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Development of Human Body CAD Models and Related Mesh Processing Algorithms with Applications in Bioelectromagnetics

Janakinadh Yanamadala
Worcester Polytechnic Institute

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Development of Human Body CAD Models and Related Mesh Processing Algorithms with Applications in Bioelectromagnetics

by

Janakinadh Yanamadala

A Dissertation

Submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Doctor of Philosophy

in

Electrical and Computer Engineering

April 2016

APPROVED:

Dr. Sergey Makarov
Worcester Polytechnic Institute

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Beth Israel Deaconess Medical Center, Harvard Medical School

Dr. Gregory Noetscher
US Army Natick Soldier Research, Development and Engineering Center

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MathWorks, Inc.

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[Signatures]
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Dr. Shashank Kulkarni
MathWorks, Inc.

Dr. Vishwanath Iyer
MathWorks, Inc.
To my family and friends
Abstract

Simulation of the electromagnetic response of the human body relies heavily upon efficient computational CAD models or phantoms. The Visible Human Project® (VHP)-Female v. 3.1 – a new platform-independent full-body electromagnetic computational model is revealed. This is a part of a significant international initiative to develop powerful computational models representing the human body. This model’s unique feature is full compatibility both with MATLAB and specialized FEM computational software packages such as ANSYS HFSS/Maxwell 3D and CST MWS. Various mesh processing algorithms such as automatic intersection resolver, Boolean operation on meshes, etc. used for the development of the Visible Human Project® (VHP)-Female are presented.

The VHP – Female CAD Model is applied to two specific low frequency applications: Transcranial Magnetic Stimulation (TMS) and Transcranial Direct Current Stimulation (tDCS). TMS and tDCS are increasingly used as diagnostic and therapeutic tools for numerous neuropsychiatric disorders.

The development of a CAD model based on an existing voxel model of a Japanese pregnant woman is also presented. TMS for treatment of depression is an appealing alternative to drugs which are teratogenic for pregnant women. This CAD model was used to study fetal wellbeing during induced peak currents by TMS in two possible scenarios: (i) pregnant woman as a patient; and (ii) pregnant woman as an operator. An insight into future work and potential areas of research such as a deformable phantom, implants, and RF applications will be presented.

I defend-

1. Development of the full-body CAD model VHP-Female v. 3.0 and 3.1
2. Creation of a full-body pregnant Japanese female CAD model (1st/2nd/3rd trimesters)
3. Comparison of cephalic vs extracephalic electrode locations for tDCS. Conclusion about the potential advantage of the extracephalic electrode location
4. Preliminary upper estimate of peak currents in Transcranial Magnetic Stimulation at distant locations from a TMS coil
5. Estimates of peak currents induced by Transcranial Magnetic Stimulation in pregnant women as patients or operators
List of Papers


Contents

Chapter 01 – ................................................................................................................................. 1
Computational Human Models (Phantoms) For Electromagnetic Field Exposures .................. 1

I. DEVELOPMENT OF HUMAN MODELS AND MATERIAL PROPERTIES ........................................... 1
   A. Use of computational human models ................................................................................. 1
   B. Major bioelectromagnetic applications .............................................................................. 1
   C. Development history .......................................................................................................... 1
   D. Development of electromagnetic tissue properties ............................................................ 2

II. HUMAN MODEL CONSTRUCTION ............................................................................................. 3
   A. CAD and voxel computational human models .................................................................. 3
   B. Other CAD models ............................................................................................................. 5
   C. Anisotropic and fiber models ............................................................................................. 6

REFERENCES ................................................................................................................................ 7

Chapter 02 – .................................................................................................................................. 10
Mesh Processing Algorithms ........................................................................................................ 10

I. Triangular mesh and its quality............................................................................................. 10
   A. Arrays of vertices and faces. Structured meshes ............................................................ 10
   B. A 3D triangular mesh. 2-Manifold meshes .................................................................... 13
   C. Triangle quality and mesh quality .................................................................................. 14
   D. Triangle size and mesh uniformity ................................................................................. 16

REFERENCES .............................................................................................................................. 16

II. Delaunay triangulation. Three-dimensional volume and surface meshes ......................... 17
   A. Structured vs. unstructured meshes................................................................................ 17
   B. Mesh generation and its properties................................................................................. 17
   C. Delaunay triangulation in two dimensions ................................................................. 17
   D. Algorithm ....................................................................................................................... 17
   E. Example of Delaunay triangulation. Incremental vertex addition ................................... 19
   F. Example of constrained Delaunay triangulation ............................................................. 20
   G. Delaunay triangulation in three dimensions (tessellation or tetrahedralization) .......... 21
   H. Three-dimensional surface mesh generation .................................................................. 21
   I. Algorithms for three-dimensional surface mesh generation ........................................... 22

REFERENCES .............................................................................................................................. 23
III. Mesh operations and transformations ................................................................. 25
   A. Topology-preserving mesh transformations ....................................................... 25
   B. Necessity of mesh smoothing .......................................................................... 25
   C. Topology-preserving Laplacian smoothing ....................................................... 26
   D. Laplacian smoothing with re-triangulation. Iterative algorithm ......................... 28
   E. Weaknesses of Laplacian smoothing ................................................................. 29
   F. Collision algorithms for 3D surface meshes ...................................................... 29
   G. Checking in/out status and finding outer normal vectors for 2-manifold 3D surface meshes ........................................................................................................... 32
REFERENCES ................................................................................................................. 33

IV. Mesh processing Algorithms ...................................................................................... 35
   A. INTRODUCTION .................................................................................................... 35
   B. Separation of distinct body regions .................................................................... 35
   C. Visual mesh processing tool .............................................................................. 36
   D. Mesh intersection algorithm ............................................................................. 36
   E. Edge collapse of intersection chains ................................................................... 39
   F. Deformation in the normal direction .................................................................... 40
   G. Deformed meshes .............................................................................................. 40
   H. About general mesh deformation methods in computer graphics ....................... 42
REFERENCES ................................................................................................................. 43

Chapter 03 – Development of VHP-Female CAD model – VHPC 3.0 and VHPA 3.0 .......... 45
I. Introduction .............................................................................................................. 45
   A. Source .................................................................................................................. 45
   B. Construction ....................................................................................................... 45
   C. Proof of surface reconstruction accuracy .......................................................... 46
   D. Model anatomy and materials .......................................................................... 46
   E. Model topology: Model objects ........................................................................ 46
   F. Model topology: List of fully enclosed objects .................................................. 47
   G. Example of assigning material properties for Finite-Element Method (FEM) and Finite-Difference Time-Domain (FDTD) Methods ......................................................... 47
   H. Scalp Topology .................................................................................................. 48
   I. Muscular System ............................................................................................... 48
J. Extension datasets ............................................................................................................... 48
   a) Bone Composition and Addition of Large Orthopedic Implants ............................... 48
   b) Variable BMI ................................................................................................................. 48
   c) Improved Resolution in the Cranium ............................................................................. 48

II. Three model versions ...................................................................................................... 55
   A. VHP-Female version 3.0 BASE (DOI:10.20298/VHP-Female-V.3.0-BASE) .................. 55
   B. VHP-Female version 3.0 SMOOTH (DOI:10.20298/VHP-Female-V.3.0-SMOOTH) ....... 55
   C. VHP-Female version 2.2 (DOI:10.20298/VHP-Female-V.2.2) ................................... 55

III. Context of Use .................................................................................................................. 58
   A. Low-Frequency Electromagnetic Simulations of Injected Electric Currents ............... 58
   B. Low-Frequency Electromagnetic Simulations of Induced Electric Currents ............... 58
   C. RF Simulations of on-body, in-body, and near-body antennas (dipoles, loops, patches, meanders, etc.) .......................................................................................................................... 62

IV. Strength of Evidence ...................................................................................................... 63
   A. Tool validity ................................................................................................................... 63
   Anatomical accuracy .......................................................................................................... 63
   Computational accuracy, cross-platform comparison, and efficiency .............................. 63
   Comparison with experiment ............................................................................................. 66
   B. Plausibility .................................................................................................................... 67
   Outcome ............................................................................................................................ 67

   Devices to be tested with regard to safety evaluation according to [17, 18] .................. 67
   C. Extent of prediction ....................................................................................................... 67
   D. Capture .......................................................................................................................... 68

V. Assessment of Advantages and Disadvantages ............................................................... 68
   A. Advantages (type, magnitude, and likelihood) ............................................................. 68
   B. Disadvantages (types, magnitude, mitigation) .............................................................. 69

VI. Improved Head Model of VHP-Female Version 3.0 Smooth (100% compatible with the rest of model) ........................................................................................................ 70
   A. Tissues .......................................................................................................................... 70
   B. Model Topology ........................................................................................................... 71

REFERENCES ......................................................................................................................... 74
VI. APPENDIX A. LIST OF TISSUE OBJECTS.................................................................76

Chapter 04 – ......................................................................................................................87

Applications of VHP-Female Computational Phantom ................................................87

SECTION A – ......................................................................................................................87

SAFE UPPER ESTIMATE OF PEAK CURRENTS IN TRANSCRANIAL MAGNETIC STIMULATION AT
DISTANT LOCATIONS FROM A TMS COIL.................................................................87

I. Introduction.....................................................................................................................87

II. Upper analytical estimate of eddy currents in a heterogeneous conducting body......89
   A. Neglecting the secondary magnetic field of eddy currents – thin limit condition ....89
   B. Neglecting the effect of free surface charges .........................................................90
   C. Neglecting the effect of tissue permittivity .............................................................90
   D. Analytical estimate of eddy current density ..............................................................90

III. Base FEM computational human phantom and computational testbed ....................91
   A. Computational phantom .........................................................................................91
   B. Computational test bed ..........................................................................................92

IV. Simulation setup. Qualitative results .......................................................................94
   A. Basic geometry setup .............................................................................................94
   B. Full body coverage ..................................................................................................96
   C. Sensitivity analysis setup ......................................................................................96
   D. Qualitative eddy current behavior at distant locations from the coil ...............96

V. Comparison between analytical and numerical results in frequency domain ..........96
   A. Transfer function for the numerical solution .........................................................96
   B. Transfer function for the analytical solution .........................................................97
   C. Comparison of numerical and analytical solutions ..............................................97

VI. Comparison between analytical and numerical results in time domain ...............99
   A. Coil current pulse form ..........................................................................................99
   B. Converting frequency-domain solution to time domain .....................................101
   C. Comparison between analytical and numerical results .....................................102
   D. Sensitivity analysis ...............................................................................................103

VII. Testing a different coil geometry .............................................................................103

VIII. Interpretation of results ........................................................................................107
   A. Why is the analytical model working? .................................................................107
   B. Exceptions ............................................................................................................109
IX. Guaranteed upper estimate ........................................................................................................ 110
   A. Guaranteed upper estimate ............................................................................................... 110
   B. Results for full-body coverage ..................................................................................... 110
X. Conclusions .............................................................................................................................. 112
REFERENCES ................................................................................................................................ 113
SECTION B – ................................................................................................................................ 118
COMPARISON OF CEPHALIC AND EXTRACEPHALIC MONTAGES FOR TRANSCRANIAL DIRECT CURRENT STIMULATION – A NUMERICAL STUDY ................................................................. 118
I. Introduction ............................................................................................................................ 118
II. Problem Statement and Organization .................................................................................. 120
III. VHP-F Model and Simulation Description ......................................................................... 122
   A. Data Acquisition ............................................................................................................... 122
   B. Segmentation .................................................................................................................. 122
   C. Mesh Conditioning and Registration ............................................................................... 122
IV. Simulation Setup .................................................................................................................. 124
   A. Material Properties ......................................................................................................... 124
   B. FEM Software and Numerical Accuracy ......................................................................... 124
   C. Boundary Conditions and Excitations .......................................................................... 124
V. Electrodes and their Montages .............................................................................................. 125
   A. Electrode Model .............................................................................................................. 125
   B. Electrode Montages ........................................................................................................ 125
VI. Simulation Results for Total Current Density ................................................................. 126
   A. Total Current Density for Surfaces of Cerebral Cortex and White Matter .................. 126
   B. Total Current Density for Sagittal Cut Planes .............................................................. 126
   C. Total Current Density for Axial Cut Planes .................................................................. 128
   D. Total Current Density for 45 Degree Cut Planes ......................................................... 128
   E. Invariance of Relative Current Densities to Changes in Skin Properties ..................... 128
VII. Quantitative Evaluation of Vertical and Horizontal Average Current Densities .......... 129
VIII. Expected Electrode Voltages and Their Variations ............................................................ 129
IX. Discussion ............................................................................................................................. 136
   A. General Observations ...................................................................................................... 136
   B. Invariance of Extracephalic Montages .......................................................................... 136
   C. Validation of Results across Configurations ................................................................. 138
D. VHP-F Model Limitations and Extensions ................................................................. 139
X. Conclusions .................................................................................................................. 139
REFERENCES .................................................................................................................. 139
SECTION C – ................................................................................................................. 143
ANTENNA APPLICATION EXAMPLES ........................................................................ 143
SECTION D – ................................................................................................................. 147
COMPUTATIONAL PERFORMANCE USING HIGH-FREQUENCY FEM SIMULATOR ANSYS HFS 147
  A. Plane Wave Test ........................................................................................................ 147
  B. MRI-Coil Modeling ................................................................................................... 148
REFERENCES .................................................................................................................. 149
Chapter 05 – ................................................................................................................. 150
Development and Applications of Japanese Pregnant Model ........................................ 150
  I. INTRODUCTION ......................................................................................................... 150
  II. Problem Statement .................................................................................................. 151
      A. CAD model construction ....................................................................................... 151
      B. Tissue properties, coil construction, and pulse excitation ..................................... 155
  III. Computational Results – Pregnant Woman as a Patient ........................................ 157
      A. Coil positioning ..................................................................................................... 157
      B. Frequency-domain results .................................................................................... 158
      C. Time-domain results and extracting maximum peak values ................................ 158
      D. Comparison with safe values of eddy current density ......................................... 159
  IV. Computational Results – Pregnant Woman as Operator ......................................... 160
      A. Coil positioning for standard operation ................................................................. 160
      B. Accidental coil discharge ...................................................................................... 161
      C. Time domain results – second trimester ............................................................... 161
      D. Comparison with safe values of eddy current density ......................................... 162
      E. Case of accidental discharge ................................................................................ 162
      F. Results for other trimesters and eddy currents in the fetal area ............................ 162
  V. Comparison with Upper Analytical Estimate for Eddy Current Density ................. 163
  VI. Control of Accuracy of Simulation Results .......................................................... 166
  VII. Summary of Results ............................................................................................... 168
      A. Pregnant patient .................................................................................................... 168
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Chapter 01 –
Computational Human Models (Phantoms) For Electromagnetic Field Exposures

I. DEVELOPMENT OF HUMAN MODELS AND MATERIAL PROPERTIES

A. Use of computational human models

Computational modeling combines mathematics, physics, and computer science to study the behaviors and reactions of complex biomedical problems in-silico. The National Institutes of Health states that “modeling can expedite research by allowing scientists to conduct thousands of simulated experiments by computer in order to identify the actual physical experiments that are most likely to help the researcher find the solution to the problem being solved” [1]. Among other computer simulation tools, computational human models or “virtual humans” are becoming a significant component of modern biomedical research.

In application to general medical device development, relevant study areas include fluid dynamics (e.g., shear stress and stagnation calculations in ventricular assist devices), solid mechanics (e.g., maximum stress locations in a hip implant), electromagnetics and optics (e.g., radiofrequency dosimetry in magnetic resonance imaging), ultrasound propagation (e.g., absorbed energy distribution for therapeutic ultrasound), and thermal propagation (e.g., radiofrequency and laser ablation devices) [2]. Computational human body models are also widely used for biomechanical and automotive safety research and design and as such have significantly contributed to a reduction of traffic injuries and fatalities [3]-[5].

B. Major bioelectromagnetic applications

In application to bioelectromagnetics, computational human models are used to perform safety and performance evaluation of the following medical devices: electrophysiology monitoring devices, magnetic resonance imaging (MRI) systems, MR conditional passive or active implanted devices (e.g., orthopedic devices, stents, pacemakers, and neurostimulators), devices for radiofrequency ablation, optical coherence tomograph devices, fluorescence spectroscopy devices, laser surgery devices, and optical therapy devices [2]. Along with this, research is geared toward implanted active antennas [6]-[9] and microwave imaging systems [10]-[12].

C. Development history

First computational human models were stylized computational phantoms comprising of (many) simple mathematical shapes (mathematical phantoms) and mostly used for ionizing radiation dosimetry studies [13]-[15]. The breakthrough occurred with the development of computed tomography (CT) and magnetic resonance imaging (MRI). It has been found that the stacks of two-dimensional tissue images could be transformed into 3D virtual yet realistic tissue models in voxel (volumetric pixel) format or FEM CAD (finite-element computer aided design) format [15],[16]. In 1990, the U.S. National Library of Medicine started the first such project, building a digital image library of volumetric data representing a complete normal adult human male and female [17]-[19]. This “Visible Human Project®” (VHP) included digitized photographic images from cryosectioning, digital images derived from computerized tomography, and digital
magnetic resonance images of two cadavers. Even today, the corresponding cryosection images (24 bits of color, resolution of 0.33 mm-female) provide state-of-the-art resolution of muscle and other soft tissues, as well as bone matter. The datasets have applied to a wide range of virtual reality uses by over 3500 licensees in 64 countries [20]. In 1996, the first full-body voxel model based on MRI data of an adult male has been developed in Great Britain by Dimbylow [21] and was further augmented with the female model [22]. In 2004-2007, a voxel model of Japanese male and female has been developed from MRI images (Nagaoka man & Nagaoka woman) [23] including a pregnant female model [24]. In 2004, Chinese Visible Human project (males and females) has been completed with the world-best resolution color images (0.17mm) [25]. The corresponding voxel models have been developed in 2008 [26] and 2011 [27], and a deformable CAD model has been developed in 2015 [28]. In 2006, a voxel model of Korean male has been developed [29] followed by the child model in 2009 [30]. Finally, the voxel “Virtual Family” [31],[32] of Switzerland was created over 2010-2014 in voxel with the whole-body MRI images having in many cases the resolution of 0.5x0.5x0.5 mm. Four voxel full-body datasets have been further converted to surface meshes [32]. To date, the Virtual Family includes 13 members [32]. The number of voxel full-body human models developed so far approaches 40 [16].

Fig. 1. a) – CAD model and b), c) – voxel model of the left patella.

D. Development of electromagnetic tissue properties

Every human model must be augmented with the electrical tissue properties. In the isotropic case, they usually include relative permittivity and conductivity of a tissue as functions
of frequency. Pioneering work of C. Gabriel and colleagues from UK [34]-[37] supported by US Air Force Research Laboratory and their latest results [38] reported the corresponding data from 10 Hz to 100 GHz for the bulk of tissues. Most of the measurements performed were on animal tissues carried out in vitro, at 20 and 37°C. Measurements were also made, in vivo, on accessible parts of the human body such as palm, sole and forearm skin, tongue, etc. This data is considered standard and is also replicated in IT’IS database for electromagnetic parameters [39]. Wagner et al [40] recently measured in-vivo impedances of human cranial tissues (skull, grey matter, white matter) from 10 Hz to 50 kHz.

II. HUMAN MODEL CONSTRUCTION

A. CAD and voxel computational human models

**Triangular CAD models.** In CAD models suitable for 3D printing (an example is an STL or STereoLithography format), every individual tissue is characterized by its closed surface. In its most common form, the surface is fully described by a set of small adjacent triangles defined through an array of nodes \( P \) and an array of triangles \( t \) shown in Fig. 1a. Every \( m \)-th row of array \( P \) gives Cartesian coordinates of a nodal point \( m \) on the surface, for example

\[
P(m=1,:) = [0.11 \quad 0.05 \quad -0.41],
\]

\[
P(m=2,:) = [0.06 \quad 0.03 \quad -0.21],...
\]  

(1)

Every \( n \)-th row of array \( t \) gives three numbers of nodal points forming \( n \)-th triangle, i.e.

\[
t(n=1,:) = [1 \quad 2 \quad 3],
\]

\[
t(n=2,:) = [1 \quad 2 \quad 5],...
\]  

(2)

**Voxel models.** To construct a voxel model from the CAD model, we subdivide the entire 3D space into many small equal brick cells (unit cells) with the size \( \Delta x, \Delta y, \Delta z \) shown in Fig. 1b. If the cell center lies within the tissue object, the cell is assumed to be a part of the object. As a result, the entire tissue volume is approximated by a staircase set of a large number of cells seen in Fig. 1b. Every such cell has a unique value of a tissue property (or properties), which is a voxel. For example, we can mark all cell centers with starts, use blue starts for centers inside the tissue, use red stars for centers outside the tissue, and finally obtain a voxel plot of the tissue shown in Fig. 1c. Frequently, the entire unit cell along with the assigned tissue properties is designated as a voxel.
Comparison between CAD and voxel models. First, the CAD model of the tissue is potentially more accurate since it provides a linear (in case of triangles or plane quadrilaterals) or polynomial (in case of a spline representation) surface approximation in contrast to the staircase approximation of the voxel grid. For voxel models, the modeling accuracy, which is the deviation from the true surface in its normal direction, is equal to the size of the unit cell. Second, the CAD model is inherently deformable including both free-form deformations and affine transformations while the voxel model is essentially “cast in stone”. However, the 3D CAD model of the tissue surface (in case of Fig. 1a it is called a triangular mesh) is more difficult to construct. Furthermore, triangulated human models with a very large number of triangles in excess of $1-5 \times 10^6$ [32],[41] require extremely large FEM (or BEM, MoM) simulation times, which could make their use prohibitive for a number of optimization tasks. For the purposes of comparison, Fig. 2 shows (an artistically assembled) voxel model of the VHP project (female) [20] while Fig. 3 demonstrates the same model but in the triangular-mesh CAD format with about 600,000 triangles total [42].

**Manifoldness of CAD models.** The following two conditions are required for the true CAD model:

1. A 3D triangular mesh represents a physical solid object. Therefore, it must be watertight, i.e. do not have missing triangles (surface holes).

2. The surface of a well-behaved triangular mesh in 3D must satisfy one critical condition, which is the so-called manifold condition. A mesh is 2-manifold if every node of the mesh has a disk-shaped neighborhood of triangles. This neighborhood can be continuously deformed to an open disk. Every edge of a 2-manifold mesh is a manifold edge – with only two attached triangles.

All other meshes are non-manifold meshes. Fig. 4 gives examples of a non-manifold mesh with a non-manifold edge and a non-manifold mesh with a non-manifold node.
**Multi-tissue voxel and CAD models.** Constructing a multi-tissue voxel model is straightforward. Different cells are simply assigned different tissue properties. Construction of a multi-tissue CAD model is more difficult. First, different tissue meshes seen in Fig. 4 must not intersect. Fully-enclosed meshes are still allowed. Another well-known problem with multi-object models is object matching in a contact region. Usually, the contact region is not explicitly defined in a CAD model to be imported, so that it has to be discovered separately by testing for face-to-face overlaps and matching CAD faces/edges in the contact region [43]. This may create problems for certain CAD kernels such as ACIS. In order to prevent CAD import errors, a thin gap may be introduced between different tissue objects filled with “average body properties” of an outer enclosing shell. If this gap is reasonably small, it provides a close approximation to reality.

**B. Other CAD models**

*Quadrilateral surfaces.* Although the triangle is a simplest polygon in two dimensions (a simplex), subdivision of the surface into a set of quadrilaterals (each of which is formed by two adjacent triangles) is used too. Some E&M software packages such as WPIL-D operate with quadrilaterals and higher-order basis functions. Quadrilateral CAD models are also common in FEM biomechanical human models [3]-[5], [44]. Fig. 5 demonstrates stress distributions and locations of rib fractures for one such model – a Toyota full-body FEM human model [44].

*NURBS.* Another alternative common in manufacturing and computer graphics is the subdivision of the original surface into a smaller
number of relatively-large curved spline surfaces – NURBS (Non-uniform Rational Bezier Spline surfaces) or B-splines [45]. NURBS surfaces enable (adaptively) refined meshes without sharp edges (where charge density becomes singular) in specific areas of interest. Furthermore, they can be used for deformation purposes [16]. Otherwise, the NURBS surfaces have a limited value for an FEM E&M solver, which internally operates with geometry primitives: triangular facets and tetrahedra. A double conversion, from segmented triangular surfaces → NURBS surfaces → FEM triangular surfaces, may require a (very) significant additional meshing time. Fig. 6 shows a conversion from the original triangulated surface of a skin shell (about 7,000 triangles) to a set of NURBS surfaces (about 60 B-splines) done with SpaceClaim Direct Modeler of ANSYS.

C. Anisotropic and fiber models

Many tissues such as the central nervous system with the brain and the spinal cord, the peripheral nervous system, and the muscles, are fibrous in nature. When the organization of a tissue follows a complex fibrous structure within a conducting medium, two approaches are possible. The first one is in using an electric permittivity tensor (usually diagonal) instead of the scalar permittivity and/or an electric conductivity tensor (diagonal) instead of the scalar conductivity. This method is not without its limitations. For instance, crossing fibers might generate an anisotropic conductivity structure that is not described by a tensor. A second, significantly more flexible approach would be in modeling individual fibers.
REFERENCES


8


Chapter 02 –
Mesh Processing Algorithms

I. TRIANGULAR MESH AND ITS QUALITY

A. Arrays of vertices and faces. Structured meshes

Arrays of vertices and faces

A triangular surface mesh is the base of any surface representation including various numerical methods in electrical and biomedical engineering, computer graphics, etc. Consider a planar rectangle on the size $a \times b$ in the xy-plane shown in Fig. 2. Our goal is to “cover” its surface with triangles – simplexes in 2D. Many ways of doing so exist. One such way is shown in Fig. 1.

Fig. 1 Mesh generation for a planar rectangle.

We first define uniformly spaced $x$-nodes and uniformly spaced $y$-nodes. Assume that there are $N_x + 1$ nodal points along the $x$-axis and $N_y + 1$ nodal points along the $y$-axis. In Fig. 1, $N_x = 3, N_y = 3$. In a general case, one has
\[ x_m = \left( \frac{m-1}{N_x} \right) a, \quad m = 1, \ldots, N_x + 1 \]

\[ y_n = \left( \frac{n-1}{N_y} \right) b, \quad n = 1, \ldots, N_y + 1 \]

(1)

There is a common way of describing triangular meshes, which originates from old NASTRAN programs written in the 1970s and 1980s. In order to define a triangular mesh, we need the array of vertices (or nodes), \( P \). This array consists of rows; every row includes three Cartesian coordinates of the corresponding nodal point. The row number in array \( P \) is simply the vertex number. Further, we need an array of faces (or triangles), \( t \). This array also consists of rows; every row includes three integer numbers of triangle vertices; each such number is simultaneously the row number of the array \( P \). The row number in the array \( t \) is the number of the triangular face. For the mesh shown in Fig. 1, there are 16 nodes and 18 triangles, giving:

\[
P = \begin{bmatrix}
  x_1 & y_1 & 0 \\
  x_2 & y_1 & 0 \\
  x_3 & y_1 & 0 \\
  x_4 & y_1 & 0 \\
  x_1 & y_2 & 0 \\
  x_2 & y_2 & 0 \\
  x_3 & y_2 & 0 \\
  x_4 & y_2 & 0 \\
  x_1 & y_3 & 0 \\
  x_2 & y_3 & 0 \\
  x_3 & y_3 & 0 \\
  x_4 & y_3 & 0 \\
  x_1 & y_4 & 0 \\
  x_2 & y_4 & 0 \\
  x_3 & y_4 & 0 \\
  x_4 & y_4 & 0 
\end{bmatrix}
\]

\[
t = \begin{bmatrix}
  1 & 2 & 6 \ \\
  1 & 5 & 6 \ \\
  2 & 3 & 7 \ \\
  2 & 6 & 7 \ \\
  3 & 4 & 8 \ \\
  3 & 7 & 8 \ \\
  \ldots \ \\
  11 & 15 & 16 
\end{bmatrix}
\]

\[ \Rightarrow t = \begin{cases} 
  \text{odd row}: & [m, m+1, m+ N_x + 2] + (n-1)(N_x + 1) \\
  \text{even row}: & [m, m + N_x + 1, m + N_x + 2] + (n-1)(N_x + 1) \\
  & m = 1, \ldots, N_x, \quad n = 1, \ldots, N_y 
\end{cases} \]

(2)

Connectivity

The triangular mesh which covers the surface of the rectangle in Fig. 1 is uniquely characterized by the array of vertices, \( P \), and the array of faces, \( t \). Thus, the mesh is simply a collection of vertices and faces. The face information is a part of the connectivity information – constructing the faces means establishing the connectivity between different vertices (or nodes). Other (secondary) parts of the connectivity information may include an array of edges, \( e \), array of edges attached to each node, array of triangles attached to each node (one or more triangles), array of triangles attached
to each edge (one or two triangles), etc. Meaning of connectivity is wide: one may reply upon face connectivity data, edge connectivity data, etc.

**Structured mesh**

The triangular mesh shown in Fig.1 is called a *structured surface mesh*. In a structured mesh, the indices of the neighbor vertices for any particular vertex could in principle be calculated using a simple addition rule – see Fig. 1 and Eq. (2). As long as this rule is known, the array of triangular patches, \( t \), is not really necessary. However, in *unstructured meshes*, studied in the next section, we must use a list of each node’s neighbors – the connectivity array, \( t \).

**Non-uniform mesh**

Often, it is desired to increase the triangle density close to the certain areas, for example, close to rectangle edges. In this case, Eqs. (1), which are the generating equations for nodal points, may be modified accordingly. For example, the generator

\[
x_m = \left[ 1 - \cos \left( \frac{m - 1}{N_x} \right) \right] \frac{a}{2}, \quad m = 1, ..., N_x + 1
\]

\[
y_n = \left[ 1 - \cos \left( \frac{n - 1}{N_y} \right) \right] \frac{b}{2}, \quad n = 1, ..., N_y + 1
\]

will create the triangular mesh shown in Fig. 2. Note that the array, \( t \), remains the same for meshes in both Fig. 1 and Fig. 2.

**Extra columns**

The array of vertices, \( t \), may also contain a fourth column, which usually indicates the *domain number*. For example, one plate of the capacitor may be designated as domain #1 and another as domain #2 with corresponding values in the fourth column of \( t \).
Fig. 2 Mesh generation for a planar rectangle with non-uniform nodes.

Mesh storage formats

Many modern mesh storage formats also include edge information. For example, the winged edge format used in computer graphics for each edge gives: two vertices of the edge, two faces attached to the edge, and the four edges attached to the edge of interest. Indeed, the edge information can be retrieved from the face information and vice versa. The above format is only valid for manifold meshes as explained in the following text.

B. A 3D triangular mesh. 2-Manifold meshes

When a third spatial coordinate $z$ is added to the array of nodes in Eq. (2), a 3D triangular mesh is obtained. Figure 3 shows a triangular mesh for a sphere. This mesh has also been generated “by hand”. The idea here is to use an initial octahedron mesh, then divide the each edge of the mesh, and push all the new vertices in the direction of their outer normal so that they all belong to the sphere surface. This process is repeated as long as necessary. A problem with the triangular mesh shown in Fig. 3 is in slightly different triangle sizes on the sphere surface, which is a deficiency if a uniform mesh is needed with the approximately equal triangles.
The 3D triangular meshes are most important for the numerical analysis, computer graphics, and pattern recognition. The following two properties are of note:

3. A 3D triangular mesh usually represents a physical solid object. Therefore, it must be **watertight**, i.e. do not have missing triangles (surface holes).

4. The surface of a well-behaved triangular mesh in 3D must satisfy one critical condition, which is the so-called **manifold condition**. A mesh is 2-**manifold** if every node of the mesh has a disk-shaped neighborhood of triangles – see Fig. 3a. This neighborhood can be continuously deformed to an open disk. Every edge of a 2-manifold mesh is a **manifold edge** – see Fig. 3b – with only two attached triangles.

5. All other meshes are **non-manifold meshes**. Fig. 3c gives an example of a non-manifold mesh with a **non-manifold edge**. A non-manifold mesh with a **non-manifold node** is shown in Fig. 3d.

The non-manifold meshes cannot be used for the numerical analysis, in most of the cases. Therefore, they must be **healed** prior to computations.

The definition of 2-manifold meshes is derived from the definition of 2-manifold surfaces. A surface is a 2-**manifold** if and only if for each point \( r \) on the surface there exists an open ball with center \( r \) and sufficiently small radius so that the intersection of this ball and the surface can be continuously deformed to an open disk [1].

C. **Triangle quality and mesh quality**

The triangular mesh should be used to obtain a solution to differential or integral equations. The long narrow triangles seen in particular in Fig. 2 are generally not desirable since electric charge
and current distributions may significantly vary along their lengths. Such a feature is in contradiction with the general idea of surface discretization, where we typically assume that the electric (or other) parameters are approximately constant for every small triangle.

The best triangle is an equilateral triangle, with all three triangle angles equal to 60 degrees, i.e. \( \alpha = \beta = \gamma = 60^\circ \). In reality, the mesh cannot consist of only equilateral triangles. The quality of the triangular element is therefore introduced, which is essentially a measure of the deviation from an equilateral triangle. One common quality measure is the ratio between the radius of the inscribed circle (times two), \( r_{in} \), and the radius of the circumscribed circle, \( r_{out} \), – see Fig. 4.

The quality factor \( q \) is expressed by \([2]\):

\[
q \equiv \frac{2r_{in}}{r_{out}} = \frac{(b+c-a)(c+a-b)(a+b-c)}{abc}
\]  

where \( a, b, c \) are the triangle sides. Another useful expression is \([2]\):

\[
q = \frac{2r_{in}}{r_{out}} = 2(\cos \alpha + \cos \beta + \cos \gamma - 1)
\]  

The overall quality factor of the entire triangular mesh may be the lowest \( q \)-factor of an individual triangle in the mesh. For example, the mesh shown in Fig. 3 has an overall mesh quality 0.83. Alternatively, a histogram may be used, which displays the number of triangles of a certain quality. The quality factor introduced above is not the only quality measure. Many other measures of element quality exist \([3]\), \([4]\), \([5]\).
D. Triangle size and mesh uniformity

A natural measure of triangle size is the radius of the circumscribed circle, $r_{\text{out}}$, in Fig. 4 [6]. Such a radius is half of the longest edge for degenerate triangles. For equilateral triangles with side $a$,

$$r_{\text{out}} = \frac{a}{\sqrt{3}}$$

(6)

Mesh uniformity is a measure of closeness of all triangle sizes to a certain constant value. For uniform meshes, all triangles should have approximately the same size. However, for non-uniform meshes with smaller triangles close to boundaries, the triangles may have quite different sizes. At the same time, the triangle quality is still required to be high, irrespectively of the size.

References

II. DELAUNAY TRIANGULATION. Three-DIMENSIONAL VOLUME AND SURFACE MESHES

A. Structured vs. unstructured meshes

Triangular meshes can be categorized as structured or unstructured. Figs. 1 and 2 of the previous section illustrate examples of structured meshes. According to Ref. [1], “structured meshes exhibit a uniform topological structure that unstructured meshes lack. A functional definition is that in a structured mesh, the indices of the neighbors of any node can be calculated using simple addition, whereas an unstructured mesh necessitates the storage of a list of each node’s neighbors.” An example of the structured mesh is given by Eqs. (2). There, all rows of the triangle array $t$ are computed analytically.

The structured surface meshes might be used for simple geometries (a rectangle or a brick in 3D). Inclusion of non-trivial (polygonal or curved) boundaries usually results in the unstructured meshes. Furthermore, the unstructured meshes can provide finer resolution in certain (sometimes not a priori known) domains of interest. In what follows, we will study the unstructured meshes.

B. Mesh generation and its properties

Surface mesh generation is a creation of an array of nodes, $P$, for a geometrical structure and subdivision of this structure into small planar triangles described by the array $t$. There are several desirable properties of mesh generation [1]:

1. The triangles should not intersect the boundaries (in other words, should “respect” the boundaries). Consecutive triangle edges should approximate actual curved boundaries by closest piecewise-linear boundaries.
2. The mesh generation should offer as much control as possible over the sizes of elements in the mesh. This control means the ability to grade from small to large triangles over a relatively short distance.
3. A third (most difficult) goal of mesh generation is that all the triangles should have a relatively high quality as described in section I.

C. Delaunay triangulation in two dimensions

According to Ref. [2], "despite an abundance of recent work on procedures of generating good triangulation, none of the modern approaches compare in elegance and generality to a procedure developed over fifty years ago by the Russian mathematician Delaunay. Delaunay [3] derived a simple procedure for triangulating an arbitrary set of points on a plane in such a way that the sum of minimum angle(s) in each triangle would be maximized. Since finite-element solutions are most accurate with nearly equilateral triangle grids, and since the Delaunay triangulation procedure comes as close as possible to this, it is an excellent method to use with the finite-element method." More precisely, the Delaunay triangulation in two dimensions maximizes the minimum angle of all the angles of the triangles in the triangulation [1]. In other words, it maximizes minimum triangle quality of the mesh according to Eq. (5).

D. Algorithm

Assume that we have the array of nodes, $P$ shown by small circles in Fig. 5. We need an array of nonintersecting triangles, $t$. The following theorem applies:
Three points \( p_1, p_2, p_3 \) are vertices of the same triangle of the Delaunay triangulation of \( P \) if and only if the circle through \( p_1, p_2, p_3 \) (the circumcircle of triangle \( p_1, p_2, p_3 \)) contains no point, \( P \), in its interior. Points on the circle boundary are permitted.

This theorem is illustrated in Fig. 5. Triangle \( p_1, p_2, p_3 \) in Fig. 5a is Delaunay since its circumcircle is empty (does not contain any other nodes of \( P \)). However, triangle \( p_1, p_2, p_3 \) in Fig. 5b is not Delaunay since its circumcircle contain another node.

One popular algorithm for computing Delaunay triangulation is the edge flip demonstrated in Fig. 5. It begins with an arbitrary triangulation. Then, we use the following theorem:

An edge of the triangulation is Delaunay if and only if there exists an empty circle that passes through its vertices.

Thus, we check every edge in an arbitrarily-created mesh. If this edge is not Delaunay, such as the edge \( p_1, p_3 \) in Fig. 5b, we simply flip it as shown in Fig. 5a. The flipped edge \( p_2, p_3 \) is Delaunay.

The algorithm requires \( O(n^2) \) edge flips for a set \( P \) of \( n \) points [1].

The Delaunay triangulation of a given vertex set is unique. The triangle test or the edge test uniquely determines if the triangle or an edge is a part of the triangulation. The known exceptions are a line with more than two points on it, a circle with more than three points on it and no points inside, a sphere with more than four points on its surface and no points inside, and a few other similar cases [1].

Many algorithms compute the Delaunay triangulation by a fast check of whether there is a node inside a triangle in question or not. Today, the Delaunay triangulation of set \( P \) of \( n \) points in the plane can be computed in \( O(n \log(n)) \) expected time, using \( O(n) \) expected storage – see, for example Ref. [4]. The online documentation, algorithms, and examples are given in Ref. [5].

![Fig. 5. a) – Delaunay triangulation of a set of four nodes; b) – non-Delaunay triangulation of the same node set.](image-url)
E. Example of Delaunay triangulation. Incremental vertex addition

The Delaunay triangulation should respect the boundary of the object. Therefore, it may start with the explicit definition of the boundary nodes and the boundary edges. Fig. 6 shows an example of a circle with eight boundary nodes and eight boundary edges. These edges will always be present in the mesh. Our goal is to construct Delaunay triangulation when one node (a black circle in Fig. 6a) is added to its interior at a time.

Figure 6a shows the initial triangulation of the circle. This triangulation is Delaunay since the circumcircle for every triangle is simply the original boundary circle. However, it is not unique. Further, a vertex marked black is introduced into a triangle \( p_3, p_4, p_8 \), say, at its center. This triangle is subdivided into three new sub-triangles. Then, the edge flip algorithm is applied to all three edges of the old triangle. The result is the mesh shown in Fig. 6b. This triangulation is still not Delaunay. We perform another edge flip and arrive at Fig. 6c. And yet, the result is not Delaunay. Two extra edge flips are necessary to obtain Delaunay triangulation in Fig. 6d.

Fig. 6. Delaunay triangulation for a circle obtained by edge flips.
Next, we add another vertex somewhere in the interior and continue until the desired triangle size is reached. The method of *incremental vertex addition* discussed here is slow; other faster methods exist [1].

**F. Example of constrained Delaunay triangulation**

Once the nodes \( p_1, p_2, \ldots, p_8 \) in Fig. 6a are given, the Delaunay triangulation will always include the circle boundary edges since the object boundary geometry is convex. In other words, it will respect the boundary. The explicit inclusion of boundary edges is not necessary. However, for non-convex boundaries and multiple boundaries the explicit inclusion of the boundary edges is a must. The Delaunay triangulation constructed in this manner is the *constrained Delaunay triangulation*. Figure 7 shows an example [6]. In Fig. 7a, we have a polygon with eight boundary nodes and no interior nodes. An application of Delaunay triangulation gives the mesh in Fig. 7b, which does not respect the boundaries. When, however, the boundary edges are forced to be a part of the Delaunay triangulation, the mesh of Fig. 7c is obtained. The boundary is respected, but there are extra triangles. They are excluded from the mesh by checking the in/out (Boolean) status with respect to a closed boundary of an object – see Fig. 7d. Hence, the mesh of a non-convex polygon has been created.

![Fig. 2.7. Top – unconstrained Delaunay triangulation of a non-convex polygon; bottom - constrained Delaunay triangulation with boundary edges included into the mesh and removal of unnecessary triangles.](image)

Note that reference [7] discusses edge-flip based algorithms for updating and constructing constrained Delaunay triangulations and constrained regular triangulations. A large collection of mesh generators can be found in online Ref. [8].
G. Delaunay triangulation in three dimensions (tessellation or tetrahedralization)

Given a set of points \( P \) in 3D one can build a tetrahedralization of the convex hull (or a convex envelope) that is, a partition of this convex volume into tetrahedra, in such a way that the circumscribing sphere of each tetrahedron does not contain any other point of \( P \) than the vertices of this tetrahedron. Such a tetrahedralization is called a 3D Delaunay triangulation or tessellation or tetrahedralization \([10]-[12]\). Under non degeneracy assumptions (no three points on a line, etc.) it is unique. Many different techniques have been proposed for the computation of Delaunay triangulation in 3D \([10]-[12]\). One flipping-based algorithm is as follows \([12],[13]\). At the beginning the triangulation is initialized as a single tetrahedron, with vertices “at infinity”, that contains all points of \( P \). At each step a new point from \( P \) is inserted and the corresponding tetrahedron, in which this point lies, is split. Then, the Delaunay property is re-established by “flipping” tetrahedra. This method is thus similar to the 2D triangulation method described above in Fig. 6.

H. Three-dimensional surface mesh generation

This problem is perhaps one of the most complicated task of mesh generation. An example is biomedical imaging, which is a very large area of research. A workflow for computational biomedical phantoms is a set of 3D mathematical surface meshing algorithms for anatomical structures, which are extracted from medical imaging data. This data includes Computed Tomography (CT), Magnetic Resonance Imaging (MRI), etc. A typical sequence for three-dimensional triangular surface mesh generation in particular includes:

1. Algorithm(s) for image registration and segmentation – creating a dense point cloud in the form of a shell corresponding to the boundary of a 3D shape from a stack of images \([14]-[17]\) – see Fig. 8a.
2. Algorithm(s) for surface reconstruction/extraction – creating a triangular surface mesh corresponding to this point cloud \([15],[18]-[22]\) – see Figure 8b.
3. Algorithms for healing the resulting mesh \([15],[18]-[22]\), mesh smoothing and coarsening \([23][30]\) – see Fig. 8b.

Fig. 8. Illustration of three-dimensional surface mesh generation for a pelvic bone from the stack of images for Visible Human (VHP) Project using MATLAB tools. Only a part of the original point cloud, which is the starting point of mesh generation, is shown in Fig. 2.8a.
I. *Algorithms for three-dimensional surface mesh generation*

A naïve but straightforward and simple way is to apply a 3D Delaunay triangulation to a point cloud, create a tetrahedral mesh, and extract the surface (boundary) faces. Unfortunately, most of the tessellation methods create a final convex tetrahedral mesh, which will mask all non-convex details in Fig. 8b. Therefore, this method works only for convex objects without holes. For arbitrary bodies, the problem greatly complicates. A very important concern is noise present in real scanned 3D data. Given that the entire problem is very complex, we will only briefly outline the concepts of two methods: a sculpting-based volumetric method [12] and a region-growing surface method – *the ball-pivoting method* [20].

**Volumetric method**

In sculpting-based methods, a volume tetrahedralization is computed from the data points, typically the 3D Delaunay triangulation. This may be done by surrounding the original dataset by a shield of extra points – see Fig. 9a where a projection of a 3D problem is shown. Tetrahedra are then removed from the convex hull to extract the original shape. It is easy to remove the tetrahedra, which contain boundary nodes. It is difficult to remove other tetrahedra under question. The concept is shown in Fig. 2.15a. A certain distance function from the domain surfaces [12] should be available and applicable.

![Volumetric method](image)

Fig. 9. Two methods of three-dimensional mesh generation. In Fig. 9a, only a projection of a tetrahedral mesh is shown.

**Region-growing surface method (ball pivoting)**

The principle of the Ball-Pivoting Algorithm (BPA) is shown in Fig. 9b. We will essentially cite Ref. [20]. Three points form a triangle if a ball of a user-specified radius \( \rho \) touches them without containing any other point. Starting with a seed triangle, the ball pivots around an edge \( ab \) (i.e. it
revolves around the edge while keeping in contact with the edge’s endpoints) until it touches another point $c$, forming another triangle. The rotation direction is shown in Fig. 9b. The process continues until all reachable edges have been tried, and then starts from another seed triangle, until all points have been considered. Parts of the surface mesh so created are then stitched together.

**Oversampling**

For both methods, oversampling (creation of a very large dense nodal set $P$) may be a big plus. However, the oversampling is limited by the image resolution and other factors.

**REFERENCES**


III. MESH OPERATIONS AND TRANSFORMATIONS

A. Topology-preserving mesh transformations

Once created, a triangular mesh (more specifically, the array of vertices \( P \)) as a whole could be subject to a shift, rotation, or other operations in 3D. A third coordinate may need to be added to a 2D surface mesh; the starting point is to use \( P(:, 3) = 0 \). Most common mesh operations include mesh translation, rotation, scaling, and deformation. Those operations correspond to the translation, rotation, or deformation of the original object. It is important to emphasize that the array of faces or triangles, \( t \), always remains the same. Mesh translation (shift) is the movement as a whole, along some vector \((x, y, z)\). It is given by:

\[
P(:, 1) \rightarrow P(:, 1) + x, \quad P(:, 2) \rightarrow P(:, 2) + y, \quad P(:, 3) \rightarrow P(:, 3) + z
\]  

(7)

Another simple operation is mesh scaling. Mesh scaling by the factor \((S_x, S_y, S_z)\) in three dimensions is done according to:

\[
P(:, 1) \rightarrow S_x P(:, 1), \quad P(:, 2) \rightarrow S_y P(:, 2), \quad P(:, 3) \rightarrow S_z P(:, 3)
\]  

(8)

Mesh rotation about the \( x \)-axis by angle \( \theta \) following the right-hand rule is given by:

\[
P(:, 1) \rightarrow P(:, 1) \\
P(:, 2) \rightarrow + \cos \theta P(:, 2) + \sin \theta P(:, 3) \\
P(:, 3) \rightarrow - \sin \theta P(:, 2) + \cos \theta P(:, 3)
\]  

(9)

Mesh rotation about the \( z \)-axis by angle \( \varphi \) is given by a similar expression:

\[
P(:, 3) \rightarrow P(:, 3) \\
P(:, 1) \rightarrow + \cos \theta P(:, 1) + \sin \theta P(:, 2) \\
P(:, 2) \rightarrow - \sin \theta P(:, 1) + \cos \theta P(:, 2)
\]  

(10)

A very efficient method for rotating an entire mesh in space, given an arbitrary axis with the unit vector \( k \) and an angle of rotation \( \theta \) is the Rodrigues’ rotation formula. It has the form (noting that rotation about the axis follows the right-hand rule):

\[
P_{\text{new}} = P \cos \theta + (K \times P)_{\text{rowwise}} \sin \theta + K \circ K’(1 - \cos \theta)
\]  

(11)

Here, \( \times \) denotes the cross-product and array \( K \) has the same dimensions as \( P \); it consists of the required number of copies of the vector \( k \). Array \( K’ \) also has the same dimensions as \( P \); it contains three identical columns of dot products \((K \cdot P)_{\text{rowwise}}\). The symbol \( \circ \) means Hadamard product (element by element multiplication).

B. Necessity of mesh smoothing

Often, the (constrained) 2D Delaunay triangulation algorithm for planar meshes is readily available, in particular using the basic MATLAB platform. Then, a mesh for a 2D convex or any
non-convex *complicated shape* represented by multiple polygonal boundaries might be created as follows:

1. We select a *larger domain* of a simple shape that contains all boundaries (a rectangle, circle, etc.). We fill this domain with nodes distributed either randomly or, alternatively, as nodes of equilateral triangles of a certain size *excluding* the nodes outside the domain. The resulting node array is $P$.

2. We analytically specify the *required* boundary nodes, $P_b$, and the corresponding boundary edges, $C$. The boundary nodes are put *up front* in the resulting node array, $P \rightarrow [P_b;P]$, in order to keep the connectivity in $C$. This is a very critical step.

3. We apply the constrained Delaunay triangulation with constrains on $C$ and select subdomains if necessary. Such subdomains may be, for example, electrodes attached to a conducting object.

Fig. 10c indicates a problem with multidomain meshes, which may be solved using different mesh improvement algorithms [1]-[7], many of which are based on a simple topology-preserving *Laplacian smoothing*.

---

**C. Topology-preserving Laplacian smoothing**

According to Ref. [5], “there are mainly three types of mesh improvement methods: (1) refinement or coarsening, (2) edge swapping, and (3) mesh smoothing. The refinement and the coarsening mainly try to optimize the mesh density, while edge swapping and mesh smoothing mainly aim to optimize the shape regularity. There are mainly two types of smoothing methods, namely Laplacian smoothing and optimization-based smoothing.”

We will apply the *Laplacian smoothing* to the problem indicated in Fig. 10c. In its basic form, Laplacian smoothing implies moving each vertex to the arithmetic average of the neighboring vertices while keeping the boundary nodes and boundary connectivity (boundary edges) *unchanged* as shown in Fig. 11. In other words, a free vertex of the mesh is simply relocated to the
centroid of the vertices connected to that vertex. In Fig. 11, the node $p_5$ is moved to the average of the neighboring points $p_1, p_2, p_3, p_4$. In this way, the local triangle quality greatly improves. Indeed, in order to perform the smoothing we should know the neighboring nodes (neighboring triangles) for every mesh node.

![Fig. 11. Concept of Laplacian smoothing.](image)

Reference [7] provides a comprehensive overview of different Laplacian smoothing methods. Some of them are briefly listed below.

**Standard Laplacian smoothing** shown in Fig. 11:

$$p^* = \frac{1}{k} \sum_{p_j \in \Omega_i, p_j \neq p_i} p_j$$

(12)

where $\Omega_i$ is the “star” of the vertex $p_i$ having $k$ points and $p^*$ is the new location of $p_i$. Note that this formulation can also be interpreted as a torsion-spring system where a central node in a star polygon is located at the centroid of the polygon balancing out the system to stay in (local) equilibrium.

**Lumped Laplacian smoothing**,

$$p^* = \frac{1}{3} p_i + \frac{2}{3} \frac{1}{k} \sum_{p_j \in \Omega_i, p_j \neq p_i} p_j$$

(13)

**Centroid Voronoi Tessellation (CVT) smoothing** utilizing attached triangle centers $t_j$ and areas $A_j$,

$$p^* = \sum t_j A_j / \sum A_j$$

(14)

**Weighted Centroid of Circumcenters (WCC) smoothing** utilizing attached triangle circumcenters $c_j$ and areas $A_j$,
\[ p^* = \frac{\sum c_j A_j}{\sum A_j} \] (15)

Equally-weighted versions of Eqs (14), (15) may be used. They have been found to perform reasonably well and might perform even better when applied to adaptive mesh refinement.

**D. Laplacian smoothing with re-triangulation. Iterative algorithm**

After the creation of the initial mesh in Fig. 10c, the Laplacian smoothing of any type is applied to all free nodes except the boundary nodes. After that, the constrained Delaunay triangulation with the constraints is repeated. Then, the Laplacian smoothing is applied again and the process repeats itself iteratively. One measure of convergence is the control of the resulting mesh quality. The process may be stopped when the triangle quality ceases to increase or it oscillates about a certain value. Note that the Laplacian smoothing does not necessarily converge except for the algorithm **Lumped Laplacian smoothing** [3].

There are two techniques to calculate new positions, \( p^* \). The first method is to modify all positions by one step. This method is called the *simultaneous version*. The second variant is to update the new positions of \( p^* \) immediately. This variant is called the *sequential version*. In this latter case, a position \( p^* \) may not solely depend on the “set” of old positions but can also depend on previously calculated new positions. The Laplacian algorithm **Standard Laplacian smoothing** has the property that the limits of the two techniques are the same if they exist [3].

Applied to the mesh shown in Fig. 10c, algorithms **CVT** and **WCC** will show the best performance. They reach the highest mesh quality in a shortest number of iterations. Algorithm **WCC** is slightly faster than algorithm **CVT**. Figure 12 shows two meshes. One is the initial mesh from Fig. 10c and another is the mesh obtained at 9th iteration using the iterative method described above with algorithm **WCC**. The minimum mesh triangle quality increases from \( 4 \times 10^{-5} \) to 0.69, which is a remarkable result given the simplicity of the algorithm. The usefulness of the Laplacian smoothing thus speaks for itself.

![Fig. 12. Results of Laplacian smoothing with algorithm **WCC** after 9th iteration.](image-url)
E. Weaknesses of Laplacian smoothing

The Laplacian smoothing algorithm described in previous subsections has the following weaknesses:

1. It is a local algorithm with fixed boundary nodes; therefore it is not able to provide high-quality uniform meshes from an arbitrary set of given nodes. If the nodes were initially concentrated in one local area, they will stay there forever.
2. For the same reason, it is not well suited for creating high-quality, non-uniform meshes with different triangle sizes from a given set of data points;
3. The proper handling of boundary nodes is an important subject. The method of fixed nodes used in this subsection is definitely not the best; it only works when the length of a boundary edge is approximately the best anticipated edge length for a given mesh. Also, intersecting boundaries require special care.

Laplacian smoothing in three dimensions

An undesirable effect is also the obvious “shrinkage” of 3D triangular surface meshes after Laplacian smoothing; the entire 3D mesh actually becomes smaller that it is in reality after several iterations [3]. A simple algorithm to avoid the shrinkage has been developed [3]. The idea is to push the modified points toward the previous points and the original points of the mesh.

F. Collision algorithms for 3D surface meshes

Algorithms that find ray-triangle, segment-triangle, and triangle-triangle intersections [13]-[22] are in particular a basic component of all collision detection data in computer animation. As far computational purposes are concerned, these algorithms allow us:

1. Find outer surface normal vectors for a 3D shell;
2. Perform the inside/outside check for an arbitrary node;
3. Perform Boolean operations (union, subtraction, intersection) on realistic 3D surface meshes [13]-[17] including meshes of various tissues obtained from the medical image data.

Ray-triangle intersection with Möller and Trumbore algorithm [18]

This algorithm is perhaps most common. We define a ray, \( \mathbf{R}(t) \) as \( \mathbf{R}(t) = \mathbf{O} + t \cdot \mathbf{d} \) where \( \mathbf{O} \) is the origin of the ray and \( \mathbf{d} \) is the normalized direction vector. We define a triangle by three vertices: \( \mathbf{p}_1, \mathbf{p}_2, \text{ and } \mathbf{p}_3 \). We define the point, \( \mathbf{T}(u, v) \) on the triangle as:

\[
\mathbf{T}(u, v) = (1 - u - v)\mathbf{p}_1 + u\mathbf{p}_2 + v\mathbf{p}_3
\]

where \((u, v)\) are barycentric coordinates, which, by definition, meet the following conditions:

\[
u \geq 0, \quad u \geq 0, \quad u + v \leq 1.\]
To find the intersection point between the ray and the triangle, Eqs. (16), (17) are to be solved simultaneously, which yields

\[
\mathbf{O} + t \cdot \mathbf{d} = (1 - u - v)\mathbf{p}_1 + u\mathbf{p}_2 + v\mathbf{p}_3
\]  

(18)

Rearranging the terms leads to the matrix equation,

\[
\begin{bmatrix}
-d
& \mathbf{e}_1
& \mathbf{e}_2
\end{bmatrix}
\begin{bmatrix}
t \\
u \\
v
\end{bmatrix}
= \mathbf{O} - \mathbf{p}_2
\]  

(19)

where \( \mathbf{e}_1 = \mathbf{p}_2 - \mathbf{p}_1 \) and \( \mathbf{e}_2 = \mathbf{p}_3 - \mathbf{p}_1 \). By solving Eq. (19), we can find the barycentric coordinates \((u, v)\) and the distance, \(t\), from the ray origin to the intersection point. The solution to Eqs. (19) is obtained using the Cramer’s rule:

\[
\begin{bmatrix}
t \\
u \\
v
\end{bmatrix}
= \frac{1}{(\mathbf{d} \times \mathbf{e}_2) \cdot \mathbf{e}_1}
\begin{bmatrix}
(\mathbf{tvec} \times \mathbf{e}_1) \cdot \mathbf{e}_2 \\
(\mathbf{d} \times \mathbf{e}_2) \cdot \mathbf{tvec} \\
(\mathbf{tvec} \times \mathbf{e}_2) \cdot \mathbf{d}
\end{bmatrix}
= \frac{1}{\det}
\begin{bmatrix}
\mathbf{qvec} \cdot \mathbf{e}_2 \\
\mathbf{pvec} \cdot \mathbf{tvec} \\
\mathbf{qvec} \cdot \mathbf{d}
\end{bmatrix}
\]  

(20)

where \( \mathbf{tvec} = \mathbf{O} - \mathbf{p}_1 \), \( \mathbf{pvec} = (\mathbf{d} \times \mathbf{e}_2) \), \( \mathbf{qvec} = (\mathbf{tvec} \times \mathbf{e}_1) \) and \( \det = \mathbf{pvec} \cdot \mathbf{e}_1 \). Fig. 13 illustrates the implementation of this algorithm in MATLAB for a ray passing through a human eye surface mesh. The intersection points are marked.

![Fig. 13. a) – Ray-triangle intersection for a human eye; b) – segment-triangle intersection.](image-url)
**Segment – triangle intersection**

The same algorithm applies (an alternative is algorithm of Ref. [19], which projects the point and triangle onto a 2D coordinate plane where inclusion is tested.). Similar to the ray-triangle intersection, we find the point(s) of intersection but the distance of this point from the origin should be less than or equal to the length of the segment. Fig. 13b illustrates the implementation of this algorithm in MATLAB for a ray passing through a human eye surface mesh. The intersection points are marked.

**Triangle – triangle intersection and mesh – mesh intersection**

A triangle is a set of three segments. Therefore, a triangle-triangle intersection problem can be reduced to the segment-triangle intersection problem considered previously. However, separate fast algorithms may be developed too [17], [20]. Next, the mesh-mesh intersection problem can be reduced to the triangle-triangle intersection problem since every mesh is a combination of triangles. Consider a master mesh X and a slave mesh Y. Both meshes are 2-manifold. For any triangle from the master mesh there exist three different intersection cases as shown in Fig. 14. Cases #1 and #3 in Fig. 14 would become equivalent if we could treat the master and slave meshes as one set of triangles.

![Fig. 14. Three types of intersection of a triangle from a master mesh X with various triangles of a slave mesh Y. Cases #1 and #3 are equivalent if we treat the master and slave meshes as one set of triangles.](image)

With reference to Fig. 14 we could formulate one possible mesh-mesh intersection algorithm as follows:
1. For each triangle of the master mesh under question, we find intersecting edges \( e_i, i = 1, 2, 3, \ldots \) in Fig. 14.
2. Next, we apply a constrained 2D Delaunay triangulation to triangle’s plane and subdivide the master triangle into sub-triangles, which respect intersections.
3. The same procedure is applied to each triangle under question of the slave mesh.
4. We construct refined master and slave meshes, which respect all intersections.
5. Boolean operations on meshes are performed by checking in/out status of separate triangles.

Apparently, the above algorithm is quite slow. However it is simple and makes use of the existing constrained 2D Delaunay triangulation in MATLAB.

**Other algorithms implemented in MATLAB**

MATLAB central provides a number of related vectorized scripts – see [21], [22] – for ray-triangle and segment-triangle intersection.

**G. Checking in/out status and finding outer normal vectors for 2-manifold 3D surface meshes**

**In/out status**

Assume that an observation point in Fig. 15a lies outside a 2-manifold shell in three dimensions. A ray emanating from this point may or may not intersect this mesh. However, the number of intersections will always be even: 0, 2, 4, etc. Similarly, if the observation point lies inside the shell, the number of intersections will always be odd: 1, 2, 3, etc.

![Fig. 15. Checking in/out status for a 2-manifold mesh. Only mesh cross-section is shown.](image)

The critical point is that we do not have to check all rays; this can be done only once (excluding some degenerate cases). Therefore, a single ray can be constructed that points toward the center of a selected triangle in the mesh. Then, the ray-triangle intersection algorithm shown in Fig. 13 is applied and the number of intersections is counted.
Finding outer normal vectors

The normal vector is defined as the normalized cross-product of two triangle edges; its sign is important. To select only the outer normal vector, we may set an observation point slightly above each triangle center, in the direction of the normal vector. If this point is outside the mesh, the normal direction is correct. If not, the direction is reversed.

REFERENCES


IV. MESH PROCESSING ALGORITHMS

A. INTRODUCTION
The basic MATLAB platform already has a number of built-in and open-source features that make it a unique and useful starting resource for interactive mesh processing. These features include interactive mesh processing operations such as selection of individual vertices or faces of a 3D surface mesh and visualization of multiple meshes in many different formats. However, the MathWorks, Inc. has not yet developed the true mesh processing functions, which are capable of performing Boolean operations with triangular surface shells – union, intersection, and difference. The VHP-Female phantom has been augmented with a unique and interactive mesh processing toolbox, which performs these and other related operations, and which may be used to create deformable human body models required in modern diagnostic and therapeutic applications [13]-[17].

B. Separation of distinct body regions
In general, whole-body phantoms are used primarily for MRI studies of specific absorption rates. For many other purposes, only a portion of the phantom is required [1]-[6], [13]-[17]. Studies on cellphone safety, for example, usually only require the head. For studies in EEG, ECG, and EMG, only the head, or a part of the torso, or arms/legs are needed. The same is valid for implanted devices, body-worn sensors, wireless capsule endoscopy, and electromagnetic therapeutics. A very important function of the mesh processing toolbox is the separation of distinct body regions via the tissue mesh intersection with predefined meshes for geometry primitives (brick, sphere, plane). Thus, the toolbox enables the isolation of any part of the phantom of interest, as shown in Fig. 16 which depicts the separated phantom head and shoulders. The method for this separation has two key characteristics. First, the method keeps all enclosed tissues as separate closed watertight triangulated surfaces without coincident faces, by intersecting them with a family of slightly different primitives. This is important for cross-platform compatibility and many numerical methods such as MoM. Second, it creates a mesh of arbitrary resolution on flat faces, which is important for any number of custom academic electromagnetic and acoustic solvers without the option of adaptive mesh refinement.

Fig. 16 Part separation of the cut VHP-Female phantom in MATLAB. The final result for head and shoulders is shown (two different views).
C. Visual mesh processing tool

The mesh processing toolbox includes a visual mesh processing tool for interactive mesh healing, stitching, refinement, and smoothing. Fig. 17a shows sequential construction and addition of individual triangles between two tissues via mouse input. The new triangle is constructed by selecting its three vertexes. Each vertex must be a vertex of another existing triangle on the figure’s current object. The selection is made by clicking with the mouse on the existing triangle close to that vertex, using zoom in/out as necessary. After the three vertexes are selected, the triangle will be created and added to the mesh; the next desired selection will be requested. The resulting stitched mesh is shown in Fig. 17b.

![Visual mesh processing tool](image)

Fig. 17 Visual mesh processing tool for mesh operations such as the stitching of two separate meshes.

D. Mesh intersection algorithm

Generally, it is rather straightforward, albeit very time consuming, to perform image segmentation and create an initial individual triangular tissue. What is more difficult, however, is to properly resolve multiple potential intersections between such meshes and between the meshes and the primitives when only a part of the phantom is needed. It is for this reason that a new mesh intersection algorithm for triangular surface meshes has been developed specifically for human tissues with numerous irregular intersections [12] and has been included with the mesh processing toolbox. In contrast to the classic paper [18] and other relevant sources [19]-[21], chains and loops of intersection line segments, which may be very complicated for multiple tissue meshes, are not
explicitly constructed. Instead, all individual intersection line segments are collected randomly and then a constrained 2D Delaunay triangulation (already available and implemented in MATLAB) is applied to each triangle with the intersection line segments separately. Note that the constrained 2D Delaunay triangulation was also used in Ref. [21], but still augmented with the construction of intersection chains. The algorithm steps are as follows (the goal is to subtract a manifold mesh \( Y \) from a manifold mesh \( X \) in Fig. 18):

- For every edge of mesh \( X \), find the triangle(s) of mesh \( Y \) intersected by this edge and the corresponding intersection points via the Möller & Trumbore algorithm [7]. Store the results in two distinct cell arrays. Swap meshes \( X \) and \( Y \) and perform the same operation – see Fig. 18a.

- For every triangle of mesh \( X \) falling into the intersection list, collect all extra line segments (node pairs) to be added. Three scenarios are possible. The first is when two edges of a triangle in \( Y \) intersect the triangle in \( X \). A line segment \( Q_{11}Q_{12} \) in Fig. 19 has to be added. The second scenario is when only one edge of a triangle in \( Y \) intersects the triangle in \( X \). Then, an edge of the triangle in \( X \) must also intersect this triangle in \( Y \). A line segment \( Q_{21}Q_{22} \) in Fig. 19 has to be added. The last scenario is when two edges of the triangle in \( X \) intersect a certain triangle in \( Y \). A line segment \( Q_{31}Q_{32} \) in Fig. 19 has to be added. Finally, store all results in a cell array. Swap meshes \( X \) and \( Y \) and perform the same operation – see Fig. 18b.

- For every triangle of mesh \( X \) falling into the intersection list, perform constrained 2D Delaunay triangulation in the triangle plane – see Fig. 18c.

- Using the original manifold \( X \), determine all triangles \( YX \) of the refined mesh \( YR \) in \( X \). To do so, again use the Möller & Trumbore algorithm [7] for triangle centers – see Fig. 18d. Perform the same operation for mesh \( Y \).
Fig. 18. Triangular mesh intersection algorithm with constrained 2D Delaunay triangulation for individual triangles
Fig. 19 Three types of triangle intersections between a master mesh X and various triangles of a slave mesh Y.

E. Edge collapse of intersection chains

For a typical intersection problem shown in Fig. 20a, the surface-preserving algorithm described above creates many triangles of a low quality at the intersection boundary as seen in Fig. 20b. To eliminate this deficiency, we will consider performing the shortest edge collapse for all edges with at least one node on the intersection chain/loop (one shared node). The last operation creates high-quality meshes with the minimum boundary complexity as show in Fig. 20c. Those meshes are not exactly surface-preserving. However, if we will perform the shortest edge collapse only for the edges belonging to the intersection chains and only for the new nodes, the results of all three Boolean operations will be surface-preserving.

Fig. 20. Mesh intersection with and without edge collapse.
F. Deformation in the normal direction

The mesh processing toolbox for the VHP-Female phantom implements this basic deformation type. It is understood as an expansion or shrinkage of the triangulated 2-manifold surface in its normal direction, \( \mathbf{n} \), where the scalar amount of deformation \( d \) in the direction \( \mathbf{n} \) is a function of the local position. Given normal vectors \( \mathbf{n}_i \) of \( N \) triangular facets surrounding vertex \( \mathbf{p}_i \), the expansion may be in particular performed as

\[
\mathbf{p}_i = \mathbf{p}_i + d \frac{\mathbf{n}_0}{|\mathbf{n}_0|^2}, \quad \mathbf{n}_0 = \sum_{i=1}^{N} A_i^{-1} \mathbf{n}_i / \sum_{i=1}^{N} A_i^{-1}
\]

where \( A_i \) are triangle areas. This method is exact for the case of a cone—see Fig. 21. Different local definitions of the vertex normal vector [9],[10] may be used to alter the expansion formula for special cases of highly-irregular meshes.

![Fig. 21. Schematic mesh expansion in the normal direction.](image)

\( G. \) Deformed meshes

**Variable BMI**

The mesh processing toolbox has a few extra sets of deformed meshes for the VHP-female phantom. Fig. 22 shows the variation of a fat shell. This shell is located between the fixed outer thin skin shell and a variable average body container, which is usually assigned the properties of muscle tissue. Fig. 22a shows the actual segmented fat shell. The fat volume for the original VHP Female model indicates a high BMI (Body Mass Index). Fig. 22b shows the fat shell deformed in the normal direction so that fat volume is reduced by 50%. Fig. 22c shows a fat shell with a minimal volume (about 5%). Varying the fat volume will allow us to modify the original VHP-Female phantom in order to model the average BMI of the US population or a varying BMI, which expands the applicability of the VHP-Female phantom.
Variable bone composition

Fig. 23 shows the meshes for a constrained expansion in an osteoporotic bone. Fig. 23a is the generic normal femur with the cortical bone matter (light gray), the cancellous bone matter (gray), and the yellow bone marrow (black). Fig. 23b is a typical pathogenic osteoporotic female femur with the dominant yellow bone marrow obtained via mesh deformation and justified anatomically.

Fig. 22. Deformable fat shell for the VHP-Female computational model – variable BMI.

Fig. 23. Normal and pathological femur bones.
H. About general mesh deformation methods in computer graphics

Computational human phantoms with tissue deformation capabilities are perhaps the most innovative research subject in meshing today [13]-[17]. General mesh deformation algorithms include

**Linear multi-resolution deformation** [22]

The deformation algorithm moves a set of mesh nodes, \( P \), of a surface \( S \), according to \( P = P + d \), with some nodes having the prescribed deformations \( d_{\text{fixed}} \). All deformation vectors \( d \) are found from the minimization of the “elastic energy” of the shell, which is the combination of shell stretching and bending. The final linearized result is an Euler-Lagrange PDE, which is discretized via finite differences on triangles and leads to a linear system of equations for deformations (sparse, symmetric, and positive-definite) in the form

\[
(-k_s L + k_b L^{-1} L) \cdot d = 0, \quad (L)_{ij} = \begin{cases} 
    \sum_{p_j \in N_i(p_i)} w_{ij} & i = j \\
    w_{ij} & p_j \in N_i(p_i) \\
    0 & \text{otherwise}
\end{cases}, \quad w_{ij} = \frac{1}{2} (\cot \alpha_{ij} + \cot \beta_{ij}), \quad M_{ij} = A_i
\]

(22)

where \( N_i(p_i) \) are the incident one-ring neighbors of vertex \( p_i \), \( A_i \) is the Voronoi area of vertex \( p_i \), \( \alpha_{ij} \) and \( \beta_{ij} \) are the two angles opposite to the edge \( p_ip_j \), and \( k_s \) and \( k_b \) are free stretching and bending constants, respectively. The boundary conditions are incorporated by moving each column corresponding to the constrained vertex to the right-hand side. Linearization causes geometric details and protruding features to be distorted. Therefore, a multi-resolution step is employed, which applies Eq. (22) to a smoothed base surface \( B \), \( S = B \oplus D \), with \( D \) being the “high-frequency” surface constituent. In practice, \( B \) and \( D \) may have the same connectivity. The deformed surface is then obtained as \( S_{\text{def}} = B_{\text{def}} \oplus D \), where the addition is made in the local frame, according to the normal vector field \( n \) to \( B_{\text{def}} \).

**Linear deformation based on differential surface representation** [23]-[26]

The original and deformed surfaces are assumed to have surface gradients (gradients of the surface coordinate functions \( x, y, z \)) as close as possible to each other in the integral sense. For example, the gradient of the \( x \) function is the projection of the unit x-axis vector \( \hat{x} = [1,0,0] \) onto the triangle plane. The corresponding local formulation gives us the Poisson equation for the new surface coordinates. Its discrete version is similar to Eq. (22). However, the result of such an editing approach is not satisfactory, because it tries to preserve the original mesh gradients, with their orientation in the global coordinate system. This ignores the fact that in the deformed surface the gradients should rotate, since they always lie in the triangles’ planes, which transform as a result of the surface deformation. Different approaches to handle this problem exist [22].
**Nonlinear surface deformation** [27]

In [26], a thin layer of prismatic volumetric elements is formed around the shell mesh. The prisms are kept rigid, but they are connected along their common faces by elastic joints, which are stretched under deformation. The amount of stretching then yields the desired deformation energy to be minimized by an *iterative procedure*.

**REFERENCES**


Chapter 03 –
Development of VHP-Female CAD model – VHPC 3.0 and VHPA 3.0

I. INTRODUCTION

A. Source

The VHP-Female CAD model is a platform-independent full-body computational human model extracted from the open-source Visible Human Project®-Female cryosection dataset of the U.S. National Library of Medicine (NLM) [1] with a pixel resolution of 0.33 mm is in full color. The Visible Human Project®-Female has been led by Dr. M. Ackerman (National Institute of Health/NLM/LHC). Fig. 1 shows an example of the color cryosection image.

![Abdominal image avf1438a of the cryosection dataset.](image)

B. Construction

The model includes 26 individual tissues and 216 separate tissue parts that were visually differentiated from the dataset with a high degree of accuracy. Manual and semi-automatic image segmentation has been performed, in particular via ITK-SNAP [2, 3]. Surface extraction and mesh healing has been done using classic mesh processing algorithms [4-6]. Our surface deviation error (from anatomical data) does not generally exceed 0.5-3 mm in the head and 7 mm in the main body. The VHP-Female v.3.0 is distributed in the form of 3D CAD objects with ~160,000 facets in total. Every 3D CAD object is a triangular surface mesh characterized by an array of triangle vertices, \( t \), and an array of vertex coordinates, \( P \). Fig. 2 shows an example of the 3D CAD intestine object with an underlying triangular mesh.
C. Proof of surface reconstruction accuracy

The triangular surface meshes have been superimposed onto the original images. An in-plane deviation between CAD object boundaries and the original object boundaries has been evaluated for multiple cross-sections. In approximately 25% of all cases, the CAD objects were reconstructed the second time. The major source of this error has been a relatively large size of triangular facets optimized for fast FEM computations.

D. Model anatomy and materials

The model is based on the dataset of a ~60 year old female subject with a heart pathology. The resulting model has a height $h$ of 162 cm measured from the top of the scalp to the average center of both heels. The model should be used in conjunction with electromagnetic and other tissue properties from the IT’IS database, which covers the frequency band from 10 Hz to 100 GHz: Hasgall PA, Di Gennaro F, Baumgartner C, Neufeld E, Gosselin MC, Payne D, Klingenberg A, Kuster N, “IT’IS Database for thermal and electromagnetic parameters of biological tissues,” Version 2.6, January 13th, 2015. www.itis.ethz.ch/database. The body mass $M$ computed using tissue densities from www.itis.ethz.ch/database and assigning the average body object the density of muscle is 88 kg. The computed BMI is 33.5 (corresponding to moderately obese).

E. Model topology: Model objects

Connectivity
- Each model object is a strictly 2-manifold triangular surface mesh that consists of no non-manifold faces, no non-manifold vertices and has no self-intersections.
- Each model object is watertight (i.e., sealed) enabling representation as a 3D object in all CADs and simulation tools.
**Implicit subtraction approach**

In order to prevent CAD import errors, for example to avoid object mismatching in a contact region, the following rules have been implemented:

- If object \( O_1 \) is fully inside another object \( O_2 \), there is no explicit geometrical Boolean subtraction. An example of this scenario is shown in Fig. 3 that follows – here the fat object (or any other object) is situated entirely inside of the skin object.

- If object \( O_1 \) is not fully inside another object \( O_2 \), both objects neither touch each other nor intersect. There is always a small volume between these objects; thus these objects are separated from geometrical point of view. The separation volume is filled (occupied) by tissue of an object \( O_3 \) that completely surrounds objects \( O_1 \) and \( O_2 \). This situation is also shown below in Fig. 4, which displays the cerebellum, the white matter, and the ventricular object (any pair of them forms objects \( O_1 \) and \( O_2 \)), all within the grey matter object \( O_3 \). Here, we can see (often very small) gaps between the inner objects. These gaps are filled with grey matter material.

**F. Model topology: List of fully enclosed objects**

If a CAD or simulator software does not support implicit subtraction feature, model end-user should perform explicit subtraction of objects included in a list of fully enclosed objects given below. This additional operation results in obtaining ready to simulate model without object intersections.

**List of fully enclosed objects:**

- All model objects are surrounded by skin object (Fig. 3)
- All model objects except the skin object are surrounded by fat object (object directly under skin in Fig. 3)
- All model objects except the skin and fat objects are surrounded by average body object (object directly under fat in Fig. 3).
- White matter object is surrounded entirely by grey matter object (Fig. 4)
- Cerebellum object is surrounded entirely by grey matter object (Fig. 4)
- Ventricular object is surrounded entirely by grey matter object (Fig. 4)
- Grey matter object is surrounded entirely by CSF object (Fig. 4)
- Cauda Equina object is surrounded entirely by CSF object (Fig. 4)
- Bone marrow object(s) is surrounded entirely by femur object(s) (Fig. 5)
- Trabecular bone object(s) is surrounded entirely by femur object(s) (Fig. 5)

**G. Example of assigning material properties for Finite-Element Method (FEM) and Finite-Difference Time-Domain (FDTD) Methods**

The outermost object in Fig. 3 is a closed surface of skin for which “skin” material properties should be assigned. The object just below the skin is a closed fat surface, for which “fat” material properties should be assigned. The final object below the fat is an “average body” surface, for which “muscle” material properties should be assigned. All three objects follow an “onion” topology. The 3D closed shell skin object has a thickness of 1mm±0.25 mm uniformly around the entire body.
H. Scalp Topology

The original Visible Human Project®-Female dataset indicated an artificial volume around the skull which appeared as a result of the freezing procedure as shown in Fig. 6a. This artificial volume was eliminated numerically by creating a fixed scalp structure around the bulk of the neuro-cranium. This scalp structure always has the total thickness of 5 mm ±0.5 mm and formed by (mentioned above) enclosed outer body objects: skin, “fat”, and “average body” as shown in Fig. 6b. After explicit/implicit subtraction fat thickness is about 2 mm and the “average body” thickness is about 2 mm.

I. Muscular System

Except for the heart muscle, the muscular system includes major skeletal muscles (32 in total) in the form of separate objects shown in Fig. 7a. All muscle objects are contained within the average body object.

J. Extension datasets

Extension datasets for the model include
- Large femoral implants
- Variable body composition
- Variable CSF topology

a) Bone Composition and Addition of Large Orthopedic Implants

Special attention motivated by ongoing osteoporotic studies has been paid to the vertebral column and to large femoral bones shown in Fig. 7b. In the latter case, surface extraction has been accurately performed for three distinct bone tissues: cortical, trabecular, and bone marrow. Based on current practices, the femoral and neighboring meshes have been deformed in order to register and embed three large orthopedic metallic implants shown in Fig. 7b. The implants originate from the Center for Advanced Orthopaedic Studies, Beth Israel Deaconess Med. Center (BIDMC), Harvard Med. School. All other tissue meshes remain the same. Large implants may cause extra MRI heating; this effect is an active area of research today.

b) Variable BMI

Estimation of the Body Mass Index (BMI) using the original phantom outer fat shell predicts a value of approximately 36, classifying the patient as obese. This fat shell and the average body container have been simultaneously deformed in order to model medium and low BMI values – see Fig. 7c. All other tissue meshes remain the same. This method is indeed somewhat inaccurate anatomically, but may perhaps be used for integral MRI estimates and assessing fat layer impact on Specific Absorption Rate (SAR).

c) Improved Resolution in the Cranium

One unique feature of the VHP-F head is an anatomically-correct continuous shell of the highly-conductive cerebrospinal fluid (CSF) with a variable thickness of 1-7 mm, which maintains a shorting path of electric current. Further, the CSF shell has a direct connection with the spinal cord. The shell continuity is critical for accurate direct-current modeling such as EEG, electric impedance tomography, and direct-current stimulation. Therefore, the cranium model has already been used for research purposes. Additionally, there are two 0.3 mm thick shells around the CSF shell shown in Fig. 7d, respectively. These shells may either model brain membranes or CSF expansions.
Fig. 3. a) – Three distinct enclosed body objects: skin object, fat object, and average body object; b) – the corresponding volumes occupied by three respective tissues.
Fig. 4. Detailed view of the intracranial volume of VHP-Female Model. The cerebellum, the white matter, and the ventricular objects neither touch each other nor intersect. They are fully enclosed by the grey matter object which, in its turn, is fully enclosed by a CSF object.

Fig. 5. Detailed view of two femoral bones along with the fully enclosed objects: bone marrow and trabecular bone. Bone marrow objects and trabecular bone objects neither touch nor intersect.
Fig. 6a. Image avf1079a of the cryosection head dataset.

Fig. 6b. Scalp structure around the neuro-cranium.
Fig. 7a. Muscular system of the VHP-Female V. 3.0 SMOOTH.
Fig. 7b. Bone composition and femoral implants registered with the VHP-Female phantom. Soft tissues are not shown.
Fig. 7c. Three different fat shells used with the VHP-Female phantom.

Fig. 7d. Cranium of the VHP-Female v. 2.0 phantom. CSF shell and additional thin layers are shown.
II. THREE MODEL VERSIONS

A. VHP-Female version 3.0 BASE (DOI:10.20298/VHP-Female-V.3.0-BASE)
This is the master model version used to create the two remaining versions. It includes 26 individual tissues and 216 separate tissue parts with approximately 160,000 triangular facets in total. A complete list of tissue meshes is given in Appendix A.

B. VHP-Female version 3.0 SMOOTH (DOI:10.20298/VHP-Female-V.3.0-SMOOTH)
The origin of the SMOOTH version is entirely the BASE version but with all tissue objects smoothed in order to avoid sharp edges and corners. The total number of triangular facets is about 600,000. We have not used any finer segmentation results in the SMOOTH Model. This version is primarily intended for more accurate SAR calculations. The smoothing procedure is as follows:

[1]. Barycentric triangle subdivision (1:4) was applied to all facets of an object with an area less than 25% of the average facets’ area for the same object.
[2]. Object expansion by 0.4 mm in the local outer normal direction was performed. This expansion is intended to compensate for the following smoothing operation.
[3]. The Centroid Voronoi Tessellation (CVT) version of Laplacian smoothing with two iterations was applied to the entire object [7,8].
[4]. Extra mesh smoothing (lumped Laplacian with alpha=2/3, two iterations [7,8]) was applied to sharp corners/edges only (to nodes for which the magnitude of the vector sum of unit normal vectors for all adjacent triangles was less than 0.5 times the number of adjacent triangles).
[5]. Resulting object intersections (which are “shallow” intersections) were resolved by a local moving of intersecting surfaces in their respective normal directions with a step of 0.2 mm until the intersection was resolved [9].

The corresponding surface deviation between the two models does not exceed 0.2-1 mm on flat surfaces, but may be as high as 2-7 mm for sharp edges and corners. To illustrate the differences between BASE and SMOOTH models, Fig. 8 shows the same view of the cardiovascular system for both models, respectively. Additionally, Fig. 9 shows the thorax of the SMOOTH model version 3.0 with some objects removed for clarity. A full-body snapshot is given in Appendix B.

C. VHP-Female version 2.2 (DOI:10.20298/VHP-Female-V.2.2)
This is VHP-Female version 3.0 BASE with all individual muscles removed (except for the heart muscle). The average body object may now be assigned muscle properties.
Fig. 8. Top – triangular objects (meshes) of the BASE model v. 3.0; bottom – the same triangular objects (meshes) depicted above within the SMOOTH model v.3.0.
Fig. 9. Thorax of the SMOOTH model V. 3.0 with some objects removed for clarity.
III. Context of Use

A. Low-Frequency Electromagnetic Simulations of Injected Electric Currents

Simulation Methods

The VHP-Female V. 3.0 and V. 2.2 Cross-Platform Full-Body Computational Human CAD Models have been carefully optimized for efficient modeling of low-frequency electromagnetic devices via the accurate Finite Element Method (FEM), the Boundary Element Method (BEM), and the Finite-Difference Time-Domain Method (FDTD). The case in point is a low-frequency electromagnetic simulator Maxwell 3D of ANSYS, Inc., a commercial FEM software package with adaptive mesh refinement. The software takes into account both conduction and displacement currents (as well as free and polarization charges), and solves the full-wave Maxwell equation for the magnetic field, $\mathbf{H}$, in the frequency domain

$$\nabla \times \frac{1}{\sigma + j\omega\varepsilon} \times \mathbf{H} = -j\omega\mu\mathbf{H}$$

where $\sigma$ is the local medium conductivity; $\varepsilon$, $\mu$ are the local permittivity and permeability, respectively. The major difference from the full-wave case is that the phase is assumed to be constant over the volume of interest. The local FEM error is the error of the divergence-free magnetic flux, $\nabla \cdot \mathbf{B}_{\text{solution}} \neq 0$. This term acts as a source and produces some energy. Tetrahedra with the largest local error energy (30% or so per adaptive pass) are automatically refined. The total error energy in the volume divided by the total energy of the electromagnetic field and multiplied by 100% is the energy error, which is returned with each adaptive pass along with the total energy. The energy error measures the convergence of the adaptive mesh refinement process.

Simulation Example

A simulation example shown in Fig. 10 includes testing various current electrode configurations for transcranial Direct-Current Stimulation (tDCS) [10] in order to manipulate the locality and depth of the stimulated area. Fig. 10 shows (i) contralateral supraorbital, (ii) extracephalic contralateral shoulder, and (iii) extracephalic ipsilateral shoulder cathode montages.

Total current density normalized by the input current density at the electrodes is evaluated in surrounding body structures using a logarithmic scale. The corresponding local and integral predictions of the corresponding electric field can be made. These results were obtained with an early version of the VHP-Female model, which did not yet include the detailed lower body.

B. Low-Frequency Electromagnetic Simulations of Induced Electric Currents

A similar computational setup can be applied to study induced current in the body. Fig. 11 and Fig. 12 demonstrate one such setup intended for the evaluation of safety levels over the entire body for Transcranial Magnetic Stimulation (TMS) [11]. Legacy model V. 2.0 and both present models V. 3.0 and V. 2.2 have been tested for this purpose.
Fig. 10. Simulation of the volumetric current density in the brain with electrodes configured in a cephalic (C3-Fp2) versus extracephalic manner with evaluations of cathode placement on both ipsilateral and contralateral shoulder locations [10].
Fig. 11. Coil setup and observation point setup for the human model VHP-Female in Maxwell 3D FEM simulator [11].
Fig. 12. Eddy current density at large separation distances from the coil at 30 and 300 kHz respectively, in the three observation planes (at 50 mm, −300 mm, and −700 mm) for coil configuration #1 in Fig. 11. Note the different color scales for each plane. The color scale for 300 kHz is exactly twenty times the color scale for 30 kHz [11].
C. RF Simulations of on-body, in-body, and near-body antennas (dipoles, loops, patches, meanders, etc.)

The VHP-Female V. 3.0 and V. 2.2 Cross-Platform Full-Body Computational Human CAD Models have been carefully optimized for efficient modeling of high-frequency electromagnetic devices via the accurate Finite Element Method (FEM), the Boundary Element Method (BEM), and the Finite-Difference Time-Domain Method (FDTD). The case in point is on-body, in-body, and near-body antenna simulation with a most accurate FEM electromagnetic simulator ANSYS HFSS of ANSYS, Inc., which uses efficient adaptive mesh refinement.

An example for patch antenna modeling is shown in Fig. 13 [12]. The skin surface has been slightly deformed to have a precisely planar shape at the antenna location. At present, it is impossible to accurately model the full-body human model with antenna(s) at 5.8 GHz or at higher frequencies via the FEM since the resulting mesh easily exceeds $20 \times 10^6$ tetrahedra. Therefore, we suggest to cut out a tissue volume, remove minor tissues, and perform the simulations only in the primary domain of interest (i.e., the neck of the femur in Fig. 13). Such a simplification is expected to be reasonably accurate given the accurate boundary conditions (a PML or an IE boundary in ANSYS HFSS).

Fig. 13. a) A 5.8 GHz patch antenna with test surfaces on a VHP Female V. 2.0 model (equivalent to the VHP-Female V. 3.0 model in the domain of interest); b) - a more realistic setup used for modeling [12].
IV. STRENGTH OF EVIDENCE

A. Tool validity

Anatomical accuracy
Anatomical accuracy of the VHP-Female v.3.0/2.2 model has been evaluated at different times during 2013-2015 by a number of medical experts and doctors from

- Beth Israel Deaconess Med. Center, Harvard Med. School, Boston, MA
- Massachusetts General Hospital, Boston, MA
- Saint Vincent Hospital, Worcester, MA
- Max Planck Institute for Human Cognitive and Brain Sciences, Leipzig, Germany, and other entities

For the evaluation, we provided model slides and/or the entire CAD model in MATLAB format or in ANSYS HFSS format available for visual 3D inspection including selective zoom of different systems. In summary, we have corrected about 20 anatomical flaws (kidneys, digestive system, vertebral column, rib cage, muscular system, bladder, scalp, etc.).

Computational accuracy, cross-platform comparison, and efficiency

Accuracy and cross-platform compatibility [13]
Computational accuracy of the VHP-Female version 3.0 model and its beta versions has been evaluated at different times during 2013-2015 by a number of computational experts. A benchmark example is a scattering problem at plane wave incidence at 300 MHz shown in Fig. 14. The incident wave polarization is vertical. To obtain quantitative estimates, we will evaluate PLD (power loss density, also called volume loss density) in W/m³ along certain paths within the body (for the total field which is the scattered field plus the incident field). Those paths are two line segments shown in Fig. 14:

(1) x=0mm, y = 25mm, -60mm<z<140mm;
(2) x=0mm, z = -12.5 mm, -87mm<y<91mm.

Fig. 15 shows a comparison between two leading EM software packages: CST MWS and ANSYS HFSS for this particular problem. Given the overall observed PLD dynamic range of 60 dB, the quantitative agreement seems to be excellent.
Low-frequency efficiency [9-11]
Low-frequency computational efficiency of the VHP-Female version 3.0 model and its beta versions has been evaluated at different times during 2013-2015 by a number of computational experts. Table 1 demonstrates typical FEM performance of VHP-Female version 3.0 for eddy current computations due to a TMS coil with five adaptive passes. The run time for the SMOOTH version is approximately ten times longer.
Fig. 15. Power loss density along two line segments. Continuous curve – CST MWS simulations; stars – ANSYS HFSS simulations. VHP-Female version 3.0 BASE has been used.
### Table 1
**Computational hardware/software for typical low-frequency simulations with the VHP-Female Model Version 3.0 BASE.**

<table>
<thead>
<tr>
<th>System</th>
<th>18 Node Super Cluster, 2 Intel(R) Xeon(R) CPU E5-2680 2.8GHz per Node, 128 GB per Node, 56GB/s FDR Infiniband, Rocks Cluster 6.1.1 with Red Hat Enterprise Linux 2.6.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPC options</td>
<td>One task, twelve cores</td>
</tr>
<tr>
<td>Project adaptive frequency</td>
<td>10 kHz (the frequency at which the tetrahedral mesh is constructed and adapted)</td>
</tr>
<tr>
<td>External boundary conditions</td>
<td>Neumann (H-field is tangential to the boundary) or radiation</td>
</tr>
<tr>
<td>Execution time for five adaptive mesh refinement passes</td>
<td>Meshing time: 60-70 min</td>
</tr>
<tr>
<td>Sim. time for 22 frequencies: 5-6 hr</td>
<td></td>
</tr>
<tr>
<td>Convergence history</td>
<td>Energy error percentage (typical): 27, 0.8, 0.12, 0.036, 0.014</td>
</tr>
<tr>
<td>Max RAM per node</td>
<td>100-120 Gbytes</td>
</tr>
<tr>
<td>Initial/final FEM mesh</td>
<td>(typical) 450,00/1,400,000 tetrahedra</td>
</tr>
</tbody>
</table>

**High-frequency efficiency [12-16]**

High-frequency computational efficiency of the VHP-Female v.3.0 model and its beta versions has been evaluated at different times during 2013-2015 by a number of computational experts. In summary, on an ordinary 8 core computer, the corresponding ANSYS simulation at 300 MHz with 5 adaptive passes and 1.2 M tetrahedra takes a few hours. About half of this time is meshing time (VHP-Female version 3.0 BASE). A CST-MWS simulation with a comparable accuracy might be somewhat faster. The run time for the SMOOTH version is approximately ten times longer.

**Comparison with experiment**

We have compared numerical modeling data generated with VHP-Female version 3.0 BASE with some experimental data for on-body antenna reflection coefficient and back lobe level [12]. An acceptable agreement has been obtained.
B. Plausibility

Outcome

1. For low-frequency electromagnetic simulations of injected electric currents, the models VHP-Female version 3.0 BASE and SMOOTH, and model VHP-Female version 2.2 output electric current density and electric voltage at any location within the human body for any electrode (array) position [10] – see Fig. 10. According to guidelines from the International Commission on Non-Ionizing Radiation Protection [17, 18], these are two major variables of interest.

2. For low-frequency electromagnetic simulations of induced electric currents, the models VHP-Female version 3.0 BASE and SMOOTH, and model VHP-Female version 2.2 output induced electric current density and induced electric field at any location within the human body [11] – see Figs. 11, 12. According to guidelines from the International Commission on Non-Ionizing Radiation Protection [17, 18], these are two major variables of interest.

3. For on-body, in-body, and near-body antennas, the models VHP-Female version 3.0 BASE and SMOOTH, and model VHP-Female version 2.2 output antenna impedance and antenna near- and far-fields (and derived from them SAR or specific absorption rate) at any location on, within, or near human body [12] – see Fig. 13 and Fig. 15. According to guidelines from the International Commission on Non-Ionizing Radiation Protection [17, 18], in-body SAR is the major antenna characteristic of interest for safety evaluations.

Devices to be tested with regard to safety evaluation according to [17, 18]

1. Devices for tDSC (transcranial direct current stimulation) therapy including safety evaluations (electrodes and electrode array injecting DC or low-frequency AC currents). We are able to accurately resolve near-electrode fields and the associated PLD (power loss density).

2. Devices for TMS (transcranial magnetic stimulation) therapy including safety evaluations (TMS coils such as Neuronetics Neurostar coils, Brainsway H-coils, Magstim BC-70 series and similar coils of Magstim Company, etc., as well as rotating magnetic field devices).

3. Power electronics devices (power lines, transformers, relays, solid-state converters).

4. Wearable and implanted patch, loop, and meander antennas, and other antenna types (SAR). We are able to accurately resolve near- and Fresnel-zone fields in the vicinity of a metal antenna.

5. High-power devices for tumor-treating fields and RF therapy.

C. Extent of prediction

The data generated by the models VHP-Female version 3.0 BASE and SMOOTH, and model VHP-Female version 2.2 predict the true outcome of interests for at least three scenarios given in Section IV.B above. This fact is evidenced by the materials of Sections III and IV, respectively, and is reflected in selected subsequent publications [9-16].
The evidence data has been partially obtained using earlier legacy model versions (VHP-Female versions 2.0/2.1), which are in fact not significantly different from the models VHP-Female version 3.0 BASE and SMOOTH, and the model VHP-Female version 2.2. The local differences are in correcting isolated anatomical and numerical flaws identified during the model use. In our opinion, every reliable human model should go through this refinement process.

D. Capture
Given its resolution, the models VHP-Female version 3.0 BASE and SMOOTH, and model VHP-Female version 2.2, accurately resolve electric and magnetic fields variations

i. at the scales of 1-3 mm within the head (see Fig. 15);

ii. at the scales of 3-10 mm within the body (see Fig. 11, 12).

The accuracy of the simulation results is guaranteed by FEM adaptive mesh refinement procedures and by cross-platform agreement established in Section IV.A. At smaller separation distances, the simulation results should be considered as averaged.

V. ASSESSMENT OF ADVANTAGES AND DISADVANTAGES

The assessment will be given by comparison with the World leader – the voxel-based Virtual Family/Population of IT’IS Foundation, Switzerland [19].

A. Advantages (type, magnitude, and likelihood)

1. For all three VHP-Female models, the original high-resolution image dataset is available to any model user or to any interested person through the US National Library of Medicine. There is no other full-body human model with such properties. For example, all full-body models distributed by IT’IS Foundation, Switzerland do not make the original datasets available to users.

2. All three VHP-Female models are fully compatible with the leading commercial FEM software packages. On the other hand, CAD models distributed by IT’IS Foundation Switzerland do not possess such a compatibility.

3. All three VHP-Female models allow a high computational speed of most accurate FEM simulations (a few hours for full-body simulations with the VHP-Female version 3.0 BASE or VHP-Female version 2.2 with adaptive mesh refinement) and, as a result, a great acceleration in the development and evaluation of the aforementioned devices by running multiple parametric sweeps. In Ref. [11], for example, over 200 different model runs have been performed, each with 21 individual frequency runs. This made it possible to establish a general upper estimate related to safety of the TMS devices.

4. All three VHP-Female models possess cross-platform compatibility with leading software packages whereas the full-body models distributed by IT’IS Foundation, Switzerland run on the Foundation’s affiliated software Sim4Life/SEMCAD. The cross-
platform compatibility makes it possible to communicate effectively between different user groups and independently validate the accuracy of the results.

5. All three VHP-Female models have been evaluated by a board of medical and computational experts, both domestic and international.

6. A simplified VHP-Female model version 2.2 is made available to all users free of charge.

B. Disadvantages (types, magnitude, mitigation)
1. The VHP-Female model versions 3.0 BASE/SMOOTH and 2.2, though being anatomically/numerically validated and 100% FEM-compatible, have generally a lower resolution as compared to the full-body models distributed by the IT’IS Foundation, Switzerland, at least at first sight. As stated above, the models’ resolution is limited by 1-3 mm within the head and by 3-10 mm within the rest of body.

2. However, since the original datasets of the full-body models distributed by the IT’IS Foundation, Switzerland are not available, we cannot independently evaluate their true anatomical accuracy after image voxelization and surface extraction. We also cannot independently evaluate the corresponding computational accuracy. The use of a proven FEM software with adaptive mesh refinement may assure that our final accuracy with VHP-Female version 3.0 SMOOTH is the same.

3. One potential risk relates to using VHP-Female version 3.0 BASE for PLD/SAR estimations in certain domains where sizes of triangular facets are large. In this case, artificial surface charges might appear and change local electric fields considerably (by 100% or so), at distances on the order of 10 mm.

4. In order to avoid this issue, the VHP-Female version 3.0 SMOOTH has specifically been developed and tested. This version is topologically similar, but possesses smooth surfaces and has a larger (by the factor of four) number of triangular facets. However, it runs considerably slower. Therefore, when there is doubt with respect to SAR computation in a local area, the VHP-Female version 3.0 SMOOTH must be used to validate the results.
VI. IMPROVED HEAD MODEL OF VHP-FEMALE VERSION 3.0 SMOOTH (100% COMPATIBLE WITH THE REST OF MODEL)

A. Tissues
About 20 tissues have been re-segmented to achieve a finer accuracy. The list of improvements is given below.

1. Grey matter has been re-segmented to achieve a better accuracy (see Fig. 16).
2. White matter has been carefully re-segmented and refined to achieve a much better anatomical accuracy (see Fig. 16).
3. The CSF shell around the grey matter has been made a more realistic (of a smaller thickness). *The new CSF shell still fully encloses the brain*, which is the major anatomical/numerical advantage of the present model (see Fig. 17).
4. Skull has been re-segmented to achieve a much better accuracy (see Fig. 18).
5. Upper jaw has been added (see Fig. 18).
6. All missing teeth have been added (see Fig. 18).

The re-segmented and/or affected tissues are listed in Table II.

<table>
<thead>
<tr>
<th>Mesh no</th>
<th>Tissue name</th>
<th>Triangular mesh size</th>
<th>Mesh quality</th>
<th>Min. Edge Length</th>
<th>Tissue Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VHP A Arteries</td>
<td>39132</td>
<td>0.098572312</td>
<td>0.373846732</td>
<td>Blood</td>
</tr>
<tr>
<td>2</td>
<td>VHP A Average Body Shell Small</td>
<td>23786</td>
<td>0.052403111</td>
<td>0.743942341</td>
<td>Average Body</td>
</tr>
<tr>
<td>3</td>
<td>VHP A Cerebellum</td>
<td>10000</td>
<td>0.034484754</td>
<td>0.360138582</td>
<td>Cerebellum</td>
</tr>
<tr>
<td>4</td>
<td>VHP A CSF Outer Shell Head Spinal Cord</td>
<td>7140</td>
<td>0.001038162</td>
<td>0.186723604</td>
<td>Cerebro-spinal Fluid</td>
</tr>
<tr>
<td>5</td>
<td>VHP A CSF Ventrices</td>
<td>2240</td>
<td>0.039189284</td>
<td>0.47134255</td>
<td>Cerebro-spinal Fluid</td>
</tr>
<tr>
<td>6</td>
<td>VHP A Eye left</td>
<td>400</td>
<td>0.554489018</td>
<td>1.730596771</td>
<td>Eye (Vitrous Humor)</td>
</tr>
<tr>
<td>7</td>
<td>VHP A Eye right</td>
<td>394</td>
<td>0.593413535</td>
<td>1.538214589</td>
<td>Eye (Vitrous Humor)</td>
</tr>
<tr>
<td>8</td>
<td>VHP A Fat Shell</td>
<td>23582</td>
<td>0.031914935</td>
<td>0.163607014</td>
<td>Fat (Average Infiltrated)</td>
</tr>
<tr>
<td>9</td>
<td>VHPA GreyMatter</td>
<td>41790</td>
<td>0.007597268</td>
<td>0.160422659</td>
<td>Brain (Grey Matter)</td>
</tr>
<tr>
<td>10</td>
<td>VHP A Jaw</td>
<td>2700</td>
<td>0.014602226</td>
<td>0.177909713</td>
<td>Bone</td>
</tr>
<tr>
<td>11</td>
<td>VHP A Skin Shell</td>
<td>23442</td>
<td>0.012658969</td>
<td>0.186124441</td>
<td>Skin</td>
</tr>
<tr>
<td>12</td>
<td>VHP A Skull</td>
<td>29946</td>
<td>8.53183E-05</td>
<td>0.08800212</td>
<td>Bone</td>
</tr>
<tr>
<td>13</td>
<td>VHP A Spinal Cord Cauda Equina</td>
<td>2094</td>
<td>0.009146166</td>
<td>0.568865681</td>
<td>Nerves</td>
</tr>
<tr>
<td>14</td>
<td>VHP A All teeth</td>
<td>Combined with upper jaw and lower jaw</td>
<td>0.028701087</td>
<td>Bone</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>VHP A Tongue</td>
<td>698</td>
<td>0.090962864</td>
<td>1.379264633</td>
<td>Tongue</td>
</tr>
<tr>
<td>16</td>
<td>VHP A Trachea Sinus</td>
<td>3368</td>
<td>0.055020857</td>
<td>0.346894047</td>
<td>Air</td>
</tr>
<tr>
<td>17</td>
<td>VHP A Veins upper</td>
<td>24570</td>
<td>0.001773154</td>
<td>0.028701087</td>
<td>Blood</td>
</tr>
</tbody>
</table>
**B. Model Topology**

The model topology is the same as for VHP-Female V. 3.0[20]:

- All model objects are surrounded by 1 mm thick skin object
- All model objects except the skin object are surrounded by fat object (object directly under skin) [20].
- All model objects except the skin and fat are surrounded by average body object (object directly under fat)[20].
- White matter is surrounded entirely by grey matter object.
- Cerebellum is surrounded entirely by grey matter object.
- Ventricular object is surrounded entirely by grey matter object.
- Grey matter is surrounded entirely by cerebrospinal fluid (CSF) object.
- Cauda Equina is surrounded entirely by CSF object.

The scalp structure always has the total thickness of 5 mm±0.5 mm and consists of three fully enclosed objects (or shells): outermost skin (1 mm distance from fat), then fat (2 mm distance from average body), and then average body (2 mm distance from skull). The material between the skin object and the fat object is “skin”, the material between the fat object and the average body object is “fat”, and the material between the average body object and the skull is “average body”.

The skin, fat, and average body shells can be cut axially at any desired location, for example, near the shoulders or neck as per the application. The Boolean cut using commercial software tend to create large triangles along the surface of the axial cut. We have custom procedures to control and create desired number of triangles on the axial cut after Boolean operations are performed [21]. The algorithms can also be used to create an onion structure of shells with the gap restored at the axial cut.
Fig. 16. Re-segmented white matter and grey matter.
Fig. 17. Continuous watertight CSF shell surrounding the grey matter shell.

Fig. 18. Cranium structure without body shells.
REFERENCES


VII. APPENDIX A. LIST OF TISSUE OBJECTS

VHP Female Computational Model version 3.0 BASE (with some shells/muscles made invisible)
Table 1. List of tissue objects (VHP-Female Computational Model version 3.0 BASE)

Note: VHP-Female version 3.0 SMOOTH has the same list of tissues.

Legend:
- Hard tissues
- Soft tissues
- Individual muscles
- Cartilage

<table>
<thead>
<tr>
<th>#</th>
<th>Tissue object name (alphabetical)</th>
<th>Number of triangular facets</th>
<th>Tissue type (IT'IS database)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VHPC_Abdominals_left_bot.mat</td>
<td>944</td>
<td>Muscle</td>
</tr>
<tr>
<td>2</td>
<td>VHPC_Abdominals_left_mid.mat</td>
<td>762</td>
<td>Muscle</td>
</tr>
<tr>
<td>3</td>
<td>VHPC_Abdominals_left_top.mat</td>
<td>894</td>
<td>Muscle</td>
</tr>
<tr>
<td>4</td>
<td>VHPC_Abdominals_right_bot.mat</td>
<td>876</td>
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</tr>
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<td>VHPC_Abdominals_right_mid.mat</td>
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<td>Muscle</td>
</tr>
<tr>
<td>6</td>
<td>VHPC_Abdominals_right_top.mat</td>
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<td>Muscle</td>
</tr>
<tr>
<td>7</td>
<td>VHPC_Arteries.mat</td>
<td>10004</td>
<td>Blood</td>
</tr>
<tr>
<td>8</td>
<td>VHPC_Average_Body_Shell_Small.mat</td>
<td>8390</td>
<td>Muscle</td>
</tr>
<tr>
<td>9</td>
<td>VHPC_Bicep_left.mat</td>
<td>774</td>
<td>Muscle</td>
</tr>
<tr>
<td>10</td>
<td>VHPC_Bicep_right.mat</td>
<td>792</td>
<td>Muscle</td>
</tr>
<tr>
<td>11</td>
<td>VHPC_Bladder.mat</td>
<td>274</td>
<td>Bladder</td>
</tr>
<tr>
<td>12</td>
<td>VHPC_C01.mat</td>
<td>1034</td>
<td>Bone Cortical</td>
</tr>
<tr>
<td>13</td>
<td>VHPC_C02.mat</td>
<td>840</td>
<td>Bone Cortical</td>
</tr>
<tr>
<td>14</td>
<td>VHPC_C03.mat</td>
<td>644</td>
<td>Bone Cortical</td>
</tr>
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<td>15</td>
<td>VHPC_C04.mat</td>
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</tr>
<tr>
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<td>VHPC_C05.mat</td>
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</tr>
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<td>VHPC_C06.mat</td>
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<td>Bone Cortical</td>
</tr>
<tr>
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<td>VHPC_C07.mat</td>
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<td>Bone Cortical</td>
</tr>
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<td>VHPC_Calcaneous_left.mat</td>
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<td>Muscle</td>
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<td>22</td>
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<td>Muscle</td>
</tr>
<tr>
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<td>VHPC_Cerebellum.mat</td>
<td>504</td>
<td>Cerebellum</td>
</tr>
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<td>Tissue type (IT'IS database)</td>
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<td>-----------------------------</td>
<td>-----------------------------------</td>
</tr>
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<td>Bone Cortical</td>
</tr>
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<td>VHPC_Clavicle_right.mat</td>
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</tr>
<tr>
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<td>VHPC_Coccyx.mat</td>
<td>150</td>
<td>Bone Cortical</td>
</tr>
<tr>
<td>27</td>
<td>VHPC_CSF_Outer_Shell_Head_Spinal_Cord.mat</td>
<td>4054</td>
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Appendix B. VHP-Female version 3.0 SMOOTH

VHP Female Computational Model version 3.0 SMOOTH (shown with 3D printing options)
Chapter 04 –
Applications of VHP-Female Computational Phantom

SECTION A –
SAFE UPPER ESTIMATE OF PEAK CURRENTS IN TRANSCRANIAL MAGNETIC STIMULATION AT DISTANT LOCATIONS FROM A TMS COIL

I. INTRODUCTION

Recent studies confirm the efficacy of Transcranial Magnetic Stimulation (TMS) as a non-invasive treatment of medication-resistant depression [1]-[4]. To date, a number of TMS devices such as the Neuronetics Neurostar Stimulators, the Brainsway H-Coil system, Magstim Magnetic Stimulators, and MagVenture Stimulators have been cleared by the Food and Drug Administration (FDA) for this purpose. The use of TMS might cause exposure to undesired induced currents throughout the entire body of a patient and/or a TMS operator or a nurse. Therefore, it is important to develop rigorous numerical and possibly fast analytical techniques that can safely estimate and predict the eddy current level not only in the brain, but also at various distant locations. As an example, using TMS may cause fetal exposure to undesired induced currents in pregnant patients. A considerable percentage of women experience symptoms of depression during pregnancy and develop clinical depression requiring medical intervention, and TMS has been proposed as a method to treat maternal depression while avoiding fetal exposure to drugs [5]. TMS for treatment of depression during pregnancy is an appealing alternative, but there are not enough studies to date to ensure the safety of TMS treatments for a pregnant mother and her fetus [6].

To find what are the acceptable levels of induced currents for different tissues, we refer to two guidelines [17],[18] from the International Commission on Non-Ionizing Radiation Protection. These guidelines provide recommendations on safe limits of exposure to time-varying electric, magnetic, and electromagnetic fields up to 300 GHz for both the general public and occupational cases. Induced electric fields and currents at levels exceeding those of endogenous bioelectric signals present in tissue have been shown to cause a number of physiological effects that increase in severity as the induced current density is increased. In the current density range of 10-100 mA/m², tissue effects and changes in brain cognitive functions have been reported. When induced current density exceeds 100 mA/m² for frequencies between 10 Hz and 1 kHz, thresholds for neuronal and neuromuscular stimulation are exceeded. At a higher level of exposure, severe and potentially life-threatening effects such as cardiac extra systoles, ventricular fibrillation, muscular tetanus, and respiratory failure may occur. The severity and probability of irreversibility of tissue effects becomes greater with chronic exposure to induced currents densities. The 1998 ICNIRP basic restrictions for general exposure to time-varying electric and magnetic fields for CW frequencies in the band 1-100 kHz recommend that the current density for head and trunk
should be below $f/500$ mA/m$^2$ where $f$ is the signal frequency measured in Hz. According to these guidelines, at 5 kHz frequency, the minimum exposure threshold recommended is 10 mA/m$^2$. For pulses of (effective) duration $\tau$, the equivalent frequency to apply in the basic restrictions should be calculated as $f' = 1/(2\tau)$. The above estimate can be also formulated in terms of the induced electric field by dividing the current density by local conductivity.

Induction currents in the entire human body (or bodies) caused by a TMS coil can be established in every particular case via numerical electromagnetic modeling. A numerically-accurate procedure adopted to model eddy currents within complex biological shapes is the finite-element method (FEM) or boundary element method (BEM) [9]-[16]. FEM or BEM has been previously applied specifically to TMS effects on the human head/body [13]-[15], [17]-[22]. Recently, detailed FEM computational phantoms of the head [23] and of the entire body [24] have been developed and made available. Other methods are the finite-difference (or finite-volume) time-domain (FDTD) [25]-[35] method and quasi-analytical techniques [36]. A typical high-fidelity full-body FEM simulation with controllable accuracy and adaptive mesh refinement performed on a multiprocessor server currently requires about 5-10 hours of elapsed time for one particular geometry.

As far as the numerical computations are concerned, different body compositions (e.g. a different BMI, or a different age, or pregnancy, as well as large or metallic implants) and poses will require different human body phantoms. There is also a growing variety of different TMS coil designs [37],[38],[36],[39]-[42], each of which in principle needs to be accurately modeled separately. Any particular coil position will require a new simulation as well. All this leads to a nearly infinite number of simulations to be performed in order to obtain general and reliable eddy current estimates. But what if we apply a simple analytical result, which uses the Bio-Savart law for the coil and Faraday’s law in an unbounded homogeneous conductor, for an eddy current estimate at any particular location? Such a result is the early transcranial magnetic stimulation model [43],[44], which has been recently revisited [45].

At first sight, this analytical model seems to be useless since it severely overestimates the eddy currents in a bounded conductor with the relative dielectric constant of one. It can be shown that, for body-like conductor sizes, such a model may overestimate the true value by a few hundred times. The hidden aspect, however, is a quite large dielectric permittivity of realistic human tissues, especially at lower frequencies [46], [47]. When the actual displacement (polarization) currents and the associated polarization charges at the boundaries are taken into account, the situation may change drastically. The present study is aimed to show that the analytical model outlined above works surprisingly well as a general upper estimate of the eddy current density. This estimate applies to different body shapes, coil compositions, and different distant locations within the body, although the line integral over the coil contour still needs to be computed numerically for every observation point. The study is organized as follows:

Section 2 formulates the analytical eddy current model and discusses all the assumptions made.

Section 3 presents an FEM computational human phantom and the computational testbed.
Section 4 describes an FEM analysis setup to be performed in order to prove the model and presents qualitative results.

Section 5 presents frequency-domain results over the band 300 Hz – 3 MHz and compares the FEM solution with the analytical estimate.

Section 6 presents time-domain results for a generic monophasic TMS pulse and compares the FEM solution with the analytical estimate.

Section 7 compares the FEM solution with the analytical estimate for a different coil model.

Section 8 explains the reason why the analytical model performs reasonably well.

Section 9 states the guaranteed upper estimate for the peak TMS currents and full body coverage.

Section 10 concludes the study.

II. UPPER ANALYTICAL ESTIMATE OF EDDY CURRENTS IN A HETEROGENEOUS CONDUCTING BODY

The upper analytical estimate of eddy currents excited in a heterogeneous non-magnetic conducting object (a human body) is based on three well-known simplifications and is rather straightforward. Recall that, after introducing the magnetic vector potential \( A \) so that \( \mu_0 H = \nabla \times A \), Faraday’s law of induction is transformed to [48]

\[
E = - \frac{\partial A}{\partial t} - \nabla \phi \tag{1}
\]

Here, \( H(r, t) \) is the total magnetic field in the body or outside, \( E(r, t) \) is the total electric field in the body or outside, and \( \phi(r, t) \) is the electric potential due to surface charges residing on interfaces separating tissues with different conductivities and/or different permittivities. The electric current excited in a tissue is a combination of the conduction current, \( \sigma E \), and the displacement current \( \varepsilon \partial E / \partial t \),

\[
J = \sigma E + \varepsilon \frac{\partial E}{\partial t} \tag{2}
\]

where \( \sigma(r) \) is the local tissue conductivity and \( \varepsilon(r) \) is the local permittivity. Equation (2) will have a more complicated integral form when conductivity and permittivity are frequency-dependent. Equations (1) and (2) are exact expressions without simplification.

A. Neglecting the secondary magnetic field of eddy currents – thin limit condition

Metals are highly-conducting materials. Therefore, the skin effect becomes dominant even at low frequencies. Human tissues, on the other hand, have conductivities six to seven orders of magnitude less than metals. Therefore, they could be considered as weakly-conducting media compared to metals. In a weakly-conducting medium, the eddy currents are small and their own
(secondary or internal) magnetic field $\mathbf{H}$ is also small as compared to the known external large magnetic field, $\mathbf{H}^{\text{inc}}$, of the TMS coil. Thus, one has in terms of the magnetic vector potential,

$$\mathbf{A} = \mathbf{A}^{\text{inc}} + \mathbf{A}^{s} \approx \mathbf{A}^{\text{inc}}$$  \hspace{1cm} (3)

Physically, (3) means that the skin layer depth, $\delta$, is large compared to a typical body size, $L$, i.e. $\delta = \sqrt{2/(\omega \mu \sigma)} \gg L$. This expression is also known as the thin limit condition.

**B. Neglecting the effect of free surface charges**

The free surface charges (and the associated electric field) will not appear if and only if the magnetic vector potential $\mathbf{A}^{\text{inc}}$ of an exciting coil current is always parallel to the interfaces, i.e. when $\mathbf{n} \cdot \mathbf{A}^{\text{inc}} = 0$, where $\mathbf{n}$ is the normal vector to any interface of interest. This is indeed not the case in the human body although important analytical solutions without surface charges do exist [49]-[51]. The effect of surface charges was specifically studied in [52] and [44]. In the last paper, it was shown that a no-surface charge model always overestimates eddy currents in a body when compared to the more realistic situation. Further development is given in [45]. Numerical simulations indicate that the free surface charges always reduce the eddy current magnitude (“shorten the path” for the eddy currents). The no-surface charge model effectively makes the conducting medium homogeneous and unbound, and gives it a certain average conductivity value. In the absence of free surface charges, and in a medium with the relative dielectric permittivity equal to one, one has

$$\varphi = \text{const} \text{ and } \nabla \varphi = 0$$  \hspace{1cm} (4)

in (1).

**C. Neglecting the effect of tissue permittivity**

Human tissues have varying values of the dielectric permittivity, which may be very large at low frequencies [46],[47]. As soon as the electric field is excited, the tissue medium will become polarized. This means that the bound (polarization) charges on dielectric-dielectric interfaces and the associated volume polarization currents, $(\varepsilon - \varepsilon_0)\hat{\mathbf{E}} / \hat{t}$, in the dielectric volume have to be taken into account, along with the free charges and conduction currents. The simplified model given by (4) neglects this effect entirely and does not involve the relative dielectric permittivity either.

**D. Analytical estimate of eddy current density**

This estimate was perhaps first formulated in an early transcranial magnetic stimulation model by Grandori and Ravazzani (the GR model) [43],[44], which disregards the secondary magnetic fields of eddy currents, neglects the free surface charges residing on conductor-conductor interfaces, and neglects the polarization charges as well as polarization currents. As a result of these assumptions, (1)-(4) allow us to express the eddy current density in the body directly through a time-varying lumped coil current, $f(t)$, in the form
\[
\mathbf{J} = -\sigma \frac{\partial \mathbf{A}^{\text{inc}}}{\partial t}, \quad \mathbf{A}^{\text{inc}}(\mathbf{r}, t) = \frac{\mu_0 f(t)}{4\pi} \oint_C \frac{d\mathbf{l}}{||\mathbf{r} - \mathbf{r}'(l)||}
\]  

The second expression in (5) is the Bio-Savart law written in the form of a line integral for a coil having a contour \( C \). Equations (5) are general results, which may in principle be applied to any coil geometry and any position in space, or to an array of coils using superposition. Calculation of the line integral for a given coil geometry may be accomplished via a Riemann sum or trapezoidal integration. Reference [53] presents the text of a MATLAB script that performs such a calculation for a current-carrying conductor arbitrarily oriented in space or for a number of such conductors which might form, for example, a Figure-8 coil. This script has been tested via an exact analytical solution and demonstrated an error in the eddy current magnitude below 2%. To complete the estimate in (5), the effective medium conductivity should be given. A widely-used average body conductivity value of 0.5 S/m will be employed in the following study.

III. BASE FEM COMPUTATIONAL HUMAN PHANTOM AND COMPUTATIONAL TESTBED

A. Computational phantom

The full-body computational phantom employed in this study is the VHP-Female FEM phantom [24],[53]. Its version 2.1 shown in Fig. 1 has been used with the properties described in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
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<tr>
<td>FEM COMPUTATIONAL HUMAN PHANTOM VHP-FEMALE V. 2.1.</td>
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</tbody>
</table>

| NAME AND HUMAN SUBJECT | VHP-Female – a 60 years old female; a few known pathologies; BMI of approximately 32 |
| IMAGE SOURCE | Visible Human Project®-Female dataset (The Visible Human Project® [54]) of the National Library of Medicine |
| RELEASE AND VENDOR | VHP-Female v.2.1 2015/NEVA Electromagnetics, LLC and ECE Dept., Worcester Polytechnic Inst., USA |
| INDIVIDUAL TISSUES | 25 |
| INDIVIDUAL TISSUE PARTS | 203 |
| TRIANGULAR FACETS | 139,450 |
| MATERIAL PROPERTIES | 50 Hz – 60 GHz [46],[47], isotropic tissue materials only |
| COMPATIBILITY | ANSYS Maxwell 3D, ANSYS HFSS, CST MWS, FEKO, REMCOM, WIPL-D, COMSOL |

The phantom has an improved resolution in the cranium including the continuous CSF shell around the grey matter and it has been optimized for accurate FEM modeling.

The phantom also possesses a set of characteristics necessary for cross-platform compatibility and computational efficiency. Each triangular surface mesh of the original tissues is strictly 2-manifold (no non-manifold faces, no non-manifold vertices, no holes, and no self-intersections). No tissue mesh has triangular facets in contact with other tissue surfaces. Between tissue surfaces,
there is always a small gap representing thin membranes separating distinct tissues and numerically characterized as an “average body” tissue(s), and guaranteeing compatibility between CAD formats. At the same time, there exist tissues fully enclosed within each other.

**B. Computational test bed**

Maxwell 3D of ANSYS, Inc., a commercial FEM software package with adaptive mesh refinement, has been used for eddy current computations, similar to the earlier studies [14],[15].

The software takes into account both conduction and displacement currents (as well as free and polarization charges), and solves the full-wave Maxwell equation for the magnetic field, $\mathbf{H}$, in the frequency domain

$$\nabla \times \frac{1}{\sigma + j\omega \varepsilon} \nabla \times \mathbf{H} = -j\omega \mu \mathbf{H}$$

(6)

where $\sigma$ is the local medium conductivity; $\varepsilon$, $\mu$ are the local permittivity and permeability, respectively. The major difference from the full-wave case is that the phase is assumed to be constant over the volume of interest. Although Maxwell 3D also has a transient FEM solver, this solver does not take into account the displacement currents and was therefore not used. The local FEM error is the error of the divergence-free magnetic flux, $\nabla \cdot \mathbf{B}_{\text{solution}} \neq 0$. This term acts as a source and produces some energy. Tetrahedra with the largest local error energy (30% or so per adaptive pass) are automatically refined. The total error energy in the volume divided by the total energy of the electromagnetic field and multiplied by 100% is the energy error, which is returned each adaptive pass along with the total energy. The energy error measures the convergence of the adaptive mesh refinement process. Table II describes the computational test bed and major numerical parameters used in this study. A logarithmic frequency sweep covering the band from 300 Hz to 3 MHz has been employed over the total 22 discrete frequencies.
Fig. 1. FEM computational phantom VHP-Female v. 2.1
TABLE II  
COMPUTATIONAL HARDWARE/SOFTWARE AND MAJOR PROJECT  
PARAMETERS.

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<th>Parameter</th>
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</tr>
<tr>
<td>Initial/final FEM mesh</td>
<td>450,00/1,400,000 tetrahedra</td>
</tr>
</tbody>
</table>

Such a frequency band is sufficient to model TMS pulses with the typical magnetic field rise time on the order of 0.1 ms [55],[56]. The time-pulse domain solution is constructed based on the interpolated frequency-domain data.

IV. SIMULATION SETUP. QUALITATIVE RESULTS

A. Basic geometry setup

The base coil is a Figure-8 coil with a loop radius of 25 mm for each loop and a solid conductor (copper) diameter of 8 mm. The coil is bent so that the loop angle versus a horizontal plane in Fig. 2 is 15 degrees, similar to the Magstim BC-70 (a commercial Figure-8 coil). Five different coil positions around the motor cortex have been investigated; four of them are shown in Fig. 2a-d. In the frequency-domain solution, the sinusoidal AC coil current has the amplitude of 1 kA, although the specific amplitude value is irrelevant due to problem linearity. As shown in Fig. 2 and in Table IIIa, ten observation base points have been selected within the body: 1, 2, 2a, 2b, 3, 3a, 3b, 4, 4a, and 4b. In the majority of the cases, we attempted to locate the points at such positions where the eddy currents may be expected to have relatively large values (in highly-conducting tissues and close to the boundaries). The analysis which follows is mainly represented as tables and plots for the ten observation points defined in Table IIIa.
TABLE IIIA - LOCATIONS OF TEN OBSERVATION POINTS.

<table>
<thead>
<tr>
<th>Location in space (local coordinates as shown in Fig. 2, mm)</th>
<th>Tissue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1 ([0, 0, 50])</td>
<td>Grey matter, near CSF ventricles</td>
</tr>
<tr>
<td>Point 2 ([0, 0, -300])</td>
<td>Avg. body, close to heart, lungs</td>
</tr>
<tr>
<td>Point 3 ([0, 0, -500])</td>
<td>Avg. body, close to large intestine</td>
</tr>
<tr>
<td>Point 4 ([0, 0, -700])</td>
<td>Uterus, close to avg. body</td>
</tr>
<tr>
<td>Point 2a ([80, 0, -300])</td>
<td>Heart</td>
</tr>
<tr>
<td>Point 2b ([-80, 0, -300])</td>
<td>Right lung, close to a rib</td>
</tr>
<tr>
<td>Point 3a ([80, 0, -500])</td>
<td>Stomach, close to avg. body</td>
</tr>
<tr>
<td>Point 3b ([-80, 0, -500])</td>
<td>Large intestine, near boundary</td>
</tr>
<tr>
<td>Point 4a ([80, 0, -700])</td>
<td>Avg. body, close to pelvic bone</td>
</tr>
<tr>
<td>Point 4b ([-80, 0, -700])</td>
<td>Avg. body</td>
</tr>
</tbody>
</table>

Fig. 2. Coil setup and observation point setup for the human phantom VHP-Female in Maxwell 3D FEM simulator.
B. Full body coverage

An additional three-dimensional uniform rectangular grid of observation points shown in Table IIIb is introduced to effectively cover the entire human body in the solution space. This uniform grid includes 500,000 nodes.

<table>
<thead>
<tr>
<th></th>
<th>Minimum (mm)</th>
<th>Maximum (mm)</th>
<th>Step size (mm)</th>
<th>Points along the axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>X axis</td>
<td>-250</td>
<td>525</td>
<td>7.75</td>
<td>100</td>
</tr>
<tr>
<td>Y axis</td>
<td>-125</td>
<td>300</td>
<td>4.25</td>
<td>100</td>
</tr>
<tr>
<td>Z axis</td>
<td>-900</td>
<td>150</td>
<td>21</td>
<td>50</td>
</tr>
</tbody>
</table>

Among these nodes, 102,189 nodes are located inside the human body and generate meaningful current/field values. The outer nodes may be used to compute the electric field around the body.

C. Sensitivity analysis setup

The sensitivity analysis was performed by varying the conductivity and permittivity of an average body object of the model (a shell), which contains all inner organs/tissues and presumably has the largest influence on the final results. Both permittivity and conductivity were varied by ±20%.

D. Qualitative eddy current behavior at distant locations from the coil

While within the motor cortex itself the eddy current of the Figure-8 coil indeed peaks underneath the intersection of the two wire loops, this is certainly not the case at distant locations from the coil. As an example, Fig. 3 shows the eddy current density (A/m²) magnitude distributions in the three different transverse planes depicted in Fig. 2 for sinusoidal coil currents at 30 kHz and 300 kHz respectively, with an amplitude of 1 kA each (coil configuration #1). Note the different color scales for each plane. The eddy current density behavior varies with changes in frequency. Compared to the 30 kHz excitation, the color scale for 300 kHz is multiplied by 20, which approximately accounts for a linear frequency increase (the factor of 10) as well as a conductivity increase.

V. COMPARISON BETWEEN ANALYTICAL AND NUMERICAL RESULTS IN FREQUENCY DOMAIN

A. Transfer function for the numerical solution

First, the eddy-current problem is solved numerically in the frequency domain. Next, the time-domain solution will be constructed, given the computed frequency response of the linear system (or its transfer function, which is the same) and the spectrum of the initial pulse, via the inverse discrete Fourier transform. In the frequency domain, the current coil excitation is given by a harmonic function

\[
 f(t) = I_0 \cos \omega t, \quad \omega = \Omega_m, \quad m = 0, \ldots, M - 1
\]  

(7)
for a number of discrete (and generally non-uniformly spaced) angular frequencies $\Omega_n$. The problem is solved via Maxwell 3D, an FEM frequency-domain solver, for every such frequency using a frequency sweep. This operation gives us an eddy current density at any point in space within the body in the form of a complex phasor vector, $\mathbf{J}(\mathbf{r}, \omega)$, with the units of A/m\(^2\). The vector transfer function $\mathbf{H}_y(\mathbf{r}, \omega)$ per unit area is simply given by

$$\mathbf{H}_y(\mathbf{r}, \omega) = \frac{\mathbf{J}(\mathbf{r}, \omega)}{I_0}$$

(8)

This transfer function can be thought of as an eddy current passing through an area of 1 m\(^2\) which is perpendicular to its direction with the current density given by the local expression $\mathbf{J}(\mathbf{r}, \omega)$ when the amplitude of the coil current is 1 A.

**B. Transfer function for the analytical solution**

Once converted to the frequency domain, (5) predicts the following form of the transfer functions (note the separation of variables):

$$\mathbf{H}_A(\mathbf{r}, \omega) = -j \omega \frac{\mu_0 \sigma}{4\pi} \mathbf{h}(\mathbf{r}), \quad \mathbf{h}(\mathbf{r}) = \frac{1}{4\pi} \int \frac{d\mathbf{l}}{|\mathbf{r} - \mathbf{r}'(I)|}$$

(9)

**C. Comparison of numerical and analytical solutions**

The magnitude ratio, $R(\mathbf{r}, \omega)$, of two vector transfer functions, given by (8) and (9), respectively, is equal to the ratio of two eddy current magnitudes at point $\mathbf{r}$ when the operating frequency is $\omega$, that is to say

$$R(\mathbf{r}, \omega) = \frac{|\mathbf{H}_A(\mathbf{r}, \omega)|}{|\mathbf{H}_y(\mathbf{r}, \omega)|}$$

(10)

Fig. 4 shows the ratio of eddy current density magnitudes as a function of frequency found from (10) for four different coil configurations. This ratio is always greater than one and does not exceed 23. A dip at lower frequencies is due to a rapid decrease of the relative dielectric constant in this band; it will be discussed separately in Section 8. The fifth coil configuration (which is configuration #1 with the coil rotated by 90 degrees) generated similar results. Therefore, it will not be discussed in the following text.
Fig. 3. Eddy current density at large separation distances from the coil at 30 and 300 kHz respectively, in the three observation planes (at 50 mm, –300 mm, and –700 mm) for coil configuration #1. Note the different color scales for each plane. The color scale for 300 kHz is exactly twenty times the color scale for 30 kHz.
VI. COMPARISON BETWEEN ANALYTICAL AND NUMERICAL RESULTS IN TIME DOMAIN

A. Coil current pulse form

A variety of different TMS pulse forms has recently been suggested [55],[57],[58]. We will model a simple monophasic (monopolar) TMS pulse. Its form is aimed to approximate some common experimental monophasic TMS coil current forms [55],[56]. A biphasic pulse or a pulse of a more complicated shape can be studied similarly, using the superposition principle.

![Graphs showing ratio of eddy current density magnitudes for different coil configurations.](image)

Fig. 4. Ratio of eddy current density magnitudes for different coil configurations. Different curves correspond to different observation points.
The present pulse form is characterized by two parameters: rise time $\tau$ and peak current $I_0$. The coil current pulse over time interval $0 \leq t < 10\tau$ is expressed in the form:

\[
f(t) = I_0 \begin{cases} 
\frac{4\tau(t-\tau)}{t^2} & \tau \leq t < 2\tau \\
\left(\frac{t-2\tau}{2\tau}\right)^2 & \tau \leq t < 10\tau, \\
e^{-\left(\frac{t-2\tau}{2\tau}\right)^2} & t \geq 2\tau
\end{cases}
\]

\[I(t) = 0 \text{ for } 0 \leq t < \tau \tag{11}\]

The derivative of the coil current pulse approximates eddy currents/electric fields induced in the body; it is given by

\[
df(t) / dt = I_0 \begin{cases} 
\frac{4\tau(2\tau-t)}{t^3} & \tau \leq t < 2\tau \\
\left(\frac{t-2\tau}{2\tau}\right)^2 & \tau \leq t < 10\tau, \\
\frac{2\tau-t}{2\tau^2}e^{-\left(\frac{t-2\tau}{2\tau}\right)^2} & t \geq 2\tau
\end{cases}
\]

\[I(t) = 0 \text{ for } 0 \leq t < \tau \tag{12}\]

The second pulse derivative is a continuous function of $t$ and is equal to $I_0 / 2\tau^2$ at $t = \tau$. Fig. 5a shows the coil current pulse normalized by $I_0/\tau$. The negative phase of the pulse derivative is approximately four times longer than its positive phase. Let $f(t_n), n = 0, \ldots, N-1$ be pulse values at $N$ sampling points $t_n = \Delta T n, n = 0, \ldots, N-1$ uniformly distributed over the time interval of interest from 0 to $10\tau$ so that $\Delta T = 10\tau/N$. After introducing the standard form of the discrete Fourier transform, implemented, for example, in the standard MATLAB package ($\text{fft}$ is an acronym for the "fast Fourier transform"),

\[F[m] = \text{fft}(f[n]) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi mn}{N}} f[n], m = 0, \ldots, N-1\]

\[f[n] = \Delta Tf'(t_n) \tag{13}\]

the energy spectral density $S_f$ of the current pulse (or of its derivative) is found as

\[S_f[m] = F[m]F^*[m], \quad m = 0, \ldots, N-1 \tag{14}\]
where the star denotes the complex conjugate. Fig. 5c shows the energy spectral density of the pulse derivative per 1 Hz normalized by \((I_0 / \tau)^2\). As expected, the spectral density peaks at about \(0.2f_0\) where \(f_c = 1/\tau\) is the characteristic pulse frequency. At the same time, the spectrum has a significant high-frequency content due to a discontinuity of the pulse derivative. Therefore, the corresponding frequency domain analysis should include all frequencies at least up to \(10f_c\) or so.

**B. Converting frequency-domain solution to time domain**

The transfer function given by (8) or (9) is applied to every harmonic component of the input coil current pulse \(f(t)\) separately. Those harmonics are described by the Fourier spectrum of this pulse, \(F(\omega)\). The Fourier spectrum of the eddy current density, \(F(r, \omega)\), is given at any point \(r\) in space by

\[
F(r, \omega) = H_{\omega, \omega}(r, \omega)F(\omega)
\]  

(15)

Fig. 5. (a) – Coil current pulse normalized by \(I_0\); (b) – pulse derivative normalized by \(I_0 / \tau\); (c) – energy spectral density of the pulse derivative normalized by \((I_0 / \tau)^2\) per 1 Hz.
The eddy current density itself, \( J_{N,A}(r,t) \), is found via the inverse Fourier transform. When moving toward inverse discrete Fourier transform, (15) becomes quite a nontrivial operation. Given the DFT in the form of (13), the discrete version of (15) must have the form (we omit the sub index for the transfer function)

\[
\begin{align*}
H[0]F[0], H[1]F[1],
\end{align*}
\]

since the standard DFT describes a set of data for the following non-monotonic frequency list:

\[
0, \omega_0, ..., \frac{N}{2} \omega_0, (1 - \frac{N}{2}) \omega_0, (2 - \frac{N}{2}) \omega_0, ..., -\omega_0.
\]

Here, \( \omega_0 = 2\pi/(N\Delta T) \) is the fundamental frequency.

The necessary frequency data in (16) has been extracted using linear interpolation (and sometimes extrapolation) of the transfer function \( H_N(r, \omega) \) previously computed over the band 300 Hz – 3 MHz. The next step is given by the ifft (iffit stands for the "inverse fast Fourier transform"):

\[
J_{N,A}(r, t_n) = \text{iffit}(H_{N,A}F)/\Delta T, \quad n = 0, ..., N - 1
\]

Note that the factor \( \Delta T \) may be omitted in both fft (13) and ifft (17). While all three components of \( J_{A}(r, t) \) are exactly synchronized in time according to (9), the three components of \( J_{N}(r, t) \) may be slightly offset since the phases of three components of \( H_N(r, \omega) \) are not necessarily the same. To avoid this issue, we have synchronized the two smaller pulse components with the largest one (slightly shifted them in time). This operation might slightly overestimate the resulting vector magnitude, which is in line with our upper-estimate task.

C. Comparison between analytical and numerical results

We select \( \tau = 0.1 \text{ ms} \) in (11), (12) which is the typical magnetic field rise time for monophasic TMS pulses [55],[56] shown in Fig. 5a,b. Fig. 7a and Fig. 7b show the simulated (smaller) pulse form and estimated (larger) pulse form for configuration #1 and configuration #4 respectively. The ratio of two peak values is also given. It can be seen that the numerically obtained pulse form is quite similar to the analytical result and to Fig. 5b. This is because the transfer function of the numerical solution in the frequency domain rather closely follows the derivative transfer function, \(-j\omega\), although some significant deviations have been observed at very low frequencies (below 5 kHz).
Table IVa reports numerically obtained peak eddy current densities for all coil configurations and all observation points in Fig. 2. The peak value of the coil current pulse in Fig. 5a is 1 kA and $\tau = 0.1 \text{ ms}$. Table IVb reports the ratio of two peak pulse values (analytical versus numerical) for the same 40 datasets, respectively. Along with the pulse rise time of 0.1 ms, we also present the result for a smaller value of 0.01 ms, which can be used as an excitation in TMS coils too [57],[58]. Remarkably, the ratio of analytical and numerical pulse peaks never becomes less than one. The average value of this ratio in Table IVa is 5.7.

Fig. 6. (a) Peak eddy current densities for all configurations and observation points for the bent coil at $\tau = 0.1 \text{ ms}$; (b) – Peak eddy current densities for all configurations and observation points for the straight coil at $\tau = 0.1 \text{ ms}$;

**D. Sensitivity analysis**

The numerical sensitivity analysis has been evaluated for coil configuration 1 at the observation points given in Table IIIa. Table IVc reports numerically obtained peak eddy current densities for all observation points in Fig. 2. Table IVd gives the similar results for the ratio of the analytical and numerical peak pulse amplitudes. Variations in permittivity have almost negligible impact on the final result, whereas conductivity variations are somewhat more important. Overall, all results stay in line with the previous observations and indicate that the peak ratio always exceeds one and does not approach it.

**VII. TESTING A DIFFERENT COIL GEOMETRY**

Next, a straight Figure-8 coil with a loop radius of 52.5 mm for each loop (Magstim D70² Coil [59]) will be studied. We do not model the presumably secondary effect of the stranded conductors
and replace all 11 coil windings by a solid conductor (copper) with a diameter of 8 mm. Further, we repeat the previous FEM simulations for all coil configurations and all observation points shown in Fig. 2. The corresponding simulation and comparison data is reported in Tables IVa and Vb, respectively, which are organized identical to Tables IVa and IVb. The peak eddy current densities for both the bent coil and the straight coil are also graphically represented in Fig. 6.

### Table IVa
**Peak eddy current densities (mA/m²) for all coil configurations and all observation points in Fig. 2 for the bent coil obtained numerically for \( \tau = 0.1 \text{ ms} \).**

<table>
<thead>
<tr>
<th>Peak current (mA/m²)</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
<th>Configuration 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>4.30</td>
<td>3.06</td>
<td>2.78</td>
<td>2.36</td>
</tr>
<tr>
<td>Point 2</td>
<td>0.914</td>
<td>1.22</td>
<td>1.21</td>
<td>0.916</td>
</tr>
<tr>
<td>Point 3</td>
<td>0.238</td>
<td>0.296</td>
<td>0.313</td>
<td>0.376</td>
</tr>
<tr>
<td>Point 4</td>
<td>0.176</td>
<td>0.209</td>
<td>0.201</td>
<td>0.178</td>
</tr>
<tr>
<td>Point 2a</td>
<td>0.601</td>
<td>0.835</td>
<td>0.736</td>
<td>0.874</td>
</tr>
<tr>
<td>Point 2b</td>
<td>0.320</td>
<td>0.396</td>
<td>0.424</td>
<td>0.955</td>
</tr>
<tr>
<td>Point 3a</td>
<td>0.285</td>
<td>0.352</td>
<td>0.364</td>
<td>0.648</td>
</tr>
<tr>
<td>Point 3b</td>
<td>0.197</td>
<td>0.248</td>
<td>0.250</td>
<td>0.487</td>
</tr>
<tr>
<td>Point 4a</td>
<td>0.251</td>
<td>0.265</td>
<td>0.305</td>
<td>0.128</td>
</tr>
<tr>
<td>Point 4b</td>
<td>0.196</td>
<td>0.243</td>
<td>0.264</td>
<td>0.152</td>
</tr>
</tbody>
</table>

### Table IVb
**Ratio of analytical and numerical peak pulse values (bent coil) for \( \tau = 0.1 \text{ ms} \) and \( \tau = 0.01 \text{ ms} \), respectively.**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>( \tau = 0.1 \text{ ms} )</th>
<th>( \tau = 0.01 \text{ ms} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Config.1 Config.2 Config.3 Config.4</td>
<td>Config.1 Config.2 Config.3 Config.4</td>
</tr>
<tr>
<td>Point 1</td>
<td>1.74  13.5  16.0  4.51</td>
<td>1.43  10.3  10.7  1.95</td>
</tr>
<tr>
<td>Point 2</td>
<td>4.36  5.00  4.97  5.35</td>
<td>2.93  3.36  3.38  3.85</td>
</tr>
<tr>
<td>Point 3</td>
<td>8.68  8.58  7.99  3.86</td>
<td>5.64  5.53  5.35  2.72</td>
</tr>
<tr>
<td>Point 4</td>
<td>7.12  6.49  6.63  3.38</td>
<td>5.55  5.12  5.24  2.61</td>
</tr>
<tr>
<td>Point 2a</td>
<td>6.84  6.91  7.81  5.29</td>
<td>3.89  4.11  4.65  3.15</td>
</tr>
<tr>
<td>Point 2b</td>
<td>12.2  14.5  13.2  4.76</td>
<td>7.41  8.99  8.03  3.01</td>
</tr>
<tr>
<td>Point 3a</td>
<td>7.14  6.59  7.09  2.21</td>
<td>4.82  4.43  4.76  1.62</td>
</tr>
<tr>
<td>Point 3b</td>
<td>10.3  10.5  9.07  2.92</td>
<td>6.49  6.64  5.66  1.95</td>
</tr>
<tr>
<td>Point 4a</td>
<td>4.95  4.68  4.62  4.68</td>
<td>3.53  3.32  3.27  3.36</td>
</tr>
<tr>
<td>Point 4b</td>
<td>6.34  5.86  4.62  3.95</td>
<td>4.44  4.1  3.23  2.83</td>
</tr>
</tbody>
</table>

### Table IVc
**Sensitivity analysis with variation of material properties of average body shell for the bent coil set up.**

**Peak eddy current densities (mA/m²) for configuration 1 at all observation points in Fig. 2 obtained numerically for \( \tau = 0.1 \text{ ms} \).**

<table>
<thead>
<tr>
<th>Peak current (mA/m²)</th>
<th>Configuration 1 (( \varepsilon ) factored by 1.2)</th>
<th>Configuration 1 (( \varepsilon ) factored by 0.8)</th>
<th>Configuration 1 (( \sigma ) factored by 1.2)</th>
<th>Configuration 1 (( \sigma ) factored by 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>4.276</td>
<td>4.319</td>
<td>4.093</td>
<td>4.552</td>
</tr>
<tr>
<td>Point 2</td>
<td>0.926</td>
<td>0.901</td>
<td>1.052</td>
<td>0.774</td>
</tr>
<tr>
<td>Point 3</td>
<td>0.242</td>
<td>0.234</td>
<td>0.281</td>
<td>0.194</td>
</tr>
<tr>
<td>Point 4</td>
<td>0.177</td>
<td>0.175</td>
<td>0.182</td>
<td>0.169</td>
</tr>
<tr>
<td>Point 2a</td>
<td>0.602</td>
<td>0.601</td>
<td>0.612</td>
<td>0.588</td>
</tr>
<tr>
<td>Point 2b</td>
<td>0.321</td>
<td>0.319</td>
<td>0.325</td>
<td>0.314</td>
</tr>
<tr>
<td>Point 3a</td>
<td>0.287</td>
<td>0.282</td>
<td>0.308</td>
<td>0.259</td>
</tr>
<tr>
<td>Point 3b</td>
<td>0.197</td>
<td>0.196</td>
<td>0.200</td>
<td>0.193</td>
</tr>
<tr>
<td>Point 4a</td>
<td>0.252</td>
<td>0.245</td>
<td>0.301</td>
<td>0.201</td>
</tr>
<tr>
<td>Point 4b</td>
<td>0.199</td>
<td>0.193</td>
<td>0.225</td>
<td>0.166</td>
</tr>
</tbody>
</table>
Fig. 7a. Estimated (large pulse form) and computed (smaller pulse form) eddy current density for configuration #1 of the bent coil. The ratio of two peak values is also given. Other coil positions generate similar results.
Fig. 7b. Estimated (large pulse form) and computed (smaller pulse form) eddy current density for configuration #4 of the bent coil. The ratio of two peak values is also given. Other coil positions generate similar results.
TABLE IVd

**RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES FOR τ = 0.1 ms AND τ = 0.01 ms, RESPECTIVELY FOR SENSITIVITY ANALYSIS CORRESPONDING TO TABLE IVc**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>(τ = 0.1) ms</th>
<th>(τ = 0.01) ms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Config.1 (ε factor 1.2)</td>
<td>Config.1 (ε factor 0.8)</td>
</tr>
<tr>
<td>Point 1</td>
<td>1.75</td>
<td>1.73</td>
</tr>
<tr>
<td>Point 2</td>
<td>4.30</td>
<td>4.42</td>
</tr>
<tr>
<td>Point 3</td>
<td>8.54</td>
<td>8.82</td>
</tr>
<tr>
<td>Point 4</td>
<td>7.08</td>
<td>7.15</td>
</tr>
<tr>
<td>Point 2a</td>
<td>6.47</td>
<td>6.49</td>
</tr>
<tr>
<td>Point 2b</td>
<td>12.2</td>
<td>12.2</td>
</tr>
<tr>
<td>Point 3a</td>
<td>7.08</td>
<td>7.20</td>
</tr>
<tr>
<td>Point 3b</td>
<td>10.3</td>
<td>10.4</td>
</tr>
<tr>
<td>Point 4a</td>
<td>4.92</td>
<td>5.06</td>
</tr>
<tr>
<td>Point 4b</td>
<td>6.25</td>
<td>6.43</td>
</tr>
</tbody>
</table>

TABLE VA

**PEAK EDDY CURRENT DENSITIES (mA/m²) FOR ALL COIL CONFIGURATIONS AND ALL OBSERVATION POINTS IN FIG. 2 FOR THE STRAIGHT COIL OBTAINED NUMERICALLY FOR \(τ = 0.1\) ms.**

<table>
<thead>
<tr>
<th>Peak current (mA/m²)</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
<th>Configuration 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>21.6</td>
<td>18.2</td>
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<td>3.87</td>
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<td>2.81</td>
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<td>0.24</td>
<td>0.72</td>
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<td>Point 4b</td>
<td>0.50</td>
<td>0.23</td>
<td>0.13</td>
<td>0.88</td>
</tr>
</tbody>
</table>

TABLE VB

**RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES FOR (STRAIGHT COIL) \(τ = 0.1\) ms AND \(τ = 0.01\) ms, RESPECTIVELY.**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>(τ = 0.1) ms</th>
<th>(τ = 0.01) ms</th>
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<td></td>
<td>Config.1</td>
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<tr>
<td>Point 2</td>
<td>3.12</td>
<td>1.73</td>
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<td>Point 3</td>
<td>5.18</td>
<td>5.27</td>
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<td>Point 4</td>
<td>3.37</td>
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<td>Point 2a</td>
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<td>Point 2b</td>
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</tr>
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<td>Point 3a</td>
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</tr>
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<td>Point 3b</td>
<td>5.78</td>
<td>2.87</td>
</tr>
<tr>
<td>Point 4a</td>
<td>2.42</td>
<td>10.1</td>
</tr>
<tr>
<td>Point 4b</td>
<td>2.78</td>
<td>4.67</td>
</tr>
</tbody>
</table>

VIII. INTERPRETATION OF RESULTS

A. Why is the analytical model working?

Routine FEM simulations indicate that the analytical model given by (5) could very significantly overestimate the eddy currents in a bounded conductor with the relative dielectric constant of 1. As an example, we consider here observation point 1 for coil configuration #1 in Fig. 2, assuming a “homogenized” VHP-Female v.2.1 phantom with constant parameters \(σ = 0.5\) S/m, \(ε_r = 1\). The ratio of eddy current magnitudes for the analytical and numerical solutions in the frequency
domain is given in Fig. 8a by an upper curve. The analytical eddy current now exceeds the computed FEM current density by a factor between 30 and 270 in the entire frequency domain of interest. Next, we still assume the homogenous phantom but assign to its volume the frequency-dependent muscle properties (which are often considered as the “average body” properties) following references [46],[47]. The ratio of the eddy current magnitude decreases drastically (but still exceeds one) as shown by the second lower curve in Fig. 8a. The reason for such a considerably better agreement is a very large muscle permittivity $\varepsilon_r$ at lower and intermediate frequencies as shown in Fig. 8b. Other tissues possess a similar frequency behavior. The large permittivity values imply large displacement currents and, consequently, large polarization (or bound) charges at the dielectric-dielectric interfaces. These charges have the opposite polarity as compared to the free charges due to conduction currents. Hence, the two charge types essentially cancel each other so that the dielectric-dielectric (and conductor-conductor) interface becomes essentially neutral, which is exactly the free-space condition used by the analytical model.

![Fig. 8. (a) – Ratio of eddy current magnitudes (analytical versus numerical solution) in a “homogenized” VHP-Female phantom at different conditions and (b), (c) – electromagnetic muscle properties at low and moderate frequencies.](image)

Interestingly, a similar situation occurs in the antenna design field for metal patch antennas printed on high-permittivity dielectric substrates. Thus, the analytical model given by (5) provides a reasonable upper estimate of eddy currents thanks to its simplicity: the model neglects both free charges and polarization charges simultaneously. An attempt to improve the model by the inclusion of only free charges would probably fail. A further step toward an even better agreement compared to the second curve in Fig. 8a is probably facilitated by multiple irregular interfaces within a realistic human body.
B. Exceptions

While Table IVb always reports the ratios of the peak pulse values greater than one, Table Vb for the straight coil indicates two special situations where numerical peak pulse values exceed the corresponding analytical results. A detailed analysis has shown that these special locations happen to be close to a locus of the magnetic vector potential given by (5), i.e. to a curve (or a surface) where the corresponding line integral vanishes, as does the eddy current density \( \mathbf{J} \). For example, the locus of a loop of current coincides with its axis. Fig. 9a shows the magnitude of the line integral in (5) for the bent coil tested in Section 6 using a color scale.

![Fig. 9a](image)

Fig. 9a. Locus of the magnetic vector potential for (a) – the bent Figure-8 coil with the radius of 52.5 mm (bending angle is 15 degrees); (b) – the straight Figure-8 coil with the same radius.

The coil radius is now 52.5 mm. The dark (blue) color corresponds to its zero values. The locus is a curve which does not penetrate into the body deeper than two coil diameters. Therefore, Table IVb reports the meaningful results. Fig. 9b, on the other hand, shows the absolute values of the line integral in (5) for the straight coil tested in Section 7.

The two loci seen in this figure may penetrate the entire body and hit an observation point anywhere in the body when the coil is rotated around the head. This is what happens with
observation points 2a and 2b in Table Vb for configurations #2 and #3, respectively. The numerical solution predicts a significant eddy current, but the analytical result does not.

IX. GUARANTEED UPPER ESTIMATE

A. Guaranteed upper estimate

In order to eliminate the loci effect for any coil type, it is suggested to find the absolute maximum of the line integral magnitude over a sphere surface, which is centered at the geometrical center of the coil. The sphere radius is the distance to the observation point. Fig. 10 shows the observation points over the sphere surface, centered at the geometrical center of the coil. Even with a few thousand test points on the sphere surface, the corresponding numerical task requires on the order of 1 s of CPU time (with a vectorized MATLAB script [53]). This maximum is then substituted in (5) instead of the local integral value and the estimate is performed. Table VI given below is a replica of Table IVb for the bent coil obtained using this method, and so is Table VII, which is a replica of Table Vb for the straight coil. The method clearly overestimates the peak pulse value close to the coil (observation point 1), but otherwise it works reasonably well.

Using this method, no situation for ten base points has been found where the analytical model underestimates the induced eddy current density, either in the frequency domain or in the time domain. The average value of the ratio of analytical and numerical peak eddy currents in the time domain is approximately 10 if we exclude observation point 1 located within the cranium.

B. Results for full-body coverage

For coil configuration 1, we evaluated 102,189 extra observation nodes located within the body as described in Section IVB. We used the guaranteed upper-estimate method described above. For 134 nodes (0.13%), the ratio of analytical and numerical peak pulse values went below one. For these noncompliant nodes, the minimum peak ratio was 0.465 and the mean peak ratio was 0.817. For the remaining set of compliant nodes, the minimum peak ratio was 1.0031, the maximum peak ratio was 5210 and the mean peak ratio was 26.5407.

We explain these results by a pure numerical error close to sharp edges/corners present in the model, especially in the spinal cord and at the interface of CSF and grey matter where the conductivity is the largest. Fig. 11 illustrates one such point denoted by a red circle present in the spinal cord close to vertebra L3.
Fig. 10. Observation sphere around the coil center for the guaranteed upper estimate.

For sharper edges and large adjacent triangles, this local (electrostatic) effect becomes quite significant and leads to non-physical fields/current peaks.

On the other hand, the maximum peak values occur very close to the interface between skin and air where the computed value of the current in air is not exactly zero due to a numerical smoothing effect.

Fig. 11. Presence of a sharp edge and corresponding noncompliant point nearby in the spinal cord shown by a red circle.
X. CONCLUSIONS

In order to validate the analytical estimate for TMS eddy currents given by (5) we have performed numerically-accurate FEM simulations with one human phantom, two coil types, five coil positions, ten observation points, and two distinct pulse rise times, thus providing two hundred different data sets for comparison. In addition to that, we have processed the results for about 100,000 observation points of a rectangular grid containing the entire body. We have also generated about a hundred other datasets with an earlier version of the phantom using somewhat different coil positions. Our simulations reveal that in 98% of the cases, the local analytical model does overestimate the peak pulse eddy current density. However, in the remaining 2% of the cases the analytical model underestimates the peak pulse eddy current density. The reason is the loci of the analytical solution discussed above.

In order to obtain the guaranteed upper estimate in every case, we have to modify (5) using the absolute maximum of the line integral magnitude over a sphere surface centered at the geometrical center of the coil with the radius $R$ equal to the distance to the point of interest. The maximum value is to be used instead of the local value. This method neglects geometrical coil features, but still takes into account a general $1/R^3$ decay of the magnetic field from a local current source in the near-field region. It appears that this method overestimates the peak eddy currents at distant locations from a coil by a factor of 10 on average. The simple analytical model explained and tested in the present study may be valuable as a rapid method to safely estimate levels of TMS currents at different locations within the human body.

TABLE VI
RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES FOR THE BENT COIL AT $\tau = 0.1$ ms AND $\tau = 0.01$ ms, RESPECTIVELY.

<table>
<thead>
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<th>$\tau = 0.1$ ms</th>
<th>$\tau = 0.01$ ms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Config.1</td>
<td>Config.2</td>
</tr>
<tr>
<td>Point 1</td>
<td>56.7</td>
<td>91.5</td>
</tr>
<tr>
<td>Point 2</td>
<td>6.56</td>
<td>7.41</td>
</tr>
<tr>
<td>Point 3</td>
<td>11.5</td>
<td>13.4</td>
</tr>
<tr>
<td>Point 4</td>
<td>8.81</td>
<td>10.6</td>
</tr>
<tr>
<td>Point 2a</td>
<td>9.63</td>
<td>11.2</td>
</tr>
<tr>
<td>Point 2b</td>
<td>18.1</td>
<td>20.4</td>
</tr>
<tr>
<td>Point 3a</td>
<td>9.44</td>
<td>11.4</td>
</tr>
<tr>
<td>Point 3b</td>
<td>13.7</td>
<td>12.0</td>
</tr>
<tr>
<td>Point 4a</td>
<td>6.14</td>
<td>8.41</td>
</tr>
<tr>
<td>Point 4b</td>
<td>7.85</td>
<td>8.83</td>
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</table>
TABLE VII
RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES FOR THE STRAIGHT COIL AT
\( \tau = 0.1 \text{ ms} \) AND \( \tau = 0.01 \text{ ms} \), RESPECTIVELY.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>( \tau = 0.1 \text{ ms} )</th>
<th>( \tau = 0.01 \text{ ms} )</th>
</tr>
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<tbody>
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<td></td>
<td>Config.1</td>
<td>Config.2</td>
</tr>
<tr>
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<td>536</td>
</tr>
<tr>
<td>Point 2</td>
<td>6.74</td>
<td>5.78</td>
</tr>
<tr>
<td>Point 3</td>
<td>10.7</td>
<td>12.6</td>
</tr>
<tr>
<td>Point 4</td>
<td>6.88</td>
<td>17.3</td>
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<td>Point 2a</td>
<td>10.3</td>
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<td>Point 2b</td>
<td>17.8</td>
<td>10.7</td>
</tr>
<tr>
<td>Point 3a</td>
<td>9.21</td>
<td>13.4</td>
</tr>
<tr>
<td>Point 3b</td>
<td>12.5</td>
<td>12.4</td>
</tr>
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<td>Point 4a</td>
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</tr>
<tr>
<td>Point 4b</td>
<td>5.82</td>
<td>14.1</td>
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</table>

REFERENCES


SECTION B –

COMPARISON OF CEPHALIC AND EXTRACEPHALIC MONTAGES FOR TRANSCRANIAL DIRECT CURRENT STIMULATION – A NUMERICAL STUDY

I. INTRODUCTION

Transcranial Direct Current Stimulation (tDCS) has been used for the treatment of various neurological and psychiatric disorders, including depression, anxiety, and Parkinson’s disease [1], [2]. Studies have shown that patients undergoing the procedure experience positive behavioral modifications with minimal negative effects that may include skin irritation, mild redness and itching under the electrode, headache, nausea, dizziness or a slight tingling sensation. Additionally, generation of toxins induced by an electrochemical reaction at the electrode-tissue interface are possible and application of tDCS above skull defects or inadequate electrode contact may produce focused current flow that has the potential to cause damage to skin and nerve tissue. Use of tDCS remains a very active area of research with the potential to non-invasively treat many of humanity’s long standing disorders.

A number of electrode configurations, known as montages, are in use to control the application of current and concentrate the current density onto a particular area of the brain. These montages have been traditionally constructed based on knowledge of human anatomy and physiology. For example, if stimulation of the visual cortex is desired, an anode and cathode would be placed at the rear center and top center of the head, respectively. In this way, a particular area of the brain is identified for stimulation via tDCS and a montage that activates that area in a targeted manner is selected. One would assume that a large portion of the current leaving the anode would pass through the visual cortex while traveling to the cathode. In-vivo measurements have been reported [3] in the brain of a monkey and were used as the basis of constructing a model of the head that may be used to predict current flow in the brain from surface electrodes. However, real-time measurements and evaluation of individual anatomy remain challenges. To this end, extensive application of modeling and simulation techniques, including the Finite Element Method (FEM) have been used to characterize and understand the impacts of electrode arrangements on the human form, along with other factors including electrode size, the number of anode and cathode locations and current density [4-8]. The FEM has become such an important tool in the realm of tDCS that proposed general and customized, patient specific, and experimental tDCS protocols are examined and optimized using computational tools [9]. The models themselves have even acquired specific terminology and are known as ‘forward models’ with particular procedures on construction and usage [10]. Clearly, the level of effort demonstrated in model construction and the prevalence of research based on modeling and simulation techniques indicates that conscientious use of FEM and other numerical method based solvers coupled together with anatomically accurate and predictive forward models [11] represents a realistic and efficient means that provides scientists, engineers and medical personnel detailed information on the performance of tDCS hardware in the very complex and multi-variant human body environment. Studies that take into account the effects of anisotropic conductivity in human tissues, including the skull and white matter, have
demonstrated the importance of considering this physical condition when using forward models [12-13].

Despite existing studies and techniques that seek to manipulate the locality and depth of the stimulated area [14-17], open questions remain on the role of the cathode in terms of placement on the body. Field localization is strongly desired to provide tDCS practitioners the ability to treat certain disorders through precise targeting of specific areas or structures of the brain. Extracephalic locations (e.g., neck) of the cathode have been examined [18], along with the efficacy of a fronto-extracephalic montage in treatment of depression [19]. The impact of extracephalic montages on the brain stem and associated organs and tissues remains a concern [20], though the influence of these montages on cardio-vascular and autonomous functionality has been discussed in [21].

Fig. 1. Estimate for separate and distinct components of computational model employed for all tDCS simulations, each with individual material properties, including: i) – Artificial skin shell (2 mm thick); ii) – fat layer shell; iii) – muscle volume; iv) bones (left and right acromion, left and right humerus, jaw, ribs, left and right scapula, skull, spine), v) – closed CSF shell; vi) – cerebral cortex (grey matter); vii) – white matter; viii) – cerebellum; ix) – CSF ventricles; x) – eyes and tongue (separate tissues); xi) - sinus cavity, lungs and trachea; xii) – aorta and cava superior.
II. PROBLEM STATEMENT AND ORGANIZATION

The focus of this work is the simulation of the volumetric current density in the brain with electrodes configured in a cephalic (C3-Fp2) versus extracephalic manner with evaluations of cathode placement on both ipsi- and contralateral shoulder locations.

The FEM phantom used for this purpose is shown in Fig. 1 and thoroughly described in Section III. The anatomical brain segmentation included grey matter, white matter, ventricles, and cerebellum – see Fig. 1. The numerical simulation setup is described in Section IV. Electrode constitution and assembly is reported in Section V.

The cerebral cortex has been numerically defined as the Boolean difference between the grey matter and the white matter meshes. The cerebral cortex has further been artificially subdivided into individual lobes and cortices as shown in Fig. 2.

Section VI provides qualitative and quantitative results for the local current density magnitude within the brain volume. We visualize the total current density by plotting current on the surfaces of both the cerebral cortex and white matter. Alternatively, we visualize the total current density using a series of cut planes, each of which is accompanied by the corresponding cut plane atlas. Section VI also discusses the invariance of the relative current density magnitudes to significant changes in skin properties. We considered two extreme cases of wet and dry skin, respectively. Along with its primary goal, this study indirectly addresses the effect of changes in the contact between electrode and skin throughout the course of an extended tDCS treatment.

Section VII reports quantitative results for both the average vertical current density magnitude and the average horizontal current density magnitude in every individual lobe and/or cortex shown in Fig. 2 along with the global coordinate system employed. The corresponding current densities are defined by

\[
J_{z}^{\text{avg}} = \frac{1}{V} \int |J_z(r)| dr,
\]

\[
J_{xy}^{\text{avg}} = \frac{1}{V} \int \sqrt{J_x^2(r) + J_y^2(r)} dr
\]

where \(V\) is the volume of the tissue under study and \(J(r)\) is the spatial vector current density in this volume.

Section VIII estimates the expected electrode voltages and quantifies voltage responses due changes in properties of individual anatomical tissues.

Section IX also compares the results of the present study with the previous numerical simulations.

Finally, Section X concludes the study.
Fig. 2. Separation of the cerebral cortex into computational subregions including the frontal lobe, the occipital lobe, the parietal lobe, the primary motor cortex, the somatosensory cortex, and the temporal lobe.
III. VHP-F MODEL AND SIMULATION DESCRIPTION

Construction of the forward model used in this study was enabled by the processes of medical image data acquisition, manual segmentation, mesh conditioning, and model registration. All surfaces describing a particular geometry must be 2D manifold and possess a sufficiently high triangle quality, as element quality has been proven to be vital to the accuracy of the simulation [22].

A. Data Acquisition

The model utilized in this study was constructed using anatomical cryosection images of the axial plane provided by the Visible Human Project® (VHP) established in 1989 by the U.S. National Library of Medicine (NLM) [23]. Male and female data sets became available in November of 1994 and 1995, respectively. Each consisted of MRI, CT and cryosection images taken predominantly in the axial plane of the bodies of various resolutions. Anatomical cryosection image data from the female patient, consisting of 2048 by 1216 pixels with each pixel measuring 0.33mm per side, was used exclusively in the construction of the model for the present study, producing the VHP-F nomenclature. The original VHP-NLM model resolution in the axial plane is limited by image pixel density, and if we assume the segmentation is legitimate and accurate, produces a resolution value of 0.33mm by 0.33mm. Since every third image in the dataset was utilized, resolution along the vertical axis of the body is limited to 0.99mm. A voxel produced by images used in this manner would have x-, y-, and z-dimensions of 0.33mm by 0.33mm by 0.99mm.

B. Segmentation

Image segmentation is an area of active research with many dynamic and varying methodologies [24-33]. Despite this diversity in implementation, no one singular technique has proven to be suitable in all applications or as accurate as manual segmentation by a human operator. Though extremely time consuming, it is for this reason that manual segmentation was employed almost exclusively for the development of the models used in this study.

One of the major tools included in the development of the VHP was the open source program Insight Toolkit (ITK), which enables the analysis of three dimensional image stacks and simultaneous segmentation of images in the axial, coronal, and sagittal body planes via manual and automatic methods [34]. Image stacks are read into ITK for segmentation and the user may manually trace organs, tissues and other structures, thus isolating these regions from other image areas. The end result is a stereolithography (STL) file describing the surface of the segmented region as a series of dense triangular elements (surface Delaunay triangulation) defined by a node point cloud.

C. Mesh Conditioning and Registration

The results of the segmentation process are very fine and dense meshes that contain a large number of nodes. Typical numbers of nodes are on the order of between $10^6$ and $10^{10}$. These meshes, while accurate with respect to the image dataset, would be too unwieldy to be used in an FEM simulation. Mesh conditioning is required to reduce the number of nodes to a
computationally reasonable number and ‘clean’ the mesh, eliminating defects and discrepancies that could reduce the accuracy and utility of the model.

Much of the mesh conditioning process has been accomplished via MeshLab [35]. Example operations include selective reduction of the number of nodes via Quadric Edge Collapse Decimation [36] which was accomplished to remove elements in areas of relatively coarse features (e.g., top of the chest) while retaining elements in regions that require finer detail. Relative triangle sizes in those areas may vary by as much as 1:5 while keeping a reasonably high triangle quality everywhere. Smoothing functions were achieved using HC Laplacian Smoothing [37] in order to retain the original shape volume. Additional defects, including redundant nodes and edges, non-manifold edges, and intersecting faces can be eliminated via Delaunay triangulation [38]. In certain cases, rebuilding the mesh by Poisson Surface Reconstruction [39] has proven to eliminate element folding and produce smooth and contiguous surfaces suitable for simulation purposes.

<table>
<thead>
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<tr>
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<td>Blood</td>
<td>0.7</td>
</tr>
<tr>
<td>Superior Vena Cava</td>
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<td></td>
</tr>
<tr>
<td>Acromion</td>
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<tr>
<td>Humerus</td>
<td></td>
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<tr>
<td>Jaw</td>
<td></td>
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</tr>
<tr>
<td>Rib</td>
<td></td>
<td></td>
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<tr>
<td>Scapula</td>
<td>Homogeneous Bone</td>
<td>0.0756/0.02</td>
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<tr>
<td>Skull</td>
<td>(a combination of</td>
<td></td>
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<tr>
<td>Spine</td>
<td>cancellous/cortical types)</td>
<td></td>
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<td>0.0275</td>
</tr>
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<td>White Matter</td>
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<td>Cerebellum</td>
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<tr>
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<tr>
<td>Skin Layer</td>
<td>Tongue</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Following the segmentation and conditioning processes, all individual components of the VHP-F model were registered to ensure proper position, size and shape. Registration was accomplished by overlaying the digitized structures over the original cryosection images and any required adjustments were made on a node by node or element by element basis. In this way, any distortions, rotations or imperfections created by the operations mentioned above were addressed.
IV. SIMULATION SETUP

The results of the mesh generation process described above can be seen in Figs. 1-2. Each structure was converted to the NASTRAN file format to facilitate importation into commercial numerical solvers. Final assembly of all components that make up the model required verification that no structures were overlapping or intersecting. Additionally, each structure required assignment of appropriate material properties.

A. Material Properties

Electromagnetic modeling of the human body requires meticulous and cautious definition of the associated material properties resident within the simulation. A wealth of research on the subject is available [40-45] demonstrating the variability of values across multiple types of tissues and a high dependence on frequency. For low frequency and static simulations such as the ones described in this work, material conductivity is paramount. A summary of tissue types and conductivity values is given in Table I [45].

B. FEM Software and Numerical Accuracy

Static electromagnetic simulations were conducted using ANSYS’ Maxwell 3D version 16 product. This software numerically obtains a unique solution to Maxwell’s equations at DC via the FEM and user specified boundary conditions. The Maxwell product has extensive mesh analysis and healing capabilities. Most important, solution convergence and the ultimate accuracy is controlled through a rigorous adaptive mesh refinement procedure. For the results presented below, five iterations of adaptive mesh refinement were employed, each with a refinement level of 30% per pass. This process grew the total number of tetrahedral elements from approximately 200,000 to over 600,000 with total runtimes on the order of about six hours on a server with 192 Gbytes of RAM. When solving a DC current conduction problem, the Degrees of Freedom (DoF) are the electric scalar potentials at each node of the tetrahedral mesh. Typical values of mesh size per iteration and energy loss are given in Table II and demonstrate a reduction of the residual error through successive refinement steps, increasing the accuracy of the calculation as it converges.

C. Boundary Conditions and Excitations

The default boundary conditions used by ANSYS MAXWELL 3D during DC conduction type simulations are as follows. Standard (or “natural”) boundaries are enforced at inner object interfaces and ensure the continuity of the normal component of the direct electric current density through the interfaces. Homogeneous Neumann boundaries imposed on all outer boundaries do not allow the normal electric current to pass. The electric field within the conductor is indeed tangential on this outer boundary. We have found that MAXWELL 3D does not implement an ideal hypothetical current source with a fixed current density and a variable voltage across the electrode surface. Instead, a more realistic voltage source model with Dirichlet boundary conditions of a fixed surface electrode voltage is internally used for both voltage and current excitations. After completing the simulations, the required total current may be related to voltage. A user can define the total current a priori, which is the current source implementation. The well-known current singularity at the voltage electrode edges is eliminated via matching sponges – see the next Section.
TABLE II
CONVERGENCE OF A TYPICAL CASE BASED ON TETRAHEDRAL MESH SIZE

<table>
<thead>
<tr>
<th>Adaptive Pass</th>
<th>Number of Tetrahedra</th>
<th>Total Loss (mW)</th>
<th>Loss Error (%)</th>
<th>Delta Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>183,113</td>
<td>0.6</td>
<td>1.10</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>248,020</td>
<td>0.607</td>
<td>0.33</td>
<td>1.17</td>
</tr>
<tr>
<td>3</td>
<td>335,946</td>
<td>0.60915</td>
<td>0.19</td>
<td>0.35</td>
</tr>
<tr>
<td>4</td>
<td>455,431</td>
<td>0.61047</td>
<td>0.13</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>616,856</td>
<td>0.61137</td>
<td>0.09</td>
<td>0.15</td>
</tr>
</tbody>
</table>

V. ELECTRODES AND THEIR MONTAGES

A. Electrode Model

Electrodes were simulated using rectangular blocks of material (sponges) with the conductivity of saline solution (2 S/m). Electrodes were sized consistent with existing procedures and protocols [1] such that the rectangular surface in conformal contact with the skin was 5 cm on a size with a total surface area of 25 cm². Each electrode was constructed in a virtual environment by subtracting larger geometric blocks that intersected the surface of the VHP model with the skin of the model. In this way, the contacting surface of the electrode was made conformal such that, even in areas of high curvature on the model, full contact was maintained and no space existed between the electrode and the skin. A total electrode current of 2 mA (with the equivalent uniform density of 0.08mA/cm² [1]) was employed as the source in all cases described below.

B. Electrode Montages

Electrodes were arranged in three different configurations: following the Modified Combinatorial Nomenclature (MCN) of the International EEG 10-20 system, we simulate the C3-Fp2 montage, which has been traditionally used to stimulate the primary motor cortex (M1) by placing the stimulation electrode at the top-left portion of the head and the reference electrode at the contralateral supraorbital position; an extracephalic contralateral shoulder montage which retains the original excitation electrode position but shifts the reference electrode to the opposite side shoulder; and extracephalic ipsilateral shoulder montage which also retains the original excitation electrode position but shifts the reference electrode to the shoulder on the same side of the body as the excitation. Electrode positions and a cut plane atlas may be viewed in the upper 2 rows of Figs. 4-8. The cut plane atlases show that the layer of muscle around the head is essentially non-existent (see Figs. 4-6) and only contributes to the model at regions midway and below the skull – see Figs. 7 and 8. Skin and fat layers mostly contribute to conduction around the head.
VI. SIMULATION RESULTS FOR TOTAL CURRENT DENSITY

A. Total Current Density for Surfaces of Cerebral Cortex and White Matter

The total current density magnitude is plotted on the surfaces of the cerebral cortex and white matter in rows 2 and 3 of Fig. 3, respectively. The traditional cephalic montage in column 1 demonstrates a significant amount of current at the anterior of the grey matter as it passes to the supraorbitally located cathode. The surface plots of the extracephalic configurations exhibit much lower current densities in this region of the brain with higher concentrations toward the posterior.

When stimulated with an extracephalic configuration, the current on the surface of the white matter appears to be slightly more concentrated directly below the anode, suggesting a relatively deeper penetration with less current passing through the anterior of the frontal lobe.

B. Total Current Density for Sagittal Cut Planes

A series of plots portraying the current densities experienced with each montage are presented in Figs. 4-8 and all figures are displayed with the same scale for comparison purposes. Figs. 4-6 (similar to Fig. 3) are divided into columns i – iii which depict current density results from the traditional, contralateral and ipsilateral shoulder montages projected onto a sequence of sagittal dissecting planes that progressively shift from the anode through the head and towards the right side of the model. In all cases, the shunting nature of the high conductivity CSF is quite apparent as high proportions of the total current are seen passing through this layer surrounding the brain. This characteristic is plainly seen on the third rows of Figs. 4-6, which provide images of the brain that include the surrounding structures. The high amount of current shown in the third row of Fig. 5 is due to the presence of the CSF ventricles at the center of the head.

A sagittal plane passing through the anode is depicted in Fig. 4. Relatively higher proportions of the current are observed in the primary motor cortex of the extracephalic cases. Additionally, the depth of stimulation appears relatively greater in both extracephalic configurations.

A second sagittal plane located at the approximate midpoint between the anode and the contralateral supraorbital cathode position is shown in Fig. 5. The current levels within the brain depicted in this plane provide evidence that the higher levels of current are present in the frontal lobe in the cephalic arrangement. Both extracephalic arrangements again indicate a slightly deeper level of stimulation. Current density values in Fig. 5 indicate that there is some minor stimulation of the brain stem when using extracephalic anode locations versus essentially no stimulation when using the traditional montage with the ipsilateral shoulder arrangement performing marginally better than the contralateral design. While current is present in the brain stem, the values are low and approximately 5 times less than current values in the area of desired stimulation.
Fig. 3. Surface plots of the total current density on the cerebral cortex (row 2) and white matter (row 3). Total current density normalized by the input current density at the electrodes is shown. Column 1 provides results for the cephalic configuration while columns 2 and 3 display the contra-lateral and ipsi-lateral extracephalic results, respectively, using a logarithmic scale.
A final sagittal plane passing through the contralateral supraorbital cathode position is presented in Fig. 6. Again, the cephalic configuration depicts a relatively larger percentage of the current passing through the prefrontal lobe as it moves toward the cathode. This would suggest stimulation of this region of the brain, which has been associated with planning and consciousness rather than body movement and coordination.

Special consideration should be given to the extracephalic configurations shown in Fig. 4-5. The depth of stimulation when using extracephalic cathode locations is visibly greater than that of the traditional cephalic arrangement. This would indicate that a greater percentage of the brain volume would be covered through extracephalic means.

This last observation about stimulation depth raises an interesting question of why electrodes placed in close proximity to what is essentially a highly conductive sphere of CSF encompassing the brain perform less efficiently than electrodes placed at farther locations.

C. Total Current Density for Axial Cut Planes

Fig. 7 depicts an axial plane located approximately midway through the brain and intersects with the supraorbital cathode location. Results displayed in this plane indicate relatively deep stimulation regions for both extracephalic designs as compared with the traditional montage. Conversely, the traditional electrode configuration shows a much higher level of current flowing into the cathode.

D. Total Current Density for 45 Degree Cut Planes

A diagonal cut-plane traversing the space between the anode at the top of the head and the traditional location for the cathode is shown in Fig. 8. A cephalic cathode configuration seems to show a relative shift in current density from the motor cortex to the frontal lobe. Virtually no stimulation beyond the parietal lobe is seen and while extracephalic configurations seem to somewhat better target the motor cortex, some stimulation of the rear of the brain is evident. This behavior is consistent in previous figures.

E. Invariance of Relative Current Densities to Changes in Skin Properties

Relatively higher total current densities in the brain for extracephalic montages may be observed in Figs. 3-8. And yet, one potentially critical configuration would correspond to a very highly-conducting skin layer so that the bulk of current might be expected to flow closer to the surface, irrespective of the particular electrode montage (cephalic or extracephalic). As a test case, we consider here a hypothetic isotropic skin layer with the extreme conductivity of 0.25 S/m (wet epidermis) compared with dry skin from Table I in Fig. 9, which would model the electrode/skin interface as the electrode dries during an extended tDCS treatment session. Despite the expected overall decrease of the absolute current density in the brain, the relative patterns of current density distribution remain approximately the same for all three tested montages shown in Fig. 9! We think that these results may be directly extrapolated to the anisotropic case. Another justification of this result will be discussed further with reference to Fig. 10.
VII. QUANTITATIVE EVALUATION OF VERTICAL AND HORIZONTAL AVERAGE CURRENT DENSITIES

Results presented thus far rely upon a visual inspection of the total current density distribution in the different observation planes or on surfaces. It may be useful to separate the total current density into two partial components (vertical and horizontal), and then find the average values of these components, $J_{z}^{avg}, J_{xy}^{avg}$ for every lobe/cortex following Eq. (1). Fig. 10 reports numerically found average vertical and horizontal components of current densities in the brain including its individual subregions defined in Fig. 2. It can be seen from Fig. 10a that the vertical component of current density in every subregion increases when extracephalic montages (the results for both of them are nearly identical and cannot be distinguished in the figure) are used. However, the horizontal component of current density in Fig. 10b either decreases or remains nearly the same compared to the cephalic case. This may be instrumental when stimulating brain regions with cell structures that are biased in either the vertical or horizontal directions. The results for extremely wet skin shown in Fig. 10c, d, respectively, follow a similar tendency although the vertical current components in the primary motor cortex and in the somatosensory cortex become close to each other for both competing montages. Note that the cephalic montage is much less sensitive to variations in skin properties.

VIII. EXPECTED ELECTRODE VOLTAGES AND THEIR VARIATIONS

Cephalic and extracephalic configurations require different electrode voltages for the same amount of current. This section provides the corresponding estimates including voltage variation margins. To investigate this and related problems we introduce the voltage response of a tissue/organ to small changes in tissue conductivity. Given the independent current source $I_{in}$ as an excitation, the dimensionless (dynamic or small-signal) voltage response may be defined as

$$S = -\frac{dV/V_0}{d\sigma/\sigma_0} > 0$$

(2)

where $d\sigma$ is the tissue conductivity variation and $dV$ is the corresponding electrode voltage variation about the unperturbed state $V_0$, $\sigma_0$. If a particular tissue carries a significant current, its corresponding voltage response should be large. This fact follows from the local form of Ohm’s law. Table III summarizes unperturbed electrode voltages and voltage responses of individual tissues for the three electrode configurations. The C3-Fp2 montage possesses a very small voltage response for the muscle tissue since there is virtually no such tissue within the main current path (only the skin, and the fat are two layers around the skull). In this analysis, we consider the brain as one entity. Based on the results of Table III for individual voltage responses, we estimate the electrode voltages and their extremes given maximum ±20% conductivity variations for various montages as: (i) cephalic: ~300 mV±60 mV; (ii) extracephalic ipsilateral: ~720 mV±120 mV; (iii) extracephalic contralateral: ~710 mV±120 mV.
TABLE III
ELECTRODE VOLTAGES AND DIMENSIONLESS VOLTAGE RESPONSES. ALL
DIMENSIONAL VOLTAGE DATA ARE RELATED TO A CURRENT SOURCE WITH
THE ELECTRODE NORMAL CURRENT DENSITY OF $J_n = 0.08 \text{ mA/cm}^2$ AND
THE TOTAL SOURCE CURRENT OF $I_s = 25 \text{ cm}^2 \times J_n = 2 \text{ mA}$

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Skin</th>
<th>Fat</th>
<th>Muscle</th>
<th>Skull</th>
<th>CSF (GM, WM, Cer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductivity</td>
<td>0.0002</td>
<td>0.038</td>
<td>0.2</td>
<td>0.076</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.048</td>
</tr>
</tbody>
</table>

C3-Fp2 Montage

| $V_0$, mV | 19.334 |
| $V$ for 20% cond. change, mV | 19.19 | 19.29 | 19.33 | 19.32 | 19.32 | 19.32 |
| $S \times 10^3$ | 813 | 11 | 0 | 5 | 3 | 1 |

C3-Extracephalic Ipsilateral Shoulder Montage

| $V_0$, mV | 21.99 |
| $S \times 10^3$ | 802 | 17 | 10 | 2 | 3 | 0 |

C3-Extracephalic Contralateral Shoulder Montage

| $V_0$, mV | 21.84 |
| $V$ for 20% cond. change, mV | 18.41 | 21.84 | 21.88 | 21.91 | 21.91 | 21.92 |
| $S \times 10^3$ | 801 | 18 | 9 | 2 | 2 | 0 |
Fig. 4. Depictions of the contralateral supraorbital (i), extracephalic contralateral shoulder (ii), and extracephalic ipsilateral shoulder (iii) cathode montages. Total current density normalized by the input current density at the electrodes is shown projected onto three sagittal planes that progressively travel from the left to right on the model using a logarithmic scale.
Fig. 5. Contralateral supraorbital (i), extracephalic contralateral shoulder (ii), and extracephalic ipsilateral shoulder (iii) cathode montages. Total current density normalized by the input current density at the electrodes is shown with surrounding body structures using a logarithmic scale.

132
Fig. 6. Contralateral supraorbital (i), extracephalic contralateral shoulder (ii), and extracephalic ipsilateral shoulder (iii) cathode montages. Total current density normalized by the input current density at the electrodes is shown with surrounding body structures using a logarithmic scale.
Fig. 7. Current densities projected onto an axial plane located halfway down the brain for the contralateral supraorbital (i), extracephalic contralateral shoulder (ii), and extracephalic ipsilateral shoulder (iii) montages. Total current density normalized by the input current density at the electrodes is shown using a logarithmic scale.
Fig. 8. Diagonal cut-plane between the anode and supraorbital cathode displaying normalized total current densities for the contralateral supraorbital (i), extracephalic contralateral shoulder (ii), and extracephalic ipsilateral (iii) montages using a logarithmic scale.
IX. DISCUSSION

A. General Observations

Given the results described in the previous sections, the following observations can be made:

I. Extracephalic montages might create larger total current densities in deeper brain regions, specifically in white matter as compared to an otherwise equivalent cephalic montage.

II. Extracephalic montages might create larger average vertical current densities in the primary motor cortex and in the somatosensory cortex. At the same time, the horizontal current density either remains approximately the same or decreases.

III. The previous observation becomes significantly less apparent for a very wet skin.

IV. The extracephalic montages may reduce the large percentage of the applied current passing through the frontal cortex when the cathode is located at the contralateral supraorbital location.

Indeed, the data presented in Figs. 3 through 10 is related to only one particular cephalic electrode configuration studied in this paper. Furthermore, it is clearly dependent on both the model construction and tissue conductivity values.

To address the last concern, we have compared our findings with simulation data from a similar problem (cephalic versus extracephalic configurations) solved numerically in Ref. [46]. The FEM model used in this work did not include any layer of fat (which has a significantly lower conductivity value [45]) around the skull. Instead, a homogeneous thick skin layer with a high conductivity value of 0.43 S/m has been used. The authors stated that “the use of extracephalic montage does not significantly increase the amount of current penetration through the skull.” The figure of merit was apparently the current density map at the surface of the cerebral cortex. Such a result is in agreement with our data presented in Fig. 10c, d where we see that some potential advantages of the extracephalic configuration may rapidly disappear when the skin conductivity becomes very high.

B. Invariance of Extracephalic Montages

The density of current flow within the body during extracephalic stimulation is weakly dependent on the choice of the shoulder location. In both ipsi- and contralateral cathode montages, current densities in the human head are nearly identical. This makes intuitive sense, as current needs to flow toward the cathode through the neck. Above the neck, the current distribution is insensitive to shoulder electrode positions. This is similar to water flow in a closed container in the form of a human body where the anode is a source, the cathode is a sink, and the neck is acting as a choke point. Thus, there is freedom in choosing the extracephalic electrode location. This may alleviate concerns regarding extraneous stimulation of other body areas (i.e., disrupting the autonomic nature of heart muscle regulation by the sinoatrial node, etc.).
Fig. 9. Comparison of tDCS simulation results from Fig. 5 as a function of skin conductivity. Total current densities projected onto an axial plane located halfway down the brain for the contralateral supraorbital (i), extracephalic contralateral shoulder (ii), and extracephalic ipsilaterial shoulder (iii) montages. Note that the images are employing a logarithmic scale.
C. Validation of Results across Configurations

Along with the previously reported extreme case, the current density distribution behavior observed in Figs. 3-10 has been confirmed for:

i. Different tissue conductivities (every value was separately varied by ±20%);

ii. Different body mass values (scaling the entire structure by $5 \times 5 \times 5\%$ while keeping the electrode size the same);

iii. Homogeneous versus non-homogeneous brain structures (assigning average conductivity values to white/grey matter/cerebellum).

Fig. 10. Averaged values of the vertical and horizontal current density components for the brain substructures shown in Fig. 2 and total white and grey matter. Rows 1 and 2 depict results for dry and wet skin, respectively.
D. VHP-F Model Limitations and Extensions

While the present VHP-F model has proven to be useful for numerical studies such as the one described in this work, it does have its limitations. In particular, the CSF flow present in the subarachnoid space is greatly simplified: only the thin yet non-uniform closed CSF shell and the ventricles are considered, as shown in Figs. 4-9. The minimum thickness of this shell was artificially set to 1 mm in order to avoid numerically inaccurate results. The skull is modeled by a homogeneous bone structure. All present calculations use lower-definition meshes with the typical resolution (surface deviation) ranging from 1mm to 3 mm, which significantly suppresses the fine sulci and gyri structures. Also, the brain membranes, including the pia mater, arachnoid, and dura mater, are not explicitly included in the VHP-F model. These membranes present a direct layer between the brain and stimulating electrodes and likely should be characterized for enhanced tDCS simulation accuracy. The balloon representation of the skin with the fixed uniform skin shell thickness of 2mm is indeed another simplifying approximation. However, the variable (and typically much thicker) fat layer of a greater conductivity just beneath the skin is anatomically correct to within 1 mm segmentation accuracy, everywhere in the phantom. Together, the skin and fat layers may still form a reasonable modeling approximation for the surface current flow. The impact of anisotropic materials on the performance of the VHP-F model is another item we wish to consider in a future work. Our host FEM software (MAXWELL 3D of ANSYS) allows for a diagonal Cartesian conductivity tensor $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ through every tissue.

X. CONCLUSIONS

It has been shown that extracephalic montages might create larger total current densities in deeper brain regions, specifically in white matter as compared to an equivalent cephalic montage. Extracephalic montages might also create larger average vertical current densities in the primary motor cortex and in the somatosensory cortex. At the same time, the horizontal current density either remains approximately the same or decreases. The metrics used in this paper include either the total local current density through the entire brain volume or the average vertical and horizontal current densities for each individual lobe/cortex.

REFERENCES


[35] Online: http://meshlab.sourceforge.net/


SECTION C —

ANTENNA APPLICATION EXAMPLES

The first full-body example (ANSYS EM Suite® 16.2) evaluates radiated fields of a 1 GHz point dipole source (a Hertzian dipole). The source is located inside the body, just below the heart muscle. Integral-Equation (IE) boundary conditions have been used. Fig. 1 shows the computed electric field distribution to scale. Three adaptive mesh refinement passes are required to achieve a Delta Mag Energy of 0.01. The total simulation time is 2 hr and 15 min, with 70 GBytes of RAM required. For five adaptive passes, the total simulation time increases to 3 hr 38 min (mesh time: 1 hr 22 min, adaptive passes: 2 hr 16 min), and 122 Gbytes of RAM will be required. The server used was an Intel(R) Xeon(R) CPU E5-2697 V2 with 256 GB running 64-bit OS Windows Server 2008 R2 Enterprise.

The second full-body example (CST MWS) evaluates volume power loss density (PLD) in W/m³ due to a cloth-mounted arm antenna with face port excitation shown in Fig. 2. This is a thin-foil antenna with felt between the ground plane and excitation (a printed blade dipole). Fig. 3 shows the PLD map for two cases: a) homogenized body with only a skin layer and fat inside and b) the realistic V.3.0 BASE model. The differences are quite significant. On an ordinary 8-core computer, the corresponding full-body simulation takes a few hours.

![E Field [V/m]](image)

Fig. 1. Vertical Hertzian dipole at 1 GHz just below the heart muscle and the associated electric field. VHP-Female V.3.0 BASE has been used.
Other examples for on-body and in-body antennas and small antenna arrays have been computed and documented. In a number of cases, comparisons with experiments have been made. These results will be reported separately in a master thesis.

Fig. 2. Cloth-mounted arm antenna. VHP-Female V.3.0 BASE with all tissue objects has been used.

Fig. 3. Power loss density generated by the arm antenna for both a) homogenized and b) realistic VHP-Female V.3.0 BASE models.
iv. A third example (benchmark) is a scattering problem at plane wave incidence at 300 MHz shown in Figure 4. The incident wave polarization is vertical. The VHP-Female V.3.0 BASE model was simulated simultaneously in ANSYS HFSS and CST MWS frequency-domain solver, respectively. On an ordinary 8-core computer, the corresponding full-body simulation with 5 adaptive passes and 1.2 M tetrahedra takes a few hours. To obtain quantitative estimates, we have evaluated volume power loss density in W/m$^3$ along certain paths within the body (for the total field which is the scattered field plus the incident field). Those paths are two line segments shown in Figure 4: (a) $x=0$mm, $y = 25$mm, $-60$mm$<z<140$mm; (b) $x=0$mm, $z = -12.5$ mm, $-87$mm$<y<91$mm. Figure 5 shows results of the CST MWS and ANSYS HFSS simulations for this particular problem. To the best of our knowledge, this is the first comparison result of such kind for a full-body model. Although the initial overall agreement is reasonable, we work on its improvement in certain domains by using a better spatial resolution.

Fig. 4. Paths within the body chosen for PLD evaluation. A subfigure presents a PLD map obtained with CST MWS.
Fig. 5. Power loss density along two line segments.
SECTION D — 
COMPUTATIONAL PERFORMANCE USING HIGH-FREQUENCY FEM SIMULATOR 
ANSYS HFSS

A. Plane Wave Test

An ANSYS HFSS project has been created for a plane wave incident upon the phantom at 300 MHz using 5 passes of adaptive mesh refinement and integral-equation boundary conditions for a box which tightly surrounds the body. Table I reports simulation benchmarks for four representative computational servers. Final relative energy error is less than 0.015. Two conclusions can be made based on these and similar results: (i) Intel processors are more beneficial for use with ANSYS HFSS and (ii) the use of distributed or high-performance computing decreases the elapsed time for accurate full-body FEM simulations related to radar cross-section/antenna modeling to about 4 hours total. Note that the classic mesher, not the τ-mesher, was enforced in HFSS (HFSS → Mesh operations → Initial mesh settings → Meshing Method). The τ-mesher causes prohibitively large execution times and should perhaps be not used for this application.

Table I - Simulation Benchmarks for Plane Wave Test – ANSYS Electromagnetic Suite Release 15/16

<table>
<thead>
<tr>
<th>System #1 (one task, one core)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intel(R) Xeon(R) CPU E5-2697 V2, 256 GB, 64-bit OS Windows Server 2008 R2 Enterprise ANSYS EM Suite® 16.0.0</td>
</tr>
<tr>
<td>Tetrahedral mesh size &amp; total RAM (start/stop)</td>
</tr>
<tr>
<td>Execution time for 5 passes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System #2 (one task, one core)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 AMD OPTERON 6174 12 core processors, 192 GB, 64-bit OS, Windows Server 2008 R2 Enterprise ANSYS EM Suite® 15.0.2</td>
</tr>
<tr>
<td>Tetrahedral mesh size &amp; total RAM (start/stop)</td>
</tr>
<tr>
<td>Execution time for 5 passes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System #3 (one task, one core)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intel(R) Xeon(R) CPU E5-2690, 192 GB, 64-bit OS Red Hat Enterprise Linux 2.6.32 ANSYS EM Suite® 15.0.2</td>
</tr>
<tr>
<td>Tetrahedral mesh size &amp; total RAM (start/stop)</td>
</tr>
<tr>
<td>Execution time for 5 passes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System #4 (one task, eight cores, HPC option)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intel(R) Xeon(R) CPU E5-2697 V2, 256 GB, 64-bit OS Windows Server 2008 R2 Enterprise ANSYS EM Suite® 16.0.0</td>
</tr>
<tr>
<td>Tetrahedral mesh size &amp; total RAM (start/stop)</td>
</tr>
<tr>
<td>Execution time for 5 passes</td>
</tr>
</tbody>
</table>

a. Systems 1 and 4 differ by the HPC option only.
B. MRI-Coil Modeling

One MRI coil utilized was a 64 MHz high pass 16 rung birdcage design with dimensions relevant to clinical 1.5 T scanners: coils of diameter 604 mm and length 650 mm as described in Ref. [1]. Yet another similar coil operated at 128 MHz and 3 T. The coil is using 48 excitation ports because this setup allows one to obtain near-field results for any kind of coil tuning (high pass, low pass, band pass) without re-running 3D EM simulations. Table II reports selected simulation benchmarks for head and full-body scans, with two adaptive meshing passes. Manual meshing in critical areas has been used prior to the adaptive mesh refinement. This manual meshing guarantees that the final relative energy error is less than 0.002, even with only two adaptive passes. This value favorably compares with the error value of 0.015 obtained previously and underscores the importance of manual meshing for modeling the human phantom augmented with external electromagnetic hardware.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Tetrahedral mesh size &amp; total RAM (start/stop)</th>
<th>Execution time for 2 passes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 T coil, 64 MHz, 48 excitations, head only</td>
<td>3,200,000/4,000,000 3.4GB/91.6 GB</td>
<td>Meshing time: 3 h 15 min Sim. time: 6 hr 8 min</td>
</tr>
<tr>
<td>3 T, 128 MHz, 48 excitations, head only, interpolating sweep</td>
<td>2,600,000/3,300,000 3.2GB/62.7 GB</td>
<td>Meshing time: 2 h 6 min Sim. time: 7 hr 42 min</td>
</tr>
<tr>
<td>3 T, 128 MHz, 48 excitations, whole body</td>
<td>4,000,000/5,000,000 4.5GB/134 GB</td>
<td>Meshing time: 4 h 30 min Sim. time: 4 hr 39 min</td>
</tr>
<tr>
<td>3 T, 128 MHz, 48 excitations, whole body</td>
<td>5,400,000/6,400,000 5.9GB/121 GB</td>
<td>Meshing time: 5 h 20 min Sim. time: 14 hr 22 min</td>
</tr>
</tbody>
</table>

Similar estimates have been obtained for the low-frequency EM simulator Maxwell 3D of ANSYS. For a plane wave test (and similar antenna/array tasks), the one-core run time is about twelve hours on average while the multi-core run time is less than four hours, yielding a final relative energy error of less than 0.015 (after five adaptive passes). For the most complicated MRI-related simulations with manual meshing, the total multicore run time is about nine to twenty hours, while maintaining a very good solution accuracy. Thus, the VHP-Female v. 2.0 full-body phantom will provide a reasonably fast yet accurate and flexible computational platform for multi-purpose...
electromagnetic modeling of a multi-tissue human body. We believe the phantom is suitable for thermal and acoustic modeling as well.

REFERENCES

Chapter 05 –
Development and Applications of Japanese Pregnant Model

I. INTRODUCTION

Recent studies confirm the efficacy of Transcranial Magnetic Stimulation (TMS) as a non-invasive treatment of medication-resistant depression [1],[2] and in the US, four different devices, the Neuronetics Neurostar Stimulator, Brainsway H-Coil system, Magstim Magnetic Stimulator, and MagVenture Stimulator have been cleared by the Food and Drug Administration (FDA) for the treatment of medication-resistant depression [3],[4].

Even though TMS coil holders and even robots have been developed that might make the application of TMS more spatially precise and efficient, to date, TMS is often applied by an operator who holds the TMS coil over the subject’s head. A potential safety concern is thus generated when the operator is a woman and is pregnant. There are no studies to date that assess the safety of TMS for a fetus. In the case of a pregnant woman as a TMS operator we must consider two possibilities:

- Standard operation with the TMS coil held distances of approximately 1-2 ft. from the belly;
- Accidental TMS coil discharge right on the belly or in its immediate vicinity.

In addition to the scenario of a pregnant woman as a TMS operator, the possibility of a pregnant woman as TMS patient is also important to consider. TMS can cause a generalize tonic seizure and of course a seizure can pose a significant risk for the integrity of a pregnancy. Therefore, in most instances pregnancy will be an exclusion criterion for TMS. However, a considerable percentage of women experience symptoms of depression during pregnancy and develop clinical depression requiring medical intervention. TMS has been proposed as a method to treat maternal depression while avoiding fetal exposure to drugs [5],[6] and the risk-benefit profile is argued to be better for TMS than for medications and yet TMS may cause fetal exposure to high induced currents.

In estimating acceptable levels of induced currents, we refer to guidelines [17],[18] from the International Commission on Non-Ionizing Radiation Protection. The 1998 ICNIRP basic restrictions for general exposure to time-varying electric and magnetic fields for CW frequencies in the band 1-100 kHz recommend that the current density for head and trunk should be below 0.002 f [mA/m²] where f is the signal frequency measured in Hz. According to these guidelines, at 5 kHz frequency, the minimum exposure threshold recommended is 10 mA/m². For pulses of (effective) duration τ, the equivalent frequency to apply in the basic restrictions should be calculated as f = 1/(2τ). We assume that those estimates apply to the fetal brain and trunk as well.

Induction currents in the entire human body (or bodies) caused by a TMS coil can be established in every particular case via numerical electromagnetic modeling. An accurate procedure adopted to model eddy currents within complex biological shapes is the finite-element method (FEM) or boundary element method (BEM) [9]-[16]. FEM or BEM has been previously applied specifically to TMS effects on the human head/body [13]-[15], [17]-[22].
Our FEM model construction begins with a voxel-based computational phantom of a 29 year old pregnant Japanese female [24] (second trimester). We first convert this model into a CAD model with triangular surface meshes using standard surface extraction algorithms [24],[25]. Next, we scale the uterus in order to approximately describe the first and the last trimesters. After that, we apply the commercial eddy current FEM simulator MAXWELL 3D of ANSYS to obtain representative numerical results and, finally, generalize those simulations using a generic analytical upper estimate of eddy current density in a human body due to the TMS coil [26].

II. PROBLEM STATEMENT

A. CAD model construction

Fig. 1e shows three CAD models constructed for this study using surface extraction on the base of the voxel model [24]. The original pregnant female voxel model [24] was developed from MRI data collected on a non-pregnant Japanese woman who was 160 cm tall and weighed 53 kg. Further, abdominal MR images of a 26-week-pregnant woman were segmented and inserted into this full-body model [24].

We have carefully converted this voxel model to the CAD FEM model in the form of triangular surface meshes with approximately 50 tissues shown in Fig. 1- center (~6 months of pregnancy). After that, mesh decimation, smoothing, and intersection resolution have been performed for every tissue separately. The resulting surface tissue meshes are 2-manifold, do not have coincident (or touching) faces, and are all included into the average-body container – the outer shape of the model. Fig. 2 demonstrates the corresponding fetal volume on a larger scale.

Finally, we artificially deformed the uterus and amniotic fluid (free-form deformations) using SpaceClaim of ANSYS, Inc. to model pregnancy during the first and second trimesters, respectively. We did not include the fetus or placenta into consideration since their positions change over time.

Table I. Material properties used in mother/fetus models [29],[30].

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$\sigma$ (S/m)/$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMNIOTIC FLUID</td>
<td>Cerebrospinal fluid</td>
</tr>
<tr>
<td>FETUS</td>
<td>Mean of muscle, uterus, and blood</td>
</tr>
<tr>
<td>FETAL BRAIN</td>
<td>$\left(\frac{\sigma_{\text{fetal brain}}(64 MHz)}{\sigma_{\text{fetus}}(64 MHz)} + \frac{\sigma_{\text{fetal brain}}(127 MHz)}{\sigma_{\text{fetus}}(127 MHz)}\right)/2 \times \sigma_{\text{fetus}}$</td>
</tr>
<tr>
<td></td>
<td>$\left(\frac{\varepsilon_{\text{fetal brain}}(64 MHz)}{\varepsilon_{\text{fetus}}(64 MHz)} + \frac{\varepsilon_{\text{fetal brain}}(127 MHz)}{\varepsilon_{\text{fetus}}(127 MHz)}\right)/2 \times \varepsilon_{\text{fetus}}$</td>
</tr>
<tr>
<td>PLACENTA</td>
<td>Average muscle</td>
</tr>
</tbody>
</table>

A MATLAB script is used to extract the point cloud and form the triangular mesh using ball pivoting algorithm based on the respective material ID for each tissue object. The point nodes which form each voxel is labelled with its corresponding material ID. Each point slice is evaluated to find the boundary points of the tissue by analyzing the neighboring nodes for different material ID. For example, liver has a material ID of 26, so a point with material ID of 26 surrounded by a point having any
other material ID is classified as a boundary node. These extracted boundary nodes are used to create the triangular mesh using surface reconstruction techniques. The point cloud can be extracted using the MATLAB code shown below:

```matlab
if (slices(x,y,z)==ID) &&... 
  ((slices(x+1,y,z)==ID)|... 
  (slices(x-1,y,z)==ID)|... 
  (slices(x,y+1,z)==ID)|... 
  (slices(x,y-1,z)==ID))
  P(row,:) = [x y (size(slices,3)-z)]; % Save to P-matrix
  row = row + 1; % go to next row
end % end if
```

The final matrix P is scaled by a factor of two to account for voxel grid scaling.
This technique works only with fully connected point clouds (e.g. liver) and works best with large smooth models. It does not work well for thin branching models such as blood vessels. Figure 1a shows the point cloud extracted in blue and the corresponding surface mesh created after surface reconstruction for a liver object.

Fig. 1a. The point cloud extracted (in blue) and the corresponding surface mesh created after surface reconstruction.

The resulting volume formed from extraction is a cloud of points, convex or not. It needs to be converted to surfaces and surface triangular meshes. This is done using powerful MATLAB function `isosurface`. This function computes isosurface data from the volume data V at the isosurface value equal to zero, i.e. exactly at the boundary of the solid. The vector V is defined as -1 for the points outside the boundary of the mesh and 1 for the points inside the boundary of the object. This function may be treated as an extension of the familiar contour plot to three dimensions.
MATLAB function `isosurface` outputs array of triangular faces `S.faces(FacesTotal, 3)` on the surface and the array of nodal points `S.vertices(VerticesTotal, 3)` on the same surface of the final solid object. These arrays are trivially converted to $t$ and $P$ arrays used in this text as shown below:

```matlab
S = isosurface(X, Y, Z, V, 0);
t = S.faces'; P = S.vertices';
```

Figure 1b illustrates the point cloud (all nodes including both enclosed and boundary) of the grey matter object extracted from the voxel data of the Japanese pregnant database based on material ID. Figure 1c illustrates the corresponding surface of the grey matter object generated using `isosurface`. The mesh generated shown has about 106, 208 triangles and is of high definition.

![Image of point cloud and surface generation](image.png)

Fig.1b. –Point cloud of the grey matter object extracted from the voxel data.

The surface patches are plotted using the `patch` command and thus allow us to visualize the resulting body.
Using the same surface extraction algorithm approximately 50 tissues were extracted. These meshes were then post processed to create a 2-manifold mesh object using custom made MATLAB tools and SpaceClaim of ANSYS. Fig. 1d shows the mesh object for gray matter after the post processing optimized for numerical simulations with 5,454 triangles only.

Fig. 1c. Surface of the grey matter object generated using isosurface.

Fig. 1d. 2-manifold surface mesh of the grey matter object after post processing.
B. Tissue properties, coil construction, and pulse excitation

The bulk of tissue properties were used following the Gabriel & Gabriel database [27] further replicated in the IT’IS database [28]. Fetal properties follow Refs. [29],[30] and are outlined in Table I.

Fig. 1e. Three CAD models used for the first, second, and third trimesters. The model in the center is the original derived one.

Fig. 2. Detailed view of the fetus for the second trimester model.
Similar to Ref. [31], the base coil is a Figure-8 straight coil with a loop radius of 35 mm. However, instead of a stranded conductor, we used a solid conductor (copper) with a diameter of 8 mm. A variety of different TMS pulse forms have recently been suggested [32],[34],[35]. We modeled a simple monophasic (monopolar) TMS pulse [26]. Its form is aimed to approximate some common experimental monophasic TMS coil current forms [32],[33]. A biphasic pulse or a pulse of a more complicated shape can be studied similarly, using the superposition principle. The present pulse form is characterized by two parameters: rise time $\tau$ and peak current $I_0$. The coil current pulse over time interval $0 \leq t < 10\tau$ is expressed in the form:

$$f(t) = I_0 \begin{cases} 
\frac{4\pi(t - \tau)}{\tau^2} & \text{for } \tau \leq t < 2\tau \\
-\frac{(t - 2\tau)^2}{2\pi} & \text{for } t \geq 2\tau \\
e^{-\frac{(t - 2\tau)^2}{2\tau^2}} & \text{for } \tau \leq t < 10\tau
\end{cases}$$

$I(t) = 0$ for $0 \leq t < \tau$  \hspace{1cm} (1)

The derivative of the coil current pulse approximates eddy currents/electric fields induced in the body; it is given by

$$\frac{df(t)}{dt} = I_0 \begin{cases} 
\frac{4\pi(2\tau - t)}{\tau^2} & \text{for } \tau \leq t < 2\tau \\
-\frac{(t - 2\tau)^2}{2\pi} & \text{for } t \geq 2\tau \\
\frac{(2\tau - t)}{2\pi^2}e^{-\frac{(t - 2\tau)^2}{2\tau^2}} & \text{for } \tau \leq t < 10\tau, \text{ } I(t) = 0 \text{ for } 0 \leq t < \tau
\end{cases}$$

df(t)/dt = I_0 \begin{cases} 
\frac{4\pi(2\tau - t)}{\tau^2} & \text{for } \tau \leq t < 2\tau \\
-\frac{(t - 2\tau)^2}{2\pi} & \text{for } t \geq 2\tau \\
\frac{(2\tau - t)}{2\pi^2}e^{-\frac{(t - 2\tau)^2}{2\tau^2}} & \text{for } \tau \leq t < 10\tau
\end{cases}$$  \hspace{1cm} (2)

The second pulse derivative is a continuous function of $t$ and is equal to $I_0/2\tau^2$ at $t = \tau$. Fig. 3a shows the coil current pulse normalized by $I_0$; Fig. 3b depicts the pulse derivative normalized by $I_0/\tau$. We select $\tau = 0.1$ ms in (1), (2) which is the typical magnetic field rise time for monophasic TMS pulses [32],[33]. Along with the pulse rise time of 0.1 ms, we also present the result for a smaller value of 0.01 ms, which can also be used as an excitation in TMS coils [34],[35]. We select only moderate peak coil currents of 1,000 or 10,000 A, respectively. Note that the above values apply to a 1,000/10,000 A peak coil current for a single-turn coil or to a 1,000/10,000 A-turns magnetomotive force (mmf) for a multi-turn coil. Due to the problem linearity, testing other peak current values is trivial.
For a given coil, a 0.1 ms long and 1,000 A strong current pulse produces an electric field of approximately 5 V/m at 3 cm from the coil; a 0.1 ms long and 10,000 A strong pulse – approximately 50 V/m, a 0.01 ms long and 1,000 A strong pulse – approximately 50 V/m, and a 0.01 ms long and 10,000 A strong pulse – approximately 500 V/m. The first pulse configuration is apparently useless practically; it is included for completeness only.

III. COMPUTATIONAL RESULTS – PREGNANT WOMAN AS A PATIENT

A. Coil positioning

Two coil positions for a pregnant patient have been considered shown in Fig. 4. In the first case (Fig. 4a), the straight coil is located 10 mm exactly above the top of the head. In the second case (Fig. 4b), the straight coil is moved and then tilted by 60 degrees. The first case might represent a standard TMS coil placement for studies aimed to evaluate central motor conduction, though a circular TMS coil would be generally used in such instances. The second case aims to approximate the position of the TMS for the treatment of depression.
B. Frequency-domain results

Frequency-domain results (coil excitation with a sinusoidal waveform) have been collected for multiple frequencies (a logarithmic frequency sweep) over the band from 300 Hz to 3 MHz in order to generate the required pulse forms via the FFT and IFFT as described in Ref. [26]. The corresponding method has been described in the same reference; it is time-consuming but accurate. Fig. 5 shows eddy current amplitude distribution in a coronal plane for three representative frequencies: 3 kHz, 30 kHz, and 300 kHz. The coil current amplitude is 10,000 A (10,000 A-turns mmf), peak coil current at 10 kHz. Note that the scale is multiplied by the factor of 10 for every subsequent figure. One can see that the peak current in the fetal area does not exceed 0.1 mA/m$^2$, 1 mA/m$^2$, and 40 mA/m$^2$. The increase in the current amplitude becomes a nonlinear function of frequency at 300 kHz or at higher frequencies.

C. Time-domain results and extracting maximum peak values

The time-pulse domain solution has been constructed based on the interpolated frequency-domain data as described in [26]. A uniform 5×5×5 mm grid of observation points has been used within a rectangular box, which cover the abdominal area only. The number of observation points within the body where the induced current is evaluated is approximately 150,000. For every such point, the pulse form has been restored and its peak value has been found. Complete results for two coil configurations a) and b) in Fig. 4 are given in Fig. 6.
Fig. 5. Eddy current amplitude distribution in a coronal plane for three representative frequencies: 3 kHz, 30 kHz, and 300 kHz.

Two major parameters of interest in Fig. 6 are the maximum peak eddy current density value in the volume occupied by the amniotic fluid (for three stages of pregnancy in Fig. 6a) and the maximum peak eddy current density in every fetal tissue separately (for second trimester only in Fig. 6b). Every such maximum value is the absolute maximum of pulse peak values for all observation points within a tissue of interest. It is important to emphasize that the results of Fig. 6 apply to a 10,000 A peak coil current for a single-turn coil or to a 10,000 A-turns magnetomotive force (mmf) for a multi-turn coil. Scaling for other coil current peak values can be done if necessary.

D. **Comparison with safe values of eddy current density**

According to the safety requirement discussed in the introduction (a pulse formula should be used), the current density in the abdomen including the fetal area should not exceed 10 mA/m² for the 0.1 ms long pulse and 100 mA/m² for the 0.01 ms long pulse. This condition is satisfied for all case studies in Fig. 6 except the amniotic fluid volume for the third trimester. For the third trimester using 10,000 A peak coil current and pulse durations of either 0.1 or 0.01 ms, the maximum peak induction current may exceed the safe limit by approximately 70%.
IV. COMPUTATIONAL RESULTS – PREGNANT WOMAN AS OPERATOR

A. Coil positioning for standard operation

A coil positioning map for the operator is shown in Fig. 7 for the second trimester. The closest distance from the coil center to the body is 115 mm. We consider three representative polarizations of the major current dipole of the coil:

- in the coronal plane ($z$-polarization in Fig. 7, Config. A, labeled as A1-A6);
- in the sagittal plane ($y$-polarization in Fig. 7, Config. B, labeled as B1-B6);
- in the transverse plane ($x$-polarization in Fig. 7, Config. C, labeled as C1-C4).

Fig. 6. Maximum peak eddy current density value in the volume occupied by the amniotic fluid (for three stages of pregnancy) and maximum peak eddy current density in every fetal tissue separately (for second trimester only).

For every polarization type, four to six representative coil locations are tested in the sagittal plane as shown in Fig. 7. The operator could reach any such location by moving the right arm along with
the coil holder. This gives us a total of sixteen base test cases. For each test case, we will compute the eddy current density and the corresponding electric field everywhere in the body. The setup is repeated three times: for the first, second, and third trimesters. For the first trimester, the closest distance from the coil center to the body is 45 mm (B) or 80 mm (C), for the second, and for the third – 40 mm (B) and 80 mm (C). The remaining topology is the same.

B. Accidental coil discharge

Two extreme cases have also been considered not shown in Fig. 7, when the coil is placed as close to the body as possible by moving it in the xy-plane. These cases will be labeled as B7, B8, and C5, C6, respectively.

Fig. 7. Coil positioning map for a pregnant operator (second trimester). Some tissues have been made invisible for clarity.

C. Time domain results – second trimester

Pulse construction procedure described in the previous section was again been applied. Two major parameters of interest remain the maximum peak eddy current density value in the volume
occupied by the amniotic fluid (for three stages of pregnancy) and the maximum peak eddy current density in every fetal tissue separately (for second trimester only).

Such maximum value is the absolute maximum of pulse peak values for all observation points within a tissue under interest.

Figs. 9 through 11 show the maximum peak current density values for every tissue of interest in the fetal area. Their results apply to the lowest peak coil current: a 1,000 A peak coil current for a single-turn coil or to a 1,000 A-turns magnetomotive force (mmf) for a multi-turn coil. Scaling for other coil current peak values can be done if necessary.

Fig. 9 corresponds to configuration A in Fig. 7, Fig. 10 – to configuration B, and Fig. 11 – to configuration C.

D. Comparison with safe values of eddy current density

According to the safety requirement discussed in the introduction, the current density in the abdomen including the fetal area should not exceed 10 mA/m² for the 0.1 ms long pulse and 100 mA/m² for the 0.01 ms long pulse. This condition is satisfied in fetal tissues for

- Configuration A when the coil -belly distance exceeds approximately 3 ft.
- Configuration B when the coil -belly distance exceeds approximately 2 ft.
- Configuration C when the coil -belly distance exceeds approximately 2 ft.

E. Case of accidental discharge

In these cases, the induced currents in the abdomen exceed the safety limits cited above by approximately 10 times for 1000 A (1000 A-turns) peak coil current and by approximately 100 times for 10,000 A (10,000 A-turns) peak coil current.

F. Results for other trimesters and eddy currents in the fetal area

Results for the first and third trimesters (pregnant operator) show a similar behavior; the variations in the peak eddy current density within the amniotic fluid are typically within 100%.

To obtain a better feeling of eddy current distribution within the fetal area, Fig. 8 demonstrates eddy current density magnitude distributions for the coil configuration C-1 in Fig. 7 (harmonic excitation at 10 kHz and 1,000 A coil current amplitude) in six cut planes using the same color scale. The maximum current density is typically observed in the amniotic fluid although the exact positions may vary considerably. Frequently, it can also occur in the bladder or in the spinal cord.
Fig. 8. Eddy current density magnitude distribution for configuration C-1 in Fig. 4 in six cut planes through the fetal area. Columns corresponds to a fixed horizontal cut plane; rows to a fixed vertical cut plane. The same scale is used in all six figures. The maximum current density is typically observed in the amniotic fluid.

V. COMPARISON WITH UPPER ANALYTICAL ESTIMATE FOR EDDY CURRENT DENSITY

Tables II through IV demonstrate a comparison of our simulation results with the upper analytical estimate of eddy currents in the human body [26]. The eddy current density in the body is expressed directly through a time-varying lumped coil current, \( f(t) \), in the form [26]

\[
J = -\sigma \frac{\partial A^{\text{inc}}}{\partial t}, \quad A^{\text{inc}}(r, t) = \frac{\mu_0 f(t)}{4\pi} \int_C \frac{d\mathbf{l}}{c |\mathbf{r} - \mathbf{r}'(t)|} \tag{3}
\]
The second expression in Eqs. (3) is the Bio-Savart law written in the form of a line integral for a coil having a contour $C$. To complete the estimate in Eqs. (5) [26], the effective medium conductivity, $\sigma$, should be given. In Ref. [26], the value of 0.5 S/m was sufficient for the upper estimate of eddy currents in a non-pregnant 60-year old female as a patient. In the present case, this value appears to be insufficient. In Tables II-IV, we report the ratio of analytical and numerical peak eddy current densities when the effective medium conductivity is increased to 2 S/m. One can see that this ratio never becomes less than one so that the condition of the upper estimate is always satisfied. Note that in Table II-IV the average value for the ratio of the two results is about 25.
**TABLE II. PEAK NUMERICAL EDDY CURRENT DENSITIES/RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES (SECOND TRIMESTER OF AN OPERATOR) FOR THE SET OF APPROXIMATELY 150,000 OBSERVATION POINTS IN THE ABDOMEN*.**

<table>
<thead>
<tr>
<th>Coil configuration</th>
<th>$r = 0.1$ ms</th>
<th></th>
<th>$r = 0.01$ ms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max. peak numerical eddy current (mA/m$^2$)</td>
<td>Min. ratio</td>
<td>Median of ratios</td>
<td>Min. ratio</td>
</tr>
<tr>
<td>Coil A-1</td>
<td>122.32</td>
<td>1.65</td>
<td>11.57</td>
<td>2.99</td>
</tr>
<tr>
<td>Coil A-2</td>
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<td>1.93</td>
<td>13.44</td>
<td>3.76</td>
</tr>
<tr>
<td>Coil A-3</td>
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<td>2.16</td>
<td>15.97</td>
<td>3.44</td>
</tr>
<tr>
<td>Coil A-4</td>
<td>20.55</td>
<td>2.16</td>
<td>17.23</td>
<td>3.98</td>
</tr>
<tr>
<td>Coil A-5</td>
<td>6.76</td>
<td>3.41</td>
<td>25.16</td>
<td>6.02</td>
</tr>
<tr>
<td>Coil A-6</td>
<td>5.45</td>
<td>3.59</td>
<td>27.84</td>
<td>5.74</td>
</tr>
<tr>
<td>Coil B-1</td>
<td>38.82</td>
<td>8.13</td>
<td>43.7</td>
<td>9.10</td>
</tr>
<tr>
<td>Coil B-2</td>
<td>40.76</td>
<td>3.73</td>
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<td>4.71</td>
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<td>Coil B-3</td>
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<td>326.32</td>
<td>2.39</td>
<td>11.41</td>
<td>2.98</td>
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<tr>
<td>Coil C-1</td>
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<td>2.75</td>
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<tr>
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<tr>
<td>Coil C-4</td>
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<tr>
<td>Coil C-5</td>
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<td>Coil C-6</td>
<td>258.71</td>
<td>1.72</td>
<td>12.11</td>
<td>1.87</td>
</tr>
</tbody>
</table>

* for this study the placenta is assigned blood for material property.

**TABLE III. PEAK NUMERICAL EDDY CURRENT DENSITIES/RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES (FIRST TRIMESTER OF AN OPERATOR) FOR THE SET OF APPROXIMATELY 150,000 OBSERVATION POINTS IN THE ABDOMEN.**

<table>
<thead>
<tr>
<th>Coil configuration</th>
<th>$r = 0.1$ ms</th>
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<th>$r = 0.01$ ms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak numerical eddy current (mA/m$^2$)</td>
<td>Min. ratio</td>
<td>Median of ratios</td>
<td>Min. ratio</td>
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<tr>
<td>Coil B-7</td>
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<td>15.07</td>
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<td>346.04</td>
<td>2.55</td>
<td>10.89</td>
<td>2.79</td>
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</tbody>
</table>

165
TABLE IV. PEAK NUMERICAL EDDY CURRENT DENSITIES/RATIO OF ANALYTICAL AND NUMERICAL PEAK PULSE VALUES (THIRD TRIMESTER OF AN OPERATOR) FOR THE SET OF APPROXIMATELY 150,000 OBSERVATION POINTS IN THE ABDOMEN.

<table>
<thead>
<tr>
<th>Coil configuration</th>
<th>$r = 0.1 \text{ ms}$</th>
<th>With averaging over neighboring nodes</th>
<th>$r = 0.01 \text{ ms}$</th>
<th>With averaging over neighboring nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak numerical eddy current (mA/m$^2$)</td>
<td>Min. ratio</td>
<td>Median of ratios</td>
<td>Min. ratio</td>
</tr>
<tr>
<td>Coil B-7</td>
<td>798.93</td>
<td>2.53</td>
<td>21.72</td>
<td>2.82</td>
</tr>
<tr>
<td>Coil B-8</td>
<td>438.67</td>
<td>1.52</td>
<td>21.55</td>
<td>1.79</td>
</tr>
<tr>
<td>Coil C-5</td>
<td>1140.50</td>
<td>1.60</td>
<td>9.13</td>
<td>1.79</td>
</tr>
<tr>
<td>Coil C-6</td>
<td>532.89</td>
<td>1.17</td>
<td>8.23</td>
<td>1.29</td>
</tr>
</tbody>
</table>

VI. CONTROL OF ACCURACY OF SIMULATION RESULTS

It is worth noting that maximum peak eddy current values may be affected by numerical errors.

![Graph a) showing maximum peak eddy current density values in the fetal area for every tissue and every coil position in configuration A from Fig. 7.](image)

![Graph b) showing maximum peak eddy current density values in the fetal area for every tissue and every coil position in configuration A from Fig. 7.](image)
Most dangerous might be a purely numerical error close to sharp edges/corners present in any FEM model [26]. Surface charge density formally becomes singular at any edge (not necessarily sharp) of a triangular mesh with non-planar triangles. For sharper edges and large adjacent triangles, this local (electrostatic) effect becomes quite significant and leads to non-physical fields/current peaks. In order to partially avoid this issue, we computed eddy current densities averaged over six neighbor points (over a cube with the side of 5 mm). Tables II through IV report the corresponding values in terms of the ratio of analytical to numerical peak eddy current densities.

In order to extract the absolute space-averaged maximum peak current, the result of the second row in Tables II through IV should be multiplied by the ratio of the third and fifth rows, respectively. One can see that the space averaging generally lowers the maximum peak eddy current density, but not very significantly (typically within 20%). In view of this, we consider the numerical results given in Tables III through V as reliable.

![Graph](image)

Fig. 11. Maximum peak eddy current density values in the fetal area for every tissue and every coil position in configuration C from Fig. 7.
SUMMARY OF RESULTS

CAD models of a pregnant woman have been constructed based on the existing voxel model [24]. Extensive ANSYS MAXWELL 3D FEM eddy-current simulations (taking into account polarization currents/charges) have been performed for three CAD models at different stages of pregnancy (first, second, and third trimesters) and for over twenty coil positions. Every simulation runs through a logarithmic frequency sweep from 300 Hz to 3 MHz. Based on this sweep, the time-domain solution for a monophasic TMS pulse has been obtained. Pulse durations of 0.1 and 0.01 ms and peak coil current amplitudes of 1,000 and 10,000 A (or A-turns mmf) have been considered.

In every case simulation setup, the maximum peak eddy current density in the abdominal region was estimated using about 150,000 observation points separated by 5 mm in any direction. All final results were then post-processed in MATLAB. Given the safety requirements of Refs. [17], [18] we state that the current density in the abdomen including the fetal area should not exceed 10 mA/m² for the 0.1 ms long pulse and 100 mA/m² for the 0.01 ms long pulse. Given those numbers, we can summarize our results as follows.

A. Pregnant patient

For the peak coil currents of less than or equal to 10,000 A (10,000 A-turns mmf) and for the monophasic pulse durations of greater than or equal to 0.01 ms, the maximum peak induction current in the fetal tissues does not exceed the safe limits.

An exception is the pure amniotic fluid volume for the third trimester at 10,000 A peak coil current and pulse durations of either 0.1 or 0.01 ms. Here, the maximum peak induction current may exceed the safe limit by approximately 70%.

For considerably higher peak coil currents, the safe current limit in the abdomen will not be met. Furthermore, it is important to remember that these calculations are bound to a pregnant woman of certain height. In a shorter woman, or in one in whom the pregnant uterus might rise higher due to narrower pelvis, the peak current exposure for the fetus would be expected to be higher and could exceed present guidelines.

Our prediction accuracy implies both anatomical and numerical errors. An accuracy of 100% or better in induction current maximum peak values may be expected. Typically, we overestimate the maximum values.

<table>
<thead>
<tr>
<th>Coil configuration</th>
<th>Peak numerical eddy current (mA/m²)</th>
<th>Median of ratios</th>
<th>Min. ratio</th>
<th>Median of ratios</th>
<th>Min. ratio</th>
<th>Median of ratios</th>
<th>Min. ratio</th>
<th>Median of ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil 1a [26]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td>4.93</td>
<td>15.01</td>
<td>5.5</td>
<td>18.72</td>
<td>12.21</td>
<td>5.07</td>
<td>14.88</td>
</tr>
</tbody>
</table>

TABLE V. Peak numerical eddy current densities/Ratio of analytical and numerical peak pulse values (second trimester of a patient) for the set of approximately 150,000 observation points in the abdomen.
B. Pregnant operator – standard operation

For the lowest peak coil current value of 1,000 A (1,000 A-turns mmf), the minimum safe body-to-coil separation distance is approximately 2-3 ft. given any coil configuration in Fig. 7 and both pulse durations considered. Often, the operator may use shorter distances.

For more realistic higher peak coil current values, the safe body-to-coil separation distance will exceed 3 ft. Thus, all realistic separation distances for the operator to actually hold the coil over the patients head are unsafe for any pulse lengths between 0.1 and 0.01 ms.

FDA cleared TMS devices employ a TMS coil holder. Using such coil holder the pregnant operator can keep a larger – and thus safe – distance. However, the possibility of accidental TMS coil discharge has to be considered.

C. Pregnant operator – accidental discharge

When the coil discharges very close to the belly, the induced currents in the abdomen may exceed the safe limits by approximately 10 times for 1,000 A (1,000 A-turns) peak coil current and by approximately 100 times for 10,000 A (10,000 A-turns) peak coil current. Other coil currents may be treated similarly.

D. Upper analytical estimate of eddy currents

The upper analytical estimate of eddy currents introduced in Ref. [26] retains its validity for a pregnant subject. However, the effective medium conductivity used in Eq. (3) should be increased from 0.5 S/m to 2 S/m. The average value for the ratio of the two results is about 25.

VIII. CONCLUSIONS

At present, safe limits of fetal exposure to TMS electric and magnetic fields are an open subject. Our results reveal that, for the pregnant operator, safe current limits in the abdomen will not be met in the vast majority of cases of practical importance. Therefore, pregnant women, and women of child-bearing age who think they might be pregnant, should not continue to work as TMS operators.

For the pregnant patient, safe current limits per one TMS pulse may be met. Nonetheless, given unknown biological consequences of a large number of pulses in a typical treatment sequence, the decision of whether to use TMS for treatment of depression (the only currently approved indication) has to be a risk-benefit balance. In considering the risk-benefit balance, it is important to contemplate the risk fetal risks posed by pharmacologic treatments for depression in pregnant patients [36], [37]. For more experimental and less well, evidence-supported indications, a prudent course of action would be to avoid the use of TMS in pregnant women. In any case, appropriate informed consent is critical.
REFERENCES


Chapter 06 –
Future Developments including Model Deformation Algorithms

I. PHANTOM FEATURES

A. Peripheral Nervous System and Cardiovascular System

The new version VHP-Female v.3.1 includes a state of the art peripheral nervous system which currently comprises of the radial, median, ulnar (brachial plexus) and sciatic nerves (sacral plexus) segmented over large lengths. The inclusion of cauda equina along with the brachial plexus and sacral plexus is the first of its kind, a unique feature of the VHP-F v3.1 model. This peripheral nervous system can be used to study and model various electrical stimulation systems on peripheral areas of the phantom. At the same time, the high-resolution human-head model developed in 0 has the detailed representation of twelve cranial nerves.

Figure 1 shows the graphical cartoon representations of the sciatic nerve, as well as radial, median, and ulnar nerves along with the corresponding anatomical counterparts identified in the cryosection images from the VHP-Female image dataset. The nerve tissues were registered and segmented via segmentation algorithms developed in MATLAB. The final triangular tissue mesh structure was built upon the existing point cloud, in the form of connective cylindrical elements with varying radii and lengths. The traditional ball-pivoting algorithm is not effective; it generated results with a low mesh quality since the nerves are extremely thin as compared to other tissues. Table I summarizes the existing nervous tissues and other nervous tissues considered for future development.

<table>
<thead>
<tr>
<th>Tissue Name</th>
<th>Triangle Size</th>
<th>Mesh Quality</th>
<th>Minimum Edge Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Nerve left</td>
<td>520</td>
<td>7.14E-7</td>
<td>0.39E-3</td>
</tr>
<tr>
<td>Median Nerve Right</td>
<td>492</td>
<td>5.24E-4</td>
<td>0.64</td>
</tr>
<tr>
<td>Sciatic Nerve left</td>
<td>946</td>
<td>9.63E-4</td>
<td>0.60</td>
</tr>
<tr>
<td>Sciatic Nerve right</td>
<td>932</td>
<td>7.23E-2</td>
<td>0.59</td>
</tr>
<tr>
<td>Radial Nerve left</td>
<td>762</td>
<td>8.24E-6</td>
<td>0.03</td>
</tr>
<tr>
<td>Radial Nerve right</td>
<td>476</td>
<td>2.66E-3</td>
<td>0.34</td>
</tr>
<tr>
<td>Ulnar Nerve left</td>
<td>476</td>
<td>1.90E-2</td>
<td>0.57</td>
</tr>
<tr>
<td>Ulnar Nerve right</td>
<td>540</td>
<td>4.51E-6</td>
<td>0.02</td>
</tr>
<tr>
<td>Peroneal Nerve left, Peroneal Nerve right, Femoral Nerve left, Femoral Nerve left, Saphenous Nerve left, Saphenous Nerve right</td>
<td>Under development</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**B. Muscular System**

Excluding the heart muscle, the muscular system includes major skeletal muscles (32 in total) in the form of separate objects. All muscle objects are contained within the average body object. Table II summarizes the muscle and other enhancements in VHP-Female v.3.1 as compared to VHP-Female v.2.0.

Figure 1 a) Graphical representation of sciatic nerves in the lower pelvic region b) Illustration of sciatic nerves in the cryosection image from the dataset c) Graphical and realistic illustration of radial, median, and ulnar nerves in the forearm region.
### MAJOR IMPROVEMENTS IN VHPC 3.1

<table>
<thead>
<tr>
<th></th>
<th>List of improvements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Development of sciatic nerve, ulnar nerve, radial nerve, and median nerve for the right and left side respectively (state-of-the-art). Addition of spinal cord cauda equinam.</td>
</tr>
<tr>
<td>2</td>
<td>Expansion of Systemic venous system.</td>
</tr>
<tr>
<td>3</td>
<td>Addition of erector spinae left, and erector spinae right.</td>
</tr>
<tr>
<td>4</td>
<td>Addition of pectoralis major left, pectoralis major right, pectoralis minor left, and pectoralis minor right.</td>
</tr>
<tr>
<td>5</td>
<td>Addition of abdominals left bottom, abdominals left middle, abdominals left top, abdominals right bottom, abdominals right middle, and abdominals right top.</td>
</tr>
<tr>
<td>6</td>
<td>Improvement of ribs left2, ribs left7, ribs left8, ribs left9, ribs right2, ribs right7, ribs right8, and ribs right9 along with respective cartilages. Separation of clavicle left, clavicle right from rib cage.</td>
</tr>
<tr>
<td>7</td>
<td>Development of forearm flexors left and right respectively.</td>
</tr>
<tr>
<td>8</td>
<td>Anatomical accuracy of intestine and bladder improved.</td>
</tr>
<tr>
<td>9</td>
<td>Development of spleen.</td>
</tr>
<tr>
<td>10</td>
<td>Sharp corners have been smoothed which improves the numerical accuracy.</td>
</tr>
</tbody>
</table>

### II. FUTURE APPLICATIONS

#### A. DC Excitation of Median, Radial and Ulnar Nerves

Electrical stimulation has been shown to be effective in pain relief, enhanced and accelerated nerve generation in sensory and motor neurons after peripheral nerve injury. Low-frequency pulsed current stimulation with a frequency around 20-50 Hz with varying time durations and voltages are used for electrical stimulation [2], [3].
An ANSYS Maxwell project has been created for a DC excitation of 1A using 5 passes of adaptive mesh refinement. Default boundary conditions (Natural and Neumann) are applied to a box tightly surrounding the body. Saline sponge electrodes with a conductivity of 2 S/m are used as anode and cathode (sink). The anode is a current electrode of 1A excitation with dimensions of 3.5 x 3.5 cm and with a surface charge density of 0.082 A/cm². Figure 2 shows the arrangement of electrodes in the right forearm and the magnitude of the current density in its cross-section. The volumetric current density in the nerves can also be obtained.

Similar excitation can be applied to the sciatic nerve to study the current densities for applications such as Transcutaneous Electrical Stimulation (TENS) and Spinal cord stimulation (SCS) [2]-[4]. Also any implantable pulse generator can be modeled and corresponding the stimulation can be studied in detailed. The desired parameters for such implantable devices for various locations and conditions could be also be predicted using the current state of art nervous system in VHPC v. 3.1.

![Simulation of median and ulnar nerves in the right forearm using DC excitation stimulation.](image)

**B. Real Time Modeling of Shock Wave Focusing in a Lithotripter**

A significantly simplified computational setup has been extracted from an accurate full-body VHP-F CAD model to develop some steps of a ray tracing method for shock wave modeling in a lithotripter [5]. Ray tracing performs reasonably well when the wavelength (for continuous radiation) or pulse length (for pulses) is much less than a typical geometry scale. Some preliminary simulation results have been obtained related to a shift of a focal point in the focal plane and the effect of body size on the focal pressure.
Human respiration is the exchange of air between the lung and the ambient atmosphere. Below, is the summary of some major facts pertinent to the study.

**Mechanics.** Respiratory mechanics represent a complex multi-object deformation process. It predominantly involves the non-rigid motion of the (i) diaphragm, (ii) thoracic cage including ribs, cartilage, and sternum, (iii) lungs, (iv) heart, (v) liver, (vi) kidneys, and (vii) intestine. For inhalation, the diaphragm contracts and pushes the contents of the abdomen in the inferior direction as shown in Fig. 3 [6] and Fig. 4 [7]. Simultaneously, the external intercostal muscles expand the rib cage and slightly raise it. For exhalation, the diaphragm and the external intercostal muscles relax.

Diaphragm motion. Respiration is chiefly driven by the diaphragm with primary motion in the superior-inferior direction; total travel is estimated as 10-30 mm during quiet breathing [6]. Other studies report 20±7.0 mm average [7]. A simplified 1D diaphragm motion, \( x(t) \), is non-harmonic, the exhalation portion dominates the inhalation. Given the exhalation at origin, one has \( x(t) = -A \cos \omega t \), where \( A \) is the corresponding amplitude [8],[9]. Furthermore, the respiratory motion often exhibits hysteresis in space, with the amplitude on the order of 2-4 mm [6].

Adjacent tissues. Closely adjacent structures (liver) show comparable motion amplitudes. Furthermore, the following motion amplitudes have been observed (cf. a review in Ref. [6]):

- Motion with an average amplitude of 12mm in the lung for targets not attached to rigid structures;
- 1-25 mm superior-inferior motion of the kidneys, 13 mm superior-inferior motion of the spleen, 2-8 mm motion of the heart (the heart motion is mostly a simple rigid-body translation [10],[11]), and 1-7 mm motion of the trachea;

![Figure 3](image)

![Figure 4](image)
**Thoracic cage kinematics.** During respiration, the ribs rotate about an axis through their costal necks to affect the anteroposterior and transverse diameters of the thoracic cavity as shown in Fig. 5 [10],[12].

![Figure 5](image.png)

Figure 5. Motion of the ribs during respiration, after [10],[12]. The ribs rotate about an axis through their costal neck.

**CAD B-Spline modeling.** Modeling of the breathing cycle to date has been mostly performed via deformable NURBS surfaces (B-splines) for the lungs and surrounding tissues. The changes the phantoms undergo are then typically splined over time to create time continuous 4D respiratory models [10],[13],[14], which indeed utilize free-form deformations.

**Challenges of FEM CAD Modeling.** Commercial FEM codes do not operate with B-spline surfaces but rather with triangulated surfaces and tetrahedral/hexahedral volumes.

This is in particular valid for most accurate frequency-domain E&M solvers such as ANSYS EM Suite/Maxwell 3D and CST Microwave Studio. Therefore, a free-form breathing sequence has to be ultimately converted to a (large) discrete series of separate (full-body) triangulated CAD models, even if the original data were in the form of parametric B-splines. The size of one detailed FEM full-body model is quite large (about 200-1000 Mbytes in ANSYS) and a computation with 20-30 such models would be a significant challenge from several points of view. For example, a user will need to create, run, and then post-process a number of large distinct project files, each of which must replicate his own excitation setup (e.g. a coil, an antenna, or a radar) and employ a new human model. Furthermore, a manual repositioning is necessary for any and all on-body and in-body devices at every step, which would potentially create errors.

**A. Approach**

**Built-in affine transformations.** A commercial FEM package typically includes an (incomplete) set of affine transformations:

- 3 translations (x, y, z);
- 3 rotations (about x, y, z axes);
- 3 directional scaling transformations (along x, y, z);

applicable to any object (including a triangular tissue mesh) or to a group of them and in the form of a parametric sweep. These transformations can be performed in a global or local coordinate systems. The user can initialize a global discrete time variable, \( t_n, n = 1, \ldots, N \), define object geometry parameters as certain unique functions of \( t_n \), and then move/rotate/deform every object of a multi-object structure independently within the framework of the same project file.
**Approach.** Built-in parameterized affine transformations are suggested to be applied to construct breathing cycles (quiet, deep, shallow) using only one base full-body human model and using only one project file. Along with the base static human CAD model, this project file will include a parametric sweep or sweeps modeling deformations of involved tissues. Such an approach is not exact, but it may have sufficient accuracy when the parametric sweep is carefully designed. It will allow us to employ any temporal resolution, which is impossible with discrete models.

**B. Preliminary Results**

To design an FEM-compatible and anatomically justified multi-tissue affine parametric sweep, an extensive preprocessing of a static human CAD model is necessary, which constitutes the major workload. The VHP-Female v. 3.0 full-body model has been used for this purpose. This model has been imported into MATLAB and then processed using a mesh intersection algorithm. The number of discrete time steps ranges from 11 to 33.

*Thoracic cage kinematics.* This is the first deformation step shown in Fig. 6. Since the rotation axes in Fig. 5 axes are very loosely defined for the actual anatomical data, each rib pair was rotated about fixed axes shown in Fig. 6. We have also rotated slightly the rib pairs about the vertical axis. The initial rotation angle sweep (0 to 10 or to 25 deg.) was chosen to be the same for every rib pair – see Fig. 6.

*Sternum and Cartilage.* The sternum deformation is the second deformation step shown in Fig. 7. For the present setup, the sternum is subject to a translation motion, without rotation. Control points on its surface are introduced. Those control points, along with the rib tips, form red lines, along which the corresponding cartilage parts will further be deformed (moved and expanded). The associated deformation step is shown in Fig. 8. The lower portion of the cartilage forms a single object and therefore is not deformed (moved and expanded) quite properly. This flaw is being fixed.

![Figure 6. (a) Maximum exhalation position; (b) Maximum inhalation position. VHP-Female 3.0 Base model with a reduced number of triangles.](image-url)
Lungs. Once the thoracic cage motion has been constructed, the lung deformation is processed. The initial static lung mesh is subject to all permissible variations in $\mathbb{R}^9$ space of the nine affine transformations described above. There are two major constraints: (1) anatomical, (2) FEM. According to the first constraint, the lungs should in particular occupy the maximum volume within the thoracic cage (stay as close as possible to the ribs). According to the second constraint, the FEM meshes must neither touch nor intersect. The search for an appropriate deformation sequence has been done using MATLAB parallel computing with 16-24 cores (workers) and is typically 12-16 hours long even for an FEM model with a reduced number of triangles. One such final result is shown in Fig. 9. This result is subject to improvements and other modifications under development.
Figure 9. (a) Maximum exhalation position; (b) Maximum inhalation position. VHP-Female 3.0 Base model with a reduced number of triangles.

**Muscles and body shell(s).** The next step is to deform large surrounding muscles. Abdominal muscles and pectoralis major are moved and slightly deformed (squeezed/stretch) while avoiding mesh intersections. The deformation of the static outer body shape $V_o$ shown in Fig. 10a is a more complicated process. Since this is predominantly a non-rigid transformation, we cannot apply the built-in affine transformations employed above. Instead, we suggest to use a method schematically shown in Fig. 10b,c. Two ellipsoidal-like objects $W_{1,2}$ in the form of triangular meshes of approximately the same resolution shown in Fig. 10b are created and then processed in MATLAB and ANSYS SpaceClaim. These objects are subjects to pre-defined affine transformations. At every discrete time step, $t_n$, the final outer shape $V(t_n)$ is obtained as a Boolean mesh operation

$$V(t_n) = V_0 + W_1(t_n) - W_2(t_n)$$  \hspace{2cm} (1)$$

which includes a union with $W_1(t_n)$ and the subtraction of $W_2(t_n)$. The resulting chest/abdomen outer boundary is shown in Fig. 10c by a solid black curve. This boundary evolves as time progresses.
Figure 10. Boolean operations with the outer shape of the VHP-Female model and simple ellipsoidal objects subject to affine transformations.

### IV. DEVELOPMENT OF TETRAHEDRAL MODEL OF THE VHP-F HUMAN HEAD

The VHP-F full body model currently comprises of objects represented using triangular surface mesh objects. These objects are imported into commercial software and the surface meshes undergo highly robust volumetric meshing techniques to be resolved into tetrahedral or voxels. ANSYS HFSS, for example, has an automatic adaptive mesh procedure which constructs a volumetric mesh to provide accurate and reliable results. Many of the custom made solvers and few commercial software accept only volumetric models instead of a surface based models as they lack an in-built volumetric meshing ability. Hence there is a need for accurate tetrahedral model of the VHP-F human head which can be widely used by these software.

For academic purposes and comparison studies, the volumetric mesh for the head model was extracted from ANSYS HFSS after running the model with a dipole pair for 1 pass, the model arrangement as shown in Fig. 11. The head model used only consists of average body shell, cerebellum, CSF fluid, eyes, grey matter, jaw, skull, tongue and white matter. The CSF ventricles, skin and fat have been excluded for the initial studies. Based on the tissue type, each tetrahedron in the volumetric mesh could be assigned frequency dependent materials such as conductivity and permeability.

This could be implemented by assigning a unique domain number to each tissue type. The cut model of the skin (given the material property of average body) and CSF fluid have been processed using the ANSYS SpaceClaim.
Figure 11. ANSYS HFSS project with the dipoles arranged around the VHP-F head model.

The tetrahedral mesh information including the vertices and faces is found in the result folder of ANSYS HFSS project and has the extension .ngmesh. The corresponding tetrahedral vertices and faces are extracted using the import tool in MATLAB. All the individual object tetrahedral meshes are then created using a simple inoutstatus check on centers of tetrahedral volumetric mesh with respect to the corresponding object’s surface mesh. For example, the grey matter’s triangular surface mesh is used to obtain the tetrahedral elements which are within the surface defined by the mesh. These tetrahedral volumetric meshes can at times have common overlapping tetrahedral elements; for example, the tetrahedral mesh formed for the grey matter earlier also contains tetrahedral elements from white matter and CSF ventricles. Therefore, the obtained volumetric meshes were further processed in MATLAB to delete the common overlapping tetrahedrons.

Each tissue domain can be assigned a unique domain number to distinguish it from the other tissues. The domain number can also repeat if the material property is used as the indicator; for example, the domain number for skull and jaw would be same as they are both made of cortical bone. Figure 12 shows the current tetrahedral model of the VHP-F head in MATLAB. The edges of the tetrahedron for the average body have been made transparent for better visibility of inner tissues. This tetrahedral model is currently available in .mat and can be converted to other format using custom scripts. This model is currently used for comparison studies between ANSYS HFSS and custom built MATLAB Method of Moments (MoM) electromagnetic solver.
V. CONCLUSION AND FUTURE WORK

In this chapter, we have described the future potential developments of the platform-independent full-body electromagnetic model, the Visible Human Project® (VHP)-Female V.3.1 originated from the U.S. National Library of Medicine and briefly described its new features. Further model development is underway including a complete peripheral nervous system, deformable respiratory system implemented as a parametric sweep, and heart beat sequence.

This Visible Human Project® (VHP)-Female V.3.1 has allowed us to form a platform to build a family of models, including adult male, adult woman and children with variable BMI. Various 3D custom implant models such as pacemakers/pulse generators, can be registered into the VHP-F model for device modeling studies.

REFERENCES


