scores.

**Comment on this hint**

To find the **mean**, we have to divide the **sum** of the scores by the number of scores that we have. For this problem, we have **nine** scores.

**Comment on this hint**

$$517 \div 9 = 57.444444444444$$

Rounding the answer to the hundredths place, we get **57.44**, so type in 57.44.
Comment on this hint

Type your answer below (mathematical expression):

- 57.44

Submit Answer

Correct!
During a medical study, doctors recorded the weights in pounds of all their volunteers. Some of the weights are given here. What is the average weight of the volunteers listed below?

135, 140, 102, 127, 116, 142, 132, 107, 147, 102, 140, 146

(round to the nearest hundredths place)

**Comment on this question**
**Break this problem into steps**

*Type your answer below (mathematical expression):*

- 

**Submit Answer**

Let's move on and figure out this problem.

How do we find the **average**?

**Comment on this question**

**Average** is the sum of the numbers divided by the number of numbers.

**Comment on this hint**
**Select one:**

- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The middle number in this set of numbers.
- The number seen most often in this set of numbers.

**Submit Answer**

Correct!

Before we can find the average, we must first find the sum of the numbers.
Calculate the sum of the following numbers:

135, 140, 102, 127, 116, 142, 132, 107, 147, 102, 146, 13

Comment on this question
In order to find the sum, we have to add all the numbers together.

Comment on this hint
135 + 140 + 102 + 127 + 116 + 142 + 142 + 132 + 107 + 147 + 102 + 146 + 140 = 1536

The sum is equal to 1536, so type in 1536

Comment on this hint
Type your answer below (mathematical expression):

• 1536

Submit Answer

Correct!

Now that we know the sum of the numbers is 1536, try the original problem again.

During a medical study, doctors recorded the weights in pounds of all their volunteers. Some of the weights are given here. What is the average weight of the volunteers listed below?

135, 140, 102, 127, 116, 142, 132, 107, 147, 102, 146, 140

(round to the nearest hundredths place)

Comment on this question
To find the average, we have to divide the sum by the amount of numbers that we have. For this problem, we have twelve numbers.

Comment on this hint
1536 ÷ 12 = 128 (answer is rounded to the nearest hundredths place)

The mean is 128, so type in 128.

Comment on this hint
Type your answer below (mathematical expression):

• 128

Submit Answer
Correct!
During a medical study, doctors recorded the heights in centimeters of all their volunteers. Some of the heights are given here. What is the average height of the volunteers listed below?

201, 195, 166, 159, 163, 195, 181, 175, 188, 163, 155

(round to the nearest hundredths place)

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

- 

Submit Answer

Let's move on and figure out this problem.

How do we find the average?

**Comment on this question**

*Average* is the sum of the numbers divided by the number of numbers.

**Comment on this hint**

*Select one:*

- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The middle number in this set of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!

**Before we can find the average, we must first find the sum of the numbers.**

Calculate the **sum** of the following numbers:
In order to find the sum, we have to add all the numbers together.

\[ 201 + 195 + 166 + 159 + 163 + 195 + 181 + 175 + 188 + 163 + 155 = 1941 \]

The sum is equal to 1941, so type in 1941.

To find the average, we have to divide the sum by the amount of numbers that we have. For this problem, we have eleven numbers.

\[ 1941 \div 11 = 176.45 \] (answer is rounded to the nearest hundredths place)

The mean is 176.45, so type in 176.45.
Mary works at the local grocery store and has to process all the sales at the end of the day. The list below gives the dollar amounts of all the sales made on a particular day.

What is the average amount of these sales?

6, 17, 18, 14.43, 4

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

- Submit Answer

Let's move on and figure out this problem.

How do we find the average (mean)?

Comment on this question

Average is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

- The sum of the numbers divided by the number of numbers.
- The number seen most often in this set of numbers.
- The middle number in this set of numbers.
- The difference between the largest value and the smallest value of the set of numbers.

Submit Answer

Correct!
Before we can find the average, we must first find the sum of the numbers.

Calculate the sum of the following numbers:

\[
6, \quad 17, \quad 18, \quad 14.43, \quad 4
\]

\[
6 + 17 + 18 + 14.43 + 4 = 59.43
\]

The sum is equal to 59.43, so type in 59.43

Type your answer below (mathematical expression):

\[
\text{• 59.43}
\]

Submit Answer

Correct!

Now that we know the sum of the numbers is 59.43, try the original problem again.

Mary works at the local grocery store and has to process all the sales at the end of the day. The list below gives the dollar amounts of all the sales made on a particular day.

What is the average amount of these sales?

\[
6, \quad 17, \quad 18, \quad 14.43, \quad 4
\]

(round to the nearest hundredths place)

To find the average, we have to divide the sum by the amount of numbers that we have. For this problem, we have five numbers.

\[
59.43 \div 5 = 11.886
\]
Rounding the answer to the hundredths place, we get 11.89, so type in 11.89.

Type your answer below (mathematical expression):

• 11.89

Submit Answer

Correct!
Mary works at the local hardware store and has to process all the sales at the end of the day. The list below gives the dollar amounts of all the sales made on a particular day.

What is the average amount of these sales?

17.67, 11, 1, 7.86, 3, 18.6, 8, 47.47, 21.15

(round to the nearest hundredths place)

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How do we find the average (mean)?

Comment on this question

Average is the sum of the numbers divided by the number of numbers.

Select one:

- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value of the set of numbers.
- The middle number in this set of numbers.
- The number seen most often in this set of numbers.

Submit Answer

Correct!
Before we can find the average, we must first find the sum of the numbers.

Calculate the **sum** of the following numbers:

\[ 17.67, \ 11, \ 1, \ 7.86, \ 3, \ 18.6, \ 8, \ 47.47, \ 21.15 \]

Comment on this question

In order to find the **sum**, we have to **add** all the numbers together.

Comment on this hint

\[ 17.67 + 11 + 1 + 7.86 + 3 + 18.6 + 8 + 47.47 + 21.15 = 135.75 \]

The **sum** is equal to **135.75**, so type in 135.75

Comment on this hint

*Type your answer below (mathematical expression):*

- 135.75

Submit Answer

Correct!

**Now that we know the sum of the numbers is 135.75**, try the original problem again.

Mary works at the local hardware store and has to process all the sales at the end of the day. The list below gives the dollar amounts of all the sales made on a particular day.

What is the **average** amount of these sales?

\[ 17.67, \ 11, \ 1, \ 7.86, \ 3, \ 18.6, \ 8, \ 47.47, \ 21.15 \]

(round to the nearest hundredths place)

Comment on this question

To find the **average**, we have to divide the **sum** by the amount of numbers that we have. For this problem, we have **nine** numbers.

Comment on this hint

\[ 135.75 \div 9 = 15.083333333333 \]
Rounding the answer to the hundredths place, we get 15.08, so type in 15.08.

Type your answer below (mathematical expression):

- 15.08

Submit Answer

Correct!
Mean with Missing Number and Decimal, 10 (#57314)

You are previewing content.

What number should be added to the list below to get a mean of 17.173?

10, 30.33, 13, 28, 22.14, 30.2, 14, 15.89, 1.17

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

Submit Answer

Let's move on and figure out this problem.

How is the mean of a set of numbers defined?

Comment on this hint
Select one:

- The middle number when the numbers are arranged in increasing order
- The sum of the numbers divided by the number of numbers
- The number that is seen most often
- Difference between the largest and smallest number.

Submit Answer

Correct!

We can get the mean by dividing the sum (that we do not know right now) of the numbers by the number of numbers (in this problem it is 10 numbers).

So, let's start by finding the expected sum of the numbers.

What should the sum be to get a mean of 17.173 with 10 numbers?
Remember, the list should have **ten** numbers when we add the new number.

Since we know that

$$\text{Total} \div \text{Number of Numbers} = \text{Mean},$$

then

$$\text{Total} = \text{Mean} \times \text{Number of Numbers}$$

$$\text{Total} = 17.173 \times 10 \quad \text{Total} = 171.73.$$

So, the sum of this set of numbers is **171.73**.

Type in **171.73**.

Type your answer below (mathematical expression):

- **171.73**

Submit Answer

Correct!

Now try the original problem again.

What number should be added to the list below to get a mean of **17.173**?

10, 30.33, 13, 28, 22.14, 30.2, 14, 15.89, 1.17
In order to find the value of the new number, all we have to do is subtract the nine current numbers from the **total**.

**Comment on this hint**


So, in order to get a **mean** of 17.173 we have to add 7 to the set of numbers.

Type in 7.

**Comment on this hint**

*Type your answer below (mathematical expression):*

- 7

Submit Answer

Correct!
Assistment

You are previewing content.

What number should be added to the list below to get a mean of 18.691?

10, 43.33, 16, 31, 10.71, 17.6, 7

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer
Let's move on and figure out this problem.

How is the mean of a set of numbers defined?

Comment on this question
Mean is the same as average. Average is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

○ The number that is seen most often
○ Difference between the largest and smallest number.
○ The sum of the numbers divided by the number of numbers
○ The middle number when the numbers are arranged in increasing order

Submit Answer
Correct!

We can get the mean by dividing the sum (that we do not know right now) of the numbers by the number of numbers (in this problem it is 8 numbers).
So, let's start by finding the expected sum of the numbers.

What should the sum be to get a mean of 18.691 with 8 numbers?

Remember, the list should have eight numbers when we add the new number.

Since we know that

\[
\text{Total} \div \text{Number of Numbers} = \text{Mean},
\]

then

\[
\text{Total} = \text{Mean} \times \text{Number of Numbers}
\]

\[
\text{Total} = 18.691 \times 8
\]

\[
\text{Total} = 149.53.
\]

So, the sum of this set of numbers is 149.53.

Type in 149.53.

Now that we know the total is 149.53, try the original problem again.

What number should be added to the list below to get a mean of 18.691?

10, 43.33, 16, 31, 10.71, 17.6, 7
Comment on this question
Remember the sum of the set of numbers is **149.53**.

Comment on this hint
In order to find the value of the new number, all we have to do is subtract the seven current numbers from the **total**.

Comment on this hint

149.53 - 10 - 43.33 - 16 - 31 - 10.71 - 17.6 - 7 = 13.89

So, in order to get a **mean** of **18.691** we have to add 13.89 to the set of numbers.

Type in 13.89.

Comment on this hint

*Type your answer below (mathematical expression):*

- **13.89**

Submit Answer

Correct!
Assistment

You are previewing content.

Harry bought nine bikes, eight of whose prices are given below. If the average price of the bikes was $321.203, what was the price of the ninth bike?

307, 328, 301, 309, 322.43, 316.4, 372, 328

Comment on this question

Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

How is the mean of a set of numbers defined?

Comment on this question

Mean is the same as average. Average is the sum of the numbers divided by the number of numbers.

Comment on this hint

Select one:

○ The sum of the numbers divided by the number of numbers
○ Difference between the largest and smallest number.
○ The middle number when the numbers are arranged in increasing order
○ The number that is seen most often

Submit Answer

Correct!

We can get the mean by dividing the sum of the numbers (which we do not know yet) by the number of numbers (there will be 9 numbers in the current list). So, let's start by finding the expected sum of the numbers.

What should the sum be to get a mean of 321.203?
Remember, the list should have nine numbers when we add the new number.

Since we know that

\[
\text{Total} = \text{Mean} \times \text{Number of Numbers}
\]

then

\[
\text{Total} = 321.203 \times 9
\]

\[
\text{Total} = 2890.83.
\]

So, the sum of this set of numbers is 2890.83.

Type in 2890.83.

Now that we know the total is 2890.83, try the original problem again.

Harry bought nine bikes, eight of whose prices are given below. If the average price of the bikes was $321.203, what was the price of the ninth bike?

307, 328, 301, 309, 322.43, 316.4, 372, 328

Remember the sum of the set of numbers is 2890.83.

In order to find the value of the new number, all we have to do is subtract the eight current numbers from the total.
2890.83 - 307 - 328 - 301 - 309 - 322.43 - 316.4 - 372 - 328 = 307

So, in order to get a mean of 321.203 we have to add 307 to the set of numbers.

Type in 307.

Comment on this hint

Type your answer below (mathematical expression):

- 307

Submit Answer

Correct!
Harry bought five bags, four of whose prices are given below. If the average price of the bags was $21.972, what was the price of the fifth bag?

14, 34, 16, 14

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

Submit Answer
Let's move on and figure out this problem.

How is the mean of a set of numbers defined?

Comment on this question
Mean is the same as average. Average is the sum of the numbers divided by the number of numbers.

Comment on this hint
Select one:

- Difference between the largest and smallest number.
- The number that is seen most often
- The sum of the numbers divided by the number of numbers
- The middle number when the numbers are arranged in increasing order

Submit Answer
Correct!

We can get the mean by dividing the sum of the numbers (which we do not know yet) by the number of numbers (there will be 5 numbers in the current list). So, let's start by finding the **expected sum of the numbers**.

What should the sum be to get a mean of 21.972?
Remember, the list should have five numbers when we add the new number.

Since we know that

\[
\text{Total} \div \text{Number of Numbers} = \text{Mean},
\]

then

\[
\text{Total} = \text{Mean} \times \text{Number of Numbers}
\]

\[
\text{Total} = 21.972 \times 5
\]

\[
\text{Total} = 109.86.
\]

So, the sum of this set of numbers is 109.86.

Type in 109.86.

Now that we know the total is 109.86, try the original problem again.

Harry bought five bags, four of whose prices are given below. If the average price of the bags was $21.972, what was the price of the fifth bag?

14, 34, 16, 14

Remember the sum of the set of numbers is 109.86.

In order to find the value of the new number, all we have to do is subtract the four current numbers from the total.
109.86 - 14 - 34 - 16 - 14 = 31.86

So, in order to get a mean of 21.972 we have to add 31.86 to the set of numbers.

Type in 31.86.

Type your answer below (mathematical expression):

• 31.86

Submit Answer

Correct!
The coach for the All-USA Math Team needs to pick one of two students for the team. The table below shows the number of points each of the students obtained in their last 8 tests.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>11,3,9,11,21,20,11,4</td>
</tr>
<tr>
<td>Ashley</td>
<td>18,23,8,14,6,7,17,24</td>
</tr>
</tbody>
</table>

What is the mean (average) number of points obtained by Bob?

(Round to the hundredths place)

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the mean?

The mean of a set of numbers is the average of the numbers.

This means that the mean is the sum of the numbers divided by the number of numbers.
Select one:

- The number seen most often in a set of numbers.
- The sum of the numbers divided by the number of numbers.
- The middle number in a set of ordered numbers.
- The difference between the largest value and the smallest value in a set of numbers.

Submit Answer

Correct!

Since part of finding the **mean** is finding the sum of all of the numbers, what is the sum of scores that Bob scored?

Bob 11,3,9,11,21,20,11,4

Comment on this question

First, write out all of the numbers in this plot on a piece of paper.

Comment on this hint

Your list of numbers should look something like this:

11,3,9,11,21,20,11,4

Comment on this hint

Now we have to add all the numbers in this list up.

```
11
3
9
11
21
20
11
+ 4
---
90
```

Type your answer below (mathematical expression):

- 90

Submit Answer

Correct!

Now that we know the definition of the **mean**, lets try the original problem again.
The coach for the All-Star Basketball Game needs to pick one of the two players for the team.

The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>11,3,9,11,21,20,11,4</td>
</tr>
<tr>
<td>Ashley</td>
<td>18,23,8,14,6,7,17,24</td>
</tr>
</tbody>
</table>

What is the **mean** (average) number of points scored by Bob?

(Round to the hundredths place)

Comment on this question

Remember, the sum of all the numbers is 90.

Now find the number of numbers and divide.

Comment on this hint

There are 8 different scores on the chart.

The mean is $90 \div 8 = 11.25$

Rounding the mean to the hundredths place gives 11.25. Type in 11.25.

Comment on this hint

*Type your answer below (mathematical expression):*

- 11.25

Submit Answer

Correct!
Appendix B:

Variabilized Templates for Median problems
Below is a list of numbers.

[83, 128, 10, 28, 86, 42, 9, 28, 34, 2, 84, 3, 9]

What is the **median** number in this list?

Comment on this question

Break this problem into steps

*Type your answer below (mathematical expression):*

- 

Submit Answer

Let's move on and figure out this problem.

What is a **median** value of a list of numbers?

Comment on this question

*Select one:*

- The number seen must often in a list of numbers
- The middle number in an ordered list of numbers
- The sum of all the numbers divided by the number of numbers (the average)
- The difference between the largest value and the smallest value of the set of numbers

Submit Answer

Correct!

Great, now that you know the **median** is the middle number in an ordered list of numbers, take another look at the list of numbers below.

[83, 128, 10, 28, 86, 42, 9, 28, 34, 2, 84, 3, 9]

What is the **median** number in this list?
Begin by writing out the listed numbers in order on scratch paper.

Then you can easily find the middle number.

The ordered list you wrote out should look something like this:

\[2, 3, 9, 9, 10, 12, 28, 28, 34, 42, 83, 84, 86\]

The middle number in this ordered list, 28, is the median.

Type your answer below (mathematical expression):  

• 28

Submit Answer

Correct!
What number should be added to the list below to get a **median** of 19.5?

13, 24, 5, 15, 28

Select one:

- 14
- 4
- 32
- 6

Let's move on and figure out this problem.

**Remember, the median is the middle value in an ordered list of numbers.** So first let's ordered the numbers.

Which of the following choices has the correct order of the list below from the least to the greatest?

13, 24, 5, 15, 28

Select one:

- 13, 24, 5, 15, 28
- 5, 15, 13, 24, 28
- 28, 24, 15, 13, 5
So, the order of the list of numbers is

5, 13, 15, 24, 28

What number should be added to the list below to get a \textit{median} of 19.5?

If the number 6 is added to the list, then the list would look like this:

5, 6, 13, 15, 24, 28

And the \textit{median} of this list would be 14, because

\[ 13 + 15 = 28, \text{ and} \]

\[ 28 \div 2 = 14, \text{ which is not the correct median.} \]

Next, if the number 14 is added to the list, then the list would look like this:

5, 13, 14, 15, 24, 28

And the \textit{median} of this list would be 14.5, because

\[ 14 + 15 = 29, \text{ and} \]

\[ 29 \div 2 = 14.5, \text{ which is not the correct median.} \]

Then, if the number 4 is added to the list, then the list would look like this:

4, 5, 13, 15, 24, 28

And the \textit{median} of this list would be 14, because

\[ 13 + 15 = 28, \text{ and} \]

\[ 28 \div 2 = 14, \text{ which is not the correct median.} \]

Finally, if the number 32 is added to the list, then the list would look like this:

5, 13, 15, 24, 28, 32

And the \textit{median} of this list would be 19.5, because

\[ 24 + 15 = 39, \text{ and} \]
39 ÷ 2 = 19.5, which is the correct median. So, choose the option with 32.

Select one:

- 6
- 14
- 32
- 4

Submit Answer

Correct!
Assistment

You are previewing content.

What number should be added to the list below to get a median of 17?

11, 23, 8, 17

Comment on this question
Break this problem into steps

*Select one:*

- 4
- 9
- 27
- 16

Submit Answer

Let's move on and figure out this problem.

Remember, the median is the middle value in an ordered list of numbers. So first let's ordered the numbers.

Which of the following choices has the correct order of the list below from the least to the greatest?

11, 23, 8, 17

Comment on this question
Ordering from least to greatest means the numbers are arranged from the smallest value to the highest value.

Comment on this hint
The smallest value is 8, and the highest value is 23.

Comment on this hint
The order of the list from the least to the greatest should look like this:

8, 11, 17, 23

So, choose the option with the order 8, 11, 17, 23.

*Select one:*

- 23, 17, 11, 8
- 8, 17, 11, 23
- 8, 11, 17, 23
So, the order of the list of numbers is

8, 11, 17, 23

What number should be added to the list below to get a median of 17?

If the number 9 is added to the list, then the list would look like this:

8, 9, 11, 17, 23

The median of this list would be 11, because that's the middle number. And, 11 is not the correct median.

Next, if the number 16 is added to the list, then the list would look like this:

8, 11, 16, 17, 23

The median of this list would be 16, because that's the middle number. And, 16 is not the correct median.

Then, if the number 4 is added to the list, then the list would look like this:

4, 8, 11, 17, 23

The median of this list would also be 11, because that's the middle number. And, 11 is not the correct median.

Finally, if the number 27 is added to the list, then the list would look like this:

8, 11, 17, 23, 27

The median of this list would also be 17, because that's the middle number. And, 17 is the correct median. So choose the option with 27.

Select one:

9
27
4
16

Submit Answer
Correct!
The coach for the All-Star Basketball Game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 11 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last eleven games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaun</td>
<td>32,67,26,81,15,76,61,8,73,24,50</td>
</tr>
<tr>
<td>Julia</td>
<td>16,23,8,16,46,9,7,23,30,20,14</td>
</tr>
</tbody>
</table>

What is the median number of points scored by Shaun?

**Comment on this question**

Break this problem into steps

*Type your answer below (mathematical expression):*

•

Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition of the median?

**Comment on this question**

Median is the same as the middle.

**Comment on this hint**

The middle number is the correct answer.

**Comment on this hint**

*Select one:*

- The number seen most often in a set of numbers.
The middle number in a set of ordered numbers.
The sum of the numbers divided by the number of numbers.
The difference between the largest value and the smallest value in a set of numbers.

Submit Answer
Correct!

Now that we know what the median is,
Let's try the original problem again.

The coach for the All-Star Basketball Game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 11 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last eleven games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaun</td>
<td>32, 67, 26, 81, 15, 76, 61, 8, 73, 24, 50</td>
</tr>
<tr>
<td>Julia</td>
<td>16, 23, 8, 16, 46, 9, 7, 23, 30, 20, 14</td>
</tr>
</tbody>
</table>

What is the median number of points scored by Shaun?

First we need to order the list from least to greatest.

It should look like this,

8, 15, 24, 26, 32, 50, 61, 67, 73, 76, 81

Now we need to find the middle number in this list.

The median(middle) number in the list is 50

Type in 50

Type your answer below (mathematical expression):
Submit Answer

Correct!
The coach for the All-Star Basketball Game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaun</td>
<td>43, 68, 25, 15, 78, 57, 8, 71, 24, 51</td>
</tr>
<tr>
<td>Julia</td>
<td>19, 26, 10, 12, 12, 9, 23, 29, 18, 9</td>
</tr>
</tbody>
</table>

What is the median number of points scored by Shaun?

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

- \[ \text{median of Shauns scores} \]

Submit Answer
Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition of the median?

Comment on this question
Median is the same as the middle.

Comment on this hint
The middle number is the correct answer.

Select one:

○ The number seen most often in a set of numbers.
Now that we know what the median is,

Let's try the original problem again.

The coach for the All-Star Basketball Game needs to pick one of the two players for the team.

The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaun</td>
<td>43, 68, 25, 15, 78, 57, 8, 71, 24, 51</td>
</tr>
<tr>
<td>Julia</td>
<td>19, 26, 10, 12, 12, 9, 23, 29, 18, 9</td>
</tr>
</tbody>
</table>

What is the median number of points scored by Shaun?

First we need to order the list from least to greatest.

It should look like this,

8, 15, 24, 25, 43, 51, 57, 68, 71, 78

Since there is an even number of numbers in this set, there are two middle numbers.

To find the median, we have to average these two numbers.

The two middle numbers are 43 and 51.

Sum these two numbers up and divide the total by 2 to get the median.
43 + 51 = 94
94 ÷ 2 = 47.

Type in 47.

Type your answer below (mathematical expression):

- 47

Submit Answer

Correct!
You are previewing content.

Below is a list of numbers.

[3, 8, 43, 9, 3, 26, 5, 72, 1, 42, 4, 82, 9]

What is the **median** number in this list?

The first step to solve is to put the numbers in order from least to greatest.

[1, 3, 3, 4, 5, 8, 9, 9, 26, 42, 43, 72, 82]

The last step is to identify the middle number in the list.

Type in 9

Type your answer below (mathematical expression):

- 9

Submit Answer

Correct!
You are previewing content.

Below is a list of numbers.

[74, 73, 99, 28, 85, 74, 35, 68, 32, 56, 5, 3]

What is the median number in this list?

The first step is to put the numbers in order from least to greatest.

[3, 5, 28, 32, 35, 56, 68, 73, 74, 74, 85, 99]

The second step is to identify the two middle numbers.

56 and 68

The last step is to find the mean of the two middle numbers.

\[56 + 68 = 124\]
\[124 \div 2 = 62\]

Type in 62
Submit Answer

Correct!

- 62
What number should be added to the list below to get a median of 20.5?

10, 23, 7, 18, 28

The first step to solve is to put the values in order from the least to the greatest.

7, 10, 18, 23, 28

The second step to solve is to see which of the four choices, when inserted into the list of values, will give you the desired median. This is what it would look like if the number 8 is added to the list.

7, 8, 10, 18, 23, 28

And the median of this list would be 14, because

\[10 + 18 = 28, \text{ and}\]
\[28 \div 2 = 14, \text{ which is not the correct median.}\]

33, when added to the list, would look like this:

7, 10, 18, 23, 28, 33

And the median of this list would be 20.5, because

\[23 + 18 = 41, \text{ and}\]
\[41 \div 2 = 20.5, \text{ which is the correct median.}\]

So, choose the option with 33
Select one:

- 33
- 11
- 1
- 8

Submit Answer

Correct!
What number should be added to the list below to get a median of 18?

12, 23, 6, 18

The first step to solve is to put the values in order from the least to the greatest.

6, 12, 18, 23,

The second step to solve is to see which of the four choices, when inserted into the list of values, will give you the desired median. This is what it would look like if the number 7 is added to the list.

6, 7, 12, 18, 23

The median of this list would be 12, because that's the middle number. And, 12 is not the correct median.

25, when added to the list, would look like this:

6, 12, 18, 23, 25

The median of this list would also be 18, because that's the middle number. And, 18 is the correct median. So choose the option with 25.

So, choose the option with 25.
17
2
7
25

Submit Answer

Correct!
Below is a list of numbers.

[3, 12.57, 3, 58, 15, 63.02, 33.71, 39, 37, 44, 40.04, 94, 18.13]

What is the **median** number in this list?

Comment on this question

Break this problem into steps

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

How is the median defined?

Comment on this question

*Select one:*

- The sum of the numbers divided by the number of numbers
- The middle number when the numbers are listed in increasing order
- The difference between the largest and smallest number
- The number that appears most frequently

Submit Answer

Correct!

Now that we know that the median of a set of numbers is the middle number, let us solve the problem again:

Below is a list of numbers.

[3, 12.57, 3, 58, 15, 63.02, 33.71, 39, 37, 44, 40.04, 94, 18.13]
What is the **median** number in this list?

Writing the numbers in increasing order, we get:

\[3, 3, 12.57, 15, 18.13, 33.71, 37, 39, 40.04, 44, 58, 63.02, 94\]

There are 13 numbers in the list. Hence we must pick the 7th number.

The 7th number is 37. Type 37.

*Type your answer below (mathematical expression):*

- 37

Submit Answer

Correct!
Assistment

You are previewing content.

Below is a list of numbers.

[4.0035, 40.035, 0.40035, 4.0035, 0.040035, 0.0040035, 400.35, 0.040035]

What is the **median** number in this list?

Comment on this question

Break this problem into steps

*Type your answer below (mathematical expression):*

- Submit Answer

Let's move on and figure out this problem.

How is the median defined?

Comment on this question

Select one:

- The difference between the largest and smallest number
- The number that appears most frequently
- The middle number when the numbers are listed in increasing order
- The sum of the numbers divided by the number of numbers

Submit Answer

Correct!

Now that we know that the median of a set of numbers is the middle number, let us solve the problem again:

Below is a list of numbers.

[4.0035, 40.035, 0.40035, 4.0035, 0.040035, 0.0040035, 400.35, 0.040035]
What is the **median** number in this list?

**Comment on this question**

Writing the numbers in increasing order, we get:

\[0.0040035, 0.040035, 0.040035, 0.40035, 4.0035, 4.0035, 40.035, 400.35]\]

**Comment on this hint**

There are 8 numbers in the list. Hence we median is equal to the \((4^{th}\) number + \(5^{th}\) number)/2

**Comment on this hint**

The \(4^{th}\) number is 0.40035 and the \(5^{th}\) number is 4.0035. Hence the median is:

\[(0.40035 + 4.0035)/2 = 2.201925\]

Type 2.201925.

**Comment on this hint**

*Type your answer below (mathematical expression):*

- 2.201925

Submit Answer

Correct!
What number should be added to the list below to get a **median** of 24.115?

12, 26.23, 35, 17, 63, 4.63, 49

The median is the middle number when the numbers are arranged in increasing order. Since there will be 8 numbers in the list, the median will be given by:

\[
\frac{\text{4th number} + \text{5th number}}{2}
\]

The first step to solve is to put the values in order from the least to the greatest.

4.63, 12, 17, 22, 26.23, 35, 49

The second step to solve is to see which of the four choices, when inserted into the list of values, will give you the desired median. This is what it would look like if the number 13 is added to the list.

4.63, 12, 13, 17, 22, 26.23, 35, 49

And the **median** of this list would be 19.5, because

\[
17 + 22 = 39, \text{ and }
\]

\[
39 \div 2 = 19.5, \text{ which is not the correct median.}
\]

22, when added to the list, would look like this:

4.63, 12, 17, 22, 26.23, 35, 49, 63

And the **median** of this list would be 24.115, because
22 + 26.23 = 48.23, and

48.23 ÷ 2 = 24.115, which is the correct median.

So, choose the option with 22

Select one:

- 5.63
- 1.63
- 13
- 22

Submit Answer

Correct!
Assistment

You are previewing content.

What number should be added to the list below to get a median of 23?

13, 26.53, 23, 15, 5.66, 45

Comment on this question

The median is the middle number when the numbers are arranged in increasing order. Since there will be 7 numbers in the list, the median will be the 4th number.

Comment on this hint

The first step to solve is to put the values in order from the least to the greatest.

5.66, 13, 15, 23, 26.53, 45

Comment on this hint

The second step to solve is to see which of the four choices, when inserted into the list of values, will give you the desired median. This is what it would look like if the number 14 is added to the list.

5.66, 13, 14, 15, 23, 26.53, 45

The median of this list would be 15 which is not the correct median.

Comment on this hint

34, when added to the list, would look like this:

5.66, 13, 15, 23, 26.53, 34, 45

And the median of this list would be 23, which is the correct median.

So, choose the option with 34

Select one:

- 34
Submit Answer

Correct!
James bought five chairs, four of whose prices are given below. If the median price of the chairs was $71.5, what was the price of the sixth chair?

86, 74, 57.63, 74, 79.23

The median is the middle number when the numbers are arranged in increasing order. Since there will be 6 numbers in the list, the median will be given by:

\[
\frac{\text{3rd number} + \text{4th number}}{2}
\]

The first step to solve is to put the values in order from the least to the greatest.

57.63, 69, 74, 79.23, 86

The second step to solve is to see which of the four choices, when inserted into the list of values, will give you the desired median. This is what it would look like if the number 89 is added to the list.

57.63, 69, 74, 79.23, 86, 89

And the median of this list would be 76.615, because

\[
74 + 79.23 = 153.23, \text{ and}
\]

\[
153.23 ÷ 2 = 76.615, \text{ which is not the correct median.}
\]
63, when added to the list, would look like this:

57.63, 63, 69, 74, 79.23, 86

And the **median** of this list would be 71.5, because

69 + 74 = 143, and

143 ÷ 2 = 71.5, which is the correct **median**.

So, choose the option with 63
Lisa obtained the following scores in 4 of 5 math tests. If the median of Lisa's math scores was 17, what was Lisa's math score on the fifth test?

15, 28.87, 17, 6

The median is the middle number when the numbers are arranged in increasing order. Since there will be 5 numbers in the list, the median will be the 3rd number.

The first step to solve is to put the values in order from the least to the greatest.

6, 15, 17, 28.87

The second step to solve is to see which of the four choices, when inserted into the list of values, will give you the desired median. This is what it would look like if the number 16 is added to the list.

6, 15, 16, 17, 28.87

The median of this list would be 16 which is not the correct median.

26, when added to the list, would look like this:

6, 15, 17, 26, 28.87

And the median of this list would be 17, which is the correct median.

So, choose the option with 26
Comment on this hint

Select one:

- 16
- 3
- 7
- 26

Submit Answer

Correct!
During a medical study, doctors measured the weights of all their volunteers. Some of weights (in lbs) are provided here. What is the median weight of the volunteers as listed below?

152, 157, 147, 145, 159, 171, 139, 163, 140

Median is the middle number when all the numbers are arranged in increasing order.

Select one:

- The number that appears most frequently
- The difference between the largest and smallest number
- The middle number when the numbers are listed in increasing order
- The sum of the numbers divided by the number of numbers

Now that we know that the median of a set of numbers is the middle number, let us solve the problem again:

During a medical study, doctors measured the weights of all their volunteers. Some of weights (in lbs) are provided here. What is the median weight of the volunteers as listed below?
Writing the numbers in increasing order, we get:
139, 140, 145, 147, 152, 157, 159, 163, 171

Since there are 9 numbers in the list, the median is the 5th number.

The 5th number is 152. Type 152.

Correct!
During a medical study, doctors recorded the ages of all their volunteers. Some of ages (in years) are provided here. What is the median age of the volunteers as listed below?

32, 37, 27, 23, 40, 50, 20, 44, 11, 21

Let's move on and figure out this problem.

How is the median defined?

Select one:

- The number that appears most frequently
- The difference between the largest and smallest number
- The sum of the numbers divided by the number of numbers
- The middle number when the numbers are listed in increasing order

Now that we know that the median of a set of numbers is the middle number, let us solve the problem again:

During a medical study, doctors measured the ages of all their volunteers. Some of ages (in years) are provided here. What is the median age of the volunteers as listed below?

32, 37, 27, 23, 40, 50, 20, 44, 10, 21

Writing the numbers in increasing order, we get:
20, 21, 23, 27, 32, 37, 40, 44, 50, 61

Comment on this hint

Since there are 10 numbers in the list, the median is \((5^{\text{th}} \text{ number} + 6^{\text{th}} \text{ number})/2\)

Comment on this hint

The 5\(^{\text{th}}\) number is 32, and the 6\(^{\text{th}}\) number is 37. So the median is:

\[
(32 + 37)/2 = 34.5
\]

Type 34.5.

Comment on this hint

Type your answer below (mathematical expression):

- 34.5

Submit Answer

Correct!
The coach for the All-Star Basketball team needs to pick one of two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jimmy</td>
<td>18,9,16,4,23,22,13</td>
</tr>
<tr>
<td>Nathalie</td>
<td>10,10,13,10,4,19,26</td>
</tr>
</tbody>
</table>

What is the **median** of number of points scored by Jimmy?

Comment on this question

Break this problem into steps

Type your answer below *(mathematical expression)*:

- 

Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the **median**?

Comment on this hint

The median is the middle number when the numbers are written in increasing order.

Comment on this hint

Select one:

- The sum of the numbers divided by the number of numbers.
- The middle number when the numbers are arranged in increasing order.
- The difference between the largest value and the smallest value in a set of numbers.
- The number seen most often in a set of numbers.

Submit Answer
Correct!

Now that we know what median is, let's try the problem again:

The coach for the All-Star Basketball team needs to pick one of two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jimmy</td>
<td>18,9,16,4,23,22,13</td>
</tr>
<tr>
<td>Nathalie</td>
<td>10,10,13,10,4,19,26</td>
</tr>
</tbody>
</table>

What is the median of number of points scored by Jimmy?

To find the median we must first write the numbers in increasing order as follows:

Jimmy 4,9,13,16,18,22,23

There are 7 numbers in the list, so the median is the 4th number. The answer is 16.

Type your answer below (mathematical expression):

- 16

Submit Answer

Correct!
The coach for the School Tennis Team needs to pick one of two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jose</td>
<td>12,6,15,9,6,23,23,14</td>
</tr>
<tr>
<td>Beth</td>
<td>18,8,26,5,23,22,13,15</td>
</tr>
</tbody>
</table>

What is the **median** of number of points scored by Beth?

Comment on this question
Break this problem into steps

*Type your answer below (mathematical expression):*

•

Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the **median**?

Comment on this hint

Select one:

- The sum of the numbers divided by the number of numbers.
- The difference between the largest value and the smallest value in a set of numbers.
- The number seen most often in a set of numbers.
- The middle number when the numbers are arranged in increasing order.
Now that we know what median is, let's try the problem again:

The coach for the School Tennis Team needs to pick one of two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jose</td>
<td>12,6,15,9,6,23,23, 14</td>
</tr>
<tr>
<td>Beth</td>
<td>18,8,26,5,23,22,13, 15</td>
</tr>
</tbody>
</table>

What is the **median** of number of points scored by Beth?

To find the **median** we must first write the numbers in increasing order as follows:

Beth 5,8,13,15,18,22,23, 26

There are 8 numbers in the list, so the median is \((4^{\text{th}} \text{ number} + 5^{\text{th}} \text{ number})/2\).

The \(4^{\text{th}}\) number is 15 and the \(5^{\text{th}}\) number is 18, so the median is:

\[
(15 + 18)/2 = 16.5
\]

Type 16.5
Matt runs a shoe store, and listed below are the store sales for the year 1997. What was the median of the monthly sales in 1997?

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1122</td>
</tr>
<tr>
<td>February</td>
<td>2506</td>
</tr>
<tr>
<td>March</td>
<td>1922</td>
</tr>
<tr>
<td>April</td>
<td>606</td>
</tr>
<tr>
<td>May</td>
<td>1048</td>
</tr>
<tr>
<td>June</td>
<td>905</td>
</tr>
<tr>
<td>July</td>
<td>1500</td>
</tr>
<tr>
<td>August</td>
<td>1639</td>
</tr>
<tr>
<td>September</td>
<td>1002</td>
</tr>
<tr>
<td>October</td>
<td>1018</td>
</tr>
<tr>
<td>November</td>
<td>2206</td>
</tr>
<tr>
<td>December</td>
<td>2053</td>
</tr>
</tbody>
</table>

Comment on this question

Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

Let's start with the definition. Which of the following is the right definition for the median?

Median is the middle number when the numbers are arranged in increasing order

Comment on this hint

Select one:
The difference between the largest value and the smallest value in a set of numbers.
The number seen most often in a set of numbers.
The middle number in a set of ordered numbers.
The sum of the numbers divided by the number of numbers.

Submit Answer

Correct!

Now that we know the definition of median, let us try the problem again:

Matt runs a shoe store, and listed below are the store sales for the year 1997. What was the median of the monthly sales in 1997?

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1122</td>
</tr>
<tr>
<td>February</td>
<td>2506</td>
</tr>
<tr>
<td>March</td>
<td>1922</td>
</tr>
<tr>
<td>April</td>
<td>606</td>
</tr>
<tr>
<td>May</td>
<td>1048</td>
</tr>
<tr>
<td>June</td>
<td>905</td>
</tr>
<tr>
<td>July</td>
<td>1500</td>
</tr>
<tr>
<td>August</td>
<td>1639</td>
</tr>
<tr>
<td>September</td>
<td>1002</td>
</tr>
<tr>
<td>October</td>
<td>1018</td>
</tr>
<tr>
<td>November</td>
<td>2206</td>
</tr>
<tr>
<td>December</td>
<td>2053</td>
</tr>
</tbody>
</table>

To find the median, we first need to arrange the numbers in increasing order.

Arranging the numbers in increasing order we get:

606, 905, 1002, 1018, 1048, 1122, 1500, 1639, 1922, 2053, 2206, 2506

Median is the middle number in this list.
Since there are 12 numbers in the list, the median is \((6\text{th number} + 7\text{th number})/2\)

The 6th number is 1122 and the 7th number is 1500, so the median is:

\[(1122 + 1500)/2 = 1311\]

Type 1311

Type your answer below (mathematical expression):

- 1311

Submit Answer

Correct!
Appendix C:

Variabilized Templates for Mode problems
Below is a set of numbers.

[146, 146, 78, 70, 16, 127, 25, 112, 22, 100, 29]

What is the mode?

Begin by writing out the numbers in order from least to greatest. Now it is easier to find the number that occurs the most.

The ordered list you wrote out should look something like this:

[16, 22, 25, 29, 70, 78, 100, 112, 127, 146, 146]

The most frequent number is 146.

Type in 146.

Type your answer below (mathematical expression):

• 146

Submit Answer

Correct!
You are previewing content.

What number should be added to the list below to get a mode of 20?

11, 20, 9, 18, 18, 11, 9, 20

Comment on this question

Break this problem into steps

Select one:

- 20
- 9
- 18
- 11

Submit Answer

Let's move on and figure out this problem.

What is the mode value of a set of numbers?

Comment on this question

Select one:

- The middle number in an ordered set of numbers.
- The sum of all the numbers divided by the number of numbers.
- The difference between the smallest value and the largest value of the set of numbers.
- The number seen most often in a set of numbers.

Submit Answer

Correct!

Great, now that you know that the mode is the number seen most often in a set of numbers, try the original problem again.

What number should be added to the list below to get a mode of 20?

11, 20, 9, 18, 18, 11, 9, 20

Comment on this question
First, organize the set of numbers in order on scratch paper.

Then you can easily find the most frequent number. The ordered list should look something like this:

9, 9, 11, 11, 18, 18, 20, 20

From the list, we can see that each number appeared twice.

So, the number 20 must be added to the list of numbers to get the mode to be 20. Choose the option with 20.

Select one:

- 9
- 18
- 11
- 20

Submit Answer

Correct!
What number should be added to the list below to get a mode of 13?

13, 22, 18, 13, 5, 22, 18, 13, 18

Comment on this question
Break this problem into steps

Select one:

- 18
- 22
- 5
- 13

Submit Answer
Let's move on and figure out this problem.

What is the mode value of a set of numbers?

Comment on this question
Select one:

- The difference between the smallest value and the largest value of the set of numbers.
- The sum of all the numbers divided by the number of numbers.
- The middle number in an ordered set of numbers.
- The number seen most often in a set of numbers.

Submit Answer
Correct!

Great, now that you know that the mode is the number seen most often in a set of numbers, try the original problem again.

What number should be added to the list below to get a mode of 13?

13, 22, 18, 13, 5, 22, 18, 13, 18

Comment on this question
First, organize the set of numbers in order on scratch paper.

Then you can easily find the most frequent number.
The ordered list should look something like this:

5, 13, 13, 13, 18, 18, 18, 22, 22

From the list, we can see the 5 appeared once, 13 appeared three times, 18 appeared three times, and 22 appeared twice.

Remember, we want the **mode** to be 13.

So, the number 13 must be added to the list of numbers to get the **mode** to be 13. Choose the option with 13.

Select one:

- [ ] 18
- [x] 13
- [ ] 22
- [ ] 5

Submit Answer

Correct!
The coach for the darts Game needs to pick one of the two players for the team.

The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chris</td>
<td>12, 5, 12, 20, 19, 19, 8, 2, 29, 9</td>
</tr>
<tr>
<td>Liz</td>
<td>34, 1, 15, 22, 17, 1, 30, 40, 1, 23</td>
</tr>
</tbody>
</table>

What is the mode number of points scored by Liz?

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer

Let's move on and figure out this problem.

To solve this problem, you need to know what mode means.

Which one of the following definitions describes mode the best?

Comment on this question
Select one:

- The highest number in the set.
- The middle number in the set.
- The number with the most occurrences.
- The number with the least occurrences.

Submit Answer

Correct!
Now that we know what the definition of **mode** is,

Let's try the original problem again.

The coach for the darts Game needs to pick one of the two players for the team.

The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chris</td>
<td>12, 5, 12, 20, 19, 19, 8, 2, 29, 9</td>
</tr>
<tr>
<td>Liz</td>
<td>34, 1, 15, 22, 17, 1, 30, 40, 1, 23</td>
</tr>
</tbody>
</table>

What is the **mode** number of points scored by Liz?

Remember the **mode** is the number that appears most often in a set of numbers.

We are only looking at the numbers on Liz's list which is:

34, 1, 15, 22, 17, 1, 30, 40, 1, 23

From this list:

34, 1, 15, 22, 17, 1, 30, 40, 1, 23

The number that appeared the most in this list is 1, so the **mode** is 1.

Type in 1.

Submit Answer
Correct!
Below is a set of numbers.

5, 28, 24, 14, 14, 19, 24, 29, 28, 24, 18, 10

What is the **mode** of the given set of numbers?

Comment on this hint

Begin by writing out the numbers in order from least to greatest. Now it is easier to find the number that occurs the most.

Comment on this hint

The ordered list you wrote out should look something like this:

5, 10, 14, 14, 18, 19, 24, 24, 24, 28, 28, 29

Comment on this hint

24 is the most frequent number.

Type **24**.

Comment on this hint

Type your answer below (mathematical expression):

- 24

Submit Answer
Correct!
Assistment

You are previewing content.

Below is a set of numbers.

5, 35, 23, 14, 14, 14, 18, 15, 27, 35, 35, 23, 18, 35, 9

What is the mode of the given set of numbers?

Comment on this question

The mode of a set of numbers is the most frequently occurring number.

Comment on this hint

Begin by writing out the numbers in order from least to greatest. Now it is easier to find the number that occurs the most.

Comment on this hint

The ordered list you wrote out should look something like this:

5, 9, 14, 14, 18, 18, 23, 23, 27, 35, 35, 35, 35, 38, 43

Comment on this hint

35 is the most frequent number.

Type 35.

Type your answer below (mathematical expression):

• 35

Submit Answer
Correct!
You are previewing content.

Listed below are the scores that May obtained in 10 geography tests.

7, 18, 15, 15, 18, 24, 26, 18, 18, 11

What is the mode of the given set of scores?

The mode of a set of numbers is the most frequently occurring number.

Begin by writing out the numbers in order from least to greatest. Now it is easier to find the number that occurs the most.

The ordered list you wrote out should look something like this:

7, 11, 15, 15, 18, 18, 18, 18, 24, 26

18 is the most frequent number. Type 18.

Type your answer below (mathematical expression):

18

Submit Answer
Correct!
Listed below are the scores that May obtained in 13 math tests.

7, 25, 7, 7, 18, 33, 25, 30, 21, 30, 18, 7, 33

What is the **mode** of the given set of scores?

The mode of a set of numbers is the most frequently occurring number.

Begin by writing out the numbers **in order from least to greatest**. Now it is easier to find the number that occurs the most.

The ordered list you wrote out should look something like this:

7, 7, 7, 7, 18, 18, 21, 25, 25, 30, 30, 33, 33

7 is the most frequent number. Type 7.

Type your answer below (mathematical expression):

• 7

Submit Answer
Correct!
The coach for the Basketball team has to select one of two people. The points obtained by May and Danny are listed below. What is the mode of May's points?

<table>
<thead>
<tr>
<th>May</th>
<th>5, 26, 18, 14, 18, 37, 25, 26, 22, 36, 18, 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danny</td>
<td>29, 6, 18, 18, 23, 24, 36, 37, 22, 29, 10, 10</td>
</tr>
</tbody>
</table>

The mode of a set of numbers is the most frequently occurring number.

Begin by writing out the numbers in order from least to greatest. Now it is easier to find the number that occurs the most.

Here are May's points arranged in order:

5, 10, 14, 18, 18, 18, 22, 26, 25, 26, 36, 37

18 is the most frequent number.

Type 18.
Comment on this hint

Type your answer below (mathematical expression):

- 18

Submit Answer

Correct!
The coach for the Basketball team has to select one of two people. The points obtained by Matt and Steve are listed below. What is the mode of Steve's points?

<table>
<thead>
<tr>
<th>Matt</th>
<th>29, 8, 11, 11, 16, 24, 26, 27, 21, 28,</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steve</td>
<td>7, 25, 11, 11, 19, 25, 26, 21, 11, 11</td>
</tr>
</tbody>
</table>

Comment on this question

The mode of a set of numbers is the most frequently occurring number.

Comment on this hint

Begin by writing out the numbers in order from least to greatest. Now it is easier to find the number that occurs the most.

Comment on this hint

Here are Steve's points arranged in order:

7, 11, 11, 11, 11, 19, 21, 25, 25, 26

Comment on this hint

11 is the most frequent number. Type 11.

Type your answer below (mathematical expression):

- 11
Submit Answer

Correct!
Appendix D:

Variabilized Templates for Range problems
Calculate the range of the following numbers:

59, 35, 9, 122, 95, 126, 122

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

•

Submit Answer
Let's move on and figure out this problem.

How do we find the range?

Comment on this question
Select one:

○ A. The number seen most often in this set of numbers.
○ B. The middle number in this set of numbers.
○ C. The sum of the numbers divided by the number of numbers.
○ D. The difference between the largest value and the smallest value of the set of numbers.

Submit Answer
Correct!

Now that we know how to find the range, try the original problem again.

Calculate the range of the following numbers:

59, 35, 9, 122, 95, 126, 122

Comment on this question
The range is the difference between the largest value number and the smallest value number.

Comment on this hint
59, 35, 9, 122, 95, 126, 122

From this list, the largest value number is 126, and the smallest value number is 9.

\[126 - 9 = 117\]

The range is 117, so type in 117.

Type your answer below (mathematical expression):

- 117

Submit Answer

Correct!
What number should be added to the following list to get a range of 125?

54, 32, 5, 107, 91, 55

Remember, the **range** is the difference between the **largest value** and the **smallest value** of the set of numbers.

So, if the number 106 was added to the list, then the **largest value** is 107, and the **smallest value** is 5. And, the difference between these two numbers are

107 - 5 = 102

102 is not the correct **range**.

If the number 130 was added to the list, then the **Largest value** is 130, and the **smallest value** is 5. And, the difference between these two numbers are

130 - 5 = 125

125 is the correct **range**, so choose the option with 130

Select one:

- 106
- 131
- 130
136

Submit Answer

Correct!
The coach for the baseball game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eric</td>
<td>15, 3, 14, 21, 19, 20, 7, 8, 26, 11</td>
</tr>
<tr>
<td>Alexa</td>
<td>18, 26, 11, 13, 9, 3, 22, 30, 16, 10</td>
</tr>
</tbody>
</table>

What is the range number of points scored by Alexa?

Comment on this question
Break this problem into steps

Type your answer below (mathematical expression):

- 

Submit Answer

Let's move on and figure out this problem.

To understand this problem, you must understand what range is.

Which of the following defines range the best?

Comment on this hint
Select one:

- The difference between the highest value and the lowest value.
- The highest value in the set.
- The lowest value in the set.
- The sum of all values in the set.

The range is the difference between the highest number and the lowest number.
Now that we know the definition of \textit{range}, let's try the main problem again.

The coach for the All-Star Basketball Game needs to pick one of the two players for the team. The table below shows the number of points each of the players scored in their last 10 games.

<table>
<thead>
<tr>
<th>Name of player</th>
<th>Number of points scored on the last ten games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eric</td>
<td>15, 3, 14, 21, 19, 20, 7, 8, 26, 11</td>
</tr>
<tr>
<td>Alexa</td>
<td>18, 26, 11, 13, 9, 3, 22, 30, 16, 10</td>
</tr>
</tbody>
</table>

What is the \textit{range} number of points scored by Alexa?

Remember, we are only looking at the numbers on Alexa's list which is:

18, 26, 11, 13, 9, 3, 22, 30, 16, 10

From this list:

18, 26, 11, 13, 9, 3, 22, 30, 16, 10

The highest number is 30 and the lowest number is 3.

Now we have to subtract the lowest number from the highest number.

30 - 3 = 27

So, the \textit{range} of Alexa's points scored is 27.
Type in 27.

Comment on this hint

Type your answer below (mathematical expression):

- 27

Submit Answer

Correct!
Calculate the **range** of the following numbers:

60, 33, 18, 124, 81, 141

Comment on this question

The **range** is the difference between the **largest value** number and the **smallest value** number.

Comment on this hint

From this list, the **largest value** number is 141, and the **smallest value** number is 18.

Comment on this hint

141 - 18 = 123

Type in 123

**Type your answer below (mathematical expression):**

- 123

Submit Answer

Correct!
What number should be added to the following list to get a range of 127?

68, 44, 19, 109, 84

Comment on this question

Remember, the range is the difference between the largest value and the smallest value of the set of numbers.

Comment on this hint

So, if the number 108 was added to the list, then the largest value is 109, and the smallest value is 19.
And, the difference between these two numbers are

109 - 19 = 90

90 is not the correct range so 108 is not the correct answer.

Comment on this hint

If the number 146 was added to the list, then the largest value is 146, and the smallest value is 19.
And, the difference between these two numbers are

146 - 19 = 127

127 is the correct range, so choose the option with 146

Select one:

- 108
- 147
- 146
- 164
Submit Answer

Correct!
You are previewing content.

Calculate the range of the following numbers:

187.89, 63.67, 24, 113, 10, 32, 148

The range is the difference between the largest value number and the smallest value number.

From this list, the largest value number is 187.89, and the smallest value number is 10.

Type in 177.89

Correct!
Calculate the range of the following numbers:

121, 41.33, 244.55, 21, 120, 42, 149, 2, 78.35

The range is the difference between the largest value number and the smallest value number.

From this list, the largest value number is 244.55, and the smallest value number is 2.

244.55 - 2 = 242.55

Type in 242.55

Correct!
What number should be added to the following list to get a range of 124?

53, 59, 31, 84, 22, 113, 95

Remember, the range is the difference between the largest value and the smallest value of the set of numbers.

So, if the number 91 was added to the list, then the largest value is 113, and the smallest value is 22. And, the difference between these two numbers are

113 - 22 = 91

91 is not the correct range so 91 is not the correct answer.

If the number 146 was added to the list, then the largest value is 146, and the smallest value is 22. And, the difference between these two numbers are

146 - 22 = 124

124 is the correct range, so choose the option with 146

Select one:

- 91
- 203
- 165
- 146
Submit Answer

Correct!
You are previewing content.

What number should be added to the following list to get a range of 123?

51, 40, 60, 35, 79, 120, 90, 118, 133

Remember, the range is the difference between the largest value and the smallest value of the set of numbers.

So, if the number 5 was added to the list, then the largest value is 133, and the smallest value is 5. And, the difference between these two numbers are

133 - 5 = 128

128 is not the correct range so 5 is not the correct answer.

If the number 10 was added to the list, then the largest value is 133, and the smallest value is 10. And, the difference between these two numbers are

133 - 10 = 123

123 is the correct range, so choose the option with 10

Select one:

- [ ] 10
- [ ] 9
- [ ] 5
- [ ] 17
Submit Answer

Correct!
Rachel's scores in 8 math tests are shown below. What is the range of Rachel's scores?

27, 32, 22, 30, 17, 26, 41, 48

Remember, the range is the difference between the largest value and the smallest value of the set of numbers.

The largest value is 48, and the smallest value is 17.

The range is the difference between 48 and 17.

48 - 17 = 31. Type 31

Type your answer below (mathematical expression):

• 31

Submit Answer
Correct!
Beth's scores in 5 science tests are shown below. What is the range of Beth's scores?

27, 20, 11, 47, 24

Remember, the range is the difference between the largest value and the smallest value of the set of numbers.

The largest value is 47, and the smallest value is 11.

The range is the difference between 47 and 11.

47 - 11 = 36. Type 36

Type your answer below (mathematical expression):

- 36

Submit Answer

Correct!
The basketball team coach needs to pick one of two people for the basketball team. Points obtained by Amy and Dan are given below.

What is the range of points obtained by Amy?

<table>
<thead>
<tr>
<th>Amy</th>
<th>21, 26, 20, 15, 19, 16, 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dan</td>
<td>19, 9, 16, 21, 27, 12, 23</td>
</tr>
</tbody>
</table>

Remember, the **range** is the difference between the **largest value** and the **smallest value** of the set of numbers.

The list of Amy's scores is: 21, 26, 20, 15, 19, 16, 30

The **largest value** is 30, and the **smallest value** is 15.

The range is the difference between 30 and 15.

30 - 15 = 15. Type 15

Type your answer below (mathematical expression):

- 15

Submit Answer
Correct!
The basketball team coach needs to pick one of two people for the basketball team. Points obtained by Gary and Jenny are given below.

What is the range of points obtained by Jenny?

<table>
<thead>
<tr>
<th>Gary</th>
<th>19, 9, 14, 19, 11, 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jenny</td>
<td>25, 17, 14, 20, 17, 30</td>
</tr>
</tbody>
</table>

Comment on this question

Remember, the **range** is the difference between the **largest value** and the **smallest value** of the set of numbers.

Comment on this hint

The list of Jenny's scores is: 25, 17, 14, 20, 17, 30

Comment on this hint

The **largest value** is 30, and the **smallest value** is 14.

Comment on this hint

The range is the difference between 30 and 14.

30 - 14 = 16. Type 16

Type your answer below (*mathematical expression*):

- 16

Submit Answer
Correct!