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Design of Structural Composite with Auxetic Behavior

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Design of Structural Composite

with Auxetic Behavior

A Major Qualifying Project
Submitted to the faculty of
Worcester Polytechnic Institute
In partial fulfillment of the requirements for the
Degree of Bachelor of Science

Submitted By:
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Submitted To:
Project Advisor: Nima Rahbar

Date: April 27th, 2017

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Abstract

Auxetic materials are intriguing materials with unusual capabilities. Energy absorption within the auxetic materials make for an interesting possible application for impact-resisting reinforcement for construction materials. In this project, geometric shapes were analyzed to discover the effect the internal reentrant angle has on maximizing the auxetic effect. These analyses use data from Finite Element models to determine the possibility of future use as a construction material. The results of this study can be used to develop high strength cement composite with superb mechanical properties.
Executive Summary

Construction affects every person’s life. Without it homes, places of work, and roads would not exist. Investments into structures and infrastructure are costly, and are often limited by amounts of capital available.

Intelligent construction and design allows for optimized systems with excellent performance, while also having a low cost for materials and labor. Using already existing materials, it is possible to increase the strength by utilizing nontraditional material properties. As concrete is typically weak in tension, utilizing auxetic geometries can encourage compression behaviors in concrete. The goal of this project was to explore the strength of an auxetic reinforcement system for use in concrete.

![Figure 1 Auxetic Unit Cell Geometries](image)

The first objective was to find the internal angle that produced the strongest auxetic effect. Two geometries were explored, a Reentrant Hexagon and a Tube and Sheet model. The reentrant angle of these two geometries, shown in Figure 1, was varied at intervals. Finite Element models are utilized to calculate the strains under an applied displacement. The strains are then used to calculate the Poisson’s ratio of the model. Regression equations are constructed to find the most negative Poisson’s ratio for the models. Figure 2 shows that the Reentrant Hexagon models tend to produce a less auxetic behavior than the Tube and Sheet model.
The second objective was to determine the rupture strength of the auxetic composite. This was accomplished by creating Finite Element models that used the most auxetic angle, and having the cavities filled with isotropic concrete. Using interpolation methods, rupture strengths were calculated and verified. The results from the Finite Element models are given in Figure 3. The Reentrant Hexagon unit cell increased the nominal strength of the 4 ksi concrete by over 25%. Conversely, the more auxetic Tube and Sheet unit cell composite decreased the strength by 25%. Tensile forces were promoted by the model, causing premature failures.

<table>
<thead>
<tr>
<th>Composite Unit Cell Style</th>
<th>Poisson's Ratio</th>
<th>Elastic Modulus (psi)</th>
<th>Maximum Allowable Load (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube and Sheet</td>
<td>-0.016</td>
<td>7,400,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Reentrant Hexagon</td>
<td>0.125</td>
<td>6,390,000</td>
<td>5,300</td>
</tr>
</tbody>
</table>

The third and final objective was to discover the effect of changing the dimensions of the unit cells. The volume fraction of steel was kept constant, and models were constructed to determine the change in strength. After testing a few models, general trends were able to be established. Figure 4 shows that as the length of the Tube and Sheet unit cell increases, the strength decreases. The horizontal legs of the unit cell get more slender, allowing for tensile forces to develop at lower loads. The
Reentrant Hexagon unit cell, on the other hand, seemed to increase in strength to a point, before losing strength at longer lengths.

Figure 4 Failure Loads for Variable Composite Dimensions
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1.0 Introduction

Globally, there are over 2.2 billion tons of concrete produced annually (Crow, 2008). As one of the most abundant construction materials, concrete has one glaring weakness, that it is much weaker under tensile stresses than it is under compression. This often requires reinforcement to limit the development of tensile stresses in concrete members, which increases the cost of the member. In nearly all construction projects, cost impacts design choices heavily.

Concrete and its typical reinforcement material, steel, differ strongly in cost. 60 ksi reinforcing steel ($2,240 per ton) about 30 times more expensive than concrete ($76 per ton) (Huynh, 2013). Creating a system that utilizes both the high strength of steel and the low cost of concrete makes logical sense. Typical reinforced concrete design utilizes the reinforcing steel to resist tension forces, and relies on concrete to resist compressive forces.

Under this typical design style, the concrete can still fail due to tensile forces. This comes from concrete possessing a positive Poisson’s ratio, which relates how the material deforms along two axes. As the concrete compresses, it is free to expand proportionally along a perpendicular axis. To remedy this issue, a negative Poisson’s ratio can be induced.

Materials possessing this property are said to be auxetic. The auxetic property is relatively rare for isotropic materials, making material selection difficult. Use of auxetic geometries is then the best course of action. The classical construction materials can be used with these geometries. It is then imperative that the strongest version of the auxetic geometries be used for construction purposes.

The purpose of this project is to identify the optimal geometries of an auxetic reinforcement system. This was accomplished using finite element models at a unit depth. The results from these models indicated that at the proper dimensions, the auxetic reinforcement can increase the strength of concrete compression members. With this information, designers can develop designs that can resist large amounts of pure compression.
2.0 Background

This chapter will focus on concepts that were needed to understand how auxetic materials function. Topics covered in this section include the history of the Poisson ratio, possible applications of the auxetic material geometry, and other work in the auxetic materials research world.

2.1 History of the Poisson Ratio

The focus of the research done in this project is to understand the Poisson ratio of a unit material. It is therefore necessary to understand exactly what the property is. This section will discuss the discovery of the Poisson ratio in two eras, as well as the discovery of materials with auxetic properties.

2.1.1 Early Work

Thomas Young, in his lecture “On Passive Strength and Friction,” speaks of how materials behave when subjected to a variety of actions. Of particular interest is his observation of elastic gum under compression and, as he says, extension. Young notes that under compression, the gum “extends itself in other directions,” and under tension, “its breadth and thickness are diminished” (Young, 1807). This observation is one of, if not the earliest, observations made about how materials deform along different axes than they are loaded on. The French mathematician Simeon Denis Poisson then further analyzed his initial thoughts. Poisson, though testing of cast-iron in compression, found the ratio to be a fixed value, of 1:4. Many contemporaries of Poisson, notably Baron Charles Cagniard de la Tour and Gustav Kirchoff (Lim, Auxetic Materials and Structures, 2015), experimentally measured a variety of other materials, and found the ratio to be different for each material tested. Here, it became apparent that the value of the Poisson ratio was not a constant value, but instead varied from material to material.

2.1.2 Modern Work

From a basic engineering application, the Poisson ratio has its obvious applications. For example, knowing how a given material will deform enables better performance in low-tolerance situations. The Poisson ratio also enables better understanding of other material properties. Through it, we are able to understand how the Bulk Modulus and Shear Modulus of a material are related (Lakes, Advances in Negative Poisson's Ratio Materials, 1993). Through this relationship, we are able to see that materials with a high Bulk modulus are more likely to exhibit high Poisson ratios. Figure 5 shows this relationship. It is worth noting that this diagram stops at the maximum isotropic limit for Poisson's ratio, \( v = 0.5 \) (Gercek, 2006). This limit is defined by functions of thermodynamics, and cannot be
passed by isotropic materials. Anisotropic materials, however, may pass this value, as internal geometric effects may cause the material to behave in an irregular manner.

2.1.3 Auxetic Materials

Most naturally found materials have positive Poisson ratios. Under small global deformations, these materials will deform in such a way that they attempt to maintain a semblance of a consistent volume, as shown in Figure 6. Both steel and concrete, two major isotropic construction materials, exhibit this quality, with $\nu=0.3$ and $\nu=0.15$, respectively, as calculated in Figure 7 (Lakes, Meaning of Poisson's Ratio, 2017).
Materials that exhibit the behavior shown in Figure 8 are much less common in nature. A 1992 study (Yeganeh-Haeri, 1992) found a type of naturally occurring SiO$_2$ that exhibited auxetic properties, with the average Poisson ratio coming in at -0.16. However, many manmade materials have a negative Poisson’s ratio. Foams, yarns, textiles, crystal and liquid crystal materials have all been synthesized and shown to possess auxetic properties (Lim, Auxetic Materials and Structures, 2015).

### 2.2 Application of Negative-Poisson Ratio Materials

#### 2.2.1 Current Applications

Auxetic structures are used in the installation of stents (Gatt, et al., 2015). As shown in Figure 9, the auxetic properties of the stent allow it to have a small cross sectional area under a compressive force, allowing for a simpler installation into a blood vessel. When the compressive force on the stent is released, it is free to expand into its full size, and push the constricted blood vessel into a more free-flowing shape (American Heart Association, 2015).
The use of porous auxetic materials is also useful for the creation of variable-sized filters (Mir, Ali, Sami, & Ansari, 2014). Due to the uniform deformation of the auxetic material, a known-sized hole can be created from a non-specialized base filter. The filter can be used for a variety of filtration needs, and changed to filter out different sized-particles with relative ease.

Research into the impact resistance (Qiao & Chen, 2015) has shown that auxetic shapes have a high resistance to impact loads. Arraying the auxetic meshes in something like a bulletproof vest or other personal protection equipment can provide a low-cost, lightweight solution to protecting the wearer without needing to utilize heavy ceramics or expensive Kevlar fibers.

2.2.2 Construction Applications

Similar to the impact protection capabilities that auxetic materials can give on the personal level, the auxetic materials can be utilized in the construction industry. Governments and militaries require structures that can resist small scale impacts in the form of small arms fire, as well as large-scale explosive impacts. Possible solutions to this problem can be to “overdesign” the structure, allowing for a larger resistance to the load, which can yield an expensive, nonaesthetic structure, or use of small, blast resistant container structures (Figure 10). These containers, while unregulated by the construction industry (Redguard, 2017) have a high nominal performance against blasts and impacts, making them ideal for use in the field. However, a more permanent, larger structure may be required.
Implementation of materials with auxetic properties can allow for current building techniques to have high-blast resistant properties. Auxetic reinforcement in typical reinforced concrete design can help provide concrete with the means to maximize its compressive resistance to the blast.

2.3 Other Work with Auxetic Materials
This section discusses other research done with auxetic materials. This includes the development of auxetic materials using mechanical interactions, as well as an exploration of some geometries that have been shown to have auxetic properties.

2.3.1 Mechanical Systems
In order to obtain the auxetic properties, an examination of mechanical systems was performed on rotating, rigid square elements (Gatt, et al., 2015). This study sought to utilize hinged elements to generate a hierarchical structure (Figure 11) that could possibly be tiled infinitely. The study concluded that as the system increased in size, or “levels”, the hierarchical system changed its auxetic properties. Additionally, if the stiffness of the hinges varied between the levels, the properties of the structure would vary as well. By manipulating the structure, it is possible to truly generate a unique set of properties that can be tailored to the desire application.
2.3.2 Geometry

Several geometries have been studied and shown to exhibit auxetic behaviors. The auxetic property is largely due to a controlled buckling effect. A large number of these geometries have been discovered. One study of auxetic geometries, given by Alvarez and Lantada (Alvarez Elipe & Lantada, 2012) organized them into a single list, totaling 24 auxetic geometries. The auxetic shapes can be either 2-D or 3-D, but the overall deformation mechanism remains similar for all. For example, Figure 12 demonstrates what Alvarez and Lantada call the “Re-entrant triangular” shape. All edges of the shapes are at an angle, which makes the shape reentrant. The reentrant-style geometry is found in both the 2-D auxetic shapes as well as the 3-D auxetic solids.
An example of a reentrant 3D solid is shown in Figure 13 (Lim, A 3D auxetic material based on intersecting double arrowheads, 2016). This 3D model is essentially an extrusion of the chevron model shown in Figure 12, indicating that multi-axial auxetic properties are possible.

![Figure 13 3D Auxetic Arrowhead Unit Cell](image)

A second type of auxetic geometry is the chiral variety. Figure 14 demonstrates what Alvarez and Lantada call “Chiral square.” These types of unit cells typically consist of a geometric shape with linear sides, and are connected to a second geometric shape. Triangular and hexagonal auxetic cells complement the square chiral unit cell shown in Figure 14 (Alvarez Elipe & Lantada, 2012).

![Figure 14 Square Chiral Unit Cell (Lim, Auxetic Materials and Structures, 2015)](image)

This analysis examines two reentrant-style geometries, the Reentrant Hexagon and a reentrant Tube and Sheet. These geometries are discussed in detail later in the report.
3.0 Models and Calculations

In this chapter, the methods used to analyze the various unit cells and composites will be discussed. This includes discussion of the modelling techniques, calculations of internal angles for auxetic geometries, and the determination of the Poisson ratio from model results.

3.1 Models

In this section, the various model geometries being analyzed will be discussed. The model shapes being analyzed include a control isotropic bar, a composite non-auxetic geometry, an auxetic reentrant hexagonal geometry, and an auxetic “tube-and-sheet” geometry will be discussed.

The “Control” model is a simple 2D rectangle. Isotropic materials, steel and concrete, will be used for this model. The model will be subjected to the same loading conditions as the other models in this analysis, and the same method of collecting the data will be used. Results from this model should be the same as the material properties as the material that was used in the model.

![Figure 15 Reentrant Hexagon Unit Cell](image)

The Reentrant Hexagon model (Figure 15) is derived from Yang et al (Yang, Lee, & Huang, 2003). This geometry has been shown to possess auxetic properties. The unit cell for this model consists of two central “stems” equal to half the length of the vertical walls on the left and right edges of the cell. Diagonal walls connect the stems to the walls.
The Tube and Sheet model is derived from Zhang et al. (Zhang, Hu, Liu, & Xu, 2013). The original model consists of circular tubes separated by a thin metal sheet. The sheet is fused to the tubes, and the entire structure becomes a single geometry. Due to concerns about shear stresses, an alternate model based on Zhang et al. was examined. This model, shown in Figure 16, will have a similar shape to the base Tube and Sheet model, but the plate thickness will be the same as the tube wall thickness. The two models were compared to each other to determine their auxetic effects, along with the internal stresses within the unit cell.

3.2 Variations on Models
This section will discuss the materials used, as well as the configurations of analysis used for each of the models.

3.2.1 Materials
The main structure of the models will all be made of A992 Steel, with the Elastic Modulus equal to 29,000 ksi. The Shear Modulus for the steel is equal to 11,200 ksi. Using Equation 1, we can find the Poisson Ratio for steel to be equal to about 0.30.

\[ v = \frac{E}{2G} - 1 = \frac{29000 \text{ ksi}}{2 \times 11200 \text{ ksi}} - 1 = 0.294 \approx 0.3 \]

Equation 1 Calculating Poisson’s Ratio of Steel
Additionally, a normal weight concrete was looked at for the inner cavity of each model. The concrete will have a nominal $f'_{c}$ of 4000 psi (4 ksi). The calculation for the Young’s Modulus is shown in Equation 2.

$$E_c = 57000 \times \sqrt{f'_{c}} = 57000 \times \sqrt{4000 \text{ psi}} = 3604996.5 \text{ psi} \approx 3605000 \text{ psi}$$

Equation 2 Calculation of Elastic Modulus for Concrete

Similarly, the Shear Modulus, also known as the Modulus of Rigidity, was calculated using Equation 3.

$$G_c = 24800 \times \sqrt{f'_{c}} = 24800 \times \sqrt{4000} = 1568489.7 \text{ psi} \approx 1568000 \text{ psi}$$

Equation 3 Calculation of Shear Modulus for Concrete

Substituting these values into the calculation for Poisson’s Ratio, we obtain a Poisson’s Ratio of 0.15 for the 4000 psi normal weight concrete.

$$\nu = \frac{E}{2G} - 1 = \frac{3605 \text{ ksi}}{2 \times 1568 \text{ ksi}} - 1 = 0.150$$

Equation 4 Calculating Poisson’s Ratio for Concrete

The concrete material was used to fill the cavity in each of the unit cells. This was done to see the effect that the steel structure will have on the concrete when acting as reinforcement. The performance of the auxetic geometries were examined both with and without the concrete core, in both the unit cell and the tiled configurations.

### 3.2.2 Tiling

As a single unit cell is not representative of the material, a larger model made of tiled unit cells were also examined. The model will be constructed of a 10-long by 5-high collection of the unit cell being examined. This was accomplished by using Abaqus/CAE’s built-in linear pattern tool, and then joining the created geometries into a single model. An increase in the number of unit cells examined in this model is possible, and is easily done for future examination of the geometries. The edges of the tiled geometries will be free to deform and no edge walls were added to attempt to stabilize the system. Purpose of this is to allow the system to be as representative of a system cut from a manufactured sheet as possible. It would not be feasible for the geometries used to be custom-built for each application, meaning that the free-to-deflect version of the geometry will be representative of an infield application. The unit cells were tested for both material systems, with the cavity hollow and the cavity filled with concrete.
3.3 Internal Angle

The location of the internal angle, $\alpha$, for both the Reentrant Hexagon and modified Tube and Sheet models are shown in Figure 17. The values for the angle were taken from the same reference point. The angle that the diagonal leg makes against the x-axis (horizontal) will be the value used for the internal angle measure. It is also important to note that in the Tube and Sheet model, the angle at which the exterior portion of the sheet begins to wrap around the tube section is equal to the internal angle measure, as in Zhang et al.’s analysis (Zhang, Hu, Liu, & Xu, 2013).

![Figure 17 Location of Internal Angle in Each Unit Cells](image)

3.4 Poisson Ratio Calculations

Utilizing the analytical capabilities of Finite Element Method software, it is simple to extract the displacement of the model being tested. Using Abaqus/CAE’s nodal probe tool, it is possible to quickly extract the entire set of coordinates for each node. Using this dataset, we are able to determine the locations of the edge nodes for the model. The original length, $L$, can be found by finding the distance between the nodes with the smallest (most negative) x-coordinate and the nodes with the largest (most positive) x-coordinate. Similarly, the original height, $H$, is able to be found by finding the distance between the nodes with the largest y-coordinate and those with the smallest y-coordinate.
After determining the locations of the nodes whose coordinates correspond to the minimum and maximum values along either the x- or y-axis, the average difference between the deformed set of coordinates for those is calculated. This is done by finding the average coordinate among the set of nodes, and using the difference between the coordinates to calculate the deformed length, \( l \) and deformed height, \( h \).

The difference between the original length, \( L \), and the deformed length, \( l \), is considered the change in length, \( \Delta l \). Similarly, the difference between the original height, \( H \), and deformed height, \( h \), is considered the change in height, \( \Delta h \). These two values are then used to calculate the x-direction strain \( \epsilon_x \) and the y-direction strain \( \epsilon_y \), using Equation 5.

\[
\epsilon_x = \frac{\Delta L}{L}; \epsilon_y = \frac{\Delta H}{H}
\]

Equation 5 Equation for Strain

Using the definition of Poisson’s Ratio, Equation 6, we can then find the overall Poisson’s ratio for the model being tested. This calculated value can then be compared to other values found through testing.

\[
\nu = -\frac{\epsilon_y}{\epsilon_x}
\]

Equation 6 Equation for Poisson’s Ratio
4.0 Results

This section will discuss the results of the testing of finite element models. First, a sample analysis is performed on an isotropic material, to test the data collection process and calculations. Then, analyses were performed to determine what internal angle gave the most negative Poisson’s ratio for both a single unit cell and for tiled unit cells.

4.1 Control Shapes

To test the methodology for creating the models, isotropic rectangular unit shapes were first examined. These shapes are simple, 0.5” high and 2” long 2D elements. The shapes were fixed for both rotation and horizontal displacement on one side. The shapes then had a unit strain of -0.01 applied on the free end, putting it in compression. A schematic drawing of the setup is shown in Figure 18. The model was then examined using FEM software. The longitudinal strains and transverse strains were then collected from each element’s integration point. The Poisson’s Ratio is then be calculated using Equation 7.

\[ \nu = -\frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{longitudinal}}} \]

Equation 7

Figure 18 Schematic Drawing of the Control Tests
The tests were run on two different materials, in order to verify a correct method of calculating the Poisson’s Ratio. The two materials used were 4 ksi normal weight concrete, and A992 steel using rounded values for the Elastic moduli and Poisson Ratios calculated in Figure 7. The deformed shapes for the concrete and steel tests are shown in Figure 19 and Figure 20, respectively.

The concrete bar had a uniform longitudinal strain for each element, the applied \(-0.01\). The transverse strain was similarly constant for all elements, 0.003. Using Equation 7, the calculated Poisson’s Ratio is 0.30 (Equation 8). This value is exactly equal to the value used in the material properties, indicating the parameters for calculation were chosen correctly.

\[
\nu_{\text{concrete bar}} = - \frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{longitudinal}}} = -\frac{0.003}{-0.01} = 0.30
\]

Equation 8 Calculation of Concrete Control Bar Poisson Ratio
Looking at the steel bar, we see a similar result. The longitudinal strain was constant for all elements, and had the value of the applied strain, -0.01. The transverse strain was also constant for all elements, and was equal to 0.0015. Putting these values Equation 7, we can calculate the Poisson’s Ratio to be 0.15 (Equation 9). As with the concrete bar, this value is the same as the one used in the material properties for the model.

\[
\nu_{\text{concrete bar}} = -\frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{longitudinal}}} = -\frac{0.0015}{-0.01} = 0.15
\]

Equation 9 Calculation of Steel Control Bar Poisson Ratio

### 4.2 Reentrant Hexagons

The first auxetic geometry that was looked at is a reentrant hexagon. This geometry was inspired by the work of Yang et al. (Yang, Lee, & Huang, 2003). In their paper, Yang et al. sought to apply 2D micropolar elasticity theory to find the structural Poisson’s ratio of the reentrant hexagonal honeycomb element. The study sought to see the effect of varying geometries of the reentrant hexagon model on the structural Poisson’s ratio. The values analyzed include varying the cell rib width, or wall thickness in this paper, and the reentrant angle, or the internal angle in this paper. To simplify the analysis, only the second half of the analysis that Yang et al. performed, where the angle was varied and the wall thickness remains constant, will be looked at.

#### 4.2.1 Single Unit Cell

To compare the reentrant hexagon geometries, several dimensions are constrained in each analysis. The wall thickness for all reentrant hexagon shapes tested are 0.030 inches, and half that value, 0.015,
at the left and right edges of the unit cell. The length of the unit cell remains constant at 0.70 inches. The vertical spacing in the cavity was constant at 0.20 inches. Figure 21 shows the generalized geometry.

![Generalized Geometry of Reentrant Hexagon Unit Cell](image)

Keeping these values constant, we are able to vary the angle of the sloped reentrant hexagon and the overall height of the unit cell. For the purposes of tiling the unit cell, the length of the stems of the unit cell were equal to half of the length of the flared portion of the unit cell. The unit cells were examined under an applied displacement, resulting in a strain of:

\[
\varepsilon = \frac{|\Delta l = -0.01|}{l = 0.5"} = 0.02
\]

Figure 22 shows the deformed geometries for the smallest (5 degree) and largest (50 degree) internal angle values tested, with the elemental stresses overlaid.
By observing the two extremes, it is apparent that the two different variations of the reentrant hexagon deform very differently. The two values of interest for determining the Poisson ratio and the strains along the longitudinal and transverse axes are very different.

Figure 23 summarizes the calculated strain values for the reentrant hexagon unit cells that were tested, and Figure 24 plots it. From this data, we can see that the most negative Poisson ratio seems to occur to geometries with an internal angle between 5 and 15 degrees.
Knowing this, more data points between 5 and 10-degree were analyzed to provide better resolution within the area. Eight additional data points representing this range were added to the data represented in Figure 24. As shown in Figure 26, the lowest Poisson ratios now seem to occur at about an internal angle of 11%. Equation 10 shows a third-degree regression for the “low-angle” reentrant hexagon. This equation is plotted against the low-angle data for the reentrant hexagon unit cells in Figure 26.
\[ v = -0.007 a^3 + 0.0340 a^2 - 0.4825 a + 0.3860 \]

Equation 10 Low-Angle Reentrant Hexagon Poisson Ratio Formula

The ratios of the FEM Poisson ratios for the 5- to 15-degree range to the Poisson ratios calculated using Equation 10 are given in Figure 24. From Figure 25, it is clear that the regression will provide values within 1% of the FEM values.

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Poisson's Ratio from FEM</th>
<th>Poisson's Ratio from Regression</th>
<th>FEM / Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.27</td>
<td>-1.26</td>
<td>1.002400</td>
</tr>
<tr>
<td>6</td>
<td>-1.44</td>
<td>-1.44</td>
<td>0.999900</td>
</tr>
<tr>
<td>7</td>
<td>-1.57</td>
<td>-1.57</td>
<td>0.997000</td>
</tr>
<tr>
<td>8</td>
<td>-1.66</td>
<td>-1.67</td>
<td>0.993300</td>
</tr>
<tr>
<td>9</td>
<td>-1.74</td>
<td>-1.73</td>
<td>1.006700</td>
</tr>
<tr>
<td>10</td>
<td>-1.77</td>
<td>-1.76</td>
<td>1.002800</td>
</tr>
<tr>
<td>11</td>
<td>-1.78</td>
<td>-1.77</td>
<td>1.001400</td>
</tr>
<tr>
<td>12</td>
<td>-1.76</td>
<td>-1.76</td>
<td>0.999000</td>
</tr>
<tr>
<td>13</td>
<td>-1.73</td>
<td>-1.74</td>
<td>0.997500</td>
</tr>
<tr>
<td>14</td>
<td>-1.7</td>
<td>-1.7</td>
<td>0.996700</td>
</tr>
<tr>
<td>15</td>
<td>-1.66</td>
<td>-1.66</td>
<td>1.003200</td>
</tr>
</tbody>
</table>

Figure 25 Ratio of FEM Poisson Ratio to Regression Poisson Ratio for Unit Cell Reentrant Hexagons

From Equation 10, solving for the local minimum in the range of the angles, it becomes apparent that the lowest Poisson ratio for this geometry is -1.77, and occurs with an internal angle of 10.93 degrees. This value seems to agree with the 11% internal angle value for the location of the most negative Poisson ratio.
4.2.2 Tiled Unit Cells

To create the tiled geometry, the unit cells exist in a 5 by 10 pattern. Due to their demonstration of having a more negative Poisson ratio, the low-angle unit cells were the only set analyzed. Larger-angle unit cells would exhibit less auxetic properties. 11 total angle measures were looked at, ranging from 5 degrees to 15 degrees, inclusive. As with the unit cells, the models were fixed for translation along the longitudinal axis, and a displacement of -0.01 inches applied at the free end. Due to the overall length of the models being fixed at 5 inches long, this results in an applied strain of:

\[
\epsilon = \frac{|\Delta l = -.01|}{l = 5"} = 0.002
\]

The reentrant hexagon unit cells for the 5- to 15-degree range were tiled in a 5-tall by 10-wide pattern. The analyses for the tiled cells were limited to the range of internal angles used in Figure 26. This was done to maximize provide a focus on the range of angles that gave the maximum Poisson ratios in the unit cell format. The two deformed and undeformed composite images for the minimum internal angle, 5 degrees, and the maximum internal angle, 15 degrees, are presented in Figure 27.
Figure 27 5-Degree (left) and 15-Degree (right) Tiled Reentrant Hexagon

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.37</td>
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<tr>
<td>6</td>
<td>-1.56</td>
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<tr>
<td>7</td>
<td>-1.71</td>
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<tr>
<td>8</td>
<td>-1.86</td>
</tr>
<tr>
<td>9</td>
<td>-1.94</td>
</tr>
<tr>
<td>10</td>
<td>-2.04</td>
</tr>
<tr>
<td>11</td>
<td>-1.92</td>
</tr>
<tr>
<td>12</td>
<td>-1.92</td>
</tr>
<tr>
<td>13</td>
<td>-1.87</td>
</tr>
<tr>
<td>14</td>
<td>-1.84</td>
</tr>
<tr>
<td>15</td>
<td>-1.79</td>
</tr>
</tbody>
</table>

Figure 28 Reentrant Hexagon Tiled Cells Calculated Strain Values and Poisson Ratios

Figure 28 lists the results of the 5- to 15-degree analysis of the tiled unit cells. A regression analysis results in Equation 11. Solving for the minimum value within the range of 5 to 15 degrees, the most negative Poisson’s Ratio for the tiled unit cell is -1.96 with an internal angle of 10.44 degrees. The regression equation is plotted against the data from Figure 28 in Figure 29.

\[ \nu = -0.001184\alpha^3 + 0.051353\alpha^2 - 0.685201\alpha + 0.939533 \]

Equation 11 Regression Equation for Tiled Reentrant Hexagon Unit Cells
4.2.3 Comparison

The values for the 10-unit cell long by 5-unit cell tall tiled geometries are compared against the Poisson ratios of their respective unit cells in Figure 30. On average, there is a 10% gain in the effective Poisson ratio for the tiled geometry as compared to the single unit cell.

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Unit Cell Poisson's Ratio</th>
<th>Tiled Cell Poisson's Ratio</th>
<th>Tiled / Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.27</td>
<td>-1.37</td>
<td>1.08</td>
</tr>
<tr>
<td>6</td>
<td>-1.44</td>
<td>-1.56</td>
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<tr>
<td>7</td>
<td>-1.57</td>
<td>-1.71</td>
<td>1.09</td>
</tr>
<tr>
<td>8</td>
<td>-1.66</td>
<td>-1.86</td>
<td>1.12</td>
</tr>
<tr>
<td>9</td>
<td>-1.74</td>
<td>-1.94</td>
<td>1.12</td>
</tr>
<tr>
<td>10</td>
<td>-1.77</td>
<td>-2.04</td>
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<tr>
<td>11</td>
<td>-1.78</td>
<td>-1.92</td>
<td>1.08</td>
</tr>
<tr>
<td>12</td>
<td>-1.76</td>
<td>-1.92</td>
<td>1.09</td>
</tr>
<tr>
<td>13</td>
<td>-1.73</td>
<td>-1.87</td>
<td>1.08</td>
</tr>
<tr>
<td>14</td>
<td>-1.7</td>
<td>-1.84</td>
<td>1.09</td>
</tr>
<tr>
<td>15</td>
<td>-1.66</td>
<td>-1.79</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Equation 10 and Equation 11 are both tabulated in Figure 31. On average, the tiled geometries have a 10% larger negative Poisson ratio than their respective unit cell counterparts. The percent variation between the tiled and single unit cell are the same for both the FEM results and the mathematical models.
The modified Tube and Sheet unit cell is examined with similar proportions as the reentrant hexagon unit cell. The overall length was kept at 0.5”, and the vertical cavity space was kept at 0.1”. The thickness of the tubes and legs is kept at 0.03”. The internal angle is measured from the x-axis to the angled portion of the legs. These dimensions are shown in

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Unit Cell Poisson's Ratio</th>
<th>Tiled Cell Poisson's Ratio</th>
<th>Tiled / Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.26</td>
<td>-1.35</td>
<td>1.07</td>
</tr>
<tr>
<td>6</td>
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<td>11</td>
<td>-1.77</td>
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<tr>
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<td>14</td>
<td>-1.70</td>
<td>-1.84</td>
<td>1.08</td>
</tr>
<tr>
<td>15</td>
<td>-1.66</td>
<td>-1.78</td>
<td>1.07</td>
</tr>
</tbody>
</table>

4.3 Modified Tube and Sheet Unit Cell

The modified Tube and Sheet unit cell is examined with similar proportions as the reentrant hexagon unit cell. The overall length was kept at 0.5”, and the vertical cavity space was kept at 0.1”. The thickness of the tubes and legs is kept at 0.03”. The internal angle is measured from the x-axis to the angled portion of the legs. These dimensions are shown in

Figure 31. Ratio of Calculated Poisson Ratio for Hollow Reentrant Hexagon Geometries

Figure 32. The models were all strained to 2%, as with the reentrant hexagon models.
4.3.1 Single Unit Cell

The finite element results for the smallest and largest angles tested are shown in Figure 33. The von Mises stresses are overlaid on the deformed shape. The maximum stresses in both models are in the same locations, but in the large-angled unit cell, the high-stress region extends through the thickness of the tube, indicating a higher level of stress.

The calculated Poisson’s ratios for the unit cells are given in Figure 34. Additional data points were added in the 1-to-10 degree range after noticing a parabolic pattern in the original set of models. This pattern is better realized in Figure 35.
All data points tested for the modified Tube and Sheet unit cell are shown. The shape of the plot is similar in shape to that of the reentrant hexagon data for the small angles. The Poisson’s ratio becomes positive around 21 degrees, indicating the potential for interesting properties. Additionally, the magnitude of the Poisson’s ratios seems to be larger for the modified Tube and Sheet model than for the Reentrant Hexagon.

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.52</td>
</tr>
<tr>
<td>2</td>
<td>-5.31</td>
</tr>
<tr>
<td>3</td>
<td>-6.9</td>
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<tr>
<td>4</td>
<td>-7.25</td>
</tr>
<tr>
<td>5</td>
<td>-6.93</td>
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<tr>
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<td>-6.24</td>
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</tr>
<tr>
<td>25</td>
<td>0.4</td>
</tr>
<tr>
<td>30</td>
<td>0.6</td>
</tr>
<tr>
<td>35</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Figure 34 Modified Tube and Sheet Unit Cell Calculated Strain Values and Poisson Ratios
Figure 36 shows the data from the low-angles tested. Using a regression analysis, Equation 12 was generated. This function is overlaid on the data from Figure 36, showing a strong level of agreement between the regression and the collected data. Solving for the minimum value of the function along the domain of angles tested, the most negative Poisson’s ratio is -7.25 and occurs at 4.22 degrees.

\[
v = -0.036\alpha^3 + 0.7845\alpha^2 - 4.7025\alpha + 1.3191
\]

Equation 12 Low-Angle Modified Tube and Sheet Unit Cell Poisson Ratio Formula
4.3.2 Tiled Unit Cell

Figure 37 shows the calculated Poisson’s ratios for the 10 by 5 tiled modified Tube and Sheet unit cells. After a 6-degree internal angle, the deformed geometry self-collided, making the data not useful for calculating the true Poisson’s ratio.

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.22</td>
</tr>
<tr>
<td>2</td>
<td>-5.22</td>
</tr>
<tr>
<td>3</td>
<td>-6.58</td>
</tr>
<tr>
<td>4</td>
<td>-7.28</td>
</tr>
<tr>
<td>5</td>
<td>-7.06</td>
</tr>
<tr>
<td>6</td>
<td>-6.44</td>
</tr>
</tbody>
</table>

Figure 37 Tiled Modified Tube and Sheet Unit Cell Calculated Strain Values and Poisson Ratios

Figure 38 plots this data set, along with the regression equation, Equation 13. The data has a minor amount of disagreement in the 2-to-4 degree range, with the regression equation suggesting lower values than the calculated data shows. Keeping this in mind, solving Equation 13 for the minimum Poisson’s ratio yields a minimum of -7.25 at an angle of 4.49 degrees.

\[
\nu = -0.0496\alpha^3 + 0.9569\alpha^2 - 5.4059\alpha + 2.2529
\]

Equation 13 Low-Angle Tiled Modified Tube and Sheet Unit Cell Poisson Ratio Formula
4.3.3 Comparison

The found Poisson’s ratios for the single unit cell are compared against those of the tiled unit cells in Figure 39. The values shown for the ratios are truncated to the range used for the tiled unit cells, with a maximum internal angle of 6 degrees. On average, Poisson’s Ratio for the 10 unit cell long by 5 unit cell tall tiled geometry is 2% below (less negative) that of the single unit cells.

<table>
<thead>
<tr>
<th>Internal Angle</th>
<th>Unit Cell Poisson's Ratio</th>
<th>Tiled Cell Poisson's Ratio</th>
<th>Tiled / Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.52</td>
<td>-2.22</td>
<td>0.88</td>
</tr>
<tr>
<td>2</td>
<td>-5.31</td>
<td>-5.22</td>
<td>0.98</td>
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<tr>
<td>3</td>
<td>-6.9</td>
<td>-6.58</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>-7.25</td>
<td>-7.28</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>-6.93</td>
<td>-7.06</td>
<td>1.02</td>
</tr>
<tr>
<td>6</td>
<td>-6.24</td>
<td>-6.44</td>
<td>1.03</td>
</tr>
</tbody>
</table>

The two regression equations (f for the unit cell and f for the tiled cells calculated from the low-angle geometries are tabulated in Figure 40. On average, the results of the equation for the tiled unit cells are 3% smaller (less negative) than the results of the single unit cell’s equation. The percent variations between the equations and the FEM results are similar, indicating a consistent result.
For the filled cavity models, the tiled geometries were used for each model. The internal angle for the models was given to be 10.44 degrees for the Reentrant Hexagon model and 4.49 degrees for the modified tube and sheet model. The open cavities along the perimeter of the tiled model were left empty. The cavities were filled with a normal weight, 4 ksi nominal strength concrete treated as an isotropic material. Rather than applying a unit strain to the models, a unit stress, 100 psi, was applied. This was done to allow for calculation of an elastic modulus, $E$, for each model. The models, while run in 2D, are assumed to be a unit length in depth. Thus, the equation for the elastic modulus, $E$, is given by Equation 14, where $\sigma_x$ is the longitudinal stress from the applied compressive load and $\varepsilon_x$ is the longitudinal strain. The elastic modulus then becomes equal to the result of the 100 psi applied force divided by the longitudinal strain.

$$E = \frac{\sigma_x}{\varepsilon_x}$$

Equation 14 Equation for Elastic Modulus

From this point, we can then assume a linear elastic relationship, and extend the elastic portion of the stress-strain curve defined by the 100 psi stress and found strain point. The curve was calculated to extend until one of the materials reaches their maximum allowable stress. For concrete, the maximum compressive stress was taken to be the nominal 4000 psi, and the maximum tensile stress was taken to be the calculated 474 psi (Equation 15). For the steel, both the maximum compressive and tensile stresses were taken to be the yield strength of the material, 60 ksi. This value is typical for steel used for concrete reinforcement.

$$f_{r} = 7.5\sqrt{f'_{c}} = 4000 \text{ psi} = 474.3 \text{ psi}$$

Equation 15 Calculation of Concrete's Maximum Tensile Stress
4.5.1 Reentrant Hexagon

Figure 41 shows the von Mises stresses overlaid over the undeformed filled reentrant hexagon model. It is important to note that the high stress regions (denoted by the warmer color) follow the location of the auxetic steel reinforcement. The maximum von Mises stress for the model is just over 500 psi, and is in the steel reinforcement.

![Figure 41 von Mises Stresses for Filled Reentrant Hexagon Model](image)

The longitudinal stresses are overlaid over the undeformed model in Figure 42. Excluding edge effects, the maximum stress at the “core” of the model is 45.5 psi in compression. This value is very far below the rupture stress for both the concrete and steel sections, indicating that the unit cell has capacity remaining.

![Figure 42 Longitudinal Stress for Filled Reentrant Hexagon Model](image)
Figure 43 shows the transverse stresses for the reentrant hexagon model. The loading edge, on the right side, has a heavy effect on the stress heat map due to edge effects. However, the location of the next largest stresses are in the vertical legs of the auxetic geometries. The maximum stresses in these elements is 56.3 psi in tension. Since these are made out of the tension-friendly steel, there is no present concern that they would yield to tension.

The calculated Poisson’s ratio for the filled reentrant hexagon model was 0.125. The value is positive, but less than the Poisson’s ratio of both steel ($\nu = 0.30$) and the concrete ($\nu = 0.15$). The auxetic geometry seems to have had a positive effect on the changing the overall Poisson’s ratio. As the free geometries deformed into the empty cavities, the extra constraint added by the concrete likely prevented the steel from collapsing, thus keeping the overall ratio positive.

Using Equation 14 and the found longitudinal strain of 0.0000156, the Elastic Modulus for this model is 6,390,000 psi. As the material closes to failure was the concrete in compression, at 1.125% of its capacity, the model was run again, but with a stress of 6,200 psi applied on the free edge. Here, the maximum compressive stress for the concrete was 4,680 psi, the maximum compressive stress on the steel 31,220 psi, and the maximum tension on the steel 3,940 psi. These values exceed the failure criterion for the concrete in compression. Using linear interpolation, we find the applied stress at which the concrete will fail in compression to be 5300 psi. Running the model at this value of applied stress shows a maximum compressive stress in the concrete of 3870 psi, corroborating the interpolation. Utilizing the auxetic reinforcement scheme increased the strength of the composite by just over 25% compared to the unreinforced nominal strength of the concrete.
4.5.2 Modified Tube and Sheet

The von Mises stresses are overlaid on the undeformed model in Figure 44. As with the reentrant hexagon model, the stress concentrations make it clear where the steel auxetic reinforcement is located in the model. The maximum von Mises stress is located in the steel reinforcement, and is just over 530 psi.

![Figure 44 von Mises Stresses for Filled Modified Tube and Sheet Model](image1)

The longitudinal stresses are shown in Figure 45. The horizontal component of the auxetic geometry has a much larger compressive stress (cooler colors) than the vertical component of the reinforcement and the concrete areas. The maximum compressive stress for the steel in this stress overlay is 523 psi. The maximum compressive stress for the concrete is 40 psi. Both of these stresses ignore the concentrations along the loading edge.

![Figure 45 Longitudinal Stresses for Modified Tube and Sheet Model](image2)
The transverse stresses for the modified tube and sheet model are shown in Figure 46. In the image, the darker-blue areas within the cyan areas show a boundary of the tube portion of the auxetic geometry. Removing the stress concentrations on the loading edge, the steel portion of the geometry undergoes a maximum tensile stress of 21 psi, while the concrete has a maximum tensile stress of 13 psi.

![Figure 46 Transverse Stresses for Modified Tube and Sheet Model](image)

Using the 100 psi stress data, the calculated Poisson’s ratio for the modified tube and sheet model is -0.016. Opposed to the positive value found in the reentrant hexagon model, this is intriguing. The extreme auxetic effect found in the free geometry seems to have carried over into the filled model. The stabilizing effect of the concrete, while still present in this filled geometry, is diminished.

The elastic modulus, calculated using Equation 14 and a longitudinal strain of 0.0000135, for the modified tube and sheet model was found to be 7,400,000 psi. The model was run once again using the same applied stress as the reentrant hexagon model, 6,200 psi. The maximum compressive stress for the concrete was found to be 4,007 psi, and the maximum tensile stress found to be 851 psi. The maximum compressive stress in the steel portion of the model was found to be 32,420 psi, and the maximum tensile stress found to be 1300 psi. At this stress, the concrete portion of the model failed in both compression and tension. The tensile failure was much more egregious, with the maximum tensile stress exceeding the limit by 15%, compared to the 0.002% excess stress in compression. Using interpolation methods, the material’s compressive strength seems to be around 3,000 psi of applied axial load. Running the modified tube and sheet model with this applied stress shows a maximum tensile stress of 467 psi in the concrete, confirming the interpolation. The use of the modified tube
and sheet geometry as a reinforcement scheme dropped the maximum compressive strength by 25%, compared to the nominal 4,000 psi strength of the concrete.

4.6 Experimentation

This section will discuss the effects of changing the dimensional attributes of the unit cells. The effects of changing the overall length of the unit cell as well as the wall thickness will be observed. The overall volume fraction and internal angles are kept constant and the derivation of the unit cell area will be discussed.

4.6.1 Unit Cell Areas

The areas of the individual unit cells tested are easily found using CAD software. The base models generated in ABAQUS were imported into Solidworks. Built-in evaluation tools returned the area of the auxetic geometries, which is the area of the steel. Using the known length, and the easily gotten vertical dimension, the rectangular area of the unit cell was found. These two values are then used to find the volume fraction of steel, $\rho$. For the Reentrant Hexagon model $\rho = 0.237$ and for the modified Tube and Sheet model $\rho = 0.337$.

Based off these values, models with changed dimensional attributes were created. To ensure a constant volume fraction, the geometries were set to the desired fixed value, and the variable dimension was changed. The volume fraction of this new model was found, and interpolation techniques were used to find the variable dimension that gave the desired volume fractions.

4.6.2 Length Modulation

To test the effect that changing the length of the unit cell will have, two additional composite models were created for each unit cell. The additional models examined the effect of doubling the overall length of the unit cell, to 1”, and halving the length of the unit cell, to 0.25”, on the maximum allowable applied loading. Figure 47 shows the FEM results for a composite model. The clear spacing was kept constant in all models, and the base model used in the remainder of the analyses is added for comparison.
The Modified Tube and Sheet model showed some improvement with the change in length. When the unit cell length is set to a quarter inch, the maximum allowable load applied exceeds 4,000 psi, allowing the reinforcement system to provide a beneficial effect. In the other tested configurations, the modified tube and sheet model failed to increase the strength of the concrete. In fact, as the length of the unit cell increased, the maximum applied stress decreased. This due to tensile stresses in the concrete being more pronounced in the longer unit cells. This relationship is demonstrated by the decrease in strength as the unit cell length increases.

The reentrant hexagon models, on the other hand, were worse performing in the additional composite models. The original 0.5” length unit cells possessed an additional 600 psi capacity over the next best unit cell. As the length increased, the tensile stresses became dominant, as with the modified tube and sheet model. Contrasting this, as the reentrant hexagon unit cell decreased in length, the compressive stresses became dominant.
5.0 Conclusions and Recommendations

This section will discuss the conclusions about the application of the auxetic reinforcement system, as well as suggestions about future work towards the application of the auxetic reinforcement system as a construction material. Attention is given to potential applications, constructability concerns, and methods of gathering more data.

5.1 Potential applications

The auxetic reinforcement system has been shown using Finite Element Models to be able to increase the nominal axial compressive stress for certain geometries. The only loading scheme tested in this analysis was pure axial compression along the x-axis. We can be confident that along this loading axis, the geometry has a large amount of promise for situations that require pure axial loading. As nearly all reinforced concrete compression members typically have some level of end moment applied, and the moment-resistance capabilities were not examined in this analysis, the reinforcement scheme is not usable for these members. Impact loading, on the other hand, is much more applicable for the loading resistance provided by the auxetic reinforcement. In a simplified sense, impacts can be treated as a directed point or pressure load. The system needs to be capable of resisting shear forces, but this design is easily done using the steel reinforcement as the shear resistance.

One such impact load type, as illustrated in Figure 48, is an explosive force. The explosion generates a force in the form of a shockwave, which applies the loading to the bearing member. The exact magnitude of the load is dependent on the distance at which the explosion is generated at, the amount of energy in the explosion, as well as other explosive design factors. The exact amount of required strength is dependent on the desired level of blast protection. Most structures need to be rehabilitated after any sort of blast, and using high-cost materials can make the repair process expensive. By utilizing both the variable strength capabilities as well as the low-cost of the materials, the auxetic reinforcement system can be used as an effective, low-cost way of providing blast resistance.
The simplest application of the system is use in a longitudinal load member. The reinforcement system can be used to maximize the strength of a concrete member of a given area. For a pure compression member, the auxetic system will be able to provide a smaller cross-section for any given load. This has applications for aesthetic and architectural purposes. Limiting the size of structural elements can assist with placement of utilities, and increase the usable floor space. The limitations of this analysis prevent any confident guesses about the moment resistance of the auxetic composite, requiring further study to see if the auxetic composite columns can provide any meaningful amount of bending resistance.

5.2 Constructability

Due to the size of the auxetic steel elements, creation of the reinforcement system may pose an issue. The thinnest element in any model is only 0.01” in thickness, making any sort of rolling process difficult to accomplish. The complex geometry adds an additional complexity to manufacturing. Casting the geometry using a negative mold is a strong option for creating the reinforcement system. This limits the overall size that can be created per cast, but the elements can be placed in the concrete together to provide a continuous reinforcement effect. The reinforcement elements likely will not
perform as a single unit, and most likely reduce the overall strength of the auxetic reinforcement system.

An alternative method is to use additive manufacturing techniques. Typical extrusion-based 3D printing techniques are not currently developed for the grade of steel used for construction reinforcement, requiring that other metals, such as aluminum be used with this method. However, alternative methods of additive manufacturing do exist for the grade of metal used. Direct Metal Laser Sintering (DMLS) allows for a powdered metal to be fused using a high-powered laser. This process (Figure 49) can be used to create the complex geometry required of the auxetic reinforcement. DMLS, however, is very expensive to manufacture, with individual parts costing in the range of thousands of dollars. This method is best for manufacturing prototype parts, which can later be produced through large-scale manufacturing.

![Figure 49 Direct Metal Laser Sintering Process](image)

### 5.3 Future Work
The digital models created in this report should be confirmed with physical models. The models should be created with the geometry and loading constraints detailed in this report. The suggested
experimental setup for these physical tests are using a standard force-displacement machine to measure the stress-strain curve. The models should be filled with a normal weight 4000 psi concrete at a 28 day break. The cross-section should have special attention paid to it, to ensure that failures in the concrete can be noticed when they occur. If desired, transverse strain measuring techniques can be implemented, including, but not limited to, photographic strain mapping and use of strain gauges. Due to the limitations of the software used, the concrete material was treated as an isotropic material, where in reality, concrete is not. This may cause some unexpected variances in the properties of the concrete, and the results of the physical tests.

With regards to the models, more data points for the modulations can be created. Using more data points, a stronger interpolation model could be generated for the creation of differently-dimensioned unit cells. The precision of the internal angle calculations can be increased by tightening the resolution of the angles used to the sub-degree mark.

Manufacturing of the auxetic geometries can be explored, by discovering how to create the models in a large-scale format. Suggestions for this are discussed in section 5.2. If needed, other metals can be tested to determine their feasibility of use with the auxetic geometries. If the alternate metals are used, however, careful consideration should be given to the strengths of the metals. This should also be considered for the shear and moment resistance of the alternate-metal auxetic unit cell.
References


Young, T. (1807). *Course of Lectures on Natural Philosophy and the Mechanical Arts.*