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# Data Envelopment Analysis

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**Data Envelopment Analysis**

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## Abstract

Companies use many different inputs (such as assets, employees, shareholders' equities, etc.) to generate outputs (such as profits, revenues, market values, etc.). This Project focused on a linear programming model used in performance evaluation of 25 property and casualty insurance companies as of the year of 2007. The goal is to determine the efficiency of each company compared to the peer competitors within property and casualty insurance industry. The technique is called data envelopment analysis (DEA). It is an approach based on data for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs. The emphasis was on data selection and cleanup, mathematical approach behind the data envelopment analysis model, and the application of this model to the efficiency comparison.

# Executive Summary

The goal of this project was to research the mathematical method of data envelopment analysis (DEA) model, and, starting from mathematically testing the DEA model in some simple two-input-one-output scenarios, apply the model to evaluate the relative efficiency of 25 property and casualty insurance companies in terms of total operation expenses, assets, debts, employee numbers, and liabilities as inputs and market caps, net incomes, revenues, and earnings per share as outputs. The model was applied with and without pre-set constraints in weights of inputs and outputs to further interpret how the efficient frontier would change caused by different preferences of weights.

Attempting to design and apply such strategy could be quite of a challenge. Essentially, the objective is to present how the model would work from mathematical perspective as well as the computer perspective. What and how input/output factors to select to evaluate the companies? Where to find the source data? What adjustment to make if the data don't fit the model perfectly? The team has outlined the data envelopment analysis process as a five stage process:

- Input and Output Determination
- Data Collection and Cleanup
- Weights Determination
- Mathematical Model Establishment
- Efficiency Optimization

The input and output determination is very essential for the goal of this project. As an accounting or finance student, one could be very interested at some important finance terms, such as assets, liabilities, and equities. However, in a DEA model, including these three factors

as input/output factors would cause duplicated information, because the equity is just the difference between asset and liability. Thus a careful selection of input/output factors that represent the key components of companies was the first step toward the success of this project.

Data collection was the content of this project. To be consistent, the team chose all the indexes and the associated source data from the online finance websites of 2007 annual data. As DEA model was not able to take negative numbers, the team had to make carefully adjustments to some input/output factors, to make sure that all the data can fit the DEA Model requirement and also maintain the relative consistence with each other.

The change of efficient frontier could be caused by newly-added pre-set weight constraints. The team testified by applying the model twice to the same data set, with weights constraints adding to compare the efficiencies of companies with the previous ones without pre-set weight constraints.

The result of company efficiencies were collected and presented by dividing companies into four types.

- Type I: Consistently Fully Efficient
- Type II: Fully Efficient Without Weight Constraints
- Type III: Consistently Inefficient and No Change by Weight Constraints
- Type IV: Inefficient and Further Cut by Weight Constraints

It was shown that a large proportion of companies fell into Type I and Type III. The team believes that this would change if a larger amount of data was provided with more input and output factors, and with more complicated pre-set constraints of input and out weights. From an academic standpoint, the success of the project is not only the achievement of mathematical approach behind this popularly used model and the proof of concept cases, but also the discovery of plenty of room for improvement in this model and the appliances of the excel-based model to a more broad use.

# Acknowledgement

The team would like to thank Professor Jon Abraham in mathematics department of Worcester Polytechnic Institute, for his on-going advice and help. Professor Joe Zhou in management department of Worcester Polytechnic Institute, the author of Data Envelopment Analysis—Modeling Operational Processes and measuring Productivity, also offers a lot of advice for this project.

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# 1 Introduction

There are many different ways to evaluate the performance of insurance companies in property and casualty field. The trend of stock price is one of the most intuitive ways for people to observe. Some others can include earning per share, PE ratio, and beta of a company. Financial analysis's may take a look at some key statistics, such as operating expenses and net income of the income statement of a company, or they can read the total assets, total liabilities, and the stockholder's equity of the balance sheet. However, there is no single index that can reflect the performance of a company. Some companies have higher profits but at the same time they need more employees and other resources to generate such high profits. Another simple case could be like by generating a certain amount of profit, one company is doing very well in saving labors. Another company takes good control of the budgets. It is very difficult to tell which company is doing better against the other one. Especially, among a bunch of insurance companies within property and casualty field, it is very essential to know which companies are doing better compared to their competitors, and which are performing not so well.

The team listed 25 property and casualty insurance companies, the names of which were from CNN Money and Hoover, Inc. For each company, the team assigned five different inputs, such as total operation expenses ($millions), asset ($millions), employee number, debt ($millions), and liabilities ($millions), and also four different outputs, such as market cap $millions), revenue ($millions), net income ($millions), earnings per share. The data envelopment analysis model was established to evaluate the relative efficiency of these 25 property and casualty insurance companies, and the team also took into consideration about the possible different weights assigned to inputs and outputs. The conclusions and recommendations part would provide a thorough way to interpret the performance of each company compared to the rest of the group, as well as possible room for improvement.

# 2 Background

## 2.1 Efficiency Measurement

The basic measure of efficiency is the ratio between one output and one input, which can be written as:

Efficiency = Output / Input

However, this equation is normally not adequate to be applied in the real world problem, because there often exist a numerous inputs and outputs of different categories, such as labor, time, money, and so much more. For a company, investors' concern would not limit one single output or input factor. Instead, investors pay highly attention to a lot of their financial information, including asset, liability, revenue, net income, market cap, as well as important financial ratios, including earnings per share, long-term debt ratio, liquidity ratio. One output or input can tell the information with respect to a certain field, but none of them can represent the overall financial performance of the company. An ideal way is to have all the major inputs and outputs information gathered together and develop a way to measure the efficiency of each company in terms of these factors, as well as flexible enough to put different constraints of weights.

A common measure for relative efficiency is used, by taking the weighted sum of output divided by the weighted sum of input

The principle behind this model is linear programming approach, which is definite as the problem of maximizing or minimizing a linear function subject to linear constraints. (Thomas I)

If no additional restriction is inserted, this problem could be then an unbounded one. Restriction includes that

- Each of the weights of inputs and each of the weights of outputs must be greater all equal to 0
- For each of the DMUs, the ratio of the sum of the weighted output factors divided by the sum of the weighted input factors is strictly less or equal than 1. This indicates that the efficiency score of each DMUs would be always less or equal than 1. If the ratio achieves 1, it indicates that this DMU is fully efficient, compared to rest DMUs in this group. The lower, the ratio is, the less efficient the DMU is.

Professor Srinivas Talluri of Silberman College of Business Administration, Fairleigh Dickinson University, New Jersey, discusses the mathematical model, proposed originally by Charnes in 1978, to achieve the relative efficiency score of DMUp among a set of homogenous DMUs:

$$\text{Max } \frac{\sum_{k=1}^{s} \mathrm{v_k\ y_{kp}}}{\sum_{j=1}^{m} \mathrm{u_j\ x_{jp}}}$$

s.t.

$$\frac{\sum_{k=1}^{s} \mathrm{v_k\ y_{kq}}}{\sum_{j=1}^{s} \mathrm{u_j\ x_{jq}}} \leq 1$$

$$v_k, u_{j \geq 0}$$

Where,

K = 1 to s,

J = 1 to m,

Q = 1 to n,

$y_{kq}$ = amount of output k produced by DMU q

$x_{jq}$ = amount of input j utilized by DMU q,

$v_k$ = weight given to output k,

$u_j$ = weight given to input j.

However, the formulas defined above are not a linear programming problem. It is a non-linear (linear fractional) programming problem. To evaluate the relative efficiency score of each DMUs, it will need to be transformed to a linear programming problem.

The basic idea of this transformation is to change the maximizing goal and restriction inequalities accordingly, and thus the result after linear transformation would evaluate the same issue as before the linear transformation.

The method that Professor Talluri takes, as most Data Envelopment Analysis researchers agree on, is to take the denominator, which evaluates the sum of weighed input factors, as a constant number of 1.

In this case, the maximizing goal is then been transformed from the ratio of the sum of the weighted output factors divided by the sum of the weighted input factors to just the sum of the weighted output factors. Accordingly, the restriction area has then been changed.

The restriction now includes that

- The sum of weighted input factors (previously the denominator) equals to 1
- The principle of sum of weighted output factors still less or equal to the sum of weighted input factors does not change. Instead of restricting the ratio between sum of the weighted output factors divided by the sum of the weighted input factors less or equal to 1, the new yet the same restriction is to take the difference between the sum of the weighted input factors and the sum of the weighted output factors is less or equal to 0.
- Each of the weights of inputs and each of the weights of outputs must be greater all equal to 0, which doesn't change.

The linear programming model is discussed below (Talluri):

$$\mathrm{Max} \sum_{k=1}^{s} \mathrm{v_k}\ \mathrm{y_{kp}}$$

s.t

$$\sum\nolimits_{j=1}^{m} \mathrm{u_j}\ \mathrm{x_{jp}} = 1$$

$$\sum_{k=1}^{s} \mathrm{v_k}\ \mathrm{y_{kq}} - \sum_{j=1}^{m} \mathrm{u_j}\ \mathrm{x_{jq}} \leq 1$$

$$v_k, u_{j \geq 0}$$

Where,

K = 1 to s,

J = 1 to m,

Q = 1 to n,

$y_{kq}$ = amount of output k produced by DMU q

$x_{jq}$ = amount of input j utilized by DMU q,

$v_k$ = weight given to output k,

$u_j$ = weight given to input j.

The model will need to be run one time for each DMUs. To find the relative efficiency score of n DMUs, it will need to be run n times. For each DMUs, a unique set of weights of inputs and outputs will be determined to maximize the efficiency score.

Before showing a complex real life example of multiple inputs and outputs, with weights and further restriction implemented for each factor, it is helpful to gain the idea by looking at a simple illustration from Professor Joe Zhu's book "Data Envelopment Analysis: Modeling Operational Processes and Measuring Productivity".

Assume there five different companies with the same profits for the last year. They all take some moneys and times to generate the profits. The data are shown as the chart below:

| DMU | Time (months) | Money (millions) | Profit (millions) |
|---|---|---|---|
| 1 | 1 | 5 | 15 |
| 2 | 2 | 2 | 15 |
| 3 | 4 | 1 | 15 |
| 4 | 6 | 1 | 15 |
| 5 | 4 | 4 | 15 |

Table 2-1 --- Five DMUs of Two Inputs and One Output

Intuitively, since all the DMUs have the exactly same profit of 15 millions. The less resource it takes the better performance (more efficient) it is. One can notice that DMU1 takes the least time of one month, and both DMU3 and DMU4 takes the least money of one million. The chart below shows a relation between time spent and money cost for all five DMUs

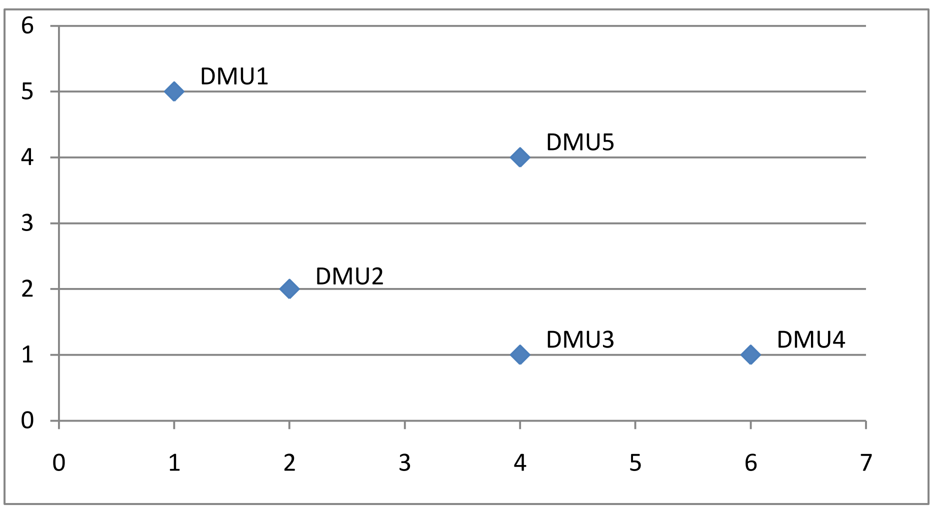


Figure 2-1 --- Scatters of Five DMUs

For instance, to find out the efficiency of the DMU 5, one can apply the linear programming model provided by Talluri. This problem can then be considered as a maximizing the DMU 5's output, subject to a few constraints:

Max 10U (Output for DMU B under evaluation)

Subject to:

$15U - P - 5Q \leq 0$ (DMU 1)

$15U - 2P - 2Q \leq 0$ (DMU 2)

$15U - 4P - Q \leq 0$ (DMU 3)

$15U - 6P - Q \leq 0$ (DMU 4)

$15U - 4P - 4Q \leq 0$ (DMU 5)

$4P + 4Q = 1$

$U, P, Q > 0$

Other than setting up the model and using excel solver, this problem could also be solved by transferring and substituting the inequality set to find to find the optimal solution and efficiency of DMU 5:

Combine the first inequality and the equation, one would get:

$U \leq 1/60 + (4/15) Q$

Combine the second inequality and the equation, one would get:

$U \leq 1/30$

Combine the third inequality and the equation, one would get:

$U \leq 1/60 + (1/5) P$

Combine the fourth inequality and the equation, one would get:

$U \leq 1/60 + (1/3)\ P$

Combine the fifth inequality and the equation, one would get:

$U \leq 1/15$

The five new inequalities will be denoted as inequality 1, 2, 3, 4, and 5.

Since the restraint of U would be the conjunction of these five new inequalities, one could get rid of inequality 4 and inequality 5.

Thus this linear programming model is further transformed to:

$U \leq 1/30$

$U \leq 1/60 + (4/15)\ Q$

$U \leq 1/60 + (1/5)\ P$

The three new inequalities above will be denoted as inequality 1, 2, and 3.

Add inequality 2 to inequality 3, one would get;

$U + U \leq 1/60 + 1/60 + (4/15)\ Q + (1/5)\ P$

It is equivalent to:

$2U \leq 1/30 + (3/15)\ (P+Q) + (1/15)\ Q$

It is equivalent to:

$2U \leq 1/30 + (3/15)\ (1/4) + (1/15)\ Q$

It is then equivalent to:

$U \leq 1/24 + (1/30)\,Q$

And

$U \leq 1/30$ also holds

Since $1/30 < 1/24 + (1/30)\,Q$,

The maximum value of U is 1/30. And the efficiency of DMU 5 is just (15) (1/30), which is 0.5. It indicates that DMU 5 is not on the efficient frontier line, and it is 50% efficient. It is easily verified by looking at DMU 2, which has the same output profit as DMU 5, but with one half time of DMU 5 and one half cost of DMU 5.

The efficient frontier in this example is line segments between DMU1, DMU2, and DMU3, which is concave up. Would the shape of efficient frontier can be concave down? We will discuss the shape of efficient frontier by illustrating another example.

## 2.2 The Shape of Efficient Frontier

Suppose there are three companies with equal product number of 10 last year. The investors are evaluating two factors as their inputs: the number of labor forces and time spent in months.

| | Time (months) | Labor (people) | Products |
|---|---|---|---|
| Company A | 1 | 100 | 10 |
| Company B | 80 | 80 | 10 |
| Company C | 100 | 10 | 10 |

Table 2-2 --- Three Companies with Same Product Capacity

What is the efficiency of Company A, B, and C?

Plot the graph of time and labor:

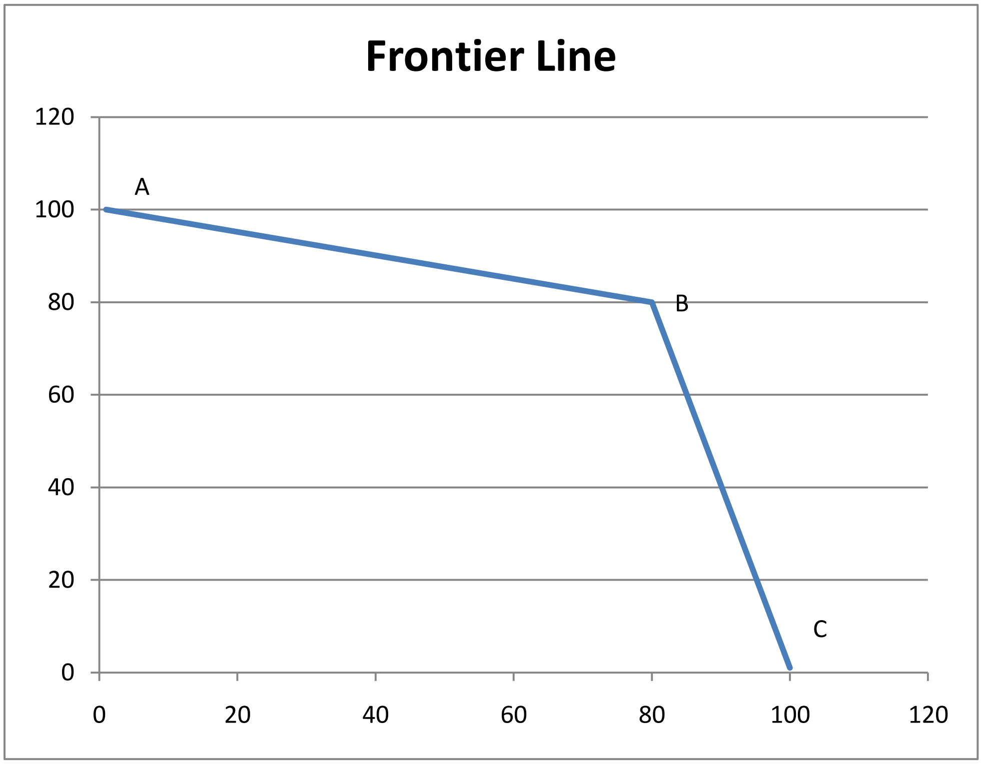


Figure 2.2 --- Frontier Line of Three DMUs without Virtual DMUs

Without further calculation, the initial frontier line above is not the efficient frontier. Before a mathematical model will be established and testified, one would just compare the inputs that company A and C take with the inputs that company B takes. Apparently, company B takes much more time and very little less labor than company A does. Company B takes just a little bit less labor and much more time than company C does. Instead of investing in company B which takes 80 months and 80 people to get 10 products, one would want to invest double the money in company A and C, which would, in combine, take 101 months and 101 people to get 20 products.

Below the team will implement the linear programming model again to approach the efficiency of company B.

Max 10U (Output for DMU B under evaluation)

Subject to

$10U - P - 100Q \leq 0$ (DMU A)

$10U - 80P - 80Q \leq 0$ (DMU B)

$10U - 100P - 1Q \leq 0$ (DMU C)

$80P + 80Q = 1$

$P, Q, U > 0$

Other than setting up the model and using excel solver, this problem could also be solved by transferring and substituting the inequality set to find to find the optimal solution and efficiency of DMU B:

Combine the first inequality and the equation, one would get:

$U \leq 1/800 + (99/10)\ Q$

Combine the second inequality and the equation, one would get:

$U \leq 1/10$

Combine the third inequality and the equation, one would get:

$U \leq 1/800 + (99/10)\ P$

The three new inequalities will be denoted as inequality 1, 2, and 3.

Add the inequality 1 to inequality 3, one would get:

$U+U \leq 1/800 + 1/800 + (99/10)\ Q + (99/10)\ P$

It is equivalent to:

$U \leq 1/400 + (99/10)\ (P+Q) = 1/400 + (99/10)\ (1/80) = 0.063125$

Thus,

$U \leq 0.1$ and $U \leq 0.063125$

The maximum of U would be 0.063125, and the maximum of output for DMU B under evaluation would be (10) (0.063125) equals to 0.63125.

The efficiency of DMU B can also be reflected in the graph below:

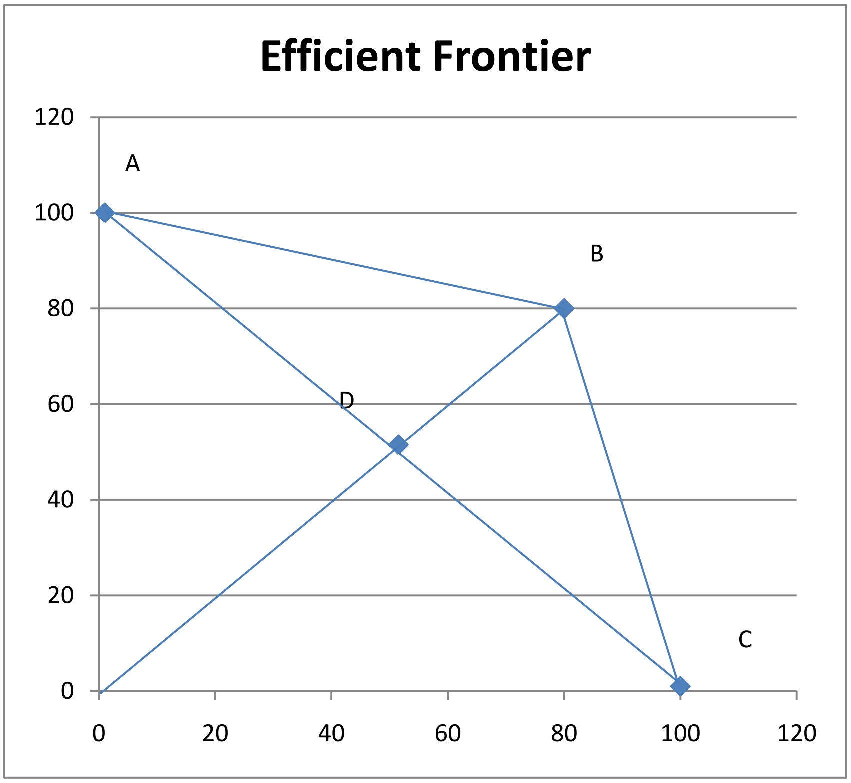


Figure 2.3 --- Frontier Line of Three DMUs with a Virtual DMU

To better interpret and visualize the efficiency of DMU B, the team would introduce point D by connecting the origin and point B, then intersecting with line segment AC at a point denoted by D. The x coordinate of point D is the average of the x coordinate of point A and point C. The y coordinate of point D is the average of the y coordinate of point A and point C. Thus the coordinate of point D is (50.5, 50.5). In this scenario, the team, based on the performance of company A and company C, assumes that there is another company D, which takes half of the sum of inputs 1 that company A and company C take, and half of the sum of inputs 2 that company A and company C take, generating the same products as company A, B, or C do.

The x coordinate of point D divided by the x coordinate of point B is 50.5/80, which equals to 0.63125. The y coordinate of point D divided by the y coordinate of point B is also 50.5/80, which equals to 0.63125. This is the same number of the efficiency of company B. This result indicates that if such a company D exists, one would rather spend 0.63125 much of inputs 1 that company B takes and 0.63125 much of inputs 2 that company B takes to generate the same products as company B does.

Company D is made up by the team to better interprets and visualizes the efficiency of company B. Although such a company doesn't exist, in reality, one could still invest in company A and C combined together for the same output and less inputs then company B consumes. As the result, the line segment ABC is not the "efficient" frontier line. Instead, the line segment AC is the efficient frontier line. The efficiency of company B is 0.63125, and the team gets the same answer when running the model in excel solver. The efficiency of company B will not change even a company D which takes (50.5, 50.5, 10) inputs/output set really exists.

To further explore this simple case, the team adds a company D which takes (40, 40, 10) inputs/output set. The new company D takes only half of the inputs 1 that company B takes, half of the inputs 2 that company B takes, and produces the same products as company D does. In this scenario, company A, C, and D are in the efficient frontier line, but company B is considered to be less efficient. The new frontier line would be as below:

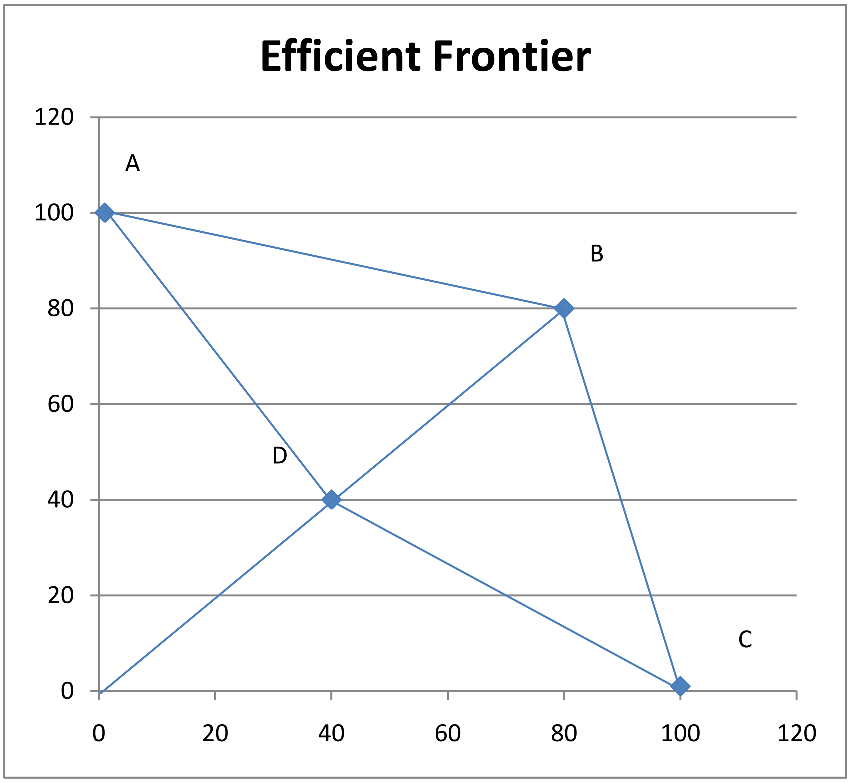


Figure 2-4 --- Efficient Frontier Line of Four DMUs

The line segment ADC would form the new efficient frontier line, and the efficiency of company B would be cut from 0.63125 to what is now 0.5, exactly one half of company D does.

## 2.3 Inputs Constraints

So far, it is examined about the efficient frontier line and the possible change of it as new DEA unit adds to the group. Although maximizing the output is very important, sometimes, it is also necessary to constraint input weights. An example of four companies will be used to interpret how to set constraint to input weights and how the constraints could possibly change the efficiency of DMUs.

Suppose there are four companies with equal product number of 10 last year. The investors are evaluating two factors as their inputs: the number of labor forces and time spent in months.

| | Time (months) | Labor (people) | Products |
|---|---|---|---|
| Company A | 1 | 100 | 10 |
| Company B | 80 | 80 | 10 |
| Company C | 100 | 1 | 10 |
| Company D | 40 | 40 | 10 |

Table 2-2 --- Four Companies with Same Product Capacity

Find the efficiency of company A.

As introduced in the previous section, one could find the optimal solution set (P, Q, U) of company A, and 10U would be the efficiency score of company A, which is 1. P and Q are the weights of input 1 and input 2. In this case, the only constraints for P and Q are that both P and Q are positive rational numbers. Without additional constraints, one would not know how large or how small the P and/or Q would be. An investor would value the factor of labor is at least important as the factor of time. A further constraint could be added to this linear programming model, such as:

$P \leq Q$

To put this model in excel and find it by solver, one would get the optimal solution set of (P,Q,U) as (0.0099, 0.0099, 0.0792). A mathematical approach will be shown below:

Max 10U (Output for DMU A under evaluation)

Subject to

$10U - P - 100Q \leq 0$ (DMU A)

$10U - 80P - 80Q \leq 0$ (DMU B)

$10U - 100P - 1Q \leq 0$ (DMU C)

$10U - 40P - 40Q \leq 0$ (DMU D)

$P + 100Q = 1$

$P \leq Q$

$P, Q, U \geq 0$

Combine the first inequality and the equation, one would get:

$U \leq 1/10$

Combine the second inequality and the equation, one would get:

$U \leq (1/10)\ (80)\ (P+Q) = 8\ (0.01\ P + Q + 0.99\ P)$

Combine the third inequality and the equation, one would get:

$U \leq (1/10)\ (100\ P+Q) = (1/10)\ (0.01\ P + Q + 99.99\ P)$

Combine the fourth inequality and the equation, one would get:

$U \leq (1/10)\ (40)\ (P+Q)=4\ (0.01\ P + Q + 0.99\ P)$

The three new inequalities will be denoted as inequality 1, 2, 3, and 4

Since the restraint of U would be the conjunction of these four new inequalities, one could get rid of inequality 2.

Thus this linear programming model is further transformed to:

$U \leq 1/10$

$U \leq (1/10)\ (100\ P+Q)=(1/10)\ (0.01\ P + Q + 99.99\ P)=0.01 + 9.999\ P$

$U \leq (1/10)\ (40)\ (P+Q)=4\ (0.01\ P + Q + 0.99\ P)=0.04 + 3.96\ P$

The value of U depends on P and the maximum value of U will be achieved as the maximum value of P is achieved.

Since

$P + 100\ Q=1$ and $P \leq Q$

$P + 100\ Q \leq P + 100\ P=101\ P \leq 1$

Thus the maximum value of P is 1/101, which equals to 0.0099.

$0.01 + 9.999\ P=0.1091$

$0.04 + 3.96\ P=0.0792$

Thus the maximum value of U is 0.0792, and the efficiency of company A is 10 U, which equals to 0.7921, not 1 anymore.

# 3 Methodology

The team has divided the Data Envelopment Analysis process as a five stage process:

- Input and Output Determination
- Data Collection and Cleanup
- Weights Determination
- Mathematical Model Establishment
- Efficiency Optimization

## 3.1 Input and Output Determination

For a data envelopment analysis model, it is very crucial to determine a set of input and output factors. The evaluating objectives are companies, a selection of inputs and outputs would be from some financial term factors. However, not all of the major financial terms can be regarded as “valid factors”.

For example, one would think for a company, the asset, liability, and equity are all important to represent the financial situation of the company. However, in a data envelopment analysis model, it is not a wise choice to put asset, liability, and equity together as three factors, since equity is the difference between asset and liability. As the result of simple linear calculation, the factor equity cannot bring up new information to the model, and thus will not be under evaluation. Instead of using equity, the team decides to pick total operating expenses and debt as other two financial term factor of input sets, as well as employee number. Although the employee number is not evaluating a company from financial perspective, it is the factor that an organization wants to minimize to achieve higher efficiency.

As for the output factors selection, the team decides to evaluate the companies from their net incomes, revenues, market caps, as well as earnings per share. The reason for the selection is that if a company could utilize as much as the limited resource and as fewer as labors they can to generate more revenues and net incomes, make itself a larger market caps and more earnings per share, the company would be considered more efficient than others.

## 3.2 Data Collection and Cleanup

The companies names in the list are from the Fortune best 25 property and casualty insurance companies. The team selects all the data as the 2007 annual data from Yahoo finance and Google finance. A requirement for the data envelopment analysis model is that all the data implemented must be positive. Unfortunately, a company could be in a very bad situation sometimes, and for some of the factors, they didn't make to be positive figures. The team would adjust the data by adding fixed positive numbers to the factors which contain negative values.

The input table is shown as below:

# Inputs

| Company | 07 Total Operation Expenses ($millions) | 07 Asset ($millions) | 07 Debt ($millions) | 07 Employee | 07 Liability |
|---|---|---|---|---|---|
| Berkshire Hathaway | 98,084 | 273,160 | 33,826 | 232,781 | 152,427 |
| AIG | 101,121 | 1,060,505 | 176,049 | 116,000 | 964,704 |
| Allstate | 30,106 | 156,408 | 5,640 | 38,500 | 134,557 |
| Travelers Cos. | 19,801 | 115,224 | 6,242 | 33,300 | 88,608 |
| ACE | 10,920 | 72,090 | 2,120 | 1,000 | 55,413 |
| Hartford Financial Services | 21,911 | 360,361 | 5,316 | 31,000 | 341,157 |
| Nationwide | 222 | 3,144 | 1,548 | 36,023 | 1,661 |
| Loews | 11,107 | 76,115 | 7,258 | 21,700 | 58,524 |
| Progressive | 12,993 | 18,843 | 2,174 | 26,851 | 13,907 |
| Selective Insurance Group | 1,653 | 5,002 | 295 | 2,200 | 3,925 |
| Chubb | 10,170 | 50,574 | 4,707 | 10,600 | 36,129 |
| Assurant | 7,442 | 26,750 | 1,514 | 14,000 | 22,640 |
| First American Corp. | 8,070 | 8,648 | 1,006 | 37,354 | 5,663 |
| MGIC Investment | 3,928 | 7,716 | 798 | 1,250 | 5,122 |
| W.R.Berkley | 4,496 | 16,832 | 1,371 | 5,494 | 13,262 |
| Fidelity National Financial | 5,347.50 | 7,556 | 1,167 | 15,500 | 4,312 |
| American Financial Group | 3,739.90 | 25,808 | 936 | 7,100 | 22,761 |
| Cincinnati Financial | 3,067 | 16,637 | 791 | 4,087 | 10,708 |
| Old Republic International | 3,712.60 | 13,291 | 64 | 5,696 | 8,749 |
| LandAmerica Financial Group | 3,787.40 | 3,854 | 579 | 11,050 | 2,653 |
| Mercury General | 2,863.71 | 4,415 | 138 | 5,200 | 25,52 |
| Unitrin | 2,676.20 | 9,405 | 560 | 7,400 | 7,107 |
| Hanover Insurance Group | 2,332.60 | 9,816 | 511 | 3,900 | 7,516 |
| HCC Insurance Holdings | 1,802.50 | 8,075 | 324 | 1,682 | 5,634 |
| Stewart Information Services | 2158.61 | 1,442 | 108 | 8,500 | 687 |

Table 3-1 --- Input Factors

The output table before adjustment is shown as below:

## Outputs (Before Adjustment)

| Company | 07 Market Cap($millions) | 07 Net Income($millions) | 07 EPS | 07 Revenue ($millions) |
|---|---|---|---|---|
| Berkshire Hathaway | 132,520 | 13,213 | 3,224 | 118,245 |
| AIG | 1,080 | 6,200 | (37) | 110,064 |
| Allstate | 8,710 | 4,636 | (3) | 36,769 |
| Travelers Cos. | 21,300 | 4,601 | 5 | 24,477 |
| ACE | 13,480 | 2,578 | 3 | 14,154 |
| Hartford Financial Services | 2,380 | 2,949 | (9) | 25,916 |
| Nationwide | 2,160 | 224 | 1 | 329 |
| Loews | 9,120 | 2,489 | (1) | 18,380 |
| Progressive | 8,390 | 1,183 | (0.12) | 14,687 |
| Selective Insurance Group | 693.52 | 147 | 1 | 1,846 |
| Chubb | 13,650 | 2,807 | 5 | 14,107 |
| Assurant | 2,540 | 653 | 4 | 8,454 |
| First American Corp. | 2,120 | (3,119) | (0.28) | 8,196 |
| MGIC Investment | 147 | (1,670) | (4.32) | 1,693 |
| W.R.Berkley | 3,430 | 744 | 2 | 5,554 |
| Fidelity National Financial | 3,670 | 130 | (1) | 5,524 |
| American Financial Group | 1,770 | 383 | 2 | 4,405 |
| Cincinnati Financial | 3,510 | 855 | 3 | 4,259 |
| Old Republic International | 2,230 | 272 | (3) | 4,091 |
| LandAmerica Financial Group | 1 | (54) | (47) | 3,706 |
| Mercury General | 1,400 | 238 | (4) | 3,179 |
| Unitrin | 710 | 218 | (1) | 2,920 |
| Hanover Insurance Group | 1,560 | 253 | 2 | 2,787 |
| HCC Insurance Holdings | 2,640 | 395 | 3 | 2,388 |
| Stewart Information Services | 302 | (40) | (13) | 2,107 |

Table 3-2 --- Output Factors before Adjustments

As it reflects in the output factor table, some companies have negative net income, and some companies have negative earnings per share. The team decides to add 3500 to the net income column, and add 50 to the earnings per share column.

The output table after adjustment is shown as below:

## Outputs (After Adjustment)

| Company | 07 Market Cap($millions) | 07 Net Income($millions) | 07 EPS | 07 Revenue ($millions) |
|---|---|---|---|---|
| Berkshire Hathaway | 132,520 | 16,713 | 3,274 | 118,245 |
| AIG | 1,080 | 9,700 | 13 | 110,064 |
| Allstate | 8,710 | 8,136 | 47 | 36,769 |
| Travelers Cos. | 21,300 | 8,101 | 55 | 24,477 |
| ACE | 13,480 | 6,078 | 53 | 14,154 |
| Hartford Financial Services | 2,380 | 6,449 | 41 | 25,916 |
| Nationwide | 2,160 | 3,724 | 51 | 329 |
| Loews | 9,120 | 5,989 | 49 | 18,380 |
| Progressive | 8,390 | 4,683 | 50 | 14,687 |
| Selective Insurance Group | 694 | 3,647 | 51 | 1,846 |
| Chubb | 13,650 | 6,307 | 55 | 14,107 |
| Assurant | 2,540 | 4,153 | 54 | 8,454 |
| First American Corp. | 2,120 | 381 | 50 | 8,196 |
| MGIC Investment | 147 | 1,830 | 46 | 1,693 |
| W.R.Berkley | 3,430 | 4,244 | 51 | 5,554 |
| Fidelity National Financial | 3,670 | 3,630 | 49 | 5,524 |
| American Financial Group | 1,770 | 3,883 | 52 | 4,405 |
| Cincinnati Financial | 3,510 | 4,355 | 53 | 4,259 |
| Old Republic International | 2,230 | 3,772 | 47 | 4,091 |
| LandAmerica Financial Group | 1 | 3,446 | 3 | 3,706 |
| Mercury General | 1,400 | 3,738 | 46 | 3,179 |
| Unitrin | 710 | 3,718 | 50 | 2,920 |
| Hanover Insurance Group | 1,560 | 3,753 | 52 | 2,787 |
| HCC Insurance Holdings | 2,640 | 3,895 | 53 | 2,388 |
| Stewart Information Services | 302 | 3,460 | 37 | 2,107 |

Table 3-3 --- Output Factors after Adjustments

After implementing adjustment to the net income column and earnings per share column, this has been transformed to a standard data envelopment analysis problem and the efficiency of each company could be obtained linear programming approach.

## 3.3 Weights Determination

Without further constraints, the only constraints for weights of the input factors and output factors are that the weights have to be positive. In reality, investors would have different preference about the weights of factors. Some would think the revenue is at least important as net income, and some other would think the earnings per share are not as much important as revenue. These preferences are arbitrary personal assumption, and the result of efficiency would change as different preferences implement in the data envelopment analysis model. The team first testified the model without any weights restrictions, and then the team implemented certain constraints of weights of input/output factors to show the change of the efficiency score by the newly-implemented constraints.

## 3.4 Mathematical Model Establishment

Since it is now a standard data envelopment analysis problem, it could solved by the method introduced in section 2.1.

For $DMU_j$, (j=1, 2, 3…25), the efficiency score is the maximum value of the weighted sum of output:

(Output will be noted as O, and Input will be noted as I)

Objective Function: $f_j\ O_{1j} + g_j\ O_{2j}\ + h_j\ O_{3j} + i_j\ O_{4j}$

It is subject to:

$$f_j\ O_{11}+g_j\ O_{21}+h_j\ O_{31}+i_j\ O_{41}-a_j\ i_{11}+b_j\ i_{21}+c_j\ i_{31}+d_j\ i_{41}+\ e_j\ i_{51}\leq 0$$

$$f_j\ O_{12}+g_j\ O_{22}+h_j\ O_{32}+i_j\ O_{42}-a_j\ i_{12}+b_j\ i_{22}+c_j\ i_{32}+d_j\ i_{42}+e_j\ i_{52}\leq 0$$

……

$$f_j\ O_{125}+g_j\ O_{225}+h_j\ O_{325}+i_j\ O_{425}-a_j\ i_{125}+b_j\ i_{225}+c_j\ i_{325}+d_j\ j_{425}+e_j\ i_{525}\leq 0$$

$$a_j\ i_{1j}+b_j\ i_{2j}+c_j\ i_{3j}+d_j\ j_{4j}+e_j\ i_{5j}=1$$

$$a_j, b_j, c_j, d_j, e_j, f_j, g_j, h_j, i_j\leq 0$$

The efficiency of $DMU_j$, as well as all the weights of the input/output factors could be achieved by using excel solver. To find other DMUs, the maximum goal will change and the model will be run 25 times in total to achieve the efficiency of all 25 DMUs.

To consider a case that certain constraints are implemented among input/output weights, the team decides to run the same model with two additional make-up conditions that for the output side the investors regard net income is at least important as market cap, and for the input side the investors regard total operating expenses are at least important as debt amount.

To achieve the efficiency, the team set up the same mathematical model with two additional constraints that:

$c_j\leq a_j$, the weight of debt amount is less or equal to the weight of operating expenses

$f_j\leq g_j$, the weight of market cap is less or equal to the weight of net income

Then, same as the method before, the team would run the model for 25 times to obtain the efficiency as well as the weights of input/output factor of all the DMUs.

## 3.5 Efficiency Optimization

The efficiency results for 25 property and casualty insurance companies as the end of year 2007 are shown as below:

| DMU | Company | Efficiency | Efficiency with Weight |
|---|---|---|---|
| 1 | Berkshire Hathaway | 1.000 | 1.000 |
| 2 | AIG | 1.000 | 0.964 |
| 3 | Allstate | 0.916 | 0.916 |
| 4 | Travelers Cos. | 0.870 | 0.867 |
| 5 | ACE | 0.970 | 0.970 |
| 6 | Hartford Financial Services | 0.858 | 0.858 |
| 7 | Nationwide | 1.000 | 1.000 |
| 8 | Loews | 1.000 | 1.000 |
| 9 | Progressive | 1.000 | 1.000 |
| 10 | Selective Insurance Group | 1.000 | 1.000 |
| 11 | Chubb | 1.000 | 1.000 |
| 12 | Assurant | 0.882 | 0.882 |
| 13 | First American Corp. | 0.963 | 0.963 |
| 14 | MGIC Investment | 1.000 | 1.000 |
| 15 | W.R.Berkley | 0.967 | 0.967 |
| 16 | Fidelity National Financial | 1.000 | 0.948 |
| 17 | American Financial Group | 0.852 | 0.852 |
| 18 | Cincinnati Financial | 1.000 | 1.000 |
| 19 | Old Republic International | 1.000 | 0.956 |
| 20 | LandAmerica Financial Group | 0.936 | 0.936 |
| 21 | Mercury General | 1.000 | 1.000 |
| 22 | Unitrin | 0.856 | 0.856 |
| 23 | Hanover Insurance Group | 0.820 | 0.904 |
| 24 | HCC Insurance Holdings | 1.000 | 1.000 |
| 25 | Stewart Information Services | 1.000 | 1.000 |

Table 3-4 --- Efficiency Table

The implementation of weight constraint changes the efficient frontier line, and thus makes some of the DMUs from fully efficient to less efficient.

To better interpret the result, the team has divided the 25 insurance companies into four different types:

- Type I: Consistently Fully Efficient

This type of companies include Berkshire Hathaway, Nationwide, Loews, Progressive, Selective Insurance Group, Chubb, MGIC Investment, Cincinnati Financial, Mercury General, HCC Insurance Holdings, and Stewart Information Services

- Type II: Fully Efficient Without Weight Constraints

The companies include AIG, Assurant, Fidelity National Financial, and Old Republic International.

- Type III: Consistently Inefficient and No Change by Weight Constraints

The companies include Allstate, ACE, Hartford Financial Services, First American Corp, W R Berkeley, American Financial Group, LandAmerica Financial Group, Unitrin, and Hanover Insurance Group.

- Type IV: Inefficient and Further Cut by Weight Constraints

The companies include Travelers Corp.

Different implementation of weight constraints will change the efficient frontier line differently, because of possibly different optimal solution set for input/output weights. Since it is a very arbitrary process depending to one's pre-judgment and preference, the team only shows one scenario to test the change of frontier line caused by change of pre-set weight constraints.

## 3.6 The Efficiency Scores in 2008

After a thorough DEA analysis of how 25 property and casualty insurance company perform against each other in 2007, the team is interested at, by taking the same input and output combination but taking the data from 2008, how the efficiency scores change for each company. The team will be following the similar procedure as the analysis of 2007 data. Below are the input table and output table for 2008 data:

# Inputs

| Company | 08 Total Operation Expenses ($millions) | 08 Asset ($millions) | 08 Debt ($millions) | 08 Employee | 08 Liability |
|---|---|---|---|---|---|
| Berkshire Hathaway | 100,212 | 267,399 | 36,882 | 232,781 | 158,132 |
| AIG | 119,865 | 860,418 | 193,203 | 116,000 | 807,708 |
| Allstate | 32,413 | 134,798 | 5,659 | 38,500 | 122,157 |
| Travelers Cos. | 20,761 | 109,751 | 6,181 | 33,300 | 84,432 |
| ACE | 12,104 | 72,057 | 3,277 | 15,000 | 57,611 |
| Hartford Financial Services | 13,810 | 287,583 | 7,431 | 31,000 | 278,315 |
| Nationwide | 267 | 3,458 | 1,491 | 36,023 | 1,697 |
| Loews | 12,660 | 69,857 | 8,258 | 21,700 | 56,731 |
| Progressive | 13,062 | 18,251 | 2,176 | 26,851 | 14,035 |
| Selective Insurance Group | 1,657 | 4,941 | 274 | 2,200 | 4,941 |
| Chubb | 10,814 | 48,429 | 3,975 | 10,600 | 34,997 |
| Assurant | 8,038 | 24,515 | 983 | 14,000 | 20,805 |
| First American Corp. | 6,170 | 8,730 | 968 | 37,354 | 6,038 |
| MGIC Investment | 2,659 | 9,183 | 1,074 | 1,250 | 6,816 |
| W.R.Berkley | 4,382 | 16,121 | 1,271 | 5,494 | 13,075 |
| Fidelity National Financial | 4,624 | 8,368 | 1,351 | 15,500 | 5,563 |
| American Financial Group | 3,977 | 26,428 | 1,030 | 7,100 | 23,938 |
| Cincinnati Financial | 3,284 | 13,369 | 840 | 4,087 | 9,187 |
| Old Republic International | 4,057 | 13,266 | 233 | 5,696 | 9,526 |
| LandAmerica Financial Group | 3,787 | 3,854 | 580 | 11,050 | 2,653 |
| Mercury General | 2,865 | 3,950 | 159 | 5,200 | 2,456 |
| Unitrin | 2,832 | 8,819 | 561 | 7,400 | 7,170 |
| Hanover Insurance Group | 2,516 | 9,230 | 531 | 3,900 | 7,343 |
| HCC Insurance Holdings | 1,843 | 8,332 | 345 | 1,682 | 5,693 |
| Stewart Information Services | 1,790 | 1,449 | 358 | 8,500 | 955 |

Table 3-5 --- Input Factors Table

The output table before adjustment is shown as below:

## Outputs (Before Adjustment)

| Company | 2008 Market Cap($millions) | 2008 Net Income($millions) | 2008 EPS | 2008 Revenue ($millions) |
|---|---|---|---|---|
| Berkshire Hathaway | 158,580 | 4,994 | 1,891 | 107,786 |
| AIG | 5,370 | (99,289) | (668) | 11,104 |
| Allstate | 16,480 | (1,679) | (4) | 29,394 |
| Travelers Cos. | 26,890 | 2,924 | 4 | 24,477 |
| ACE | 16,820 | 1,197 | 3 | 13,632 |
| Hartford Financial Services | 8,930 | (2,749) | (15) | 9,219 |
| Nationwide | 3,430 | 268 | 1 | 371 |
| Loews | 15,060 | 4,319 | (3) | 13,247 |
| Progressive | 11,590 | (70) | (0.08) | 12,840 |
| Selective Insurance Group | 867 | 43 | (0.06) | 1,696 |
| Chubb | 17,200 | 1,804 | 4 | 13,221 |
| Assurant | 3,770 | 447.80 | 3 | 8,601 |
| First American Corp. | 3,120 | (26) | 0.32 | 6,214 |
| MGIC Investment | 1,100 | (519) | (7) | 1,722 |
| W.R.Berkley | 4,000 | 281 | 0.52 | 4,709 |
| Fidelity National Financial | 3,660 | (179) | (1) | 4,329 |
| American Financial Group | 2,950 | 195.80 | 2 | 4,293 |
| Cincinnati Financial | 4,220 | 429 | 3 | 3,824 |
| Old Republic International | 3,040 | (558) | (1) | 3,238 |
| LandAmerica Financial Group | 1 | (54) | (47) | 3,706 |
| Mercury General | 1,970 | (242) | (2) | 2,414 |
| Unitrin | 1,210 | (30) | (1) | 2,742 |
| Hanover Insurance Group | 2,120 | 21 | 1 | 2,680 |
| HCC Insurance Holdings | 3,120 | 305 | 3 | 2,279 |
| Stewart Information Services | 247 | (242) | (14) | 1,555 |

Table 3-6 --- Output Factors Table Before Adjustment

As it reflects in the output factor table, some companies have negative net income, and some companies have negative earnings per share. The team decides to add 100000 to the net income column, and add 1000 to the earnings per share column.

The output table after adjustment is shown as below:

## Outputs (After Adjustment)

| Company | 2008 Market Cap($millions) | 2008 Net Income($millions) | 2008 EPS | 2008 Revenue ($millions) |
|---|---|---|---|---|
| Berkshire Hathaway | 158,580 | 104,994 | 2,891 | 107,786 |
| AIG | 5,370 | 711 | 331 | 11,104 |
| Allstate | 16,480 | 98,321 | 996 | 29,394 |
| Travelers Cos. | 26,890 | 102,924 | 1,004 | 24,477 |
| ACE | 16,820 | 101,197 | 1,003 | 13,632 |
| Hartford Financial Services | 8,930 | 97,251 | 985 | 9,219 |
| Nationwide | 3,430 | 100,268 | 1,001 | 371 |
| Loews | 15,060 | 104,319 | 997 | 13,247 |
| Progressive | 11,590 | 99,930 | 1,000 | 12,840 |
| Selective Insurance Group | 867 | 100,044 | 1,000 | 1,696 |
| Chubb | 17,200 | 101,804 | 1,005 | 13,221 |
| Assurant | 3,770 | 100,448 | 1,003 | 8,601 |
| First American Corp. | 3,120 | 99,974 | 1,000 | 6,214 |
| MGIC Investment | 1,100 | 99,481 | 993 | 1,722 |
| W.R.Berkley | 4,000 | 100,281 | 1,000 | 4,709 |
| Fidelity National Financial | 3,660 | 99,821 | 999 | 4,329 |
| American Financial Group | 2,950 | 100,196 | 1,002 | 4,293 |
| Cincinnati Financial | 4,220 | 100,429 | 1,003 | 3,824 |
| Old Republic International | 3,040 | 99,442 | 999 | 3,238 |
| LandAmerica Financial Group | 1 | 99,946 | 953 | 3,705 |
| Mercury General | 1,970 | 99,758 | 998 | 2,414 |
| Unitrin | 1,210 | 99,970 | 999 | 2,742 |
| Hanover Insurance Group | 2,120 | 100,021 | 1,001 | 2,680 |
| HCC Insurance Holdings | 3,120 | 100,305 | 1,003 | 2,279 |
| Stewart Information Services | 247 | 99,758 | 986 | 1,555 |

Table 3-7 --- Output Factors Table After Adjustment

The team inserts input and output data into the DEA model, and does two experiments: one without any constraint, one with the same constraint as the one for 2007 data. The efficiency results for 25 property and casualty insurance companies as the end of year 2008 are shown as below:

| DMU | Company | Efficiency | Efficiency With Weight |
|---|---|---|---|
| 1 | Berkshire Hathaway | 1.000 | 0.967 |
| 2 | AIG | 0.070 | 0.070 |
| 3 | Allstate | 0.745 | 0.742 |
| 4 | Travelers Cos. | 0.942 | 0.942 |
| 5 | ACE | 0.905 | 0.905 |
| 6 | Hartford Financial Services | 0.535 | 0.535 |
| 7 | Nationwide | 1.000 | 1.000 |
| 8 | Loews | 0.840 | 0.840 |
| 9 | Progressive | 1.000 | 1.000 |
| 10 | Selective Insurance Group | 1.000 | 1.000 |
| 11 | Chubb | 0.986 | 0.986 |
| 12 | Assurant | 1.000 | 0.981 |
| 13 | First American Corp. | 1.000 | 0.999 |
| 14 | MGIC Investment | 1.000 | 1.000 |
| 15 | W.R.Berkley | 0.918 | 0.918 |
| 16 | Fidelity National Financial | 0.929 | 0.906 |
| 17 | American Financial Group | 0.871 | 0.871 |
| 18 | Cincinnati Financial | 0.967 | 0.967 |
| 19 | Old Republic International | 1.000 | 0.759 |
| 20 | LandAmerica Financial Group | 1.000 | 1.000 |
| 21 | Mercury General | 1.000 | 1.000 |
| 22 | Unitrin | 0.855 | 0.855 |
| 23 | Hanover Insurance Group | 0.907 | 0.907 |
| 24 | HCC Insurance Holdings | 1.000 | 1.000 |
| 25 | Stewart Information Services | 1.000 | 1.000 |

Table 3-8 --- Efficiency Table

In 2008, many insurance companies have been hit by the recession severely. Since the DEA analysis is a way to show the relative performance compared to peer companies within the group, most companies are still able to maintain high efficiency among the group. However, there are companies that have been dramatically impacted by the economy, and the efficiency score has been going down. For instance, AIG is down from 1.000 to 0.070, and the Hartford Financial Services is down from 0.858 to 0.535. This also reflects the performance of these two companies in 2008.

## 3.7 The comparison of 2007 and 2008

To further present the "efficiency score" concept of each company, the team decides to combine the 25 companies in 2007 and the 25 companies in 2008 together. The team expects to see that the scores of the companies in 2008 will reduce further, as stronger "competitors" join in this group. The efficiency results are shown below:

| Company | 2007 | 2008 | 2007 (07&08) | 2008 (07&08) |
|---|---|---|---|---|
| Berkshire Hathaway | 1.000 | 1.000 | 1.000 | 1.000 |
| AIG | 1.000 | 0.070 | 1.000 | 0.065 |
| Allstate | 0.916 | 0.745 | 0.903 | 0.684 |
| Travelers Cos. | 0.870 | 0.942 | 0.864 | 0.570 |
| ACE | 0.970 | 0.905 | 0.965 | 0.933 |
| Hartford Financial Services | 0.858 | 0.535 | 0.853 | 0.472 |
| Nationwide | 1.000 | 1.000 | 1.000 | 1.000 |
| Loews | 1.000 | 0.840 | 1.000 | 0.785 |
| Progressive | 1.000 | 1.000 | 1.000 | 1.000 |
| Selective Insurance Group | 1.000 | 1.000 | 1.000 | 1.000 |
| Chubb | 1.000 | 0.986 | 1.000 | 1.000 |
| Assurant | 0.882 | 1.000 | 0.881 | 0.873 |
| First American Corp. | 0.963 | 1.000 | 0.961 | 0.905 |
| MGIC Investment | 1.000 | 1.000 | 1.000 | 1.000 |
| W.R.Berkley | 0.967 | 0.918 | 0.966 | 0.842 |
| Fidelity National Financial | 1.000 | 0.929 | 1.000 | 0.971 |
| American Financial Group | 0.852 | 0.871 | 0.852 | 0.777 |
| Cincinnati Financial | 1.000 | 0.967 | 1.000 | 0.901 |
| Old Republic International | 1.000 | 1.000 | 1.000 | 1.000 |
| LandAmerica Financial Group | 0.936 | 1.000 | 0.936 | 1.000 |
| Mercury General | 1.000 | 1.000 | 1.000 | 1.000 |
| Unitrin | 0.856 | 0.855 | 0.860 | 0.787 |
| Hanover Insurance Group | 0.820 | 0.907 | 0.904 | 0.838 |
| HCC Insurance Holdings | 1.000 | 1.000 | 1.000 | 1.000 |
| Stewart Information Services | 1.000 | 1.000 | 1.000 | 1.000 |

Table 3-9 --- Efficiency Comparison

After the team combines the 25 companies in 2007 and the 25 companies in 2008 together, it shows that some companies which are not fully efficient have been further reduced the efficiency score. For example, AIG has an efficiency score of 0.070 in 2008, and by bringing the 25 companies over two years together, the efficiency score of AIG has been further reduced to 0.065.

The results also indicates that there are fewer fully efficiency companies in 2008 than those in 2007, which represents the recession impact to the property and casualty insurance industry.

## 3.8 The problem left

One thing that could be improved is the number of "1"s, which means there are many companies that are fully efficient. The team notices that since AIG did so poor during the financial storm, by applying the "efficiency based on others' in the group" idea, it makes every other companies looks much better than what they really are. The employee input factor also correlates to the size of a company, which has already been represented by other financial indexes such as assets and market shares.

To improve the quality of the results, the team decides to run the model again while reducing AIG and the employee input factor. The idea is to take out a DMU that has much lower efficient score than every other DMUs, and reduce the correlated input/output factor.

The efficiency results are shown below with the modified method:

| Company | 2007 | 2007 Modified | 2008 | 2008 Modified |
|---|---|---|---|---|
| Berkshire Hathaway | 1.000 | 1.000 | 1.000 | 0.010 |
| AIG | 1.000 | | 0.070 | |
| Allstate | 0.916 | 0.516 | 0.745 | 0.756 |
| Travelers Cos. | 0.870 | 0.344 | 0.942 | 0.951 |
| ACE | 0.970 | 0.677 | 0.905 | 0.909 |
| Hartford Financial Services | 0.858 | 0.203 | 0.535 | 0.535 |
| Nationwide | 1.000 | 1.000 | 1.000 | 1.000 |
| Loews | 1.000 | 0.586 | 0.840 | 0.837 |
| Progressive | 1.000 | 1.000 | 1.000 | 1.000 |
| Selective Insurance Group | 1.000 | 1.000 | 1.000 | 1.000 |
| Chubb | 1.000 | 0.750 | 0.986 | 0.991 |
| Assurant | 0.882 | 0.606 | 1.000 | 1.000 |
| First American Corp. | 0.963 | 0.938 | 1.000 | 1.000 |
| MGIC Investment | 1.000 | 0.441 | 1.000 | 0.581 |
| W.R.Berkley | 0.967 | 0.692 | 0.918 | 0.915 |
| Fidelity National Financial | 1.000 | 1.000 | 0.929 | 0.914 |
| American Financial Group | 0.852 | 0.408 | 0.871 | 0.871 |
| Cincinnati Financial | 1.000 | 0.719 | 0.967 | 0.967 |
| Old Republic International | 1.000 | 1.000 | 1.000 | 1.000 |
| LandAmerica Financial Group | 0.936 | 0.903 | 1.000 | 1.000 |
| Mercury General | 1.000 | 1.000 | 1.000 | 1.000 |
| Unitrin | 0.856 | 0.612 | 0.855 | 0.855 |
| Hanover Insurance Group | 0.820 | 0.663 | 0.907 | 0.908 |
| HCC Insurance Holdings | 1.000 | 1.000 | 1.000 | 1.000 |
| Stewart Information Services | 1.000 | 1.000 | 1.000 | 1.000 |

Table 3-10 --- Modified Efficiency Comparison

- Modified means without AIG and removing employee as an input

The number of fully efficient companies has been reduced from 14 to 9 in 2007 and from 12 to 10 in 2008. The team also combines the data of 2007 and 2008 together, and tests in the model. The number of fully efficient companies then has been reduced from 25 to 16.

Taking 2007 for an illustration, now the model shows there are 9 companies in 2007 doing fully efficient than previously indicated 14.

The model is telling that these 9 companies are the most efficient companies in terms of given input and output factors in 2007. However, the team still faces an important question: are the 9 companies left all fully efficient? Having this doubt in mind, the team runs the model again with these 9 companies exclusively. The model shows that they are all 100% efficient.

It looks like under current setting of input and output factors, the model is not able to tell any slight different among these 9 companies. The input factors left are expenses, assets, debts, and liability. The output factors left are market cap, net income, earnings per share, and revenues. To explore any possibilities to compare these 9 companies, the team decides to take out debts factor as it relates to the liability factor, and take out revenues factor as it relates to the net income.

The efficiency results are shown below:

| **Company** | **2007** |
|---|---|
| Berkshire Hathaway | 0.724 |
| Nationwide | 1.000 |
| Progressive | 0.648 |
| Selective Insurance Group | 0.568 |
| Fidelity National Financial | 0.707 |
| Old Republic International | 1.000 |
| Mercury General | 0.564 |
| HCC Insurance Holdings | 0.476 |
| Stewart Information Services | 1.000 |

Table 3-11 --- Comparison For The Left 9 Companies

Now the results of these companies differ a lot with each other with only 3 companies left to be fully efficient.

By going through the process of reducing “1”s, the team learns to take out extreme cases and reduce input/output factors that are correlated.

# 4 Conclusions

After completing the data envelopment analysis model applying for 25 property and casualty insurance companies, the team was able to research and thus have a deeper understanding of the DEA model, applying linear programming method to some two-input-one-output DEA problems with both mathematical approach and excel solver approach, testing the change of the efficient frontier line caused by further constraints of input/out weights from both numerical prospective and graphical prospective, and eventually applying DEA model with pre-set weight constraints to evaluate 25 property and casualty insurance companies with five inputs and four outputs. Since the data was not provided, the team has also spent a fair amount of time to determine the input and output factors, a list of companies, the source data, as well as to clean up and adjust some data to fit the DEA model.

The team compared the efficiency of 25 companies under DEA evaluation with the efficiency of 25 companies of the same method but with two more pre-set constraints in the weights of input/output factors. To further present the comparison result, the team divided the companies into four types depending on if a company is fully efficient, and if the efficiency would change by pre-set weight constraints. Most of the companies fell into type I and type III.

The purpose of this project is to research about the linear programming method behind the data envelopment analysis model, generate a mathematical approach applying to basic case with computer verification, and apply the model to a more complex scenario. What the result reflects is that many companies fall into type I and type II, which indicates that a fair amount of the companies are fully efficient, and don't change by the implementation of pre-set weight constraints. Since DEA model presents the relative efficiency score of each company compared to the rest in the group, it is unable to give out a definite answer of the rather broad concept "efficiency" of a company in terms of a few input and output factors. To better and more accurately approach the efficiency of companies, one needs to come up with a large amount of DMUs, with well-selected input and output factors (recommended 5 or more for each), and determines possible pre-set constraints of input/output weights.

But for a proof of concept process, the team examined the appliance of DEA model in both simple and complicated scenarios. The team also finds out that the DEA model has its flaws. To have the DEA model work well, following problems have to be concerned:

- The amount of DMUs has to be very large, ideally over 50, or insufficient data will influence the accuracy of the results.
- The input/output factors cannot be linearly correlated, or duplicated input/output factors will influence the accuracy of the results
- The outliers need to be removed from the model, or the efficiency score of the rest DMUs will reach higher

Given the fact that there exist a few flows in the DEA model, it is still a nice model to implement to compare a large group of similar identities. With the help of solver function in Excel, one could avoid tedious calculation process which makes multi dimensional data comparison impossible for manual calculation.

# 5 References


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