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A Nonlinear Computational Model of Floating Wind Turbines

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A NONLINEAR COMPUTATIONAL MODEL OF FLOATING WIND TURBINES

by

ALI NEMATBAKHSH

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Abstract

The dynamic motion of floating wind turbines is studied using numerical simulations. Floating wind turbines in the deep ocean avoid many of the concerns with land-based wind turbines while allowing access to strong stable winds. The full three-dimensional Navier-Stokes equations are solved on a regular structured grid, using a level set method for the free surface and an immersed boundary method for the turbine platform. The tethers, the tower, the nacelle and the rotor weight are included using reduced order dynamic models, resulting in an efficient numerical approach which can handle nearly all the nonlinear wave forces on the platform, while imposing no limitation on the platform motion. Wind is modeled as a constant thrust force and rotor gyroscopic effects are accounted for. Other aerodynamic loadings and aero-elastic effects are not considered. Several tests, including comparison with other numerical, experimental and grid study tests, have been done to validate and verify the numerical approach. Also for further validation, a 100 : 1 scale model Tension Leg Platform (TLP) floating wind turbine has been simulated and the results are compared with water flume experiments conducted by our research group. The model has been extended to full scale systems and the response of the tension leg and spar buoy floating wind turbines has been studied. The tension leg platform response to different amplitude waves is examined and for large waves a nonlinear trend is seen. The nonlinearity limits the motion and shows that the linear assumption will lead to over prediction of the TLP response. Studying the flow field behind the TLP for moderate amplitude waves shows vortices during the transient response of the platform but not at the steady state, probably due to the small Keulegan-Carpenter number. The effects of changing the platform shape are
considered and finally the nonlinear response of the platform to a large amplitude wave leading to slacking of the tethers is simulated. For the spar buoy floating wind turbine, the response to regular periodic waves is studied first. Then, the model is extended to irregular waves to study the interaction of the buoy with more realistic sea state. The results are presented for a harsh condition, in which waves over $17\,m$ are generated, and linear models might not be accurate enough. The results are studied in both time and frequency domain without relying on any experimental data or linear assumption. Finally a design study has been conducted on the spar buoy platform to study the effects of tethers position, tethers stiffness, and platform aspect ratio, on the response of the floating wind turbine. It is shown that higher aspect ratio platforms generally lead to lower mean pitch and surge responses, but it may also lead to nonlinear trend in standard deviation in pitch and heave, and that the tether attachment points design near the platform center of gravity generally leads to a more stable platform in comparison with attachment points near the tank top or bottom of the platform.
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Nomenclature

$AR$  Tank aspect ratio

$A_R$  Platform response amplitude

$A_W$  Wave amplitude

$C$  Marker function

$D$  Floater tank diameter

$D_c$  Cylinder diameter

$D_l$  Half length of the flume

$Fr$  Froude number $= \frac{u}{\sqrt{gL}}$

$H$  Heave amplitude

$H_{1/3}$  Significant wave height

$I_s$  Second moment of inertia tensor for the platform tank

$K$  Wave number

$K_c$  Keulegan-Carpenter number $= \frac{U_m}{f_cD}$

$Re$  Reynolds number $= \frac{uD}{\nu}$
\( S \)  Surge amplitude

\( S(\omega) \)  Wave spectrum

\( S_R(\omega) \)  Spectral density of the floating wind turbine response in a specific direction

\( S_W(\omega) \)  Spectral density of the incoming wave

\( St \)  Strouhal number = \( f_q D_c / u_\infty \)

\( T_0 \)  Spectral peak period

\( T_1 \)  Spectral mean period

\( f_q \)  Oscillation frequency

\( h \)  Water depth

\( k_i \)  Restoring coefficient of the tethers

\( l_0 \)  Neutral length of the tethers

\( l_i \)  Tethers length in tension

\( m_s \)  Mass of platform tank

\( m_{Eq} \)  Total mass of the tower, the rotor, and the nacelle

\( p \)  Pressure

\( r_1 \)  Platform tank radius

\( r_2 \)  Half of the platform tank height

\( std \)  Standard deviation

\( t \)  Time
$u_\infty$  Far field velocity  

$x_1$  Distance from an arbitrary grid point to the platform tank axis  

$x_2$  Distance from an arbitrary grid point to the platform tank midsection plane  

$M_{Pitch}$  Gyroscopic moment in pitch direction  

$M_{Yaw}$  Gyroscopic moment in yaw direction  

$F_{Eq-F}$  Equivalent force for using reduced order models for the tower, the rotor, the nacelle, and the spokes  

$F_{Eq-M}$  Equivalent moment for using reduced order models for the tower, the rotor, the nacelle, and the spokes  

$F_{Tethers}$  Tether forces  

$F_{Wind}$  Wind force  

$a_{scg}$  Center of gravity acceleration of the platform tank  

$e_i$  Unit vector along the tether  

$g$  Gravity acceleration  

$u$  Velocity  

$u_{Corr}$  Corrected velocity by the immersed boundary method  

$u_{scg}$  Center of gravity velocity of the platform tank  

$x_{si}$  points used for tracking the wind turbine platform  

$U_m$  Maximum relative oscillation velocity
\( \Gamma \) Volume occupied by the platform tank
\( \Omega \) Total volume of the computational domain
\( \epsilon \) Smoothing parameter for the platform tank/fluid interface
\( \eta \) Free surface
\( \mu \) Dynamic viscosity
\( \nu \) Kinematic viscosity
\( \omega \) Frequency
\( \omega_H \) Natural frequency in heave
\( \omega_P \) Natural frequency in pitch
\( \omega_S \) Natural frequency in surge
\( \phi \) Level set function or potential function
\( \rho \) Density
\( \xi \) Random variable between 0 and 2\( \pi \)
\( \alpha_s \) Angular acceleration of the platform tank
\( \omega_s \) Angular velocity of the platform tank
\( \gamma \) Rotor speed
Chapter 1

Introduction

1.1 Description of floating wind turbines

In order to handle the growing rate of the world’s energy consumption and alleviate the problem of limitation of fossil fuel resources, renewable energy technologies have received renewed interest. Renewable energy can be harvested from different natural sources, such as solar, wind, waves and currents. Wind energy has emerged as a viable option due to its relatively low production costs.

Two types of wind turbines can be categorized, land-based wind turbines and offshore wind turbines. At present the cost of land-based wind turbine is less and the design is simpler than offshore wind turbine. Therefore, most of the currently developed wind energy systems are land based (Figure 1.1), but there is a lot more potential in the oceans (Figure 1.2a) to generate energy from the wind.

There are potential advantages and benefits for offshore wind turbine. Far from the coastlines there are stronger and steadier winds, which lead to more potential to harvest energy from wind. An example can be seen in Figure 1.2b which clearly represents the large resources on the east and west coast of US. Furthermore, wind
turbine noise, which is a severe challenge to construct land-based wind turbines, is not a concern for offshore wind turbines. In addition, the places best suited for the land-based wind turbines such as the plains of the Midwest United States are often far from the densely populated areas along the coasts.

![Figure 1.1: Typical land-based wind turbines. Figure from reference [1].](image)

The challenge of constructing offshore wind turbine is the complexity of design and the economical concerns. Imprecise design of the offshore wind turbine design can result in economically prohibitive cost. This emphasizes the importance of understanding the physical aspects related to the proper design of the offshore wind turbines.

Offshore wind turbines can be further categorized as fixed bottom and floating wind turbines. Fixed bottom wind turbines are more suitable for shallow waters (usually less than 30\(m\)), but for deeper waters installing fixed based wind turbines is not economically feasible. Bottom fixed offshore wind turbines have the advantages of lower maintenance cost, due to the closeness to the beach, and simpler design in comparison with floating wind turbines. One of the main negative points of the fixed based wind turbines is the negative visual impact on coastlines and beaches has delayed projects such as Cape Wind on the East Coast of US for years.

Placing floating wind turbines far offshore, in deep water, avoids many of the concerns of fixed base offshore wind turbines such as the negative visual impact and limitation in the potential wind resource. However, the complexity in design
of these structures while keeping the cost reasonable, is the main challenge facing floating wind turbines.

A floating wind turbine consists of a standard wind turbine mounted on a floating platform. The main components of a standard wind turbine are the tower, nacelle, and rotor. The size of the tower depends largely on the rotor diameter, nacelle, and the height at which the wind is to be harvested. Most design studies, simulations, and experiments of floating turbines have focused on a 5 MW turbine as a benchmark. The main challenge in installing the wind turbine on a floating platform, is the instability of the whole platform due to the wind, wave and current effects. Therefore, design of a stable platform is one of the most important considerations in the design of floating wind turbines.

The stability of a floating wind turbine can be provided by three different mechanisms. Those are the difference in the center of gravity (\(C_g\)) and center of buoyancy (\(C_b\)), water-plane area, and/or the mooring system. Based on these three mechanisms, different concepts for the design of the floater platforms have been introduced. They are mainly borrowed from the oil and gas industry. Figure 1.3 represents the
typical configuration of these three types floating wind turbines.

The first concept is the tension leg platform which remains stable mainly due to the mooring system. The mooring system for the tension leg platform consists of a number of pre-tensioned tethers which are anchored to the ocean floor. The pre-tension of the tethers are provided by the extra buoyancy of the platform compared to the wind turbine weight.

The second concept is the spar buoy which mainly remains stable by a significant difference in the position of $C_g$ and $C_b$. This difference creates a large restoring moment the keeps the platform stable. Mooring system for a spar buoy can be slack or taut tethers and are employed mainly to keep the platform in place. They do not contribute much to the stability of the platform.

The third concept is a barge type floating wind turbine which takes advantage of a large water-plane area on the free surface for stability. The mooring system is merely used to keep the platform in place. Also there are some concepts which use two or three of these three stability mechanisms such as semi-submersibles and SWAY concepts which will not be covered in the current research.

Most of the previous design studies on floating wind turbine floaters indicated that the barge type floating wind turbine, although attractive in shallow waters due to the short draft, might not be very practical because of large pitch and roll motion in response to incoming waves and wind. Therefore, the models developed in this thesis were used to study the tension leg platforms and the spar buoys.
1.2 Background

Research in the field of floating wind turbine on both experimental and numerical sides are relatively new and are based on previously described concepts for the stability of floating platforms to carry the wind turbines.

In the current research, the focus is on numerical simulations, so a literature review on floating wind turbine with more focus on the numerical model is provided first. Floating wind turbines were first envisioned by Heronemus [14], but it was not until the mid-1990s that they were examined further in Refs. [15–20]. These studies considered various aspects of the feasibility, design, and economics of the concept. The overall findings were that floating turbines are technically feasible with existing or near-term technology; that some design challenges still exist; and that reducing platform and mooring system costs is critical for economic viability. The cost does,
in turn, depend on the wind and wave loading on the floating wind turbine and the
detailed motion of the coupled platform, turbine, and mooring system.

Studies focused on modeling the response of different designs to operational and
extreme loads are relatively recent, and have used either linear frequency-domain
(LFD) analysis or time-domain dynamics (TDD) models; or a combination of both.
LFD approaches are mainly based on linear, or at most weakly nonlinear, descrip-
tions of the waves, assuming potential flow. Viscous effects are neglected, and it is
assumed that platform motions are small compare to platform size, so that radiation,
diffraction, and hydrostatic effects can be superimposed. Also, wave impact (slam-
ming) effects are neglected. In TDD modeling the forces on the platform are usually
computed using the Morison formula, where drag and inertial effects are considered
as two separate terms. Inertial effects are included as added mass, and drag forces
are included using empirically determined drag laws. The standard Morison formula
does not include radiational damping and scattering effects, nor does it account for
forces in the heave direction.

A brief summary of investigations using the above-mentioned models is given
next. Utsunomiya et al. [21] studied the optimum shape for spar type floating wind
turbine and Phuc and Ishahara [22] conducted a feasibility study of new platform
designs. Both simulations were conducted by TDD using the Morison formula. Since
this formula was originally developed for long cylinders, both papers modified
it by adding mass and damping terms in the heave direction. Karimirad et al. [23]
did a code-to-code comparison between HAWC2, developed by the Riso National
laboratory in Denmark [24], and USFOS/vpOne from USFOS Ltd. in Norway [25].
In both codes the Morison formula is used to find the forces in the direction of wave
propagation on the platform and heave forces are found by integrating the pressure.
Subsequently, Karmirad and Moan [26] used the HWC2 code to model the NREL
5 MW spar buoy floating wind turbine. Based on their verified coed, they designed a short spar buoy platform for shallow waters [27]. Bachynski and Moan [28] did a design study on a wide range of TLP sizes and displacements. They concluded that usually the overall magnitude of the TLP response will be reduced by increasing the platform displacement. Wayman et al. [29] designed a full scale tension leg platform based on frequency domain analysis to carry a 5 MW wind turbine and compared the response of the platform with a barge type floating wind turbine. Sclavanous et al. [30] did a wide range of design study with the same approach on different types of floating wind turbine platforms (tension leg, spar buoy and barge). Later they proposed two new designs for floating wind turbines [31], in order to reduce the installation cost and weight of floating wind turbine. Shim and Kim [32] performed a numerical simulation of mini-TLP platforms by considering the coupled dynamics of blade-rotor, mooring system, and hydrodynamics effects. An extensive summary of the applicability and limitation of the LFD and TDD approaches can be found in Jonkman [33] who compared predictions by both models. Jonkman and Matha [34] did a comprehensive dynamic analysis for all the three major types of floating wind turbines in time domain. Their analysis showed that the barge type floating wind turbine has larger response to wave and wind when compared with the spar buoy and TLP models. In Roddier et al. [7] the recently conceived WindFloat platform was modeled numerically and Nielsen et al. [35] performed an integrated dynamic analysis of spar buoy floating wind turbine (HYWIND).

In all these studies [7, 31–33, 35], the WAMIT code [36], which is based on using a potential function in the frequency domain, is used. However, in references [7, 33–35] the final solution is in the time domain so that other effects, such as viscous dissipation and previously radiated waves (memory effects), can be added by taking advantage of TDD techniques. The empirical nature of the Morison formula may
lead to limitations for new design shapes of floating platforms and these can be seen in Roddier et al. [7] who had to conduct experiments to find the viscous drag forces, due to the special shape of WindFloat columns. Other examples of the modeling of hydrodynamic forces on a floating wind-turbine platform, using either the time or the frequency domain, can be found in references [37–40].

On the experimental side, a few large scale models have already been built. Those include two turbines, one by Statoil called Hywind and another one called SWAY. The Hywind floating wind turbine is the first large scale wind turbine deployed off the coast of Norway and it is carrying a 2.3 MW wind turbine. SWAY is an innovative concepts which uses both difference in $C_g$ and $C_b$ and pre-tension tethers for the stability of the platform. In addition, the Blue H prototype which is based on tension leg platform concept has been tested off the coast of Italy. Energias de Portugal and Principle Power Inc. have recently deployed a full-scale 2 MW WindFloat platform off the coast of Portugal which remain stable by a semisubmersible platform. Hywind, BlueH, and WindFloat floating wind turbines can be seen in Figure 1.4.

To help overcome the limitations of LFD and currently used TDD type models, here we develop a time domain model, using the three-dimensional full Navier-Stokes equations to describe the interactions of large-amplitude waves on a floating turbine platform. Fully nonlinear waves are likely to result in maximum structural loading, which in turn is likely to drive design decisions for these systems. In this approach there is no limitation on the platform motion, the free surface motion is arbitrary and there is no dependency on experimental “corrections.” The floating platform is included using an immersed boundary (IB) method [41, 42]. Tether forces, tower and rotor weights, rotor gyroscopic effects, and wind loading on the rotor are modeled as auxiliary forces in the Navier-Stokes equations. This allows us
Figure 1.4: Three different large scale model floating wind turbines. (a) Hywind (spar buoy concept). Figure from reference [5]. BlueH (TLP concept). Figure from reference [6]. (c) WindFloat (semisubmersible concept). Figure from reference [7].

to limit the fluid solver to a region near the water-air interface, thus significantly reducing the computational cost. The present numerical formulation focuses on the effect of wave loading on platform motions. Winds effects are modeled by simply specifying a constant thrust force. Other aerodynamic loading and aero-elastic effects are not considered here.

In the next section the thesis contribution along with a summary of each chapter will be described and in the following chapter details on the computational model and results will be presented. A relatively shorter discussion on the computational model and initial results can be found in [43–46].

1.3 Outline

The rest of the thesis is organized as follows. In the next chapter, the computational model is described including; modeling the regular and irregular random waves; describing the governing equations employed to model the fluid motion; modeling
method for tracking the free surface; the method for tracking the solid platform; a model for the effect of the tethers; a model for the wind forces, nacelle and tower mass on top of the platform plus inclusion of the gyroscopic effects of the rotor. Finally, an iteration approach is described which allows for coupling the motion of the floating wind turbine platform in interaction with waves, with reduced order models employed for including the tower, nacelle, rotor, and wind load effects. In chapter three, extensive tests are performed to validate and verify the developed numerical model. The accuracy of solving the Navier-Stokes equations along with the mass conservation equation is verified by studying the lid driven cavity problem. The free surface tracking method (level set method) is verified by comparing the results with available analytical solutions. The immersed boundary method for tracking the solid-fluid interaction is verified and validated by comparing the results with both experimental and numerical data. The combination of both free surface and solid-fluid interaction model is verified and validated by performing a grid refinement study and comparing experimental and numerical data. Finally, the verified numerical approach has been used to model a 100 : 1 scale model TLP wind turbine, tested in a water flume, to compare the numerical results with the experimental data. The experimental research has been done concurrently in the our research group [47].

In chapter four, the numerical model is used for the full-scale modeling of a tension leg platform floating wind turbine. First, the response of the tension leg platform wind turbine to a single amplitude and frequency wave is studied in some detail. The flow field around the platform is examined, the validity of the linear assumption for the platform response to different amplitude waves is investigated, and it is demonstrated that linear assumption models may lead to over prediction of wave forces. Then, a simple design study is performed and the interaction of the
wind turbine with a large amplitude nonlinear waves is presented.

In chapter five the focus is on a spar buoy floating wind turbine. The responses of the spar buoy to regular periodic waves and random, irregular waves is studied. The response has been converted to the frequency domain to examine the behaviour of the platform at the various natural frequencies modes. The model has also been used for simulating an extreme condition in which waves larger than 17 meters is generated and the response of the spar buoy is measured. Parametric design studies on the tether attachment point, stiffness, and on the aspect ratio of the platform are conducted and it is shown that the tethers very close to the bottom or top of the platform are not a very attractive designs, due to the large platform response.

### 1.4 Thesis contribution

The main contributions of this research are as follows:

- Development of a nonlinear model based on Navier-Stokes equations for accurate modeling the wave forces on a floating wind turbine. To the best knowledge of the author, this is the first dedicated code based on Navier-Stokes equations for modeling floating wind turbines. The model accounts for the nonlinear free surface wave forces on the platform with no restriction on the platform motion.

- Level set and Immersed boundary method can be successfully applied to model floating wind turbine interaction with waves.

- Extensive tests have been done to verify and validate the numerical model. Those tests include grid study, comparison with experimental, and comparing with available analytical solutions. Furthermore, a 100 : 1 scale model has been simulated and the results in different modes are compared with the experimental data, conducted in our research group, to validate the numerical model.
• By using the nonlinear model, low strength vorticities are observed behind the TLP in the transient region but not in the steady state, probably due to small \( Kc \) number. The obtained result was for a single frequency wave and moderate amplitude wave.

• Results for the effects of wave height on the response of the TLP to a single frequency wave, show that assuming a linear trend for the effects of wave height on the platform, as is often assumed by using linear models, may lead to over-prediction of the platform response.

• A spar buoy is modeled for an extreme ocean wave state in which waves over 17 m are simulated. Temporary submergence of the entire spar buoy platform and pitch over 10\(^\circ\) for the spar buoy are observed. These cases cannot be captured accurately using linear models.

• Parametric design studies has been conducted on a spar buoy floating wind turbine for one short-term state of the ocean waves \((T_0 & H_{1/3})\). Effects of tether attachment points, stiffness, and aspect ratio of the platform on different response modes are investigated.

• It is observed that tether attachment points near the platform center of gravity, generally lead to a more stable platform in compare with attachment points near the top or bottom of the platform.

• It is noted that increasing the tether stiffness is not always a trustable approach for limiting spar buoy surge and heave motions.

• The simulations suggest that increase in the aspect ratio of the spar buoy platform, although could result in decrease in the platform response to some degrees but it could lead to possible increase in the wind and wave moment on the platform.
Chapter 2

Computational Model

In this chapter, the nonlinear computational model, developed in this research, will be presented. It will then be used in the next chapters to model tension leg and spar buoy floating wind turbine platforms.

In the present model, the focus is on the hydrodynamic forces on the platform and its response. Those cannot, however, be computed accurately without accounting for static and dynamic effects of the tower, nacelle and rotor, wind forces, and the forces from the tethers. A fully resolved computational model of the full platform may be possible, but would certainly be very demanding on computational resources. It is likely that the elasticity of the tower, for example, plays only a small role in determining the response of the platform to wave loading. Similarly, using simplified models for other aspects, such as the wind load and the tethers, is unlikely to change the results that we are interested in to any significant degree. Thus, we have developed a hybrid computational model where the hydrodynamics and the platform motion are fully resolved, while the wind loading, the tower, nacelle and rotor mass and inertia, rotor gyroscopic effects and the mooring system are included using reduced models that are coupled to the hydrodynamic simulation. Although
our aerodynamic and dynamic models are relatively simple at this point, we believe that the results show the potential of integrating such models with our hydrodynamic model.

The approach developed here is applicable to most platform shapes, but in this chapter a typical TLP will be used to initially describe the model. In the next chapters, we will use the numerical model for the detail studying of the full-scale TLP and spar buoy with a standard NREL 5 MW wind turbine [48].

A typical floating wind turbine (TLP) is shown in Figure 2.1. It consists of a cylindrical buoyant tank with concrete at the bottom to provide the ballast for the platform and the standard NREL 5 MW wind turbine [48]. The ballast is mainly used to lower the center of gravity of the platform, hence reducing the response of the wind turbine to the incoming ocean winds and waves. The platform is tethered to the ocean floor by four prestressed tethers that are attached to spokes extending horizontally from the bottom of the platform. The platform and tether orientations with respect to the incident waves are shown in Figure 2.1. A tower and a turbine are mounted on top of the platform and the blades are rotating, thus causing a gyroscopic effect. Surge, sway, and heave for translational and roll, pitch, and yaw for rotational motion is used to describe the floating wind turbine motion as shown in Figure 2.1.

The response of the floating wind turbine to wind and waves is modeled, on a three dimensional rectangular domain which encompasses the water, the water-air interface, and a portion of the air above the water. The density ratio of water to air is set to 1000. The domain is discretized using a regular structured grid, consisting of straight but unevenly spaced grid lines. Figure 2.2 shows the grid and the model from different view angles. The grid lines are spaced in such a way to resolve relatively well the region around the platform. The platform is located
Figure 2.1: The base line tension leg platform with a 5MW wind turbine. Notation: 1-Rotor, 2-Nacelle, 3-Tower, 4-Platform tank, 5-Tank ballast section, 6-Spoke, 7-Tether

far enough from the wave generator to minimize effects of waves reflected from the platform and to let the waves develop fully before interacting with the platform. The benchmark computational domain is $1600 \times 100 \times 175 \text{ m}$ in length, width and height, respectively.

At the upstream end, waves are generated by specifying the inlet/outlet velocities. The top boundary conditions allow in and out flow to match the velocity specified at the wave maker end, thus ensuring that mass conservation is satisfied
Figure 2.2: Three different views of the computational grid. The grid consists of straight lines that are unevenly spaced to give a fine resolution around the platform. $150 \times 64 \times 64$ grid points are used for the base line simulation.

over the entire numerical domain at every time step. Full slip, no-through flow, boundary conditions are imposed on all other sides of the flume. No-slip boundary conditions are imposed on the exterior surfaces of the buoyant tank but boundary layers are, however, not fully resolved since the grid resolution at the tank surface is not fine enough.

The computational model consists of the wave generator, flow solver, tracking methods for the free surface and solid, inclusion of reduced order models for tower, nacelle and rotor weight and inertia, considering rotor gyroscopic effects, and modeling the tethers. In the next section, the model to solve Navier-Stokes equations and immersed boundary method are described along with methods for modeling tether forces, gyroscopic effects, and wind thrust force. In section 2.2, the free surface model will be described and in section 2.3 and section 2.4, methods to generate regular and irregular waves will be summarized.
2.1 Navier-Stokes equations and the immersed boundary method

The fluid flow is described by the one-fluid Navier-Stokes equations, where one set of equations is used for the whole domain and the different fluids are identified by their different material properties. The momentum equation, with extra forces added to couple in the effect of the tower, rotor, nacelle, tethers and the wind is:

\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \mathbf{u}) \right) = -\nabla p + \rho \mathbf{g} + \nabla \cdot \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \ldots \nonumber \\
F_{Eq-F} + F_{Eq-M} + F_{Tethers} + F_{Wind}. \tag{2.1}
\]

The fluids are taken to be incompressible so the continuity equation is:

\[
\nabla \cdot \mathbf{u} = 0. \tag{2.2}
\]

The last four terms on the right hand side of Equation 2.1 will be described later. The governing equations are solved by an explicit second-order predictor-corrector method on a staggered grid. The advection terms are discretized using a second order ENO method and a simple second order centered difference approximation is used for the viscous terms. The pressure equation is solved using a Semi Coarsing Multigrid method [49].

To track the free surface we use a level set method which will be described in details in section 2.2.

The platform consists of a tank that is ballasted with a higher-density material at the bottom and to track its motion we use an immersed boundary method [41,42]. A marker function \( C \), defined such that \( C = 1 \) inside the solid and zero everywhere else, is used to identify the solid. Determining whether a point is inside or outside
a given region is, in general, a complex problem [50], but here our geometry is relatively simple (cylinder). Thus, a point is inside the cylinder if it is less than a radius away from the centerline and less than half the cylinder height from the midsection plane. We also want the marker function to transit smoothly from one value to the other, with the width of the transition being of the order of the grid spacing. Thus, we find the marker function by the following formula:

\[ c_i = 0.5 + 0.5 \left( \frac{(r_i - x_i)^3 + 1.5 \epsilon^2 (r_i - x_i)}{(r_i - x_i)^2 + \epsilon^2)^{1.5}} \right) ; \quad i = 1, 2; \quad C = c_1 c_2 \]  

(2.3)

where \( \epsilon \) can be adjusted to control the thickness of the transition zone.

The Navier-Stokes equations are solved in the whole domain, including in the region occupied by the platform tank, resulting in a velocity field that generally does not satisfy the rigid body motion constraint. To correct the velocity we first find the linear and angular momentum by integrating the velocity found by the solution of the Navier-Stokes equations, over the part of the domain occupied by the platform tank:

\[ m_s u_{s cg} = \int_{\Omega} C \rho d\Omega, \]  

(2.4)

\[ I_s cg \omega_s = \int_{\Omega} C (r \times u) \rho d\Omega. \]  

(2.5)

The components of the moment of inertia tensor with respect to center of gravity,

\[ I_{s cg} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \]  

(2.6)

are computed at every time step by integrating over the part of the domain occupied by the platform tank. After the velocity at the center of gravity, and the angular
velocity, have been found, the velocity inside the platform tank is corrected by:

\[ u_{\text{Corr}} = u + C(u_{\text{cg}} + (\omega_s \times r) - u), \quad (2.7) \]

and the rigid body constraint in the solid part will be satisfied. Since the original velocity field and the velocity given by Equation 2.7 are both divergence free, the new velocity field should be divergence free everywhere. Sometimes that is not exactly true at the interface and we iterate a few times to correct that ([41,42]).

While we capture the interaction of the floating tank with the incoming waves by solving the Navier-Stokes equations, the effects of the wind and the dynamic effects of other parts of the platform (tower, nacelle and rotor) are included using reduced models that result in forces and/or moments that act on the platform. The weight and inertia of the tower, rotor, and nacelle are modeled by placing an equivalent point mass at the proper distance from the center of gravity of the platform tank:

\[ \mathbf{F}_{\text{Eq}-F_1} = m_{\text{Eq}} \mathbf{g} + m_{\text{Eq}}(a_{\text{cg}} + \alpha_s \times r) \]

\[ = m_{\text{Eq}}(\mathbf{g} + (a_{\text{cg}} + \alpha_s \times r)). \quad (2.8) \]

This force also generates a moment equal to \( M_{\text{Eq}_1} = r \times \mathbf{F}_{\text{Eq}-F_1} \) that is added to the center of gravity of the tank. The wind thrust force on the rotor is treated in the same way as \( \mathbf{F}_{\text{Eq}-F_1} \).

Gyroscopic effects, due to rotation of the rotor, can be important when the wind turbine is in operational mode and the rotor is rotating. Their effect on the motion should, however, be small under operating conditions. Therefore, using the dynamic equations of the motion for the rotor [51,52], it can be shown that the dominant gyroscopic moments are pitch-induced yaw motion and yaw-induced roll motion.
Those can be written in the following form:

\[ M_{\text{Yaw}} = (I_{yy} - I_{xx})\gamma \omega_{\text{Pitch}} \]
\[ M_{\text{Pitch}} = (I_{xx} - I_{zz})\gamma \omega_{\text{Yaw}}. \]  

(2.9)

These moments are transferred from the rotor to the floater tank center of gravity \( (M_{E_{eq2}}) \), since the rotor cannot have independent roll or pitch motion with respect the platform (except for turbine alignment with wind direction, which happens slowly enough so that its gyroscopic effects can be neglected [53]).

To model the four pre-tensioned vertical tethers, attaching the platform to the ocean floor, we use a simple Hooke’s law for the tether tension in Equation 2.1.

\[ F_{\text{Tethers}} = k \left( \max(l_i - l_0) - l_0 \right) e_i \]  

(2.10)

Equation 2.10 shows that if the tension becomes zero, the tethers will go slack. The vertical tethers are attached to the end of horizontal spokes extending out from the bottom of the tank to keep the platform stable. The forces from the tethers are added at the surface of the tank but the additional moment from the spokes \( (M_{E_{eq3}}) \) are included in the model separately. The spokes are not modeled using an immersed boundary method but their buoyancy effects are included as a force \( F_{E_{eq-F_2}} \). The drag force on the spokes is assumed to be small and is neglected.

The sum of all the forces given by the models of the tower, nacelle, rotor and spokes results in a total body force on the floater tank (4th term on the right hand side of Equation 2.1), that is added to the center of the ballasted volume of the tank, distributed over six grid points in each direction:

\[ F_{E_{eq-F}} = F_{E_{eq-F_1}} + F_{E_{eq-F_2}}. \]  

(2.11)
A summation of the moments from the models for the tower, nacelle, rotor, and spokes gives a total moment:

\[ M_{Eq} = M_{Eq1} + M_{Eq2} + M_{Eq3}. \]  \hspace{1cm} (2.12)

The moment is represented as three force couples (\( F_{Eq-M} \) in equation Equation 2.1) in the pitch, roll, and yaw directions and smoothed over the ballasted volume of the tank, using a truncated 3D Gaussian to distribute the forces. Table 2.1 shows how the different parts of floating wind turbine are treated.

The overall numerical algorithm is shown in Figure 2.3. The outer loop is the time integration and the second loop is the iteration where the tower inertia is coupled with the Navier-Stokes solver. Each iteration consists of predicting the platform motion with the Navier-Stokes equations and correcting the prediction by linear and angular momentum conservation (Equation 2.4 and Equation 2.5). In step 1 the values from the previous time step, including position, velocity and acceleration are assumed as the initial guess. The same is done for the marker function and the velocity in the whole domain. The density and viscosity at every grid point in the whole domain, including inside the platform tank, are then assigned, based on the free surface and the platform position (step 2). Equation 2.1 is solved in step 3, and the velocity in the whole domain at the new time level is found. In step 4, the platform position is updated and used in step 5 to update the marker function identifying the solid. In step 6, the platform position from step 4, along with the marker function found in step 5, is used to calculate the linear and angular velocity of the platform, based on the conservation of linear and angular momentum. These two values are used to correct the velocity found by solving the Navier-Stokes equations in step 7 and then used for tracking the interface by solving the level set
and the reinitialization equation. Convergence is reached when the values calculated in step 4 to 7 do not change. Usually, the angular acceleration of the platform (due to the height of the tower) converges more slowly than other quantities. Once the solution has converged, the other quantities listed in the box are updated in step 9, otherwise we assign these values as the new guess and start the procedure again. After convergence, the variables shown in step 9 are updated and are used in step 10 for updating the free surface position (solving the level set and the reinitialization equation). Then we go to the next time step. As implemented, our numerical model is second order in time, but for simplicity we show only the first order version in Figure 2.3. For the initial time steps, we usually require around 10 iterations for convergence, but after that, one or two iterations are usually sufficient. For tall towers, fluctuations of the angular acceleration are large and underrelaxation is used after each iteration (step 8).

The simulations have been conducted on the Linux servers supported by Worcester Polytechnic Institute Computational Center. Single Intel(R) Xenon(R) Processor X5690 with 3.47Ghz Clock speed has been used for the simulations. The baseline simulations for 600 s real time modeling takes about 170 h of CPU time.
Table 2.1: Modeling strategies for the different components of the Flowing wind turbine platform

<table>
<thead>
<tr>
<th>Components</th>
<th>Effects considered</th>
<th>Modeling method</th>
<th>Resultant force and moment on the floater tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank</td>
<td>Fully captured</td>
<td>IB method</td>
<td>-</td>
</tr>
<tr>
<td>Tower</td>
<td>Weight and inertia</td>
<td>Lumped mass</td>
<td>$F_{E} - F_{1}$ and $M_{E}$</td>
</tr>
<tr>
<td>Nacelle</td>
<td>Weight and inertia</td>
<td>Lumped mass</td>
<td>$F_{E} - F_{2}$ and $M_{E}$</td>
</tr>
<tr>
<td>Rotor</td>
<td>Weight and inertia</td>
<td>Rotating Lumped mass</td>
<td>$F_{E} - F_2$ and $M_{E}$</td>
</tr>
<tr>
<td></td>
<td>Gyroscopic</td>
<td></td>
<td>$M_{E}$</td>
</tr>
<tr>
<td>Tethers</td>
<td>Force and moment on the platform</td>
<td>Hook’s law</td>
<td>Equation 2.10 and $M_{E}$</td>
</tr>
<tr>
<td>Spokes</td>
<td>Volume</td>
<td>Hydrostatic</td>
<td>$F_{E} - F_2$</td>
</tr>
</tbody>
</table>
\[ x_{n+1}^s = x_n^s, \quad u_{n+1}^{scg} = u_n^{scg}, \quad a_{n+1}^{scg} = a_n^{scg} \]
\[ \omega_{n+1}^s = \omega_n^s, \quad \alpha_{n+1}^s = \alpha_n^s, \quad u^* = u^n, \quad C^* = C^n \]  

\[ \rho = f(C^*, \phi), \quad \mu = f(C^*, \phi) \]  

\[ u^* = u^n - (\nabla \cdot (u^* u^*)) - \frac{\nabla \rho}{\rho} + \nabla \left( \frac{\mu}{\rho} (\nabla u^* + \nabla u^*^T) \right) + \ldots \]
\[ g + F_{Eq} + F(C^*, a_n^{scg}, \alpha_n^s)/\rho + F_{Eq - M}(C^*, a_n^{scg}, \alpha_n^s)/\rho + \ldots \]
\[ F_{Tethers}(C^*)/\rho + F_{Wind}/\rho \Delta t \]

\[ x_{n+1}^s = x_n^s + \omega_n^s \times (x_n^s - x_{n+1}^{s cg}) \Delta t \]

\[ C^{**} = \text{solid finder} \left( x_{n+1}^{**} \right) \]

\[ u_{n+1}^{**} = \frac{\int C^{**} u^{**} \rho d\Omega}{\int C^{**} \rho d\Omega} \]
\[ \omega_{n+1}^{**} = \frac{\int C^{**} (x - x_{n+1}^{**}) \rho u^{**} d\Omega}{\int C^{**} \rho d\Omega} \Delta t \]
\[ \alpha_{n+1}^{**} = \frac{v_{n+1}^{**} - v_n^{**}}{\Delta t} \]

\[ u^{***} = u^* + C^{**} (u_{n+1}^{**} + \omega_{n+1}^{**} \times (x - x_{n+1}^{**}) - u_{n+1}^{**}) \]

\[ \frac{\partial \phi}{\partial t} + u_{n+1}^{**} \cdot \nabla \phi = 0, \quad \frac{\partial \phi}{\partial t} + s(\phi)(\sqrt{\nabla \phi \cdot \nabla \phi} - 1) = 0 \]

Figure 2.3: A flowchart of the algorithm for numerical modeling of a floating wind turbine, based on the Navier-Stokes equations.
2.2 Level set method for free surface tracking

The level set method, introduced by Osher and Sethian [54], is one type of Eulerian methods for following an interface. The method is one of the most popular methods for tracking the interface due to its simplicity. Here, a brief description of the level set method will be presented. Details on level set method can be found in [55]. Also [56] presented a complete review on different numerical techniques used to model multiphase flow problems.

The main idea of the method is using a distance function, $\phi$, for tracking the interface. The value equal to zero is assigned to the interface of two phases, positive value to one phase and negative value to the other phase. These values are equal to the minimum distance from the interface.

Equation 2.13 is the mathematical representation of the level set function. A simple presentation of the level set function for a water flume, consists of water and air, is shown in Figure 2.4. In this figure values less than zero represent the air, values greater than zero represent the water, and zero is assigned for the water-air interface.

\[
\begin{align*}
\phi(x,t) &> 0 \quad \text{Fluid 1} \\
\phi(x,t) &< 0 \quad \text{Fluid 2} \\
\phi(x,t) &= 0 \quad \text{Interface}
\end{align*}
\]  
(2.13)

Since the interface is determined by $\phi = 0$, the value of the interface should not vary in time, therefore the total derivative of $\phi$ with respect to time should be zero on the interface.

\[
\frac{D\phi}{Dt} = 0
\]  
(2.14)

Equation 2.14 is a Lagrangian description. To convert to an Eulerian description,
it can be expanded in the following form:

\[ \phi_t + \mathbf{u} \cdot \nabla \phi = 0. \] (2.15)

Here subscript \( t \) shows the partial derivative with respect to time and \( \nabla \) is the gradient operator. Equation 2.15 is the level set equation. This equation is valid on the interface and from analytical point of view, only the information on the interface is required for solving this equation and tracking the interface in time.

Equation 2.15 is a hyperbolic equation, therefore discretizing advecting terms with the second order central difference method will lead to instability of the solution. In order to have a stable solution, a first order upwind method can be used. Equation 2.15 can be discretized in the following form:

\[ \frac{\phi^{n+1} - \phi^n}{\Delta t} + u^n \phi_x^n + v^n \phi_y^n + w^n \phi_z^n = 0 \] (2.16)

Assuming uniform grid for simplicity, the advection term in \( x \) direction, \( u^n \phi_x^n \), can be discretized in the following form which has a first order accuracy.
\[ u^n(i, j, k) \phi_x^n(i, j, k) = \begin{cases} u^n(i, j, k) \frac{\phi^n(i,j,k) - \phi^n(i-1,j,k)}{\Delta x} & u^n(i, j) \geq 0 \\ u^n(i, j, k) \frac{\phi^n(i+1,j,k) - \phi^n(i,j,k)}{\Delta x} & u^n(i, j) < 0 \end{cases} \] (2.17)

The advection terms in other directions can be discretized in the same manner. The main disadvantage of the simple upwind scheme is low order of accuracy which will result in insufficient precision in tracking the interface, therefore using this scheme might lead to a loss or gain of mass on each side of the interface in time.

A higher order stable discretizing method, used here for the advection terms, is called ENO (essentially none oscillatory) method which was first described by Harten et al. [57]. It allows for discretizing the advection terms with higher order accuracy. The concept can be described by using Newton interpolation polynomial for approximating the \( \phi \) function. If the data points are uniform, \( \phi \) function can be written in the following form:

\[ \phi(x) = \phi(x_0) + \frac{\Delta \phi_0}{h} (x - x_0) + \frac{\Delta^2 \phi_0}{2! h^2} (x - x_0)(x - x_1) + \ldots + \frac{\Delta^n \phi_0}{n! h^n} (x - x_0)(x - x_1)\ldots(x - x_n) \] (2.18)

where \( \Delta^n \phi_0 \) is defined by:

\[ \begin{cases} \Delta^n \phi_i = \Delta^{n-1} \phi_{i+1} - \Delta^{n-1} \phi_i, & n \in \mathbb{Z}, n > 0 \\ \Delta^0 \phi_i = \phi_i \end{cases} \] (2.19)

Equation 2.18 is accurate up to order \( N \). Equation 2.19 is the forward form of Equation 2.18, however it can also be described in the backward form. Using the
first three terms of the Newton polynomial, \( \phi(x) \) can be written:

\[
\phi(x) = Q_0 + Q_1(x) + Q_2(x)
\]  

(2.20)

Derivation of the above equation in \( x \) direction at the point \( x_i \) can be stated as:

\[
\phi_x(x_i) = Q'_1(x_i) + Q'_2(x_i)
\]  

(2.21)

In order to describe the coefficient of the Newtonian expansion, the following parameters are defined:

\[
D_i^0 \phi = \phi_i \\
D_{i+\frac{1}{2}}^1 \phi = \frac{D_{i+1}^0 \phi - D_i^0 \phi}{h}
\]  

(2.22)

Using the mentioned above definitions, the first order approximation of the \( \phi_x(x_i) \) becomes:

\[
\phi_x(x_i) = D_{k+\frac{1}{2}}^1 \phi.
\]  

(2.23)

in which \( k \) can be equal to \( i \) or \( i - 1 \). If \( k = i \) is chosen then \( \phi_x(x_i) = D_{i+\frac{1}{2}}^1 \phi \) and if \( k = i - 1 \) then \( \phi_x(x_i) = D_{i-\frac{1}{2}}^1 \phi \).

The equation derived for derivative of the function \( \phi \) in the \( x \) direction is exactly the upwind approximation for the derivatives. We can also use other terms in the Newton polynomial and derive higher order approximation for required derivatives in the desired points \( (\phi_x(x_i)) \). We can use both forward scheme or backward scheme for the third term of the Newton polynomial interpolation. In this case although both give a second order approximation for derivative of \( \phi \), due to the instability
concern of advection terms, the one that will lead to less variation will be chosen. Therefore, the absolute minimum of the two values is probably the better choice.

Supposing that forward scheme is used for the first order term, the second order none oscillatory derivative in the $x$ direction can be written as follows:

$$
\phi_x(x_i) = D_{i+\frac{1}{2}} \frac{1}{h} \phi - D_{i-\frac{1}{2}} \frac{1}{h} \phi + \min \left( | \frac{D_{i+\frac{1}{2}} \frac{1}{h} \phi - D_{i-\frac{1}{2}} \frac{1}{h} \phi}{h} |, | \frac{D_{i+\frac{1}{2}} \frac{1}{h} \phi - D_{i-\frac{1}{2}} \frac{1}{h} \phi}{h} | \right)
$$

(2.24)

The above equation is second order accurate. Higher order terms of Newton polynomial can be used for obtaining higher order accuracy. In our numerical model, we keep the precision to the second order to not increase unnecessary complication and computational cost to the numerical model while properly track the free surface.

The discretization of the level set advection terms in other directions can be done in the same manner. For marching in time, a second order predictor corrector method which is easy to implement and accurate enough will be used. Hence, all the terms of the level set equation are discretized with second order accuracy.

As described earlier for the level set method, positive and negative values are assigned to the two phases of the fluid. These values are proportional to the distance from the interface. Since they are distance function, they have the following property.

$$
| \nabla \phi | = 1
$$

(2.25)

In order to track the interface we need to have accurate values for the distance function. Although only a narrow band around the interface is essential for tracking the interface, accessing the proper distance around the interface may add complication to the numerical solution. Thus, a straightforward option is to solve the level set equation in the whole numerical domain. However, the velocity values in the
whole domain which govern the level set advection terms do not necessary keep the level set function a distance function. Therefore, an additional equation is needed to satisfy the Equation 2.25 over the entire time of the simulation.

One option is to add a time derivative term to Equation 2.25 to keep the level set function a distance function in time.

\[ \phi_t + |\nabla \phi| = 1 \quad (2.26) \]

Equation 2.26 can be solved in fictitious time until Equation 2.26 reaches the steady state, hence \( \phi_t \) will vanish and the level set function returns back to a distance function. However, the issue regarding Equation 2.26 is that it moves all the level set functions including the interface, \( \phi = 0 \). We wish to keep the interface as the reference surface and rearranging other level set functions based on the interface position to make all the level set functions in the numerical domain a distance function. To reach that, Sussman et al. [58] suggested the following equation:

\[ \phi_t + S(\phi_0)(|\nabla \phi| - 1) = 0 \quad (2.27) \]

where \( S(\phi_0) \) is a function set to +1 on the positive side, −1 for the negative side and 0 for the interface. This function smoothly varies from positive or negative values to zero. Various smoothing function are available which more or less lead to the same result. The following smoothing function is used for all simulations:

\[ S(\phi_0) = \frac{\phi_0}{\epsilon} + \frac{1}{\pi} \sin(\frac{\pi \phi_0}{\epsilon}) \quad (2.28) \]

where \( \epsilon \) is the parameter for adjusting the smoothing length. Figure 2.5 shows the \( \phi \) values for a flume which is filled with water and air. \( \phi \) remains the distance function.
in the whole domain.

Figure 2.5: $\phi$ surfaces which are distance functions in the whole domain by using Equation 2.28. Black surface is the interface which is the not relocated by Equation 2.28.

The next step is to discretize Equation 2.28. This equation can be considered in the same manner which we treated the level set equation with the only difference that the coefficient for the advection terms are not functions of the domain velocity field, but function of level set function itself.

Following Fedkiw et al. [59], Equation 2.27 can be rewrite in the following form (2D case):

$$
\phi_t + \left( \frac{S(\phi_0)\phi_x}{\sqrt{\phi_x^2 + \phi_y^2}} \right) \phi_x + \left( \frac{S(\phi_0)\phi_y}{\sqrt{\phi_x^2 + \phi_y^2}} \right) \phi_y = S(\phi_0). \quad (2.29)
$$

The denominator of the advection coefficients are always positive, therefore the nominator is important for us to select the proper direction for the second order ENO method. If $S(\phi_0)\phi_x^+ \geq 0$ and $S(\phi_0)\phi_x^- \geq 0$ then we use $\phi_x^-$, if $S(\phi_0)\phi_x^+ \leq 0$ and $S(\phi_0)\phi_x^- \leq 0$ then we use $\phi_x^+$. If $S(\phi_0)\phi_x^+ \geq 0$ and $S(\phi_0)\phi_x^- \leq 0$ then $\phi_x = 0$ and finally if none of the above hold, $S(\phi_0)\phi_x^- \geq 0$ and $S(\phi_0)\phi_x^+ \leq 0$ then
following parameter will be defined:

\[
S = \frac{S(\phi_0)(|\phi_x^+| - |\phi_x^-|)}{|\phi_x^+| - |\phi_x^-|},
\]

(2.30)

\(S\) introduces an estimation for the direction of the gradient of \(\phi\) inside the cell. If \(S > 0\) we use \(\phi_x^-\) else we use \(S < 0\) we use \(\phi_x^+\).

It should be noted that reinitialization equation does not necessarily need to be solved for every single time step. The required frequency for solving reinitialization equation depends on the particular physics that we are interested in. If there are lots of complication in the interface such as sharp curvatures, there is more chance that the level set function loses the property of the distance function, hence the reinitialization equation needs to be solved more frequently in number of fictitious time steps.

In the current simulation the fictitious time step is \(\Delta t = 0.5\Delta x\) and it is solved in five fictitious time steps in each real time step. Considering that the reinitialization equation advection terms are designed with unit velocity, therefore about 2.5 cells around the interface will be reinitialized in every real time step. The time derivative term of the reinitialization equation is discretized by a second order predictor-corrector method, therefore the reinitialization equation is solved with second order accuracy in both space and time.

Now that the interface is tracked properly, the material properties of each fluid (density and viscosity) can be assigned with enough accuracy. Material properties of two fluids can be assigned based on the level set function sign which will be assigned to each cell. But this straightforward approach add some issues. The first issue is sharp variation of density in the numerical domain will which will lead to a stiff coefficient for the Poisson equation. This issue is augmented by studying high
density ratio materials such as air and water. The second issue is regarding the fact that even in this sharp assignment, there are some cells that are assigned positive or negative values based on the value at the cell center, but are not completely filled with one fluid. In order to overcome these problems, a transient length can be assigned and smoothly vary the material properties from one phase to the other.

In the current research, a 3 cells smoothing length is used on each side of the interface and the following smoothing function is adopted.

\[
f(\phi) = 0.5 + 0.5\phi^3 + 1.5\phi^2 + \phi^0.5 \left(\phi^2 + \epsilon^2\right)^{1.5}
\]

The above function maps the level set function values by a third order polynomial to a function between zero and one. Also, \(f(\phi)\) less than 0.001 is set to zero and above 0.999 to one. This will limit the smoothing length to a reasonable distance from the interface. This function which is used for smoothing is consistent with the smoothing function of the immersed boundary method.

The material properties of the two phases of the domain are assigned in the following form:

\[
\rho(x, t) = \rho_{water} + f(\phi)(\rho_{air} - \rho_{water})
\]

\[
\mu(x, t) = \mu_{water} + f(\phi)(\mu_{air} - \mu_{water})
\]

The above material properties are used in Navier-Stokes and Poisson equations.

The contact angle is the angle that solid, fluid and gas generates at the intersection and it is shown in Figure 2.6. This angle can be imposed as a boundary condition to the numerical model. The contact angle boundary condition can be defined in terms of normal vectors of the gas-fluid interface and normal vector of
solid-gas interface. Mathematically it can be written in the following form:

\[ \mathbf{n}_{\text{Liquid-Gas}} \cdot \mathbf{n}_{\text{Gas-Solid}} = \cos(\theta) \]  \hspace{1cm} (2.34)

Figure 2.6: Contact angle on the solid-liquid-gas intersection point. The dashed line shows the extension of the level set function equal to zero (interface) inside the solid.

By using the level set and marker function, the contact angle can be defined as follows:

\[ \frac{-\nabla C}{|\nabla C|} \cdot \frac{\nabla \phi}{|\nabla \phi|} = \cos(\theta) \]  \hspace{1cm} (2.35)

where  \( \theta \) is contact angle and it is a boundary condition. In the current model,  \( \theta = 90^\circ \) is selected. For the contact angle boundary condition, we follow the method proposed by Sussman and Dommermuth [60] which uses the inactive portion of the level set function inside the solid to apply proper contact angle. The extension equation is in the form of the advection equation and it is described as follows:

\[ \phi_t + \mathbf{u}_{\text{ext}} \cdot \nabla \phi = 0, \]  \hspace{1cm} (2.36)

where  \( \mathbf{u}_{\text{ext}} \) is defined as  \( \frac{-\nabla C}{|\nabla C|} \). The coefficient of the advection equation leads to
movement of information normal the solid surface. In other words it leads to movement of level set functions inside the solid (inactive region) normal to the solid-gas interface. Therefore if we start with accurate contact angle, by solving Equation 2.36 the contact angle will remain the same if there is no-slip boundary condition on the solid-fluid interface. It should be noted that, as mentioned by [60], Equation 2.36 can be implemented in the narrow band of the free surface where there is a transient region.

2.3 Regular wave generation

When studying the hydrodynamic loads on floating wind turbine platforms, currents and free surface waves are the major concerns. Although currents are important because of the considerable drag force, the free surface waves are more likely to cause significant dynamic loads. In the current research our focus is on modeling the effect of free surface waves on the floating wind turbine motions.

There are different methods for generating free surface waves in numerical simulations. Free surface waves can be generated by simulating a piston-type wave maker in the numerical domain [61], adding a source term in the governing equations [62], or assigning the analytical solution for free surface waves to the boundary cells of the numerical domain [63]. The last alternative has been chosen in the current research due to its simplicity and sufficient accuracy. The velocities for the boundary elements can be obtained by the analytical solution of the free surface waves with inviscid and irrotational flow assumption. Since, our focus is more on the interaction of the waves with the floating wind turbine, this assumption will not highly affect the results which we are interested in. In the following, the analytical model to obtain the velocity of the free surface waves will be described in details.
The inviscid and irrotational fluid velocity is governed by the Laplace equation.
The boundary conditions for the Laplace equation will be assigned according to the
physics that we are interested in. Here, linearity for the free surface is assumed,
kinematic and dynamic boundary condition are applied on the free surface, and
no-through boundary condition is imposed on the bottom of the ocean. The result
of velocities in $x$ and $z$ directions are:

$$
\begin{align*}
  u &= \frac{gAk \cosh(kz + h)}{\omega \cosh(kh)} \cosh(kh) \cos(kx - \omega t) \\
  w &= \frac{gAk \sinh(kz + h)}{\omega \cosh(kh)} \cosh(kh) \sin(kx - \omega t)
\end{align*}
$$

For deep water the effect of water depth is negligible and even if the depth of the
water is about half of the free surface wavelength, the difference between considering
and ignoring the water depth effect will less than 4%.

Since linearity is assumed for the free surface waves, Equation 2.37 is accurate for
low amplitude waves. To consider the nonlinear effects of larger amplitude waves,
kinematic and dynamic boundary conditions should be applied on the free surface
$\eta$ using a Taylor expansion:

$$
\phi(x, y, \eta, t) = \phi(x, y, 0, t) + \eta \left( \frac{\partial \phi}{\partial z} \right)_{z=0} + \frac{1}{2} \eta^2 \left( \frac{\partial^2 \phi}{\partial^2 z} \right)_{z=0} + ...
$$

As we increase the accuracy the complexity of the problem will increase rapidly.
Figure 2.7 illustrates the recommended theories which might be used for different
wave heights ($H$), periods ($T$), and water depths ($d$).

If we neglect the third order terms in Equation 2.38, Stokes wave with second
order accuracy can be obtained. For more information on the nonlinear free surface
waves, see [64]. Velocities in $x$ and $z$ directions with second-order accuracy are as
follows
Figure 2.7: Free surface wave theory space based on water depth, wave height, and wave period. Figure from reference [8].

\[ u(x, z, t) = A \omega \cos(kx - \omega t) \frac{\cosh(K(z + h))}{\sinh(kh)} + \]
\[ \frac{3}{32} 2k \omega A^2 \cos(2kx - 2\omega t) \frac{\cosh(2k(z + h))}{\sinh^4(kh)} \]
\[ w(x, z, t) = A \omega \sin(kx - \omega t) \frac{\sinh(K(z + h))}{\sinh(kh)} + \]
\[ \frac{3}{32} 2k \omega A^2 \sin(2kx - 2\omega t) \frac{\sinh(2k(z + h))}{\sinh^4(kh)}. \]  

Figure 2.8 shows a second order Stokes wave which is composed of two sine waves, derived in Equation 2.39. The second order Stokes waves are flatter than the first order waves and are not symmetric with respect to the free surface. The other difference is that the net flow of the second order waves are not zero as for the first
order. Newman [64] shows that the net flow of second order Stokes wave is equal to

$$Q = \frac{1}{2} \omega A^2.$$  \hspace{1cm} (2.40)

This effect is called Stokes’s drift and can cause finite mean motion for floating objects due to the free surface waves.

In the next chapters we use Equation 2.39 to generate regular waves. Note that $x$ can be an arbitrarily set. In our simulations, we set $x = 0$ for simplicity.

In order to apply boundary condition on the numerical domain, it is not essential to assign both velocities in $x$ and $z$ directions. In unidirectional free surface waves, assigning the velocity in one direction, result in automatic calculation of the other velocity component based on mass conservation.

![Figure 2.8: Typical wave shape for second order Stokes waves. Second order Stokes waves are flatter than the first order waves. Figure from reference [9].](image)

Applying this boundary condition to a free-surface flow problem need some modifications. First, in our numerical simulation a finite height region of air above the free surface is included in the numerical domain. The velocity values of the air adjacent to the water cannot vary discontinuously. Considering the free surface as symmetric line for the velocities of the boundary cells can solve the problem, since
the velocity field of the air is of minor concern.

Secondly, when finite amount of mass, either water or air, is flowing inside the numerical domain, the same amount should flow out to keep the mass conserve in the whole domain. In order to conserve the mass in the domain, for every single time step the amount of inflow/outflow is calculated and the same amount is let to flow out/in from the top of the numerical domain.

2.4 Irregular random wave generation

In the previous section, a method to generate regular periodic waves is described. In this section the method will be extended to cover modeling the ocean waves. More details on modeling the irregular ocean waves can be found in [10].

Ocean waves consist of a range of frequencies with different amplitudes in time. By approximating the waves with the linear wave theory and super imposing the linear waves, irregular ocean waves can be modeled. Superposition of the linear waves can be written in the following form

\[ \sum_{j=1}^{N} A_j \sin(\omega_j t - k_j x + \xi_j) \],

(2.41)

where \( \xi_j \) is a random variable uniformly distributed between 0 and \( 2\pi \).

Ocean waves can be described as short-term and log-term status. Short-term status refers to limited time ocean free surface waves in which the ocean waves can be considered as a stationary random process, meaning that statistical random variables will not vary in time. This short-term status may vary from half to 10 hours. Short-term ocean wave status can be described using an ocean wave spectrum which has a Rayleigh shape distribution. Recommended spectrum from ISSC and ITTC are often used to calculate the spectrum of ocean waves [10]. For example
15th ITTC recommended the following spectral formulation for open sea condition and fully developed sea.

\[
\frac{S(\omega)}{H_{1/3}^2 T_1} = \frac{0.11}{2\pi} \left( \frac{\omega T_1}{2\pi} \right)^{-5} \exp \left[ -0.44 \left( \frac{\omega T_1}{2\pi} \right)^{-4} \right]
\]  

(2.42)

In this equation \( H_{1/3} \) is the significant wave height defined as mean wave height for one third of the highest waves, and \( T_1 \) is the mean wave period defined as:

\[
T_1 = 2\pi \frac{m_0}{m_1},
\]

(2.43)

where

\[
m_k = \int_0^\infty \omega^k S(\omega) \, d\omega.
\]

(2.44)

Equation 2.42 is the called the modified Pierson-Moskowitz spectrum. Figure 2.9 shows this spectrum for a significant wave height of 8 m and a mean wave period equal to 10 s. There are other spectrum formulations such as JONSWAP which can be used as well. Usually the location, in which the floating wind turbine is planned to be installed, is the main factor to select the appropriate wave spectrum profile.

So far, the discussion was limited to short-term description of ocean waves in which the mean wave period and significant wave height of the spectrum are assumed constant. For long-term description of the ocean waves, these two parameters will vary in time. The long-term description of the ocean waves is the combination of the short-term ocean waves with different \( T_1 \) and \( H_{1/3} \). Figure 2.10 shows the long-term description for the northern North-Sea. The spectral peak period which is used in Figure 2.10 can be stated as a function of mean wave period, since the spectral distribution of the ocean waves has a Rayleigh distribution. The spectral peak wave period, \( T_0 \) can be defined as
Figure 2.9: Ocean wave spectra for a significant wave height equal to 8 m and a mean wave period equal to 10 s.

\[
T_0 = 1.408 \left( \frac{m_0}{m_2} \right)^{0.5}
\]  \tag{2.45}

In order to study the effect of the wave on the floating wind turbines, different extreme and operating short-term status of the ocean waves should be considered. Conditions in which the ocean peak period is close to the one of the platform natural frequencies or the cases in which the significant wave heights are large, need to be simulated before finalizing the floating wind turbine design.
### 2.5 Response Amplitude Operator

Response amplitude operators (RAOs) are standard functions used to summarize offshore structures response to incoming waves in a range of frequencies. This concept also has been in other fields such as electrical engineering to estimate the ratio of an output signal to an input signal in a range of frequencies.

RAOs are Mathematically described by using spectral density function. Spectral density is a measure of the signal energy as a function of frequency. For example, the signal can be the ocean wave energy, or the platform motions in a particular direction. The ocean irregular waves can be described as \[10\]:

\[
\chi = \sum_{j=1}^{N} A_{Wj} \sin(\omega_j t - k_j x + \xi_j),
\]

and the corresponding spectral density with fine enough frequency segmentation is \[10\]:

---

**Figure 2.10:** Joint frequency of the spectral peak period and significant wave height. Long-term description of northern North sea. Figure reprinted from reference \[10\].
\[ \frac{1}{2} A_{Wj}^2 = S_W(\omega_j) \Delta \omega \] (2.47)

in which \( S(\omega) \) unit is m\(^2\)s. The same concept can be used to describe the response of the platform in a particular direction. The corresponding spectral density is in the form of:

\[ \frac{1}{2} A_{Rj}^2 = S_R(\omega_j) \Delta \omega \] (2.48)

Finally the RAO will described as the ratio of the platform response to the incoming wave in the frequency domain. Thus, for a particular frequency it can be written as [64]:

\[ RAO(\omega_j) = \frac{A_{Rj}}{A_{Wj}}, \] (2.49)

since, it is assumed that frequency segmentation is the same for incoming wave and platform response, Equation 2.48 to Equation 2.50 can be used to relate the RAO to the spectral density of incoming wave and the spectral density of response in a particular direction:

\[ | RAO(\omega) |^2 = \frac{S_R(\omega)}{S_W(\omega)} \] (2.50)

Figure 2.11 shows the graphical description for the relation between the spectral density and the RAO of the platform.
Figure 2.11: Graphical description for the relation between spectral density and RAO.
Chapter 3

Verification and Validation

In this chapter a number of comparisons with existing results and grid refinement studies have been conducted in order to verify and validate the numerical method and to assess the grid resolution requirements for the problems considered in the next chapters. A lid-driven cavity problem is modeled to assess the accuracy of the numerical solution for the Navier-Stokes and mass conservation equations. A free surface wave decay test is performed and compared with an analytical solution to verify the free surface tracking method. Vortex shedding behind a bluff body is studied and shedding frequency, drag and lift coefficients are compared with the numerical and experimental data to verify the immersed boundary method. Interaction of a floating object with incoming waves is studied to verify the combination of immersed boundary and level set method. Finally, the developed model is validated by simulating a 100 : 1 scale model tension leg floating wind turbine and comparing the wind turbine response with the experimental data from concurrent wave flume tests conducted by our research group.
3.1 Model verification

The accuracy of solving Navier-Stokes equations without any free surface or solid-fluid interaction is examined. The lid-driven cavity problem is solved in different directions for the developed 3D code and the results are compared with the well-verified numerical results of Ghia et al. [11]. A lid-driven cavity is a rectangular domain with no-slip boundary condition on all sides and to one side a pre-defined constant velocity is assigned. Since the developed code is in three dimensions and the Ghia et al. results are in two dimensions, limited number of grids are used in the third dimension and free slip boundary condition is imposed to resemble the 2D case. The simulations after certain amount of time (depends on the Reynolds number) will reach to steady state. The only nondimensional number contributing is Reynolds number \((Re = uD/\nu)\) in which \(\nu\) is related to the material properties of the fluid inside the cavity, \(D\) is the cavity height, and \(u\) is the lid velocity. Pressure contours for lid-driven cavity problem is shown in Figure 3.1 as it reaches the steady state condition. Figure 3.2 shows the comparison of velocity field in the midsection of the cavity with the numerical results.

![Pressure contours for lid-driven cavity problem with Reynolds number equal to 1000 as the simulation reaches the steady state.](image)

In a second test the accuracy of the free surface tracking method will be exam-
Figure 3.2: Comparison of the velocity filed components at the midsection of the cavity with the results from Ghia et al. [11] for $Re = 1000$ at steady state.

ined. Free surface oscillations of a viscous liquid in a flume with full slip walls is studied and the results are compared with the analytical solution of Wu et al. [12], for a long, small amplitude wave at $Re$ number equal to 200.

Here, the characteristics length and velocity are defined as $D_l$ and $\sqrt{gD_l}$ accordingly. We take the flume to be 2 m long and the mean depth of the liquid to be 1 m and start with a single wave with an amplitude of 10 cm. The simulation is done using a flume that is 1.2 m high, using a mesh with $240 \times 120 \times 4$ grid points in the length, depth, and width direction. The density ratio of fluid to air is 1000, so the effects of the air on the free surface motion is negligible. Figure 3.3 shows the initial condition and the amplitude of the free surface at the center of flume versus time as predicted by both the simulation and the analytical solution. The agreement is very good and we believe that the slight over-prediction of the amplitude by the numerical method is because we include advection terms which are ignored in the analytical solution.

Since we are concerned with modeling floating wind turbine platforms that are
circular in shape, for a third test, the flow field behind a circular cylinder in two dimensions is studied. The problem is solved in a $25D_c \times 6.25D_c$ domain with a $Re$ number equal to 150. The cylinder diameter and the far field velocity are defined as characteristic length and velocity of the problem. The cylinder diameter is resolved by 20 meshes. Figure 3.4 shows the vortices behind the cylinder and it is clear that the Von Karman street is captured well behind the cylinder. Table 3.1 compares the Strouhal number, the mean drag force, and oscillation of the lift coefficient (peak to peak) with experimental and numerical data. The vortex shedding frequency agrees very well with earlier results. For the lift and drag coefficients the results are close to data reported by Lai and Peskin [65] which, like the currently used approach, is based on the IB method. The reason for the difference in the lift and drag coefficients compared with the results from Henderson [66] and He and Doolen [67] is possibly due to the differences in the domain size between the studies. In addition to the
two dimensional study shown here, the vortex shedding behind an infinite circular cylinder has also been simulated to verify the immersed boundary method used in the three dimensional numerical model.

Figure 3.4: Vortex shedding behind a two dimensional cylinder for $Re$ number equal to 150. Dashed and solid lines represent negative and positive vorticity.

Table 3.1: Comparison of shedding frequency and lift and drag coefficients of a 2D cylinder at $Re = 150$ under uniform flow.

<table>
<thead>
<tr>
<th>Method</th>
<th>Strouhal number</th>
<th>Max. and Min. lift coefficient</th>
<th>Mean drag coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current method</td>
<td>0.178</td>
<td>±0.62</td>
<td>1.50</td>
</tr>
<tr>
<td>Lai and Peskin 1st order IB method</td>
<td>0.156</td>
<td>±0.4</td>
<td>1.60</td>
</tr>
<tr>
<td>Lai and Peskin formally 2nd order IB method</td>
<td>0.183</td>
<td>±0.58</td>
<td>1.45</td>
</tr>
<tr>
<td>He and Doolen (Lattice Boltzmann method)</td>
<td>0.179</td>
<td>±0.49</td>
<td>1.261</td>
</tr>
<tr>
<td>Henderson [66]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Williamson (Exp.) [68]</td>
<td>0.183</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hammache and Gharib (Exp.) [69]</td>
<td>0.176</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In a fourth test, we compare our predictions of the pitch motion of a rectangular free-floating block, shown in Figure 3.5(a), with experimental results from Jung et al. [13]. The block in [13] is made of acrylic with a uniform density of $1048 \, kg/m^3$ and dimensions of 0.1 $m$ height, 0.3 $m$ length, and 0.9 $m$ width which floats in a flume that is 0.9 $m$ wide, 1.2 $m$ deep, with a mean water depth of 0.9 $m$. The block width is equal to the width of the flume, so the problem can be taken to be two dimensional [13]. The block can pitch freely, but heave and sway motions are
The block is initially tilted at a 15 degrees angle in pitch direction about its center of gravity and is then allowed to pitch freely until its motion has decayed. Since the decay is affected more by radiation of energy by the waves rather than viscous effects, we use a relatively long computational domain of 10 m and stop the calculation before the waves reflect from the boundaries. Figure 3.5(a) shows a grid refinement study, where it is seen that the results are nearly converged on the 640 × 240 grid. The pitch frequency is relatively well captured by all but the coarsest grids, but the damping, which depends on the radiation of energy, needs relatively fine grids. A comparison between the results from the finest grid with the experimental results of [13] is shown in Figure 3.5(b). The agreement for the frequency is relatively good, but the experimental results decay faster, possibly due to the three dimensional and surfactant effects.

Figure 3.5: (a) Results from the grid resolution study for the pitch motion of a block. (b) A comparison of the pitch motion results from the finest grid with the experimental results of [13].

In the last test presented in this section, we compute the pitch response of the
same block as in the third test to imposed waves of varying frequency and compare
the results with the experimental data of [13] and linear potential flow theory. The
block is initially horizontal and the wave generator is located 1.7 wavelengths away
from the block. In Figure 3.6 the response amplitude operators (RAO) in roll is
shown. The RAO in Figure 3.6 is plotted for a range of wave frequencies. The RAO
results are in relatively good agreement, specially at higher wave frequencies where
diffraction of the waves is important. The numerical results show a slight shift
(about 2%) in the natural frequency toward higher frequencies. Apart from this
shift, the RAO responses on either side of the resonant peak are in good agreement
with both potential and experimental data. We note that flow field results, not
shown here, for the long waves (low frequencies) show vortices on the both sides of
the block in agreement with observations reported by [13]. These vortices are due to
the interaction of the large amplitude pitch motion of the block with the incoming
waves. If the pitch motion of the block is small, there are no vortices and linear
potential theory can be accurate enough.

Figure 3.6: Comparison of the response amplitude operators (RAO) given by our
numerical simulations with the experimental and theoretical results of [13].
3.2 Validation using scale-model experiments

In order to validate the numerical model, the 100 : 1 scale model experimental results of Naqvi [46, 47] are compared with the numerical results. Since the experimental research is done in parallel with the numerical models in our research group and there were active collaboration in this regard, a brief description of the experimental works will be given. Figure 3.7 and Figure 3.8 show the 100 : 1 TLP scale model in the water flume facility at Alden Research laboratory in Holden, MA. Figure 3.9 shows a schematic of the scale-model platform and Table 3.2 lists parameters used for the scale-model and simulation.

![Figure 3.7: A side view of 100 : 1 scale model tension leg platform.](image)

The scale-model was constructed using three-dimensional printing technology and ABS plastic for the main structural components. The model dimensions and weights are based on previous conceptual studies on full-scale tension leg platforms and Froude scaling is used to properly set the scale model components weights and dimensions. Aluminum plates and additional sand are used for ballast located at the tank bottom. The turbine rotors are not modeled in the experimental works, however scaled rotor and nacelle weights are added to the tower top.
Figure 3.8: A top view of 100:1 scale model tension leg platform.

Four vertical tethers (nylon, $D = 1.6\, mm$) restrain the motion of the TLP. The four tethers are equally spaced around the circumference of the platform tank (90°), and attached to horizontal legs that extend 14.5 cm from the side of the buoyant tank. The stiffness of the tethers has been estimated by the numerical simulations, since rough estimate for the tethers stiffness was prepared by the manufacturer. The natural frequency in heave reported by Naqvi [47] is 0.5 s. The proper tether stiffness which leads to the experimental natural frequency, was found by tuning the tether stiffness through multiple simulations, and it is found equal to 480 N.

Three-axis accelerometers (Analog Devices, Model ADXL335) were placed both at the models center of gravity (within a water-tight instrumentation cylinder), and at the top of the model tower (nacelle location) as shown in Figure 3.9. An inclinometer (Turck, Model B2N45H) located at the tower top measured pitch and roll angles. A lightweight USB-based wireless data acquisition system (Arduino Duemilanove, ATmega168 microcontroller; XBee RF antennae modules) placed at the model center of gravity, eliminated the need for an umbilical cable to transfer
Figure 3.9: Detailed drawings of the model tension leg platform used in the simulations and experiments. 1-Tethers; 2-Horizontal legs; 3- Buoyant tank (Air filled in experiment, density in simulation); 4 - Instrumentation cylinder (experiment only); 5- Tower, 6-Nacelle and rotor weight; 7-Nacelle accelerometer and inclinometer; 8-Center of gravity accelerometer and wireless transmitter; 9-Ballast weights (Aluminum in experiment, density in simulation). The top view details the platform and tether orientations with respect to the incident waves.

Accelerometer and inclinometer data to the data acquisition computer. An umbilical cable would significantly alter platform dynamics. The wireless system also allows for measurement of platform dynamics and motion RAOs without the need for time consuming video post-processing. A float type wave height meter was placed 0.7 m upstream of the scale models on the tunnel centreline.

Accelerometer accuracy is estimated at $\pm 1.0 \text{ cm/s}^2$, inclinometer accuracy at $\pm 0.1^\circ$, and wave height probe accuracy at 4 mm. These accuracy estimates lead to calculated error bars of $\pm 0.025$, $\pm 0.07$, and $\pm 0.015$ on pitch angle, surge, and
Table 3.2: Experiment and simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank height</td>
<td>0.21 m</td>
<td>0.21 m</td>
</tr>
<tr>
<td>Tank diameter</td>
<td>0.22 m</td>
<td>0.22 m</td>
</tr>
<tr>
<td>Draft</td>
<td>0.18 m</td>
<td>0.18 m</td>
</tr>
<tr>
<td>Buoyant Tank Mass</td>
<td>3.18 kg</td>
<td>3.18 kg</td>
</tr>
<tr>
<td>Ballast mass</td>
<td>1.8 kg</td>
<td>1.8 kg</td>
</tr>
<tr>
<td>Tower and Nacelle Mass</td>
<td>1.047 kg (incl. DAQ)</td>
<td>1.047 kg</td>
</tr>
<tr>
<td>Floating wind turbine mass</td>
<td>6.27 kg</td>
<td>6.3 kg</td>
</tr>
<tr>
<td>Buoyant displacement mass</td>
<td>7.25 kg</td>
<td>7.25 kg</td>
</tr>
<tr>
<td>Tether Pretension</td>
<td>13.5%</td>
<td>17%</td>
</tr>
<tr>
<td>Center of gravity</td>
<td>−0.08 m</td>
<td>−0.062 m</td>
</tr>
<tr>
<td>Tower height</td>
<td>0.95 m</td>
<td>0.95 m</td>
</tr>
<tr>
<td>Tether leg length</td>
<td>0.145 m</td>
<td>0.145 m</td>
</tr>
<tr>
<td>Tether stiffness</td>
<td>480 N</td>
<td>480 N</td>
</tr>
</tbody>
</table>

heave RAOs respectively. Acceleration data was post-processed to correct for pitch and roll angle inclinations. Response amplitude operators were determined using LabView System Identification Toolkit routines. Hanning windows were applied for power spectra calculations.

The testing section of the flume is 183 cm (width) × 183 cm (height) with a transparent side wall for model viewing. The nominal water height in the flume was set at 1.37 m. The entire flume is 12 m in length, and a plunger-type wave maker is located about 5 m upstream of the models. The wave maker generated waves with periods ranging from about 0.6 – 1.2 s, and peak-peak wave heights from about 0 – 7 cm. This corresponds to full-scale wave periods in a range of 6 – 12 s and wave heights of 0 – 7 m. A sloped beach with surface damping material, which damps upstream wave reflections, is located about 5 m downstream from the test section.

The input parameters for the baseline simulation are shown in Table 3.2. The simulations are run for approximately 30 wave periods with a time step of 0.001 s.
The baseline simulation is conducted in a rectangular domain (flume) with $16 \times 1 \times 1.8 \, m$ length, width, and height respectively, with water height set at $1.37 \, m$. The length of the domain is long enough in order to prevent the effects of reflecting waves. The wave period and amplitude are $1.0 \, s$ and $0.07 \, m$ respectively.

In Figure 3.10 a three dimensional view of the baseline grid and a frame of the scale-model simulations are shown. Platform motions are quantified next. In Figure 3.11, surge and heave motions of the platform center of gravity are presented, along with wave heights, and pitch angle motions. A nearly single frequency response at the incident wave frequency is observed and clearly the surge response dominate over heave. A low-frequency oscillation is observed in the wave heights due to upstream influence of the floating platform. Pitch angle amplitudes (peak-peak) are approximately $2.5^\circ$.

Figure 3.10: The numerical grid and a frame from the numerical simulation.

Figure 3.12 shows the experimental platform motions and wave heights. The incoming waves are not a pure single frequency due to the small reflection of the waves from the downstream end of the flume but the effect is small. The surge and heave responses are captured well but there is some noise in the pitch response due to the small amplitude of the pitch angle.
Response amplitude operators (as a function of wave period) for the tension leg platform are presented in Figure 3.13 from both the simulations and experiments. Response amplitude operator defines the response of the platform in specific mode per unit wave height in a particular frequency. Response amplitude operators are presented for generated periodic waves with periods and peak-peak wave heights ranging from $0.68 - 1.2$ s and $3.5 - 7$ cm respectively in the numerical simulations, and about $0.6 - 1.25$ s and $1.0 - 7$ cm for the experiments. Response amplitude operators for the dominant surge, heave, and pitch motions were measured. Heave and surge RAOs are for the platform center of gravity motions in both the simulations and experiments.

The experimental tests of Naqvi [47], shown in Figure 3.13, are for two different ballast weights (0.6 kg and 1.8 kg), however the comparison with numerical results
Figure 3.12: Experimental data for the scale model tension leg platform.

is for the heavier ballast section (1.8 kg) due to the limitation in numerical modeling of very thin ballast sections. The numerical results for the pitch RAO, shown in Figure 3.13(a), are in good agreement for wave periods 0.6 – 1 s. These corresponds to full-scale wave periods of 6.0 – 10 s. The small deviation in experimental results in comparison with numerical results might be because of very small peak to peak pitch amplitude of TLP which is the consequence of the high stiffness of the pre-tension tethers.

The experimental and numerical surge response of TLP is shown in Figure 3.13(b). Again, both the experimental and numerical data show good agreement. Numerical results shows a monotonic increase in the response with respect to wave period. The same trend can be seen for the experimental data but with slightly higher slope. The surge results in both experimental and numerical simulations are for
wave periods less than the measured natural frequency in surge which is 15 s. So, it seems reasonable to have an increasing trend in RAO as we move toward the natural frequency.

We pointed out that, previously, we tuned the tether stiffness to match the TLP experimental heave natural frequency, not necessarily the heave amplitude, however, there is a very good agreement between the experimental data and the numerical results for the heave response shown in Figure 3.13(c). Both numerical and experimental results have the same monotonic increase with respect to the incoming wave period.
Figure 3.13: Comparison of the TLP response amplitude operators (RAO) given by our numerical simulations with the experimental results (1.8 kg Exp.) for 100 : 1 scale model. 1.8 kg represents the ballast mass of the tension leg platform.
Chapter 4

Tension Leg Platform Floating Wind Turbine

In this chapter we use the method described in chapter 2 and validated in chapter 3, to study the interaction of a tension leg floating wind turbine to linear and nonlinear waves.

A tension leg floating wind turbine is a wind turbine which is mounted on a tension leg platform (TLP). The tension leg platform for the wind turbines (see Figure 4.1) mainly consists of a buoyant tank attached to the ocean floor by a number of vertical tethers. The tethers are connected to horizontal spokes extending from the tank. The tethers provide the main mechanism for keeping the TLP stable and the excess buoyancy of the tank keeps the tethers in tension. The spokes are mainly used to increase the length of the moment arm which the vertical tethers tension is applied, thus decreasing the pitch, roll, and yaw responses of the platform. TLP is a concept which is borrowed from the oil and gas industry and it has been well studied during the last three decades. Numerous papers on the stability, stiffness,
and dynamics of TLP can be found. For example, Faltinsen [70] described a linear method to analyze a full scale TLP platform in deep water. Jain [71] studied the response of a TLP to regular periodic waves. His simulations were based on calculating the stiffness matrix of the TLP and then using a modified Morison equation to study the effects of the wave forces on the platform. Chandrasekaran and Jain [72] studied a triangular shaped tension leg platform with the same approach to study the TLP response to random ocean waves. All of these studies were for application in oil and gas industry. Studies which consider TLP for carrying a wind turbine are very recent [28–30,34] and were reviewed in chapter 1.

Here, in the first section, the response of the TLP to regular periodic waves will be studied. The effects of wave height on the response of TLP will be examined. A simple design study will also be performed by changing the tank aspect ratio. Finally, we look at the nonlinear response of the platform to a large amplitude nonlinear wave.

4.1 Response under operating conditions

In this section the response of a tension leg platform shown in Figure 4.1 will be studied. The TLP properties are presented in Table 4.1 in which the parameters are approximately scaled up of the 100:1 scale model \((\lambda = 100)\), studied in chapter 3. According to the Froude scaling, if the scaling factor for length is \(\lambda\), the weight is scaled by \(\lambda^3\), mass moment of inertia by \(\lambda^5\), and wave period by \(\lambda^{\frac{1}{2}}\). The numerical domain is \(1600 \times 100 \times 175\) m in length, width and height, respectively and the generated waves are of single frequency and amplitude \((\omega = 0.2\pi \text{ rad/s} \& H = 5.3\) m\).

The floating wind turbine and its response to the incoming waves are shown at
two times (separated by a half wave period) in Figure 4.2. The incoming wave and the wind force on the rotor push the platform downstream but the prestressed tethers keep the platform nearly horizontal, and the primary response is an oscillatory surge motion. In Figure 4.3(a), we show the amplitude of the waves, measured half-way between the wave maker and the platform. The wave amplitude is slightly lower than the nominal height at the wave maker due to the presence of the platform. In the absence of the platform the waves propagate essentially unchanged downstream until they reach the coarsely resolved part of the domain and damp out. The non-
Table 4.1: The parameters used for the computations of the floating wind turbine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank mass</td>
<td>$4.97 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Ballast mass</td>
<td>$4.67 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Reserved buoyancy (including spokes)</td>
<td>$7.25 \times 10^3 \text{ m}^3$</td>
</tr>
<tr>
<td>Tethers stiffness</td>
<td>$150 \text{ KN/m}$</td>
</tr>
<tr>
<td>Pre-tension of tethers</td>
<td>22% of the buoyancy</td>
</tr>
<tr>
<td>Spoke radius</td>
<td>$1.5 \text{ m}$</td>
</tr>
<tr>
<td>Tower mass</td>
<td>$0.347 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Nacelle+rotor mass</td>
<td>$0.350 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Rotor second moment of inertia</td>
<td>$0.0047 \times 10^{10} \text{ kgm}^2$</td>
</tr>
<tr>
<td>Rotor speed</td>
<td>$12.1 \text{ rpm}$</td>
</tr>
<tr>
<td>Wave gage position</td>
<td>$\lambda/2$ upstream</td>
</tr>
<tr>
<td>Wave generator distance</td>
<td>$\lambda$ upstream</td>
</tr>
<tr>
<td>Wind thrust force</td>
<td>$600 \text{ KN}$ (correspond to 9 m/s [29])</td>
</tr>
<tr>
<td>$C_g$ in $Z$ direction</td>
<td>$-2.75 \text{ m}$ (respect to MWL)</td>
</tr>
</tbody>
</table>

zero mean water level in Figure 4.3(a) is due to asymmetry of the Stokes wave at the inlet. The platform has six degrees of freedom (translations and angles) but the main responses are in surge, pitch and heave. Yaw and roll motion may happen due to gyroscopic effects (to be discussed later) and there are essentially no net forces in the sway direction.

Figure 4.3(b) and (c) show the response of the TLP to the incoming waves. The dominant response is the surge, as seen in Figure 4.3(b), which is restricted by the pretension of the tethers. The surge motion shows a transient response for about 100 seconds after the start of the simulation and then an approximately steady state. This evolution is similar to the surge response of a freely floating structure reported by Koo and Kim [73] and, as they also point out, the frequency of the transient response is a combination of the natural frequency of the TLP in surge and the incoming wave frequency. The steady state response has, however, nearly the same frequency as the incoming wave, since radiation and viscous damping are not large enough to greatly affect the response frequency. In the steady state conditions,
the TLP response is approximately single frequency at constant amplitude. The slight variation in the surge amplitude response, even in steady state, is due to the imperfection of incoming wave amplitude described earlier.

The amplitude of the heave motion is small compared to the surge because of the high stiffness of the tethers, as seen in Figure 4.3(b), and like the surge response, the frequency of the heave at steady state is nearly the same as the incoming wave. Since the heave response is small and the natural frequency of the heave motion of the platform is usually far from the typical peak ocean waves frequencies, it is generally not a critical quantity in the design of floating wind turbines. Therefore heave is sometimes neglected in design studies, such as by Wayman et al. [29].

The pitch response, on the other hand, is very important since a rocking motion of the turbine can adversely affect the efficiency and operational conditions of the rotor. The pitch response for a TLP floating wind turbine is significantly larger than for traditional TLP structures in the oil industry since the structure is lighter, the center of gravity is higher, and there is a moment due to the wind force on the rotor.
Figure 4.3(c) shows the pitch response of the TLP. The frequency is determined by the incoming wave, as before, except at the very early time when the wind force tilts the tower but the waves have not yet reached the platform. The natural frequency of the pitch motion is significantly higher than the frequency of the incoming wave, due to the stiffness of the tethers and the relatively large length of the spokes. Therefore, its influence on the pitch response is small. Because of the high stiffness of the tethers, the restoring forces of the TLP in the pitch direction is mainly because of the tethers rather than hydrodynamic effects. At steady state, when the structure responds in pitch with a single amplitude and frequency, the mean pitch response is due to the difference of the mean moment from the wind force and the mean moment due to the wave drift force.

We have not seen a noticeable difference in the yaw and pitch response when the gyroscopic effect is added, and an order of magnitude analysis for the moments supports the numerical results, which are also in qualitative agreement with the study done by Jensen [51]. The small values for the yaw and pitch gyroscopic moments are due to the large platform second moment of inertia in the yaw direction and restoring moment of the tethers. Although gyroscopic effects are negligible in the present work, they may be important in other design studies since gyroscopic moments in yaw and pitch augment each other and can potentially lead to an instability of the whole platform.

The tether tension forces are shown in Figure 4.3(d) for the upstream and downstream tethers. Because yaw, roll and sway motions of the platforms are small, the tension in the two upstream tethers are nearly the same. The same is true for the downstream tethers, therefore we only plot one upstream and one downstream tether force. The figure shows that the tension in the upstream tethers is higher, because of the mean pitch angle of the platform toward the downstream direction.
The transient part of the tether response includes the initial motion where the platform is trying to find a stable position and the later motion where the effects of the platform natural frequency has not yet disappeared. The highest value for the tension is seen in the very beginning of the simulations when the wind force is applied impulsively. Ability to study the platform response in different modes plus measuring the tethers load in transient region is one of the advantage of using tim domain simulation where in linear frequency domain analysis, the results are limited to steady state.

Overall, observing surge, heave, and pitch response plus the tether forces suggest that this design at least for this waves and wind loads gives an acceptable response. The peak-to-peak surge response is less than $0.2D$, peak-to-peak heave is less than $0.02D$, peak-to-peak pitch is $0.3^\circ$, and tether forces are less than $10000\,KN$.

Our computational simulations can capture details of the flow field around the floating wind turbine tank that are usually not available from currently used time domain and linear frequency domain methods. As summarized by Sarpkaya [74], oscillatory flow around a smooth circular cylinder is governed by the Keulegan-Carpenter number ($KC$) and the Reynolds number ($Re$). $Re$ is based on maximum relative oscillatory velocity and $KC = U_m T/D$ number is essentially function of incoming wave amplitude and TLP tank diameter. This can be justified by noticing that the TLP oscillation velocity is completely less than oscillation velocity of the flow and with nearly the same frequency, therefore the oscillation velocity is mainly dominated by the flow oscillation velocity. Multiplying the oscillation velocity ($A 2\pi /T$) by oscillation period ($T$) will cancel out $T$ and KC number becomes ($KC = 2\pi A/D$).

At small $KC$ numbers the effects of the $Re$ number are small. For our TLP floating wind turbine tank, $U_m$ is a function of both wave and buoyant tank velocity.
Knowing that the oscillation frequency of the floating wind turbine tank is nearly the same as the incoming wave frequency, the relative maximum velocity of the oscillatory flow leads to a maximum of $KC \approx 1.25$. According to Guilmineau and Queutey [75], separation of the flow behind the cylinder will start from $KC$ numbers between 1 and 2 and this separation will remain symmetric until around $KC = 4$. But, the study performed by Guilmineau and Queutey is in the absence of the free surface. Yu et al. [76] claimed that presence of the free surface may inhibit the vortex generation near the wake of a circular cylinder. Thus, for the floating wind turbine we may expect generation of the vortices at higher $KC$ values than the values reported in [75].

We have not observed vortices in the steady state condition behind the cylinder, but for the transient region, we have seen weak vortices as shown in Figure 4.4. The reason for this can be seen in Figure 4.3(b), where near the beginning of the transient region there are two large surge motions (before and after the first peak at $t \approx 20s$ and $t \approx 30s$). The large surge motion gives the flow enough time for the formation of vortices behind the cylinder due to a temporary higher $KC$ number. As we approach steady state, the amplitude of the surge motion is reduced and the flow field does not have enough time for the formation of vortices. Note that, even for the highest wave heights for which $5MW$ TLP floating wind turbines are usually designed, the $KC$ number is not high enough for the creation of asymmetric vortex shedding behind the floating tank in the absence of currents.
Figure 4.3: (a) The wave height, measured half a wavelength upstream of tension leg platform; (b) The surge and heave response; (c) The pitch response; (d) The upstream and downstream tether forces.
Figure 4.4: Top view of the velocity vectors at the midsection of the floating wind turbine tank in the very beginning of the transient region \((t = 23 \text{ s})\). The solid line shows the floating wind turbine tank border. Note that slight deviation of the velocity field on the solid border grids with respect to the inside solid, is due to transition region from solid to fluid (see Equation 2.3).
4.2 Effects of the wave height

After studying the response of the wind turbine to moderate amplitude waves in a baseline run, we examine the effects of changing the wave height on the surge response, while keeping the wave frequency the same. As summarized in section one, many previous studies of the effects of wave amplitude on the floating wind turbine have been conducted in the frequency domain with a linear assumption for the response of the platform to different incoming wave heights [33]. On the other hand, drag coefficients in time domain results are mainly based on experimental data. Figure 4.5(a) shows the peak-to-peak surge response to different incoming wave heights for a single frequency wave. It can be seen that linear assumption is verified by the current approach up to about 10 m wave height, but above that the results show a nonlinear trend. The nonlinearity is toward the safe side and the linear assumption leads to an over-prediction of the wind turbine response. The linear trend of the TLP surge response for the moderate and low amplitude wave heights is also shown in Figure 4.5(a) which in the limit of zero wave height yields a zero surge response, as we expect. One reason for the nonlinearity in the larger amplitude waves might be due to the limited height of the platform tank above the mean water level (MWL). This will limit the platform area on which the waves can apply forces and will reduce the platform response to the wave. This effect could not well be captured by standard frequency domain analysis. The significantly higher wave disturbance around the platform in Figure 4.5(b) compared with Figure 4.2, which are for incoming wave heights 13.5 m and 5.3 m accordingly, can be related to this effect.
Figure 4.5: (a) Surge response of the TLP to different incoming wave heights. At a wave period of 10 s, a linear trend is observed for low and moderate amplitude waves, but not for large waves. (b) Wind turbine interacting with a 13.5 m wave. Waves radiated from the floater tank can be seen around the platform.

4.3 Tension leg platform design study

The design concepts developed in the oil industry are unlikely to be applicable for a TLP for a wind turbine, since the tower, nacelle, and rotor, mounted on top of the TLP platform, considerably vary the dynamic characteristics of the floating wind turbines such as $C_g$ and natural frequencies in different modes.

An extensive design study would include changing the various design parameters for the rotor, tower, horizontal spokes, and tether components. We will conduct this type of more detailed design study for spar buoy in chapter 5. Here, however, we focus on varying the buoyant tank geometry for a TLP platform. The floater displacement, mass, and mooring system are kept the same, but the buoyant tank diameter is varied by ±10%. In both cases the height of the floater tank is changed so that the volume is the same. Increasing the radius leads to a more barge-like shape whereas reducing the radius makes the tank more spar-buoy like. A comparison of the steady state peak-to-peak pitch angle and the tether forces are shown in Figure 4.6. There is a linear decrease in the peak-to-peak pitch angle as we increase
the radius of the floater tank. This decrease in the pitch angle is probably because of the increase in the stabilizing moment arm of the tether forces (floater tank radius plus spoke length). On the other hand, the tether tension peak-to-peak response has a nonlinear trend. The decrease in tether forces as the radius increase is due to the same reason as for the pitch angle, but the nonlinear trend is likely due to change in the tank shape that affects the force from the incoming wave. The changes here are unlikely to greatly affect the hydrodynamic restoring force in surge, heave, and pitch because of the high stiffness of the tethers.

![Figure 4.6: A comparison of the peak-to-peak pitch responses and the tether forces at steady state for different designs of the floating wind turbine tank. $R$ shows the radius of three different platform tanks which is normalized based on the radius of the standard tank.](image)

### 4.4 Single large amplitude wave loading

While changes in the response of the platform under operating conditions are obviously very important, these dynamics are likely to be captured reasonably well using linear models. The fully nonlinear model presented here is, however, valid for arbitrary surface waves and platform motion, and should be particularly useful in
computing what happens in off-design and extreme conditions. Figure 4.7 shows results from one simulation where we start the simulation by releasing a large mass of water at one end of the tank. The wave maker is turned off, and the elevation of the water in the first $31.2\,m$ of the tank is raised by $20\,m$. At the very beginning, the water starts to slump down, sending a large wave toward the platform (Figure 4.7). Pitch angle and surge responses along with tether tensions are shown in Figure 4.8. Note that the initial increase in pitch, surge, and tether forces on the platform are due to the effects of the wind and initial oscillation of the platform to satisfy the static stability. As the water hits the platform, the surge amplitude is still increasing, but the pitch angle starts decreasing. The decrease of the pitch response is because of platform center of gravity location in comparison with the resultant hydrodynamic force from the incoming wave. Both upstream and downstream tethers have continuously decreasing tension. This decrease is higher in upstream tethers due to the decrease in the pitch angle. The continuous reduction in the tether forces is likely due to the downward vertical load mentioned by Bea et al. [77] on the platform and reduction in reserve buoyancy.

After the first wave passes the wind turbine, due to reduction of the wave forces on the platform, the surge response decreases and the pitch angle and the tether forces increase to let the wind turbine move toward its static stability position. The second wave is not as strong as the first one, but nearly the same trend can be seen in the surge, pitch, and tether forces. The second wave occurs since the increase of the water height from the first large wave leads to a decrease in the water height behind the first wave. This decrease in height gives enough potential energy to the water for the generation of a second wave. Gradually, the waves become smaller and the wind turbine returns back to the static equilibrium position, although the turbine thrust force is still affecting the wind turbine. Overall, this simulation shows the
ability of the method to simulate completely nonlinear and non-periodic conditions for floating wind turbine interacting with large waves and modeling cases in which the tethers of TLP can go slack, thus the dynamic features of TLP is not constant. Slacking of TLP tethers cannot be predicted well by most of the currently used LFD and TDD models.

Figure 4.7: The response of the TLP to a large nonlinear wave. (a) the very beginning of the simulation, (b) after 6.25 s. Splashing of the water can be seen behind the tank.

Figure 4.8: (a) The pitch and surge response of the TLP for large nonlinear wave versus time; (b) The tether forces versus time.
Chapter 5

Spar Buoy Floating Wind Turbine

In this chapter we use the method, described in chapter 2 and validated in chapter 3, to study the interaction of a spar buoy floating wind turbine with regular and irregular random ocean waves.

A spar buoy floating wind turbine is a wind turbine mounted on a spar buoy platform. The stability of this type of wind turbine is guaranteed by placing the platform center of gravity far below the center of buoyancy. Tethers, which can be taut or slack, have less effects on the stability of the spar buoy and are mainly used for platform station-keeping.

The spar buoy concept has been applied in oceanographic metrology studies [78, 79] and there has been interests in using the spar buoy concept for drilling and production of oil [80]. This concept has gained a lot more interest in recent years due to the trend in installing floating wind turbines far offshore. Most of the investigation in the field of floating wind turbine have emphasized the possibility of using spar buoy concept to carry wind turbines. Examples of studies conducted on spar buoy floating wind turbines, reviewed in chapter 1, can be found in [21, 26, 27, 30, 34, 35]. Furthermore, a large scale spar buoy floating wind turbine, Hywind, has
been successfully deployed and has been operational since 2009.

In this chapter, the response of a spar buoy floating wind turbine to regular waves and random ocean waves will be studied, platform response in ocean harsh condition will be considered and finally, design studies on the tether properties, tether position, and spar buoy aspect ratio will be performed.

5.1 Numerical results for regular waves

The response of a spar buoy floating wind turbine with a rather large displacement, shown in Figure 5.1, will be studied in this section. The various parameters for the spar buoy floating wind turbine are listed in Table 5.1. In Table 5.1, the tower, rotor, and nacelle mass are taken to be the standard 5MW wind turbine [48]. Platform center of gravity is far enough below the center of buoyancy to guarantee the stability of the platform. Tether attachment points are very close to the floating wind turbine center of gravity in the Z direction. This design will significantly reduce the tethers load, since the platform displacement due to the induced pitch motion from the wind is minimum near the center of gravity. It is assumed that the turbine is in locked condition, therefore the wind thrust force may be assumed constant. Tethers stiffness and pretension are considerably lower than for TLP mooring system, since they are designed for station-keeping purpose. The standard numerical domain is 1200 × 162 × 162 m in length, width and height, respectively (shown in Figure 5.2). The generated waves from the upstream are single frequency (0.2π rad/s) with 5.0 m heights for the baseline run.

The spar buoy floating wind turbine initial condition and its response to a single frequency and amplitude incoming wave after 250 s are shown in Figure 5.3. The incoming wave and the wind force on the rotor push the platform downstream, while
Figure 5.1: A spar buoy platform with a 5 MW wind turbine. Notation: 1-Rotor, 2-Nacelle, 3-Tower, 4-Platform tank, 5-Tether

the taut tethers keep the platform in place and the difference in the platform center of gravity and buoyancy keeps it stable.

In Figure 5.4 the waves amplitude measured at the wave maker, is shown. The wave amplitude has a slight variation in time due to the presence of the platform. In the absence of the platform, the waves propagate essentially unchanged until they reach the coarsely resolved part of the domain and damp out.

Figure 5.5 shows the response of the spar buoy to the incoming wave in surge and pitch direction. The surge motion shows a single frequency response with a frequency
Table 5.1: The parameters used for the computations of the spar buoy floating wind turbine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank mass</td>
<td>$13.55 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Tethers stiffness</td>
<td>$401500 \text{ KN}$</td>
</tr>
<tr>
<td>Pre-tension of tethers</td>
<td>0.2% of the tethers length</td>
</tr>
<tr>
<td>Tower mass</td>
<td>$0.347 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Nacelle+rotor mass</td>
<td>$0.350 \times 10^6 \text{ kg}$</td>
</tr>
<tr>
<td>Wave generator distance</td>
<td>$\lambda$ upstream</td>
</tr>
<tr>
<td>Wind thrust force</td>
<td>$400 \text{ KN}$</td>
</tr>
<tr>
<td>$C_g$ in Z direction</td>
<td>$-46.5 \text{ m}$ (respect to MWL)</td>
</tr>
</tbody>
</table>

equal to the incoming wave, but the amplitude of the response is not uniform. This non-uniformity may be due to the reflecting waves from the platform. In the design of floating wind turbines, platform surge response should be small enough to prevent the high tension or slacking of the tethers. Slacking of the tethers can reduce the fatigue life of the mooring system and large tether tension will increase the total cost of the system.

The pitch response is very important parameter since a rocking motion of the turbine can adversely affect the efficiency and operational conditions of the rotor. Figure 5.5b shows the pitch response of the spar buoy. The frequency is determined by the incoming wave, as before, except at the very early time when the wind force tilts the tower but the waves have not yet reached the platform. The restoring moment of the spar buoy in the pitch direction is mainly provided by the difference in center of gravity and buoyancy. The mean pitch response is due to the difference of the mean moment from the wind force and the mean moment due to the wave drift force.

The forces for the upstream and downstream tethers are shown in Figure 5.6. Since the platform does not have displacement in the yaw, roll, and sway directions,
tether forces in the two upstream (downstream) tethers are nearly the same, thus forces for only one of them is shown. Since the attachment point of the tethers is designed near the platform center of gravity, as discussed earlier in section 5.1, wind force does not have a major effect on the tether tensions, although it may slightly pushes the wind turbine downstream. Wave forces on the other hand, results in a noticeable surge motion of the platform, thus variation in tether tensions. This periodic motion is limited by two tethers in upstream direction with 60° respect to incoming wave propagation and only one tether in down stream. Since the wave propagation is inline with the downstream tether, significant load will be applied on the downstream tether. The upstream tethers share the wave load according to their 60° with respect to direction of incoming wave, Therefore the upstream tethers withstand lower load from wave in comparison with the downstream tether and consequently higher load variation in downstream tether than the upstream tethers.
Figure 5.3: Spar buoy floating wind turbine (a) initial condition and (b) after 250 s.

Figure 5.4: Generated wave amplitude measured at the wave maker.

The higher mean tether forces on the upstream tethers, shown in Figure 5.6, are due to the wind thrust force plus the drift motion of the platform.
Figure 5.5: (a) Surge response and (b) pitch response of the spar buoy floating wind turbine as a function of time.

Figure 5.6: Tether forces of the spar buoy floating wind turbine as a function of time.
5.2 Numerical results for irregular waves

A spar buoy platform has been simulated for a baseline regular waves. But, floating wind turbine platform will be installed in oceans and needs to respond well in operational conditions and survive in harsh conditions. In section 5.1, a relatively large displacement spar buoy has been studied and acceptable responses in different modes were obtained. Since the platform cost is usually in proportion to the platform displacement [28], we decrease the platform displacement to examine if we still could get acceptable results for the different modes. The new design is shown in Figure 5.7 and different parameters are listed in Table 5.2. By decreasing the platform displacement the mooring system also can have lower stiffness. Furthermore, in this design the tethers angle are increased to reduce the required length of the tethers. This increase in the tethers angle will decrease the surge stiffness and increase the heave stiffness of the platform. In section 5.4, a more systematic design parameter variation will be conducted. In this section, the baseline design is presented, natural frequencies for this design in different modes is investigated, and finally the response of the platform to moderate amplitude waves is studied.

5.2.1 Spar buoy baseline model

The new design main properties can be seen in Figure 5.7 and Table 5.2. The new designed spar buoy is being simulated for random ocean waves with significant wave height equal to 8 m and mean wave period of 10 s. It is assumed that the wind turbine is in parked condition, thus the rotor is not rotating to minimize the wind thrust force.

The spar buoy floating wind turbine that is studied has three degrees of freedom in the surge, pitch, and heave directions. The tethers are taut and they are designed
Figure 5.7: A modified spar buoy platform with a 5 MW wind turbine. Notation: 1-Rotor, 2-Nacelle, 3-Tower, 4-Platform tank, 5-Tether

to remain in tension during the simulation. Although the equations of motions are coupled in all three directions, in the limit of small displacement the coupling is not severe.

One of the coupling terms in heave and pitch direction is related to dependency of pitch restoring moment to the magnitude of buoyancy force and the position of center of buoyancy. When the spar displacement in heave is large enough, both the center of buoyancy magnitude and position will vary considerably which will affect the restoring moment in pitch direction. The Mathieu instability can arise from this
coupling effect ([81,82]), but in the limit of small heave motion this coupling is not significant.

Tethers forces will couple all the three degrees of freedom of the spar buoy, but the coupling in pitch and heave direction due to relatively low stiffness of tethers (in compare with TLP) are not significant. The main duty of the tethers are limiting the spar buoy motion in the surge direction. Due to small horizontal angle of tethers (15° in this design), initially the tethers will not highly contribute in restraining the spar buoy heave motion. Pitch motion is mainly limited by the difference in center of gravity and buoyancy. Therefore, the attachment points of the tethers can not be placed far from the spar buoy center of rotation. If the attachment points are far from the center of rotation, it may lead to large oscillation of the tether forces which highly reduce the fatigue life of the tethers. On the other hand, due to relatively low stiffness of the tethers, they can not highly limit the spar buoy pitch motion.

In order to estimate the spar buoy natural frequencies, free pitch, heave, and surge decay tests have been simulated. The natural frequencies in pitch, surge, and heave determined from free decay tests will be used later to help interpret power spectra and RAO responses of spar buoy platform to incident random waves. In the
free decay tests, the platform is disturbed from its stable, equilibrium position in pitch, heave, or surge direction, then released to oscillate with its natural frequency.

For the pitch decaying test, an initial $10^\circ$ pitch angle is given to the spar buoy as the initial condition. The spar buoy is placed in a statically stable position in the heave direction to reduce the coupling in heave and pitch. The initial condition of the spar buoy is shown in Figure 5.8.

![Figure 5.8: Initial condition for free pitch decaying test.](image)

The result of the pitch response is shown in Figure 5.9a in which the decaying motion of the spar buoy in time can be observed. This free decaying test represents one of the advantages of using Navier-Stokes equations to study the wave forces on the platform. As can be seen in Figure 5.9a, the decaying trend can be captured without relaying on experimental data. In a frequency domain analysis only hydrodynamic damping is being considered and the viscous damping can be only estimated by tuning methods [29] which usually rely on experimental data.
Figure 5.9: (a) Free pitch and (b) free heave decaying test of the spar buoy floating wind turbine.

In Figure 5.9a the increase in pitch amplitude during the first half cycle is due to the coupling of heave and pitch e.g. the spar buoy has a positive motion in the heave direction, which leads to higher pitch response. This coupling become weaker at later times. Converting the response in Figure 5.9a to frequency domain yields an essentially single frequency response equal to $\omega_p = 0.23 \text{rad/s}$. This natural frequency in pitch direction is lower than the frequency range found in typical ocean waves (see Figure 2.9).

As a result the chance of exciting the pitch natural frequency is low, but it still may be desirable to further reduce the natural frequency. This cannot be achieved easily since increasing the mass of the platform to reducing the natural frequency in pitch, for example, will automatically increase the restoring moment in pitch. Decreasing the difference in center of gravity ($C_g$) and buoyancy ($C_b$) would lead to a decrease in the restoring moment, thus reducing the natural frequency for pitch. However, decreasing the difference in $C_g$ and $C_B$ will reduce the stability of the platform. Therefore, there is a trade off between higher stability and further
reduction of natural frequency in pitch direction.

Figure 5.9b represents the platform free heave decay test in which an initial displacement from the stable position in heave is given to the platform and the platform starts oscillating in heave direction. Although there are some coupling effects in the heave and pitch direction, the initial displacement in heave is small enough to considerably reduce this coupling. The heave response also shows a nearly single frequency decaying motion. Plotting the spectrum of the response yields a dominate peak at the resonance frequency of $\omega_H = 0.33 \text{ rad/s}$. Heave natural frequency, the same as pitch natural frequency, is in the lower range of ocean waves typical range of frequencies (see Figure 2.9). Decreasing the platform radius is an option for lowering the natural frequency in heave to increase the difference of heave natural frequency from typical peak periods of the ocean waves. However, this reduction will lead to decrease in platform displacement. Another option is to increase the aspect ratio of the platform which will lead to the increase in the difference between $C_g$ and $C_b$. This will increase the stability of the platform. However, the natural frequency in pitch will be increased. The same simulation for the surge natural frequency estimation, leads to $\omega_S = 0.244 \text{ rad/s}$.

Overall the pitch, heave, and surge natural frequencies are relatively far from the typical peak frequencies of the ocean waves, thus the design described in Figure 5.7 and Table 5.2 will be used as a benchmark model for the numerical simulations. Furthermore, the platform will not have any issues regarding Mathieu instability which mainly happens because of coupling in heave and pitch direction. Here, the ratio of $\omega_p/\omega_H$ is 0.7 while Mathieu instability occurs around $\omega_p/\omega_H = 0.5$ [81].
Figure 5.10: Wave height measured upstream of the spar buoy platform in (a) time and (b) frequency domain.

5.2.2 Simulation results

The simulations are for ocean random waves with significant wave height equal to 8 m and mean wave period equal to 10 s. The response in time and frequency domain are calculated and discussed next. This simulation will let us to investigate the behaviour of the platform in the range of ocean wave frequencies.

Random ocean waves are generated at the inlet of the numerical domain and are measured half mean wave length upstream of the platform. Figure 5.10 represents the measured wave heights in time and frequency domain. As can be seen from the measured wave heights in the frequency domain, the desired Pierson-Moskowitz spectrum waves profile is modeled very well. This verify the accurate segmentation technique (see section 2.4) used to generate ocean waves. The measured wave height has slightly lower amplitude in time domain (Figure 5.10a) and small deviations in the frequency domain (Figure 5.10b). These small deviations are due to downstream effects.

Figure 5.11 shows one frame from the numerical simulation. Portion of the
platform pitch angle is due to the wind thrust force which applies a considerable moment on the platform.

Figure 5.11: Spar buoy floating wind turbine in interaction with random waves and constant wind thrust force.

The surge time domain, spectral density, and the corresponding RAO are shown in Figure 5.12. It can be seen in Figure 5.12a that the maximum peak to peak surge response is less than half of the platform diameter. The surge spectral density response is shown in Figure 5.12b and two peaks are observed. The lower frequency peak on 0.25 rad/s is due to the $\omega_S$ and possible coupling with $\omega_P$ and the second one is due to the peak in the spectral density of the incoming wave (Figure 5.10b). The RAO response shown in Figure 5.12c presents only one peak near $\omega_S$ and $\omega_P$ which emphasizes on weak coupling of surge and heave, since near the $\omega_H$ a peak is not observed.

The surge, heave, and pitch natural frequencies are estimated by the free decaying test. However, this is only an estimation for the natural frequency in different
modes in the limit of zero incoming wave frequency. Added hydrodynamic mass, which can highly affect the natural frequency, is a function of incoming wave frequency. Therefore, it is completely reasonable that the surge peak response is not exactly at the estimated surge natural frequency.

The heave response in time domain is shown in Figure 5.13a, the spectral density in Figure 5.13b, and the corresponding RAO is presented in Figure 5.13c. The peak to peak heave response in Figure 5.13a is about one fourth of the platform diameter which considerably higher than heave response of TLP floating wind turbines. The spectral density shows one dominate sharp peak which is mostly due to the combination of platform response in $\omega_H$ and the peak frequency of the incoming wave. The RAO response shows several peaks. The low frequency peak which is around $0.2 \ rad/s$ is probably due to the coupling of heave and pitch motions, since the frequency in peak is close to $\omega_P$. There is another peak at $0.38 \ rad/s$ which is related to $\omega_H$.

Figure 5.14a shows the pitch response in time domain, Figure 5.14b the spectral density, and Figure 5.14c presents the corresponding RAO. The pitch response in Figure 5.14a shows a maximum pitch angle of $7.5^\circ$. Usually a maximum allowable pitch angle of $10^\circ$ is chosen for the design of floating wind turbines [30]. This results emphasize the need for simulations for larger amplitude waves to make sure that the pitch response is below the desired limit. In the next section, the same platform will be examined for an extreme wave condition. The spectral density shows two sperate peaks which is clearly due to the $\omega_P$ and the incoming wave peak frequency. The RAO response shows a dominate peak in $\omega_P$ as it is expected and there is no peak around the natural frequencies in heave direction.

The tether forces from the simulation are shown in Figure 5.15. The initial tension on the three tethers are $8000 \ KN$. It is clear from Figure 5.15 that the
Figure 5.12: Surge response of the wind turbine in (a) time domain, (b) the corresponding spectral density and (c) the RAO response in surge direction.
Figure 5.13: Heave response of the wind turbine in (a) time domain, (b) the corresponding spectral density and (c) the RAO response in heave direction.
Figure 5.14: Pitch response of the wind turbine in (a) time domain, (b) the corresponding spectral density and (c) the RAO response in pitch direction.
downstream tether tension is higher than for the upstream tethers. This is due to the fact that the incoming wave is unidirectional and in this simulation the incoming wave direction is inline with the downstream tether direction. The upstream tethers share the force and therefore experience lower load than the downstream tether. The maximum and minimum tether tensions on the tethers are $12000 \, KN$ and $3000 \, KN$.

5.3 Spar buoy response for extreme condition

According to the measured wave height for 100,001 short-term description of the Northern North Sea (see Figure 2.10), the highest recorded waves are for significant wave height equal to $14 \, m$ and mean wave period equal to $16 \, s$. In this section, the previously designed spar buoy floating wind turbine will be studied for this extreme case to determine if the wind turbine can survive this condition.

Two frames of the platform motion, in interaction with waves and wind for this
extreme condition, are shown in Figure 5.16a and Figure 5.16b. Figure 5.16a shows a time at which the whole platform is submerged due to the incoming wave. This submerging might lead to a large heave motion. Five seconds later, as can be seen in Figure 5.16b, the buoyancy forces raise the platform out of the water again. The quantified surge, heave, and pitch responses are presented next.

Figure 5.17 shows the wave height measured upstream of the wind turbine. It can be seen that the highest generated wave is more than 17\textit{m} in height. Figure 5.18 shows the surge and heave responses of the floating wind turbine. The maximum peak-to-peak surge response is about 1.3\textit{D} and the maximum peak-to-peak heave response is 1.2\textit{D}. The heave motion of the spar buoy platform is relatively large compared to the tension leg platform. For the TLP the heave response is considerably less than the surge response of the platform due to the pre-tension and high stiffness of the tethers.

The pitch response is shown in Figure 5.19a. It can be seen that the maximum pitch response goes slightly beyond 10° which is a common design constraint for floating wind turbines. Therefore, we may need to increase the stability of the platform. This may be done by increasing its displacement tonnage of the platform.

In the current simulations the wind is modeled as a constant thrust force. This thrust force is linearly increased in the first 10\textit{s} of the simulation and remains constant during the simulation. Figure 5.19a shows that the initial wind thrust force result in a 5° pitch angle. Note that as the wind is applied in the first 10\textit{s} on the platform, the surge and heave response are not significant compare to the pitch response. It can be stated that the surge and heave response are more related to wave loads, but the pitch response is due to the both wind and wave loads.

The upstream and downstream tether tensions are shown in Figure 5.19b. The tether forces are significantly higher compared to the previous simulation for signifi-
Figure 5.16: Spar buoy floating wind turbine in extreme condition at (a) $t = 55 \, s$, (b) $t = 60 \, s$. 

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The previous simulation with $H_{1/3} = 8\ m$ and $T_1 = 10\ s$ has the maximum peak-to-peak response of 9000 $KN$. In this case, the tethers go slack and the maximum tethers load reach 20,000 $KN$ which is likely to be higher than the maximum breaking load of the tethers.

This nonlinear trend is not solely due to the increase of the wave height, but it is more related to the mean wave period variation. The variation is toward the natural frequency of the platform in surge, heave, and pitch (estimated in section 5.2) which are in the lower frequency range of the ocean wave spectra. This leads to significant increase in the response of the spar buoy floating wind turbine. Based on this study, situations recorded for the Northern North sea, such as ($H_{1/3} = 13\ m$ & $T_1 = 14\ s$) and ($H_{1/3} = 12\ m$ & $T_1 = 13\ s$) shown in Figure 2.10 can also result in large responses of the platform. In these two cases, although the significant wave height is lower, the mean wave period is nearer the natural frequencies of the spar buoy platform.

The standard deviation and the mean responses in different modes for this ex-
Figure 5.18: (a) Surge response and (b) heave response of the spar buoy platform in extreme condition. $y$ axis is nondimensionalized by the platform diameter.

Figure 5.19: (a) Pitch response of the platform in extreme condition, (b) Upstream and downstream tether forces of the wind turbine in the extreme condition.
Table 5.3: The standard deviation and mean response of the spar buoy floating wind turbine in extreme condition.

<table>
<thead>
<tr>
<th></th>
<th>std</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>0.3051D</td>
<td>0.0022D</td>
</tr>
<tr>
<td>Heave</td>
<td>0.2254D</td>
<td>0.0176D</td>
</tr>
<tr>
<td>Pitch</td>
<td>4.3076°</td>
<td>3.2235°</td>
</tr>
</tbody>
</table>

treme condition are shown in Table 5.3. It can be seen that the mean surge and heave responses are small, but the mean pitch response due to the mean wind thrust force is about 3.2°. The standard deviation is particularly helpful in design studies of platforms in interaction with random waves. The standard deviation represents the average response of the floating wind turbine in a particular mode. In the next section the standard deviation will be used to compare different designs of the floating wind turbines.

Overall, the current simulation presents the robustness of the current method in modeling the extreme conditions that might affect the design of floating wind turbines. In these conditions, the linear assumption that the platform motion is small in different modes with respect to a platform characteristic length, might not be an accurate, while the current model requires no assumption on the amplitude of the platform motions. Furthermore, situations such as Figure 5.16a where the platform temporarily submerges or the tethers go slack (Figure 5.19b) cannot be accurately predicted with the current linear models.

5.4 Spar buoy design study

In this section the spar buoy design described in subsection 5.2.1 has been chosen as a baseline design and different key design parameters of the spar buoy floating
wind turbine are varied to determine the effect of these design variation on wave and wind loading. Although in this section the design study is conducted for one short-term status of the ocean waves, it can be easily extended to different ocean waves conditions. This is a commonly used approach to verify proposed designs. In this section all the design studies are conducted for random ocean waves with a significant wave height equal to 8 m, a mean wave period of 10 s, and constant thrust force.

5.4.1 Effects of tethers attachment point

A number of studies has examined the effect of tethers attachment point on the response of spar buoy floating wind turbine. For example, Lee [83] studied this effect for regular spar buoy and Utsunomiya et al. [21] studied it for a stepped-type spar buoy floating wind turbines.

In our study, taut tethers with constant stiffness and pre-tension are attached at four different attachment points (Table 5.4) on the spar buoy platform, as shown in Figure 5.20, and the results are compared and discussed. The variation is over a relatively wide distance from the bottom to near the top of the platform. In this design study, the anchoring position of the tethers on the ocean floor remain the same which results in different tethers angle with respect to the platform.

Table 5.4: parameters used to study the effects of tethers attachment points for the spar buoy floating wind turbines (see Figure 5.20).

<table>
<thead>
<tr>
<th>Tethers attachment point design</th>
<th>Angle(θ)</th>
<th>$L_1/L_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12.2°</td>
<td>4.55</td>
</tr>
<tr>
<td>B</td>
<td>15.0°</td>
<td>1.77</td>
</tr>
<tr>
<td>C</td>
<td>18.4°</td>
<td>0.85</td>
</tr>
<tr>
<td>D</td>
<td>23.3°</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Figure 5.20: Four different tethers positions for a spar buoy floating wind turbine design.

Figure 5.21a represents the mean and standard deviation of the pitch response for the four different tether positions. Design A and design D are not very attractive due to the high standard deviation. This can be related to coupling effects that happen between surge and heave with the pitch which might augment the pitch response. On the other hand, the mean pitch response of these two designs are less than other two designs. This can be explained by considering the distance of tethers attachment point position from the platform center of rotation which according to the numerical simulations is usually around the center of gravity of the platform.
Designs A and D have the highest distance, therefore higher restoring moment than designs B and C.

Design B and design C show lower standard deviation with respect to designs A and D. This may be related to the tethers attachment points being closer to the platform center of rotation thus, reducing the contribution of the tether forces to the restoring pitch moment. Note that if the platform response in pitch mode has the same phase with the surge and heave modes, the contribution of the tethers would always be positive, but this might not be true, therefore it may brings some negative effects on the platform response. Overall, it seems that design C can be a candidate for lower response in pitch.

Figure 5.21b, shows the standard deviation of the platform for the four designs in surge and heave direction. The results can be explained by the angle of the tethers on different positions. Design A, which has the lowest angle, shows the lowest response in surge and the highest response in heave as we expect. Design D, which has the highest angle, shows the highest response in heave and lowest in...
surge. The responses for designs B and C are between the range of responses for
the designs A and D, with small deviation from this trend may be due to coupling
effect of surge and heave with pitch.

From the four designs shown in Figure 5.20, it can be seen that design B and
design C are more attractive than design A and D which is consistent with the
previous spar buoy designs in this field, such as the Hywind and the OC3-Hywind
spar buoy floating wind turbines.

5.4.2 Effects of tether stiffness

In the spar buoy floating wind turbine design, the tethers are mainly used for station-
keeping but their stiffness has some effect on the response of the floating wind tur-
bine. The most important mode of response for the spar buoy is the pitch response.
Large pitch responses can lead to a large accelerations of the platform nacelle and
rotor due to the considerable tower height. This acceleration will reduce the floating
wind turbine performance and cold cause structural damage. In this design study,
the tether stiffness are varied by ±25% from the baseline to study this effect on the
response of floating wind turbine.

Figure 5.22a shows the mean and standard deviation of the platform in pitch for
various tether stiffness. There is nearly a linear trend in the mean pitch response,
as we expect. However, the standard deviation does not have a linear trend with
respect to increasing the tether stiffness. This can be highly due to the fact that the
tethers are not the main mechanism for stability of the platform in pitch and even
high stiffness of the tethers has a negative impact on the platform pitch standard
deviation. This will become more clear as we plot platform responses in surge and
have.
Figure 5.22: Effects of the tether stiffness on the response of the spar buoy floating wind turbine. (a) Pitch standard deviation and mean response, (b) surge and heave standard deviation.

Figure 5.22b, shows the standard deviation in surge and heave. The surge and heave shows their maximum standard deviation in the case where the tethers have the highest stiffness. Although this trend looks unusual, since we might expect that higher stiffness should lead to lower surge and heave motion. This can be explained by noticing that the floating wind turbine achieve stability in pitch mainly due to the difference in the center of gravity and buoyancy location and not because of the tethers. Therefore, as we can see from from Figure 5.22a varying tether stiffness does not highly affect the pitch response. On the other hand, the pitch motion of the platform may lead to considerable platform motion in the attachment points of the tethers. So, we can conclude that the pitch motion will vary with the tethers tension. As we make the tethers stiffer, the pitch motion will lead to higher unbalance tether forces on the platform. Decreasing the tethers stiffness leads to less variation in tethers tension, thus lower response in surge is observed.

Further reduction in tether stiffness may result in higher motion of the platform in surge, not due to pitch coupling, but only because of lower stiffness in surge
direction. This could be the reason that we have an increasing in the surge response for less stiff tethers. Due to the angle of the tethers, higher motion in the surge direction can lead to higher heave response as seen in Figure 5.22b.

5.4.3 Effects of tank aspect ratio

In this section the spar buoy total mass and displacement are kept the same and the aspect ratio of the platform (AR) is varied by ±15%. The three different platform aspect ratios are shown in Figure 5.23. It is expected that increasing the platform aspect ratio will affect the surge and heave responses. Figure 5.23 shows the three different designs for a spar buoy floating wind turbine with the same weight but different aspect ratio (AR). AR is defined as height over diameter of the tank.

![Figure 5.23: Three different designs for a spar buoy floating wind turbine with the same weight but different aspect ratio (AR). AR is defined as height over diameter of the tank.](image-url)
Figure 5.24: Effects of the platform aspect ratio on the response of the spar buoy floating wind turbine. (a) Pitch standard deviation and mean response, (b) surge and heave standard deviation.

Figure 5.24a shows that mean and standard deviation in pitch. The decreasing trend in the mean pitch response, due to the increase in the restoring moment, can be clearly seen. The standard deviation in pitch decreases in the first step of increasing the aspect ratio, but in the next step, we have a slight increase. Since the restoring moment is increased by increasing the aspect ratio, this increase might be due to the increase in the moment applied by wind and wave on the platform.

Figure 5.24b represents the standard deviation in surge and heave. A nearly linear decrease in the surge standard deviation with increasing aspect ratio can be seen. This decrease may be due to the reduction of the platform diameter which
decreases the effective surface on which the wave loading is applied. A second reason could be due to the overall decrease in the pitch response which decreases the part of surge response which is coupled with pitch motion. By increasing the aspect ratio of the platform, there is a nonlinear trend in heave standard deviation. The main mechanism for stability in heave direction is the buoyancy of the platform. In all these three designs, the buoyancy is kept the same. Therefore, possibly this nonlinear trend could be due to the coupling with surge and pitch.

Overall, varying the aspect ratio of the spar buoy platform suggest that by increasing the aspect ratio, mean pitch response and standard deviation in surge show a nearly linear decrease, but the standard deviation in pitch and heave show a nonlinear trend. The simulations suggest that increase in the aspect ratio although could result in a decrease in the response to some degree but it could lead to a possible increase in the wind and wave moment on the platform.
Chapter 6

Conclusion and Future Work

6.1 Conclusions

A computational model is developed to predict the response of a floating wind turbine to large amplitude waves. The method is based on solving the unsteady Navier-Stokes equations with a level set method to predict the free surface motion and an immersed boundary method for tracking the floating wind turbine. The tethers, tower, nacelle, and rotor are included using reduced order models, leading to a reasonably efficient computational approach. Wind is modeled as a constant thrust force. Various comparisons with analytical, numerical, and experimental data reveals the accuracy of the proposed method for nonlinear modeling of floating wind turbine.

The developed numerical model is used to simulate a 100:1 scale model tension leg floating platform wind turbine and the RAO results in surge, heave, and pitch direction are compared with the experimental data.

Results are extended to a full-scale tension leg platform wind turbine. Pitch, heave, and surge response, as well as the tension in the tethers are shown as a
function of time for one incident wave frequency and amplitude. Vortex shedding
behind the TLP is not observed due to the large diameter of the TLP tank and
the resulting small Keulegan-Carpenter (KC) number. Thus, the flow remains
attached to the platform for regular incoming ocean waves. The effects of varying
wave amplitude on the surge response of TLP is studied. Our results show that the
linear assumption for the response of floating wind turbines is accurate for a wide
range of wave heights but leads to over-prediction for large waves (for this case,
over 10 m wave height approximately). We also examine the difference in the TLP
motion and the tether tension when the aspect ratio of the buoyant tank is changed.
The present method allows simulations of large amplitude waves and fully nonlinear
motion of the platform. We present one such result here, for which slacking of the
tethers (zero tension) are observed. This effect is difficult to model with current
linear models.

Spar buoy floating wind turbines are studied next with the numerical model. The
spar buoy floating wind turbine interaction with regular and random ocean waves
is investigated. An extreme case in which waves over 17 m are interact with the
spar buoy is studied. In this case the linear assumption may not be accurate, since
the platform motion is not small with respect to the platform characteristic length
(diameter of the spar buoy). Furthermore, in this extreme case, platform pitch
angles higher than 10°, complete submergence of the platform tank, and tether
slacking are captured. It is difficult for linear models to accurately predict these
effects.

A parametric design study is performed on the tether attachment points, stiffness,
and tank aspect ratio of the spar buoy floating wind turbine. It is concluded
that tether positions near the top or near the bottom of the tank are not very attrac-
tive designs and may lead to higher oscillation in surge, heave and pitch directions.
Based on the design study at the tethers stiffness, it is concluded increasing tether stiffness cannot be very trustable solution to limit the platform motion in surge and heave. Finally, increasing the aspect ratio of the platform tank, results in lower mean pitch and surge motions due to the increase in restoring moment, but it may lead to nonlinear trend in standard deviation in heave and pitch because of possible increase in the wind and wave moment on the platform.
6.2 Future work

The goal of this work was the development of a nonlinear model to fully capture the linear and nonlinear wave loads on floating wind turbines. Therefore, the developed simulations concentrate on modeling wave loads on floating wind turbine and rather simple models are used for aerodynamic and structural dynamic effects. It is highly desirable to integrate the developed hydrodynamic model with more robust structural dynamic and aerodynamic models in future.

Although various tests have been performed to verify and validate the current developed numerical model, it is interesting to compare the hydrodynamic loads, calculated by the current developed model, with WAMIT [36] which is well verified code based on potential flow theory.

Adaptive mesh refinement and converting the code from serial to parallel can highly reduce the computational time and increase the robustness of the model to capture smaller scale physics, specially on the solid-fluid interface.

The current model has been used for modeling TLP and spar buoy floating wind turbine so far. Semi-submersible is another concept which can be a suitable candidate for carrying floating wind turbines. It would be interesting to study the semi-submersible concept with the current developed model.

In the current developed model due to the small $Kc$ number turbulence model is not included. A RANS turbulence model can be added to the developed approach to increase the robustness of the model in order to be able the cover broader range of the simulations in which $Kc$ number is high and not only potential flow theory may not be applicable, but also laminar assumption for flow may not be accurate enough.

In the spar buoy simulations, the tethers are considered as taut and non-ballasted.
Ballasted and catenary tethers are two other concepts for spar buoys mooring system. These concepts could be modeled in the future.

The developed model could also be extended to simulate other systems in marine environments, especially for renewable energy technologies. Proposed wave-energy systems often feature floating structures that extract energy from ocean waves. Airborne wind energy hydro-kinematic energy systems have been proposed that would be deployed from floating platforms in the deep oceans or in ocean currents. The developed model could be extended to help model these concepts in the future.
Bibliography


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