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Option Pricing Using Monte Carlo Methods

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Abstract

This paper aims to use Monte Carlo methods to price American call options on equities using the variance reduction technique of control variates and to price American put options using the binomial model. We use this information to form option positions.

This project was done a part of the masters capstone course Math 573: Computational Methods of Financial Mathematics.

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1. Introduction

The main idea of this project is to use Monte Carlo methods to price options on equities and then utilize that information to form option positions using an Interactive Brokers paper trading account[1].

2. Data

Our portfolio consists of 12 stocks, 6 call options, 3 put options, for a total of 21 assets.

	Stock	Price	Shares	Option	Shares
OIL	BP	44.5	200	CallOption	
	HES	77.41	100	BP	10
	HP	66.88	100	HES	10
	PTR	147.07	150	HP	10
	SUN	41.52	150	PTR	10
	CVX	107.95	300	SUN	10
	RDS A	74.8	200	CVX	10
				RDS A	10
Web	GOOG	522.95	300		
	SOHU	100.14	100	PutOption	
	YHOO	16.22	150	GOOG	10
	BIDU	150.19	150	YHOO	10
	SINA	135.73	100	SINA	10

Table 1

We can know the volatility of each stock as follows:

Sigma_Oil	BP	CVX	HES	HP	PTR	RDS A	SUN
	0.0009	0.0007	0.0012	0.0013	0.001	0.0008	0.001
Sigma_Web	SINA	SOHU	GOOG	YHOO	BIDU		
	0.0023	0.0016	0.0009	0.0012	0.0013		

Table 2

After modeling asset prices using a multidimensional geometric Brownian motion, we can estimate the price using[2]:

[1] Interactive Brokers, www.interactivebrokers.com, the online discount brokerage firm in the United States, founded by Thomas Peterffy, 1977

[2] Monte Carlo Methods in Financial Engineering, Paul Glasserman, August 7, 2003

$$S_i(t_{K+1}) = S_i(t_k) \times \exp((\mu_i - \frac{1}{2}\sigma^2) \times (t_{k+1} - t_k) + \sqrt{t_{k+1} - t_k} \sum_{j=1}^{d} A_{ij} . Z_{k+1,j})$$

$$\underline{Z_k} = [Z_{k,1}, Z_{k,2}, \ldots . Z_{k,d}] \longrightarrow N(\underline{0}, I)$$

(k=0,1…n-1 i=1,2…d)

Where, St is the stock price at time t, mu is the average market rate of return, sigma is the stock volatility, and matrix A is calculated from the Cholesky factorization of Σ. We get the estimated price for the Oil sector stock prices as follows:

BP	HES	HP	PTR	SUN	CVX	RDS A
45.9246	78.4288	73.7372	153.8592	42.3165	109.6646	73.9756

Table 3

Comparing to the corresponding trade prices, we can see that almost every price is likely to rise.

In order to get higher return at lower risk, I bought more shares of stocks which have lower volatility and fewer shares of stocks which have higher volatility. The results for the 5 stocks in the Web sector were similar.

3. Methodology

(1) Data processing:

Select 12 stocks (7 in the "Oil" Sector and 5 in the "Web" Sector).

Choose an interest rate. For some selected asset prices, according to the collected market data, estimate historical volatility and other parameters needed in the option pricing model.

(2) Building up the option pricing model for call options:

Choose those equities in the same "Oil" sector and model their asset prices together in a multidimensional geometric Brownian motion. Apply Monte Carlo methods to price call options using the variance reduction technique of control variates.

(3) Build up the option pricing model for American put options:

Choose those equities in the same "Web" sector and use a binomial model for pricing.

(4) Compare the computed option prices with the market prices. Form a portfolio consisting of option positions and equity positions.

	computed option price	bid prices	ask prices	trade price of the option
"OIL"	CallOptionPrice =			CallOptionPrice =
BP	4.9242	5.30	5.42	5.34
CVX	9.6637	8.02	8.31	8.3
HES	8.4279	7.54	7.81	7.6
HP	8.9748	4.08	5.12	5
PTR	53.8548	49.2	51.6	50
RDS	28.7348	28.3	28.9	28.6
SUN	4.3161	5	5.27	5.1
"Web"	PutOptionPrice =			PutOptionPrice =
GOOG	20.0441	19.7	21.85	20.7
YHOO	4.65	3.68	3.71	3.7
SINA	17.92	17.5	17.84	17.69

Table 4

We use Monte Carlo method to price options. It is very important to implement variance reduction techniques when using this method. This is because that an error will occur, which has a normal distribution N(0,sigma^2/n). Thus we need to increase n and reduce sigma to minimize the error.

As we introduce the control variates into the problem, we exploit information about the estimates of known quantities to reduce the error in the estimate of an unknown quantity.

$$Yi(b) = Yi - b \times [Si(T) - E(S(T))]$$

$$\overline{Y}(b) = \frac{1}{n} \times \sum [Yi - b \times [Si(T) - E(S(T))]]$$

$$E(S(T)) = e^{rT} \times S(0)$$

We need to solve:

$$\overline{Y}(b) = \frac{1}{n} \times \sum [Yi - b \times [Si(T) - e^{rT} \times S(0)]] \quad i = 1, 2, \ldots n$$

Where, r is the 3-month Treasury bill rate as our risk-free rate; Y is the discounted payoff:

$$Yi = e^{-rT} \times \max(Si(T) - K, 0) \qquad i = 1\,,\ 2\,, n$$

Also, the optimal b is given by:

$$b^* = \frac{\operatorname{cov}(S(T), Y)}{\operatorname{var}(S(T))}$$

Thus, $\overline{Y}(b)$ is the price of the American call option[3].

In this project, we use Y as the discounted payoff of the call option and simulate 1000 paths using the stock price at maturity as X. Then we form the control variate estimator $\overline{Y}(b)$, which has a smaller variance and is an unbiased estimator of E(Y). In this case we get our option price with this variance reduction technique.

Next, we price put options using the binomial tree model to simulate the price path of the underlying stock. We assume that the stock price moves from one time step to another by either an upward factor u with probability p or a downward factor d with probability 1-p. [4]

We calculate the up factor and down factor using:

$$\Delta X = \sqrt{\sigma^2 \Delta t + (r - \frac{1}{2}\sigma^2)^2 \times \Delta t^2}$$
$$\Delta Xu = \Delta X$$
$$\Delta Xd = -\Delta X$$

$$Pu = \frac{1}{2} + \frac{(r - \sigma^2/2) \times \Delta t}{2\Delta X}$$
$$Pd = 1 - Pu$$

Simulate the stock price path:

$$S(i,j) = S(0) \times \exp[(j-i) \times \Delta Xu + (i-1) \times \Delta Xd]$$

Then the put option price can be calculated as follows:

At every time t< T, the holder needs to choose either exercising early or holding the option until the next time step. Thus option price would be the maximum of the early exercise payoff and the holding value.

$$P(i,T) = \max(K - S(i,T), 0)$$

$$P(i,j) = \max\{\max(K - S(i,j)), e^{-r\Delta t}[Pu \times P(i,j+1) + Pd \times P(i+1,j+1)]\}$$

4. Performance

Due to the Libyan crisis, stock prices in the Oil sector tended to increase much faster in recent months. For example with CVX, one of the biggest oil companies in the world, we can earn 108.75-107.95=$0.8/share only holding it for one day, and we

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3 Monte Carlo Methods in Financial Engineering, Paul Glasserman, August 7, 2003

4 Implementing Models in Quantitative Finance: Methods and Cases, Gianluca Fusai and Andrea Roncoroni, Dec, 2007

can expect the price to reach 109.67 on May 20th based on historical data. As we expect the stock price to rise, we hold some call options on CVX. If the estimated option price is higher than the market price, then the market is undervaluing the option, and we can hope to get higher returns in the future.

For Internet companies, SOHU grows faster than the others with slight rises and falls every day. We expect the prices for SINA, GOOG, YHOO to fall, thus we hold some put options on these assets.

In general change of stock prices was more stable than changes in option prices, and there is an ascending trend as a whole; however, the option prices do fluctuate from day to day. Our option positions mainly determine whether our portfolio either earns or loses money.

5. Conclusion

As the economy fluctuates every day, some prices rise to a great height and some prices fall. In general, the Oil sector stock prices fluctuate more than those in the Web sector. On the whole it appears that we can expect an upward trend.

The major challenge in this project is that we actually had to solve a problem in a real-world setting. The use of the variance reduction technique was another challenge for us.

If I have the chance to do this project again, I would adjust the covariance matrix. We assume that asset prices are stationary stochastic processes in the beginning, which is required to use historical volatility to estimate constant volatility. The MATLAB function “cov” would not give an accurate volatility matrix; we need to verify it using other factors.

6. References

1. Interactive Brokers, www.interactivebrokers.com, the online discount brokerage firm in the United States, founded by Thomas Peterffy, 1977

2. Gianluca Fusai and Andrea Roncoroni, 2008, *Implementing Models in Quantitative Finance: Methods and Cases*, Springer, New York City, 631 P.

3. Paul Glasserman, 2003, *Monte Carlo Methods in Financial Engineering*, Springer, New York City, 596 P.

4. The stock price data used in the project was obtained through Yahoo Finance, http://biz.yahoo.com/r/ (May 2, 2011)