A Novel Approach for MAC and PHY Performance Analysis in Relay Networks in Presence of Interference and Shadow Fading

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Abstract

Relays in communication networks is a well-researched topic. Historically, relays were used in analog radio and television to extend the coverage. Using relays in wireless data networking applications is a more recent problem. In the early 2000s, relays were introduced for Micro-cellular and Wi-Fi deployments. Recently it has been considered for sensor networks and Vehicular Ad-hoc Networks (VANETs) applications.

In this dissertation we present a novel approach to determine the optimal bounds for the Medium Access Control (MAC) throughput at the target receiver in a multi-hop multi-rate wireless data network. For a given relationship between the throughput and the distance, and a given distance between the access point and the target receiver, there is a minimum number of nodes that provides the maximum throughput to the target receiver. It is always desirable to optimize the deployment from various aspects. These aspects are application-dependent and they range from energy conservation in sensor networks to throughput and coverage maximization in data networks.

We apply this novel approach to vehicular ad-hoc network (VANET) scenarios. Using multi-hop relays, we show how to determine the optimum throughput for communication between two vehicles. The optimal number of relays is chosen to maximize the throughput for point-to-point communication between a source and a destination as well as broadcast among all vehicles in the coverage area of the source.

Additionally, in the physical layer, performance issues arise from the effects of interference and fading. The physical layer performance will in turn impact medium access control performance, effectively reducing the network throughput. We evaluate the
performance of dense small cells for wireless local area networks (WLAN) and femto cells for data applications under the effects of interference and fading. We assume the network is fully saturated. We use the throughput-distance relationship to take into consideration the effects of interference, fading as well as the medium access control overheads. Using this model, we show that under certain conditions, the medium access control throughput for WLANs can outperform that of femto cells.
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This space is to express my sincerest appreciation and gratitude to those that played a foundational role during my PhD journey. Doing a PhD is not just accumulating technical knowledge and mastery of a single topic, it’s much more than that. You learn a number of valuable life lessons.

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This journey, like any other journey, had its highs and lows. More highs than lows for certain. Those lows however were tough. They made me evaluate and reflect on many decisions I made in my life. During those harsh times my family stood with me. Specifically, my mom and my dad. For the two of you, no amount of gratitude is good enough. Specially you mom.

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Chapter 1

Background and Motivation

Relays in communication networks is a well-researched topic. Historically, relays were used in analog radio and television to extend the coverage. Using relays in wireless data networking applications is a more recent problem. In the early 2000s, relays were introduced for Micro-cellular and Wi-Fi deployments. Recently it has been considered for sensor networks and Vehicular Ad-hoc Networks (VANETs) applications.

An abundance of relay research exists in information theory literature going back all the way to the 1960s. The information theory research focused on the capacity of networks using different relay network topologies. However very few literature articles are available for relay effects in communication networking performance that considers practical issues related to PHY, MAC interference.

For PHY the practical issues rise from the variation of the data rate due to distance and RSS. This in turn affects the end-user throughput. RSS is subject to shadow fading which governs statistical characteristics of the data rate and the coverage. For MAC, the available channel will be shared between the access point and the network relays. This leads to reduced user throughput wherever the coverage areas of the access point and relays overlap. Another complication arises from using different MACs whether they are centralized or distributed where the sharing mechanism is substantially different. Due to the limited available bandwidth frequencies are reused by far apart access points and base stations. This in turn causes to interference.
In this dissertation we formulate the variable PHY data rate with a centralized MAC to provide an analytical solution for the optimum throughput using a novel formulation of relay networks under the lights of linear programming. Afterwards, we extend the analysis to include the effects of shadow fading in the PHY layer. We then extend the analysis for distributed MAC with variable data rates and shadow fading. Finally, we extend the analysis to include interference for dense networks and the effect of relays in such networks.

1.1 Contribution

The dissertation consists of the following major sections and the major contribution of this dissertation has been listed as follows:

- In chapter 3, we present a novel approach for throughput analysis of multi-hop multi-rate wireless local area networks (WLANs). The contents of this chapter have been published in [46] and [47]

- In chapter 4, we extend the approach from chapter 3 to include a general analytical model for the physical layer (PHY) throughput. The contents of this chapter have been published in [45]

- In chapter 5 we extend the analysis to take into consideration decentralized medium access control in the form of carrier sensing multiple access with collision avoidance (CSMA/CA) employing request to send and clear to send frames (RTS/CTS). The contents of this chapter have been published in [86] and [88]

- In chapter 6 we explore the effect of interference and fading in the physical layer on the performance of the MAC layer. The interference effects are considered for both
femto cells and Wi-Fi for densely deployed networks with and without the presence of relays. The contents of this chapter are published in [89]

1.2 Dissertation Outline

The remainder of this dissertation is organized as follows: Chapter 2 introduces the research background for the entire dissertation. Chapter 3 presents the novel approach for throughput analysis for multi-hop multi-rate WLANs using an empirical model. Chapter 4 extends the analysis to a general analytical model for the throughput-distance relationship. The model considers fading effects. Chapter 5 considers decentralized MACs and applies the approach presented in chapter 3 to VANETs. Chapter 6 considers the effects of interference and fading in the physical layer on the MAC throughput with and without the presence of relays. Finally, Chapter 7 presents the conclusion to this dissertation and discussion of future works.
Chapter 2

Relay Research in The Literature

In this chapter we go over relevant literature concerning relay networks. We start by looking at background research for multi-hop networks. Afterwards, we explore the literature concerning modern applications for multi-hop relay networks such as first responder applications. Finally, we go over the issue of interference and medium access control effects available in the literature for next generation networks.

2.1 Multi-hop Networks in General

The advantage of multi-hop networks is their ability to deliver data from a source to a destination in a large coverage area. In such a network, when a target receiver is located beyond the coverage area of an access point, relays are added between the source and destination terminals to extend the coverage. In a multi-rate transmission data network such as a Wireless Local Area Network (WLAN), the data rate is a function of the distance from the access point or the relay. Therefore, for a given relationship between the throughput and the distance and a given distance between the access point and the target receiver there must be a minimum number of hops that provides the maximum throughput to the target receiver.

Using empirical data from simulation platforms, the impact of a multi-hop wireless channel on the performance of a WLAN has been discussed in the literature [1] [2] [3]. In these analyses, however, the transmission data rate of an AP is assumed to be fixed and hops between the transmitter and the receiver are managed so that they can carry the load provided
by the AP. In practice, all modern wireless data communication networks such as WLANs, WPANs and cellular data services are implemented on multi-rate modems. They are installed in scenarios where the distances between different elements of the network are not the same. Since in multi-rate modems the data rate is a function of distance between the transmitter and the receiver [4], the throughput of a link must be a function of the distance as well. In particular, for a realistic installation with different link lengths among the elements of the network we are interested in bounds on the performance of the network.

Results of both empirical measurements [4] [5] and analytical modeling [6] [7] of the relation between the throughput and distance in a WLAN reveal that this relationship can be shown by a continuous function of distance that is parameterized by the maximum possible throughput and decay rate of the first link (AP). On the other hand, results of empirical data for multi-hop networks [1] [2] [3] reveal that the Media Access Control (MAC) throughput in a multi-hop network with N-hops is proportional to the maximum PHY throughput that occurs without relays. The placement of a single relay station (RS) problem in IEEE 802.16j networks was explored in [11], using a cooperative relay strategy of Decode-Forward (D-F) or Compress-Forward (C-F) to find both the optimal RS location and relay time allocation in a single stage. The aim was to obtain maximum overall system capacity as well as to meet the uneven distributed traffic demand of each subscriber station (SS). Multiple accesses with collision avoidance were used in WLANs, and hidden terminal (HT) and exposed terminal (ET) problems occurred depending on the distance between stations and the carrier sensing range. An AP cooperation system was proposed that detects the HT and ET problems between stations (STAs) [12]. Focusing on the uplink and categorizing the MAC-level
problems that occur in densely deployed WLANs, the system integrated the observed information and then detected the MAC-level problems based on their connected APs. The simulation results showed that the system can detect the MAC-level problems accurately, regardless of AP density or shadowing effect. Using IEEE 802.16e module for NS-2, particularly throughput versus distance, results were obtained [8]. A quantitative analysis was conducted via extensive simulation, and the optimal modulation and coding scheme as well as channel bandwidth profiles were identified for specific distances. The obtained data was used in test bed designs intended for installation along the BNSF Railway track in Nebraska. Also, the client access coverage area provided is investigated by this module and its expected throughput was determined.

2.2 Multi-relay Network applications for next generation systems

Vehicular Ad-hoc Networks (VANETs) have emerged an active area of research, standardization, and development due to its tremendous potential for improving vehicle and road safety, traffic efficiency, and convenience as well as comfort to both drivers and passengers [1]. Recent research efforts have placed a strong emphasis on the design architectures and implementations of VANET [2]. A significant amount of these efforts have been focused on routing, broadcasting, Quality of Service (QoS), and security of the VANET [1] [3]. A VANET turns every participating vehicle into a wireless transmitter or a relay, allowing a swarm of moving vehicles to connect and, in turn, create a network with a wide range. As vehicles fall out of the signal range and drop out of the network, other cars can join in, connecting vehicles to one another so that a mobile Internet is created. This technology is expected to be integrated for emergency responders such as police and fire vehicles
These studies have been extended to interactions with roadside equipment as well. In these studies, most vehicles are restricted in their range of motion, for example by being constrained to follow a paved highway or a railroad. In recent years using Wi-Fi technology for performance evaluation and deployment of multi-hop multi-relay adhoc wireless networks for non-vehicular networks has emerged as an active area of research. Deployment of WiFi networks is gaining increasing attention with the popularity of these networks and its consecutionals emergence of new WiFi networking standards with improved throughput and availability at low-costs. Applications of great importance is for emergency responders. The first scenario is for emergency broadcasting for vehicular safety. The second is establishing a dedicated link between a source node and receiver node. When an accident occurs, information can be relayed from an autonomous robot or from nearby vehicles. This data includes the health conditions of the passengers involved and accident information. Such information is usually large in size and requires immediate delivery. The transmitted information can go to the backbone roadside infrastructure or to a first responder vehicle that is on the road. Relays are used to establish an extended communication link to a receiving vehicle located outside the normal coverage range of the transmitting vehicle.

2.3 Interference for dense networks

Even with advancements made in the speed and ease of implementation of Wi-Fi networks, the basic nature of wireless transmission is generally unchanged. Ultra-dense networks offer unprecedented capacity by bringing the network closer to mobile users. Theoretically, the overall capacity scales with the number of small cells deployed.
However, in an ultra-dense deployment, not only desired signal strength but also interference from other cells increase due to the reuse of spectrum. Thus, interference has been considered as the key limiting factor for the system capacity in ultra-dense small cell networks. Moreover, small cells are usually opportunistically and irregularly deployed in the hotspots [17]. The ultra-dense and unplanned deployment of small cells has imposed a great challenge on interference management and radio resource optimization [18], [19]. The analysis is considered extremely challenging resulting on reliance on model simulations [23]. Currently, two trends in dense deployments are emerging in Wireless networks [24]. The first is a linear one-dimensional network and the second is for a two-dimensional network. The first type is popular in transportation and vehicular networks. The latter is more commonly used for wireless access in densely populated areas. They physical layer (PHY) of both methods is very similar. They both use multi-rate modems that drops the data rate as the user gets further away from modem. The main difference is in the medium access control (MAC) in which femto cell deployments use cellular time division multiple access whereas WLANs rely on carrier sensing. Ultimately the degradation of signal to noise ratio (SNR) through fading and interference is going to significantly affect client devices. As the network utilization increases, the interference effects will become more destructive. On the other hand, if interference is present and the network is lightly utilized, the effects of interference may not even be noticeable. However, if the Wi-Fi network is highly utilized, then even a small amount of interference can have a large and noticeable effect.

In [25] MAC protocol performance in MANETs was carried out under fading conditions. A comparison was made using the standard distributed coordination function
(DCF) in the 802.11 standard and the enhanced DCF (EDCF) introduced in the 802.11e standard. However, the analysis did not take into consideration interference effects. [26] presents a framework for analyzing the performance in terms of downlink SINR, coverage, and throughput of wireless network densification and for identifying its potential and challenge. They considered a channel model that captures multi-slope pathloss and general channel power distributions, including transmit power, shadowing, fast fading, as well as associated gains such as antenna pattern and beamforming gain. However, both models did not take MAC into consideration. New MAC protocols for TDMA and CSMA/CA are proposed with the main focus being to minimize collisions under poor channel conditions. However, those models do not take multi-rate transmissions into account.
Chapter 3

A Novel Approach for Throughput Analysis of Multi-hop Multi-Rate WLANs

Nowadays the deployment of the Wireless Local Area Network (WLAN) is under the limelight of the academic community. In a multi-rate data WLAN, the data rate is a function of the distance from the access point (AP). To extend the coverage of the AP to cover a target terminal outside the AP coverage area, relay nodes are deployed between the AP and the target terminal. Therefore, for a given relationship between the throughput and the distance, and a given distance between the access point and the target receiver, there is a minimum number of nodes that provides the maximum throughput to the target receiver. It is always desirable to optimize the deployment from various aspects. These aspects are application-dependent and they range from energy conservation in sensor networks to throughput and coverage maximization in data networks. This chapter presents a novel approach to determine the optimal bounds for the Medium Access Control (MAC) throughput at the target receiver in a multi-hop multi-rate wireless data network. We use the principles of linear programming and a model for the physical layer (PHY) throughput-distance relationship to determine the optimal performance bounds for the MAC throughput of a target terminal as a function of distance from the AP and the number of hops. Additionally, we determine the physical locations for the relays to reach the optimal MAC throughput for a target terminal located at a specific distance from the AP.

3.1 Introduction

The advantage of multi-hop networks is their ability to deliver data from a source to a destination in a large coverage area. In such a network, when a target receiver is located beyond
the coverage area of an access point, we add relays between the source and destination terminals to extend the coverage. In a multi-rate transmission data network such as a Wireless Local Area Network (WLAN), the data rate is a function of the distance from the access point or the relay. Therefore, for a given relationship between the throughput and the distance and a given distance between the access point and the target receiver there must be a minimum number of hops that provides the maximum throughput to the target receiver.

Using empirical data from simulation platforms, the impact of a multi-hop wireless channel on the performance of a WLAN has been discussed in the literature [1] [2] [3]. In these analyses, however, the transmission data rate of an AP is assumed to be fixed and hops between the transmitter and the receiver are managed so that they can carry the load provided by the AP. In other words, the links between the AP and the hops along with the links between the hops all share the same performance capabilities. In practice, all modern wireless data communication networks such as WLANs, WPANs and cellular data services are implemented on multi-rate modems. They are installed in scenarios where the distances between different elements of the network are not the same. Since in multi-rate modems the data rate is a function of distance between the transmitter and the receiver [4], the throughput of a link must be a function of the distance as well. Such statements imply that the distance among different elements of the network plays an important role in the throughput analysis of the network. In particular, for a realistic installation with different link lengths among the elements of the network we are interested in bounds on the performance of the network.

Results of both empirical measurements [5] [6] and analytical modeling [27] [28] of the relation between the throughput and distance in a WLAN reveal that this relationship can be shown by a continuous function of distance that is parameterized by the maximum possible throughput.
and decay rate of the first link (AP). On the other hand, results of empirical data for multi-hop networks [1] [2] [3] reveal that the Media Access Control (MAC) throughput in a multi-hop network with N-hops is proportional to the maximum PHY throughput that occurs without relays.

The placement of a single relay station (RS) problem in IEEE 802.16j networks was explored in [30], using a cooperative relay strategy of Decode-Forward (D-F) or Compress-Forward (C-F) to find both the optimal RS location and relay time allocation in a single stage. The aim was to obtain maximum overall system capacity as well as to meet the uneven distributed traffic demand of each subscriber station (SS). Multiple accesses with collision avoidance were used in WLANs, and hidden terminal (HT) and exposed terminal (ET) problems occurred depending on the distance between stations and the carrier sensing range. An AP cooperation system was proposed that detects the HT and ET problems between stations (STAs) [31][82]. Focusing on the uplink and categorizing the MAC-level problems that occur in densely deployed WLANs, the system integrated the observed information and then detected the MAC-level problems based on their connected APs. The simulation results showed that the system can detect the MAC-level problems accurately, regardless of AP density or shadowing effect. Using IEEE 802.16e module for NS-2, particularly throughput versus distance, results were obtained [32]. A quantitative analysis was conducted via extensive simulation, and the optimal modulation and coding scheme as well as channel bandwidth profiles were identified for specific distances. The obtained data was used in test bed designs intended for installation along the BNSF Railway track in Nebraska. Also, the client access coverage area provided is investigated by this module and its expected throughput was determined.

In this chapter we use principles of linear programming to develop a mathematical framework for analysis of the behavior of multi-hop networks as a function of distance between
the source and destination terminals and the number of hops. We first analyze the behavior of the throughput as a function of distance between the intermediate hops. Then we derive bounds on the performance of a multi-hop network for a linear relation between throughput and distance of a terminal in the absence of hops. To reach the optimal user data rate, assume that there is no frame overhead and no spacing between consecutive frames. In general in a condensed area wireless nodes are visible to each other. Therefore, we also assume that all relay nodes are visible to each other.

The remainder of the chapter is divided as follows: In section 3.2 we present the general statement of the problem and formulate it as a linear programming optimization problem. Section 3.3 provides the details of the optimization process to calculate the optimum throughput-distance relationship using the linear programing approach. This section solves the problem for different number of hops. Section 3.4 provides conclusions of our research.

3.2 General Statement of The Problem

The throughput-distance relationship for 802.11b is empirically shown to be linear [6] [7]. Generalizing linear relationship, we can represent the PHY throughput as:

\[ f(r) = ar + b = a(r - R_{\text{max}}); r \leq R_{\text{max}} \]  

(1)

where the PHY throughput is \( f(r) \) and \( r \) is the distance between the transmitter and receiver terminals [6]. Parameter \( R_{\text{max}} \) is the maximum coverage of the transmitter, \( b \) is the maximum throughput, and \( a = -\frac{b}{R_{\text{max}}} \) is the decay rate of the throughput.
Fig. 1 shows the general scenario for a multi-hop network. The first device on the left side is the access point AP followed by \( N - 1 \) relays, \( H_i; i = 1, 2, \ldots, N - 1 \), where \( N \) is the number of hops. The target terminal \( T \) at the other end of the network. The distance between the target terminal and the AP is \( D = \sum_{i=1}^{N-1} r_i \). It is important to make the distinction between MAC and PHY throughputs. Let the MAC throughput be \( S(D) \), and the PHY throughput be \( f(r_i) \). The empirical results in [1] [2] [3] show that the MAC throughput at target destination terminal \( T \) is given by \( S(D) \leq \frac{f(r_i)}{N} \). That is, the MAC throughput is a function of the PHY throughput of the first link and the number of hops. This result holds if the distance between the consecutive intermediate hops is less than a threshold distance that can support the traffic. In particular, when the distance between two consecutive terminals is greater than the maximum coverage distance \( R_{\text{max}} \), the throughput \( S(D) \) approaches zero. Therefore, the most general description of the throughput in a multi-hop network is given by:

\[
S(D) \leq \frac{f(r_i)}{N} \quad \text{where} \quad D = \sum_{i=1}^{N} r_i \quad (2)
\]
This general statement is based on the empirical observations in [1] [2] [3]. In the following section we discuss the behavior of this function based on physical constraints of the medium to find the conditions for optimal performance to calculate an upper bound for a linear throughput distance relationship.

3.3 Linear Programming for Throughput Optimization

The first step in optimization is to keep the distance between any two consecutive devices less than the maximum distance $R_{max}$, that is, we must satisfy the condition $r_i; 1 \leq i \leq N$, such that $r_i \leq R_{max}$. If this condition does not hold, the communication link is broken and we have zero throughput. Additionally, relays are added to extend the coverage. Therefore, if we add $N$ relays, we wish to extend the coverage to a bounded distance $N \times R_{max}$. The second step in optimization is to ensure that all the links on the path can support the PHY throughput in the first hop, $f(r_i) \geq f(r_1)$, and to ensure that we can achieve the maximum MAC throughput of $S(D) \leq f(r_i)/N$. From this condition we can determine a relationship for distance between the relays. Using (1), we have $ar_1 + b \geq ar_i + b$. This implies that $r_1 \geq r_i$. The third step in optimization is that, since $r_1 \leq D$, for up to certain distance values, if we add a relay at the distance $Z_i$, the initial MAC throughput with no relays ($N = 1$) will be greater than throughput with a relay at a distance $D \leq Z_i$. Stated differently, from (1), the MAC throughput $S(D)$ is less than or equal to the PHY throughput. Adding any number of hops between the transmitter and the receiver will reduce the MAC throughput below the maximum PHY throughput. From (2) we also see that $S(D)$ is a function of the throughput of the first hop. Therefore, it depends on the distance of the first relay and consequently we can assume all other relays are located at equal distances. Considering (1) and the above discussion, we can describe the optimization problem as:
Max: \( S(D) = \frac{f(r_1)}{N} \) where \( D = \sum_{i=1}^{N} r_i \) subject to

\[
\begin{align*}
  f(r_i) &\geq f(r_1) \Rightarrow f(r_i) \geq \frac{f(r_1)}{N} & \quad 2 \leq i \leq N \\
  r_i &\leq R_{\text{max}} & \quad 2 \leq i \leq N \\
  r_1 &\geq r_i & \quad 2 \leq i \leq N \\
  r_i &\leq r_{i+1} & \quad 2 \leq i \leq N - 1 \\
  D &\leq N \times R_{\text{max}} \Rightarrow D \leq N \times r_1
\end{align*}
\] (3)

Using the linear throughput-distance relationship shown in (1), the objective function and set of constraints in (3) reduce to a classical linear programming problem [29].

### 3.3.1 Optimum Solution for \( N = 1 \)

To solve this linear programming problem, we start with the no relay scenario where \( N = 1 \). If we have no relay the distance \( r_1 \) will be the same as \( D \) and therefore the MAC throughput will be the same as the PHY throughput. From (1), the MAC throughput is given by \( S(D) \leq f(r_1)|_{r_1=D} = aD + b \). When we go to the \( N = 2 \) case, we add a relay between the AP and \( T \). Depending on the location of the hop, the MAC throughput from (2) will be \( S(D) \leq \frac{ar_1+b}{2} \). In the \( N = 1 \) case, \( r_1 \) is the distance from the AP to the target terminal \( T \). In the \( N = 2 \) case, \( r_1 \) is the distance from the AP to the relay and \( r_2 \) is the distance from the relay to the target terminal \( T \).

The linear relationship for \( N = 1 \) and \( N = 2 \) cases is shown in Fig. 2. In Fig. 2, the MAC throughput of the direct link up to a certain distance \( Z_1 \) is greater than the MAC throughput with a single relay. In the vicinity of the AP, the MAC throughput without hops is \( b \); if we use a relay, for \( D < Z_1 \) the maximum MAC throughput reduces to \( \frac{b}{2} \). From Fig. 2 we see that using a relay between the transmitter and the receiver will reduce the MAC throughput and the optimal throughput is attained if we avoid using any relays (i.e. \( N = 1 \)). Since the rate of reduction of throughput for direct connection is twice the rate of reduction when we use a relay between the
AP and \( T \); as \( D \) is increased the throughput of direct connection reduces faster than the throughput when we have a single relay. At \( D = Z_1 \) the throughput of the direct connection and the throughput of the connection with one relay become equal. In other words, throughput for \( N = 1 \) and \( N = 2 \) are equal at distance \( Z_1 \). At distance \( D > Z_1 \) the maximum throughput with one relay (i.e. \( N = 2 \)) is greater than the throughput without relays (\( N = 1 \)). Therefore, the first piece of the optimal solution or the bound on the performance is given by \( S_{\text{max}}(D) = aD + b; 0 < D \leq Z_1 \).

\[ S(D) \]

Fig. 2. Bounds for optimal performance of a two-hop (single relay) network

### 3.3.2 Optimum Solution for \( N = 2 \)

For \( N = 2 \) we go back to the general description given in (3) from which we have:

\[
Max: S(D) = \frac{f(r_1)}{2} = \frac{ar_1 + b}{2} \quad \text{where } D = r_1 + r_2 \text{ subject to}
\]

\[
\begin{align*}
  &ar_2 + b \geq \frac{ar_1 + b}{2} \\
  &\Rightarrow a(D - r_1) + b \geq \frac{ar_1 + b}{2} \\
  &\Rightarrow 2aD - 3ar_1 + b \geq 0 \\
  &\Rightarrow 2D - 3r_1 - R_{\text{max}} \geq 0 \quad (4)
\end{align*}
\]
This is a classical linear programming problem with a function to be maximized subject to a set of constraints. Fig. 3 illustrates the constraint lines and the shaded area in which all constraints are satisfied. Since the decay rate $a$ is a negative number, the optimization objective function $S(D)$ for a given value of $D$ will be maximized for the smallest possible value of $r_1$. Therefore, the optimum solution is provided by the lines in the first quarter plane with positive values. To determine the distance $Z_1$, we refer back to Fig. 3. The smallest value of $r_1$ that maximizes the distance $D$ is located at the lower end of the solution region using the last constraint, we have $D = 2r_1$. Substituting $r_1 = \frac{D}{2}$ and solving for $D$, we obtain $D = Z_1 = \frac{2}{3}R_{max}$. Going back to Fig. 2, for $D \leq \frac{2}{3}R_{max}$, we do not need any relays and the maximum throughput is given by $S_{max}(D) = aD + b$. For $Z_1 < D < 2R_{max}$ we have $r_1 = \frac{2aD+b}{3a}$ and $S_{max}(D) = \frac{ar_1+b}{2}$. With $Z_1 \leq \frac{2}{3}R_{max}$, we have:

$$S_{max}(D) = \begin{cases} aD + b, & \text{for } 0 < D \leq \frac{2R_{max}}{3} \\ \frac{aD}{4} + \frac{b}{2}, & \text{for } \frac{2R_{max}}{3} < D \leq 2R_{max} \\ 0, & \text{otherwise} \end{cases}$$

(5)

Fig. 2 shows the optimum throughput-distance relation for one hop. The breakpoint distance is located at distance $Z_1 = D = \frac{2}{3}R_{max}$. To determine the optimal relay distance $r_1$, we go back to the solution region shown in Fig. 3. The optimal relay is located at the minimum distance of $r_1$ that maximizes $D$. Therefore, the solution is located at the line $D = 2r_1$ or equivalently $r_1 = \frac{D}{2}$. 

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3.3.3 Optimum solution for N = 3

Referring back to the general description of the problem we have:

\[ \text{Max: } S(D) = \frac{f(r_3)}{3} = \frac{a r_3 + b}{3} \text{ where } D = r_1 + r_2 + r_3 \text{ subject to} \]

\[
\begin{align*}
  ar_3 + b & \geq \frac{a r_3 + b}{3} \\
  \Rightarrow a \left( \frac{D - r_1}{2} \right) + b & \geq \frac{a r_3 + b}{3} \\
  \Rightarrow aD & - \frac{5}{6} ar_1 + \frac{2}{3} b \geq 0 \\
  r_2 &= r_3 \leq R_{\text{max}} \\
  r_1 &\geq r_2 \\
  r_2 &= r_3 \\
  D &\leq 3r_1
\end{align*}
\]
From Fig. 4 we see from the constraints that $S(D) = \frac{ar_1 + b}{3} = \frac{aD + b}{3} = \frac{aD}{9} + \frac{b}{3}$. Recalling that the decay rate $a = -\frac{b}{R_{\max}}$, to determine the distance $Z_2$ at the second breakpoint, we set $S(D)|_{N=2} = S(D)|_{N=3}$. From the equation, the second throughput breakpoint is located at distance $D = Z_2 = \frac{6}{5}R_{\max}$. To get the optimal MAC throughput at $Z_2$, we use $S(D) = \frac{6}{5}R_{\max}|_{N=3}$ which results in $\frac{1}{5}b$. For distance $D < Z_1$, the optimal throughput is achieved when we do not use any relays. For a distance $Z_1 \leq D < Z_2$, the optimal throughput is achieved when a single relay is used. For distance $Z_2 \leq D < 3R_{\max}$, the optimal throughput is achieved when we use two relays. With $Z_1 = \frac{2}{3}R_{\max}, Z_2 = \frac{6}{5}R_{\max}$, we obtain:

$$S_{\text{max}}(D) = \begin{cases} 
  aD + b, & \text{for } 0 < D \leq \frac{2R_{\max}}{3} \\
  \frac{aD}{4} + \frac{b}{2}, & \text{for } \frac{2}{3}R_{\max} < D \leq \frac{6}{5}R_{\max} \\
  \frac{aD}{9} + \frac{b}{3}, & \text{for } \frac{6}{5}R_{\max} < D \leq 3R_{\max} \\
  0, & \text{otherwise}
\end{cases}$$

(7)

Fig. 4 shows the optimum throughput-distance relationship for the two-relay case. To determine the relay distance, we have:

$$D = r_1 + r_2 + r_3 = r_1 + 2r_2$$

(8)

which follows from the constraint $r_1 \geq r_2 = r_3$ shown in (6). To determine the optimal relay locations, we select $r_1$ and adjust $r_2$ and $r_3$ to carry to carry the load. From (8),

$$r_2 = r_3 = \frac{D - r_1}{2}$$

(9)
3.3.4 Optimum solution for k-hops

The solution can be generalized to $N = k$ hops. The solution is recursive and $R_{\text{max}}$ is the maximum distance allowed for a hop. In all cases, the values of $r_k$ depend on the previous $r_{k+1}$ value. The key is to select an optimum $r_1$ and adjust the rest of the distances so they can carry the load. Referring to Fig. 1 and the set of constraints shown in (2), we have:

\[ S_k(D) = \frac{aD + b}{k} = \frac{aD}{k^2} + \frac{b}{k} \]
\[ S_{k+1}(D) = \frac{aD + b}{k+1} = \frac{aD}{(k+1)^2} + \frac{b}{k+1} \]  

(10)

To determine the distance $Z_k$ at the $k$th breakpoint, we set $S_{k+1}(D) = S_k(D)$ and use the decay rate $a = -\frac{b}{R_{\text{max}}}$. Solving for $D$ we obtain:

\[ Z_k = D = \frac{k(k+1)R_{\text{max}}}{2k+1} \]  

(11)
To determine the throughput at the $k$th breakpoint, we substitute (11) into (10) resulting in:

$$S_k(D = Z_k) = \frac{b}{2k+1}$$  \hspace{1cm} (12)

And the optimal throughput bound is given by:

$$S_{k,max}(D) = \begin{cases} 
  aD + b & 0 < D \leq Z_1 \\
  \frac{aD}{k^2} + \frac{b}{k} & Z_{k-1} < D \leq Z_k \\
  k > 1 & k \geq 1
\end{cases}$$  \hspace{1cm} (13)

To determine the optimal relay locations, we have $D_k = r_1 + r_2 + \cdots + r_k$ subject to $r_1 \geq r_2 = r_3 = \cdots = r_k$. This results in:

$$D_k = r_1 + (k = 1)r_2$$  \hspace{1cm} (14)

Substituting $Z_k$ from (11) and solving for $r_2$ and recalling that $r_2 = \cdots = r_k$ we have:

$$r_2 = \cdots = r_k = \frac{D_k - r_1}{k-1}; \quad k \geq 2$$  \hspace{1cm} (15)

It can be noticed that (10), (11), (13) and (15) give the general form to obtain the breakpoint distance $Z_k$, the maximum MAC throughput $S_{max}$ at a distance $D$, and the relay intermediate distances $r_i$.

### 3.4 Summary

In this chapter, we present a novel approach to obtain the optimal performance bounds for a multi-hop multi-rate WLAN as a function of distance between the relays. The optimal distance is derived from the throughput-distance relationship of the access point and the relays subject to a set of constraints on the PHY and MAC throughputs. For the linear throughput distance relationship for 802.11b, we determine the number of relays and the optimal separation distance between the relays. We have shown analytical results for 1, 2 and 3 hops and generalized the solution to an arbitrary number of $k$ hops. Furthermore, we show that the general solution to achieve the optimal throughput at distance $D$ depends on two parameters. These parameters are
the coverage radius of the access point and relays, $R_{max}$, and the distance of first relay from the access point.
Chapter 4

Optimal Throughput-Distance Relationship in Multi-Relay Wireless Data Networks

The advantage of multi-hop networks is their ability to deliver data from a source to a destination in a large coverage area without the deployment of expensive infrastructure. When a target receiver is located beyond the coverage range of an Access Point (AP), relays are added between the source and destination terminals to extend the coverage. In some situations, relays are used within the coverage area of the AP to increase the throughput after a certain distance from the AP. In a multirate transmission data network such as a Wireless Local Area Network (WLAN), the data rate is a function of the distance from the access point or the relay. Therefore, for a given relationship between the throughput and the distance and a given distance between the access point and the target receiver there must be a minimum number of relays that provides the maximum throughput.

Using empirical data from simulation platforms, the impact of a multi-hop wireless channel on the performance of a WLAN has been discussed in the literature [1] [2] [3]. In these analyses, however, the transmission data rate of an AP is assumed to be fixed and hops between the transmitter and the receiver are managed so that they can carry the load provided by the AP. In other words, the links between the AP and the relays along with the links between the relays all share the same performance capabilities. In practice, all modern wireless data communication networks such as WLANs, WPANs and cellular data services are implemented on multi-rate modems. They are installed in scenarios where the distances between different elements of the network are not the same. Since in multi-rate modems the data rate is a function of distance
between the transmitter and the receiver [4], the throughput of a link must be a function of the distance as well. In particular, for a realistic installation with different link lengths among the elements of the network we are interested in bounds on the performance of the network. Results of both empirical measurements [5] [6] [33] and analytical modeling [34] [35][64] of the relation between the throughput and distance in a WLAN reveal that this relationship can be shown by a continuous function of distance that is parameterized by the maximum possible throughput and decay rate of the first link. On the other hand, results of empirical data for multi-hop networks [1] [2] [3] reveal that the Medium Access Control (MAC) throughput in a multi-hop network with N-hops is proportional to the maximum physical (PHY) layer throughput that occurs without relays.

Optimization to the deployment of multi-hop multi-relay networks has been investigated in the literature. The placement of a single relay station (RS) problem in IEEE 802.16j networks was explored in [36], using a cooperative relay strategy of Decode-Forward (D-F) or Compress-Forward (C-F) to find both the optimal RS location and relay time allocation in a single stage. The aim was to obtain maximum overall system capacity as well as to meet the uneven distributed traffic demand of each subscriber station (SS). Multiple accesses with collision avoidance were used in WLANs, and hidden terminal (HT) and exposed terminal (ET) problems occurred depending on the distance between stations and the carrier sensing range. An AP cooperation system was proposed that detects the HT and ET problems between stations (STAs) [31]. Focusing on the uplink and categorizing the MAC-level problems that occur in densely deployed WLANs, the system integrated the observed information and then detected the MAC-level problems based on their connected APs. The simulation results showed that the system can detect the MAC-level problems accurately, regardless of AP density or shadowing effect. Using IEEE 802.16e module for NS-2, particularly throughput versus distance, results were obtained [32]. A quantitative
analysis was conducted via extensive simulation, and the optimal modulation and coding scheme as well as channel bandwidth profiles were identified for specific distances. The obtained data was used in test bed designs intended for installation along the BNSF Railway track in Nebraska. Also, the client access coverage area provided is investigated by this module and its expected throughput was determined.

In this chapter, we develop a mathematical framework for the analysis of the behavior of multi-hop data networks as function of the distance between the source terminal, destination terminal and the intermediate distances between the relays and AP. First, we determine the intermediate distances between the AP and relays to obtain the optimal performance bounds as a function of the distance from the AP to the target terminal and the number of relays. We then determine the required number of relays to get the optimal throughput at a target terminal located at a desired distance. We demonstrate the results of the analysis using a general analytical model for the PHY throughput distance relation. Using the general model we show the optimal performance bounds for the network and the MAC throughput distance relation for a deployment optimized for a desired distance.

The remainder of the chapter is divided as follows: In Section II we present the network system model and general statement of the problem. In section III we show the proposed solution theorems and their proofs. Section III contains an application example for a general analytical PHY throughput-distance model. Section IV provides the conclusions of our research.

4.1 Network System Model

In section 4.1.1 we detail network system model. We start with introducing the general physical (PHY) layer model. In section 4.1.2, we then present the medium access control (MAC)
layer model. Afterwards, in section 4.1.3 we go over the general statement of the problem and provide our analysis.

4.1.1 Physical Layer Model

In wireless transmission, the physical (PHY) layer throughput is a monotonically increasing function of the signal-to-noise ratio. In practice, manufacturer data sheets specify the data rate as a discrete function of the receiver sensitivity. Table I shows the general description of the discrete relationship between the PHY throughput and the receiver sensitivity. If we assume the target terminal \( T \) is located at a distance \( r_1 \) within the Access Point (AP) coverage range, the received signal power \( P_r \) is given by

\[
P_r(r_1) = g(r_1) + \eta
\tag{1}
\]

where \( g(r_1) \) is the path loss model used for the transmission medium and \( \eta \) is a Gaussian random variable with variance \( \sigma \) representing shadow fading. The IEEE 802.11 standard defines \( g(r_1) \) as

\[
g(r_1) = P_t - L_0 - \begin{cases} 
10\alpha_1 \log_{10}(r_1), & r \leq r_{bp} \\
10\alpha_1 \log_{10}(r_{bp}) - 10\alpha_2 \log_{10}\left(\frac{r_1}{r_{bp}}\right), & r > r_{bp}
\end{cases}
\tag{2}
\]

where \( P_t \) is the transmission power, \( L_0 \) is the path-loss in the first meter, \( \alpha_1 \) and \( \alpha_2 \) are the distance power gradients and \( r_{bp} \) is the break point distance.
Equations (1) and (2) show that $P_r(r_1)$ is a Gaussian random variable with mean $g(r_1)$ and variance $\sigma$. As the mobile terminal moves away from the AP, it experiences varying data rates at each location due to the random fluctuations in the received signal. Therefore, the average spatial PHY throughput $f(r_1)$ is calculated as

$$f(r_1) = \sum_i R_i P\{R_i(r_1)\}$$

(3)

where $P\{R_i(r_1)\}$ is the probability of getting throughput $R_i$ at distance $r_1$ and is obtained from [34]

$$P(R_i|r_1) = \begin{cases} 
1 - \text{erf}c\left(\frac{g(r_1) - P_i}{\sqrt{2}\sigma}\right), & i = 1 \\
\frac{1}{2} \text{erf}c\left(\frac{g(r_1) - P_{i-1}}{\sqrt{2}\sigma}\right) - \frac{1}{2} \text{erf}c\left(\frac{g(r_1) - P_i}{\sqrt{2}\sigma}\right), & i \geq 2
\end{cases}$$

(4)

Graphically, the PHY throughput $f(r_1)$ in (3) has the general shape shown in Fig. 1. Fig. 1 shows that the PHY throughput $f(r_1)$ shown in (3) is a monotonically decreasing function of distance $r_1$. 

---

**TABLE I**

**GENERAL DESCRIPTION OF THE RELATIONSHIP BETWEEN RECEIVER SENSITIVITY AND THROUGHPUT**

<table>
<thead>
<tr>
<th>Data rate $R_i$</th>
<th>Receiver Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>$P_1$ decreasing</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$P_2$ decreasing</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$R_M$</td>
<td>$P_M$</td>
</tr>
</tbody>
</table>
4.1.2 MAC Layer Model

In wireless data network, the MAC protocol is responsible for the coordination of multiple terminals over a shared channel. Relays are used to extend the coverage of the access point and to increase the throughput to mobile terminal $T$. Fig. 2 shows a model of a network with a single relay. Let $D$ be the distance between the AP and the target terminal $T$. For a deployed single relay network, the throughput-distance relation between MAC throughput $S(X)$ for deployment distance $X$ is shown in Fig. 3. As an example, consider a target terminal $T$ located near the border of the coverage range of the AP at distance $D = 145m$. The addition of a relay to the network at distance $r_1$, increased the throughput to the receiver located at $145m$ and increased the coverage range of the AP to $250m$. The tradeoff is that the addition of the relay affects the overall MAC throughput.
Let $S(D)$ be the MAC throughput at the location of the target terminal separated by a distance $D$ from the AP. Assuming the MAC layer uses Time-Division-Multiple-Access (TDMA) for a single relay network shown in Fig. 2, the physical transmission medium will be shared between the AP and relay. Therefore, $S(D)$ will be bounded by

$$S_2(D) \leq \frac{f(r_1)}{2} \text{ where } D = r_1 + r_2$$  \hspace{1cm} (5)

Fig. 2. Topology for a single relay network.
Fig. 3. Effect of relay addition on deployment throughput-distance relation.

To extend the coverage of the AP to larger areas, additional relays are needed. The additional relays will share the physical medium and reduce the MAC throughput even further.
4.1.3 General Statement of the Problem

Fig. 4 shows the general scenario for a multi-hop network. The first device on the left side is the AP followed by $N - 1$ relays, $H_i; i = 1, 2, ..., N - 1$, where $N$ is the number of hops.

<table>
<thead>
<tr>
<th>$P_i$ (dBm)</th>
<th>RSS (dBm)</th>
<th>PHY Data Rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>-72</td>
<td>54</td>
</tr>
<tr>
<td>19</td>
<td>-73</td>
<td>48</td>
</tr>
<tr>
<td>20</td>
<td>-78</td>
<td>36</td>
</tr>
<tr>
<td>20</td>
<td>-81</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>-84</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>-85</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>-87</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>-87</td>
<td>6</td>
</tr>
</tbody>
</table>

The target terminal $T$ is located at the other end of the network. We assume that there is no frame overhead and no spacing between consecutive frames. We also assume that all relay nodes are visible to each other. Sharing the physical medium with $N - 1$ relays using TDMA, the MAC throughput $S_N(D)$ is bounded by

$$S_N(D) \leq \frac{f(r_1)}{N} \text{ where } D = \sum_{i=1}^{N} r_i$$

Fig. 4. General scenario of a multi-hop network.
where $D$ is the distance from the AP to target terminal $T$. Equation (6) indicates that the MAC throughput at distance $D$ is a function of the PHY throughput of the first link $f(r_1)$ and the number of hops $N$.

### 4.2 Performance Analysis

In section 4.2.1, we start by determining the intermediate distance between the relays and access point. Then, in 4.2.2 we determine minimum number of relays and the optimal performance bounds for the MAC throughput. Finally, in 4.2.3, we apply our results using 802.11a and compare our analytical results with that of a network simulator.

#### 4.2.1 Intermediate Distance Between Relays and Access Point

We start the analysis by determining the intermediate physical separation distances between the AP and relays to maximize the MAC throughput for a target terminal located at a distance $D$ from the AP.

**Theorem 1:** Given a distance $D$ in a wireless data network with $N-1$ relays, the optimal intermediate distance between the relays and the AP is

$$r_1 = r_k \text{ for } 2 \leq k \leq N$$

### Proof:
From (3) and (4), the PHY throughput $f(r_1)$ is a monotonically decreasing function of distance $r_1$. Mathematically this is represented as $f'(r_1) \leq 0$. In order to carry the traffic through a network with $k-1$ relays, all relay nodes must at least carry the PHY throughput of the AP. That is $f(r_k) \geq f(r_1); 2 \leq k \leq N$.

Since $f(r_1)$ is a monotonically decreasing function, this implies that $r_k \leq r_1$. Therefore, the distance $D$ separating the AP and $T$ is bounded by $D \leq kr_1$. From $f'(r) \leq 0$, we require the
minimum value of \( r \) that maximizes the PHY throughput. From the distance bound we have \( r_1 \geq \frac{D}{k} \) with a minimum value of \( \frac{D}{k} \). This implies that all the intermediate \( r_k \) distances are equal, resulting in (7).

For a network with \( N - 1 \) relays, from (7) we have \( r_1 = \frac{D}{N} \). Then the upper bounds for \( S(D) \) can be represented as a function of distance \( D \) and the number of relays \( N - 1 \) as

\[
S_N(D) = \frac{f\left(\frac{D}{N}\right)}{N}
\]

(8)

For a general monotonically decreasing function, this provides the maximal bounds on the MAC throughput as a function of target distance \( D \) and the number of hops \( N \). Fig. 5 shows the optimal MAC throughput bound in (8) for different number of relays. In all cases of Fig. 5, the AP and relays are equi-distant from each other.

### 4.2.2 Optimal MAC Throughput Bounds and Minimum Number of Relays

Given the MAC throughput bound in (11), we determine the maximum achievable throughput at a distance \( D \) and the required number of relays to reach that throughput.

**Theorem 2:** In a relay network for any given distance \( D \), there is an optimal number of relays \( N - 1 \) that can attain the optimal MAC throughput \( S(D) \) given by

\[
S_{\text{max}}(D) = \frac{f\left(\frac{D}{N}\right)}{N} \text{ where } Z_{N-1} \leq D \leq Z_N
\]

(9)

And

\[
Z_N = D|_{S_{N-1}(D)=S_N(D)} \text{ for } N = 1, 2, 3, \ldots, k.
\]
Proof: The Theorem can be proven by induction. We present a base case and an inductive step to prove it. From (8), the MAC throughput for the direct connection is $S_1(D)$ and the MAC throughput for a single relay network is $S_2(D)$. They can be represented as

$$\begin{cases} 
S_1(D) = f(D) \\
S_2(D) = \frac{D}{2}
\end{cases} \quad (10)$$

The PHY throughput function $f(\cdot)$ is a monotonically decreasing function. From Fig. 5, $S_1(D)$ decreases faster than $S_2(D)$. However $S_1(D)$ has a larger initial values than $S_2(D)$. Therefore $S_1(D) \cap S_2(D)$ at distance $D = Z_1$. For $D \leq Z_1$ the MAC throughput for the direct connection to the AP $S_1(D)$ is greater than the MAC throughput for the single relay $S_2(D)$. Therefore we do not need any relays and the upper bound is $S_1(D)$. For $D > Z_1$ we need a single relay and the upper bound is $S_2(D)$ then the optimal performance bounds are

$$S_{\text{max}}(D) \begin{cases} 
S_1(D) = f(D) & D \leq Z_1 \\
S_2(D) = \frac{f(D)}{2} & D > Z_1
\end{cases} \quad (11)$$

Consider the case for $k-1$ relays and $k$ relay networks, from (8) we have

$$\begin{cases} 
S_k(D) = \frac{f(D)}{k} \\
S_{k+1}(D) = \frac{f(D)}{k+1}
\end{cases} \quad (12)$$

$S_k(D)$ has larger initial values and has a faster rate of decay than $S_{k+1}(D)$. Therefore $S_k(D) \cap S_{k+1}(D)$ at $D = Z_k$. For $D \leq Z_k$ The MAC throughput with $k-1$ relays is greater than the MAC throughput for a network with $k$ relays. Therefore, we do not need any relays and the upper bound
Fig. 5. MAC throughput bounds as a function of distance and number of relays.
is $S_k(D)$. For $D > Z_k$ we need $k$ relays and the upper bound is $S_{k+1}(D)$ and the optimal performance bounds are

$$
S_{\text{max}}(D) \begin{cases} 
S_k(D) = \frac{f(D)}{k} & D \leq Z_k \\
S_{k+1}(D) = \frac{f(D_{k+1})}{k+1} & D > Z_k
\end{cases} 
$$

(13)

By induction, we can conclude that for $k \geq 1$

$$
S_{\text{max}}(D) = \frac{f(D)}{k} \text{ with } Z_{k-1} \leq D \leq Z_k
$$

(14)

where

$$
Z_k = D|_{S_k(D)=S_{k+1}(D)}.
$$

In summary, given $N - 1$ relays, the optimal relay deployment to maximize the throughput for a target receiver located $D$ away from the AP is $r_k = \frac{D}{N}; 1 \leq k \leq N - 1$. To determine the number of relays for optimal deployment for a given $D$, we use the distance bound in (9).

4.2.3 Application to 802.11a

In this subsection we show the application to the IEEE 802.11a standard. We want to optimize the deployment throughput of the network for a distance $D = 400$m. Given the manufacturer data sheet in Table II and the path-loss model in (3) and (4) we obtain the PHY throughput distance relationship shown in Fig. 1. We show two scenarios. The first is with the restriction of using a specified number of relays. The second is with an arbitrary choice of relays.
For the 3 relays situation, let us assume we are restricted with the use of 3 relays \((N = 4)\). From Fig. 5, the maximum achievable MAC throughput at a distance of 400m using 3 relays is 3.547 Mbps. The optimal deployment is a direct result of (7) where the relays are placed 100m apart. Fig. 6 shows the optimal MAC throughput distance relation for the 3 relay network case. In [37], software simulation results have been provided without an analytical process and it shows that the maximum throughput for a 3 relay network is approximately 3.75Mbps. Our analytical model results match those of the simulation results.

We now consider the unspecified number of relays case. From Fig. 5, we see that the maximum throughput at a distance of 400m is 5.268Mbps using 6 relays \((N = 7)\). Using (7), the relays are placed 57.14m apart. The 6 relays deployment is shown in Fig. 6. The results in Fig. 6 reveal that the 6 relay network delivers higher throughput than the 3 relay network at distances greater than 371m. This holds for the desired distance at 400m. The first step in the optimization is to determine the minimum number of relays for optimal achievable MAC throughput at \(D = 400\)m. Due to the nonlinear nature of (3) and (4), we can perform numerical analysis to determine the optimal achievable throughput at \(D = 400\)m with the minimum number of relays required to achieve that throughput. Alternatively, we can determine the solution graphically using Fig. 5. Fig. 5 shows that the optimal throughput at \(D = 400\) is 5.268Mbps and it requires 6 relays \((N = 7)\).

4.3 Summary

In this chapter, we presented a novel approach to obtain the optimal performance bounds for a multi-hop multi-rate wireless data network. First, we determined the optimal relay placements for a target terminal located at a distance \(D\) away from the access point. Second, for a general analytical PHY layer throughput model, we determined the maximum achievable MAC throughput as a function of the number of relays for a target located at distance \(D\). Finally, we determined the
required number of relay nodes to maximize the MAC throughput for the target receiver. To validate the analysis, we applied the approach to IEEE802.11 standard and compared the analytical results to the results of a network simulator in two scenarios.
Fig. 6. Optimal MAC throughput-distance relation.
Chapter 5

A Novel Performance Analysis for Throughput of a VANET using Multi-hop Wi-Fi Relays

Since in wireless data networks throughput is a function of distance between communicating nodes, wireless devices deployed in vehicles can serve as relays for other vehicles to extend their coverage range and to optimize their throughput for wireless networking with neighboring vehicles. Using multi-hop relays one is able to find the optimum throughput for communication between two vehicles. For a given distance among communicating nodes and a specific throughput-distance relationship governing operation of a wireless data device, there is a minimum number of nodes that provides the maximum throughput to the target receiver. This chapter presents a novel performance analysis for to determine the optimal throughput in a multi-hop multi-rate Vehicular Adhoc Network (VANET) using WiFi signals for networking. The optimal number of relays is chosen to maximize the throughput for point-to-point communication between a source and a destination as well as broadcast among all vehicles in the coverage area of the source.

5.1 Introduction

Vehicular Ad-hoc Networks (VANETs) have emerged an active area of research, standardization, and development due to its tremendous potential for improving vehicle and road safety, traffic efficiency, and convenience as well as comfort to both drivers and passengers [17]. Recent research efforts have placed a strong emphasis on the design architectures and implementations of VANET [38]. A significant amount of these efforts have been focused on routing, broadcasting, Quality of Service (QoS), and security of the VANET [17] [48]. A VANET
turns every participating vehicle into a wireless transmitter or a relay, allowing a swarm of moving vehicles to connect and, in turn, create a network with a wide range. As vehicles fall out of the signal range and drop out of the network, other cars can join in, connecting vehicles to one another so that a mobile Internet is created. This technology is expected to be integrated for emergency responders such as police and fire vehicles [17][38][48]. These studies have been extended to interactions with roadside equipment as well [39][1]. In these studies, most vehicles are restricted in their range of motion, for example by being constrained to follow a paved highway or a railroad. In recent years using Wi-Fi technology for performance evaluation and deployment of multi-hop multi-relay adhoc wireless networks for non-vehicular networks has emerged as an active area of research [2][6]. Deployment of WiFi networks is gaining increasing attention with the popularity of these networks and its consecuitional emergence of new WiFi networking standards with improved throughput and availability at low-costs [1] [5].

In this chapter we discover the bounds on the throughput of the multi-hop multi-rate VANET networks using WiFi technology. We consider two scenarios of great interest for emergency responders. Fig. 1 illustrates the general concept behind our analysis. The first scenario is for emergency broadaccasting for vehicular safety. The second is establishing a dedicated link between a source node and receiver node. When an accident occurs, information can be relayed from an autonomous robot or from nearby vehicles. This data includes the health conditions of the passengers involved and accident information. Such information is usually large in size and requires immediate delivery. The transmitted information can go to the backbone roadside infrastructure or to a first responder vehicle that is on the road. Relays are used to establish an extended communication link to a receiving vehicle located outside the normal coverage range of the transmitting vehicle. In certain cases relays are used within the coverage area of the transmitter.
to increase the throughput past a certain distance from the transmitting vehicle at the cost of additional transmissions for a single packet. In a multi-rate transmission data network such as a VANET, the data rate is a monotonically function of the distance from the relay or access point. As such, for a known throughout-distance relationship and a given distance between the transmitting vehicle and the target vehicle, there must be a minimum number of relays that provides the maximum throughput. We start by determining the intermediate distances between the transmitter node and relays to obtain the optimal performance bounds as a function of the distance from the transmitting vehicle to the target receiver and the number of relays. Following that, we then determine the

![Communication scenarios for first responder application](image)

**Fig. 1.** Communication scenarios for first responder application. Blue links indicates a broadcasting scenario. Green links indicates dedicated vehicle-to-vehicle link

required number of relays to get the optimal throughput for broadcasting scenarios as well as a dedicated link between two specific vehicles. Using a general analytical model for the PHY throughput distance relation, we demonstrate the results of our analysis. We use the IEEE 802.11 RTS/CTS protocol for the MAC layer. Using the general PHY model we show the optimal
performance bounds for the network and the MAC throughput distance relation for a deployment optimized for either coverage or specific distance.

The rest of the chapter is organized as follows: In Section II we detail related research available in the literature. In Section III we present the physical (PHY) and Medium Access Control (MAC) network system model for VANETs. In section IV we formulate the optimal solution using the principles of mathematical programming. Section V shows the performance analysis for two application examples. Finally, Section VI provides the conclusions of our research.

5.2 VANETs Research

Recently, the analysis of multi-hop vehicular networks is garnering the interest of researchers [48][31]. The deployment and optimization of multi-relay and multi-hops networks has been well investigated in the literature [17][38][48]. For example, [39] explored the placement of a single relay station (RS) problem in IEEE 802.16j networks to find both the optimal RS location and relay time allocation in a single stage by using a cooperative relay strategy of Compress-Forward (C-F) or Decode-Forward (D-F). The goal was to get the maximum overall system capacity as well as to meet the uneven distributed traffic demand of each subscriber station (SS). The analysis however did not take MAC effects into consideration. The statistical and empirical performance of a WLAN using simulation platforms had been discussed in the literature [1] [2] [6][5]. In these analyses the operating principle is that the transmission data rate of an AP is to be fixed and hops between the transmitter and the receiver are managed so that they can carry the load provided by the AP. That is, the links between the AP and relays as well as the links between the relays all share the same performance capabilities. In practice, all modern wireless data communication networks such as VANETs, WLANs, WPANs and cellular data services are implemented on multi-rate modems. They are installed in scenarios where the distances between
different elements of the network are not the same. Since the data rate is a function of distance between the transmitter and the receiver in multirate modems[17][34][3], the throughput of a link must be a function of the distance as well. In particular, for a realistic installation with different link lengths among the elements of the network we are interested in bounds on the performance of the network. Both, analytical modeling [39] [1] [2] and empirical measurements results [5] [17] [34] of the relation between the throughput and distance in a WLAN reveal that this relationship can be shown by a continuous function of distance that is governed by the maximum possible throughput and decay rate of the first link. On the flip side, empirical data results for multi-hop networks [3] [28] reveal that the Medium Access Control (MAC) data-rate in a multi-hop network with $N$-hops is proportional to the maximum physical (PHY) layer throughput without relays. Multiple access with collision avoidance was used in WLANs, and hidden terminal (HT) and exposed terminal (ET) problems occurred depending on the distance between stations and the carrier sensing range. An AP cooperation system was proposed that detects the HT and ET problems between stations (STAs) [49]. Focusing on the uplink and categorizing the MAC-level problems that occur in densely deployed WLANs, the system integrated the observed information and then detected the MAC-level problems based on their connected APs. MAC-level problems were shown to be detected accurately using simulations independent of AP density or shadowing effect. Using IEEE 802.16e module for NS-2, particularly throughput versus distance, results were obtained [32][68][69]. An extensive simulation quantitative analysis was conducted to identify the bandwidth profiles, optimal modulation and coding scheme for specific distances. The simulation results were used in test bed designs intended for installation along the BNSF Railway track in Nebraska. Also, the client access coverage area provided is investigated by this module and its expected throughput was determined. In our analysis, we include the effects of fading, MAC
effects in addition to multi-rate transmissions in the network.

5.3 Physical and MAC Layer Models

We start our analysis by looking at the general physical layer model in 5.3.1. We determine the outage rate along with the probabilistic propagation model for the throughput-distance relationship. Then in 5.3.2, we introduce carrier sensing with RTS/CTS packets as an example of decentralized MAC.

5.3.1 Data Rate, Distance and Outage Rate

In wireless transmission, the physical (PHY) layer throughput is a monotonically increasing function of the signal-to-noise ratio. In practice, manufacturer data sheets specify the data rate as a discrete function of the receiver sensitivity. Table I shows the general description of the discrete relationship between the PHY throughput and the receiver sensitivity. If we assume the target vehicle T is located at a distance $r_1$ within the transmitting vehicle coverage range, then the general expression for the received signal power $P_r$ can be expressed as [17]

$$P_r(r_1) = g(r_1) + \eta$$

(1)

where $g(r_1)$ is the propagation path loss model used for the transmission medium and $\eta$ is a Gaussian random variable with variance $\sigma$ representing shadow fading. The function $g(r_1)$ is sometimes broken into multiple pieces. The distance partitioned pathloss models are the most popular for the 802.11 and JTC communities. These models can generally be represented by [47]

$$g(r_1) = \begin{cases} 
  P_t - L_0 - 10\alpha_1 \log(r_1), & r \leq r_{bp} \\
  P_t - L_0 - 10\alpha_1 \log(r_{bp}) - 10\alpha_2 \log\left(\frac{r_1}{r_{bp}}\right), & r > r_{bp}
\end{cases}$$

(2)

where $P_t$ is the transmission power, $L_0$ is the pathloss in the first meter, $\alpha_1$ and $\alpha_2$ are the distance power gradients, and $r_{bp}$ is the break point distance. Channel models for VANETs are starting to
emerge in the literature [70-75]. We note that our analysis is applicable to any pathloss model. The most recent pathloss model that is popular in the literature is given by [39]

\[ g(r_1) = \begin{cases} 
Pt - L_0 - 10\alpha_1 \log\left(\frac{r}{r_0}\right), & r_0 \leq r \leq r_{bp} \\
Pt - L_0 - 10\alpha_1 \log\left(\frac{r_{bp}}{r_0}\right) - 10\alpha_2 \log\left(\frac{r_1}{r_{bp}}\right), & r > r_{bp} 
\end{cases} \quad (3) \]

where \( r_0 \) is the reference distance used in the measurement data [39]. Equations (1) and (3) show that \( P_t(r_1) \) is a Gaussian random variable with mean \( g(r_1) \) and variance \( \sigma \). We note that this model is very similar to the 802.11 model [75]. As the distance between the receiving vehicle and transmitting vehicle changes, the receiving vehicle experiences varying data rates at each location due to the random fluctuations in the received signal. Therefore the average spatial PHY throughput \( f(r_1) \) is calculated as

\[ f(r_1) = \sum_i R_i P\{R_i(r_1)\} \quad (4) \]

<p>| TABLE I |
| MANUFACTURER DATA SHEET PARAMETERS FOR IEEE 802.11P FOR THE GENERAL DESCRIPTION OF THE THE RELATIONSHIP BETWEEN RECEIVER SENSITIVITY AND THROUGHPUT |</p>
<table>
<thead>
<tr>
<th>( P_t ) (dBm)</th>
<th>RSS (dBm)</th>
<th>PHY Data Rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>-72</td>
<td>27</td>
</tr>
<tr>
<td>19</td>
<td>-73</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>-78</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>-81</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>-84</td>
<td>6</td>
</tr>
<tr>
<td>20</td>
<td>-85</td>
<td>4.5</td>
</tr>
<tr>
<td>20</td>
<td>-87</td>
<td>3</td>
</tr>
</tbody>
</table>

where \( P\{R_i(r_1)\} \) is the probability of getting throughput \( R_i \) at distance \( r_1 \) and is given by [34][27]
\[ P(R_i[r]) = \begin{cases} 
1 - \text{erfc}\left(\frac{g(r_1) - P_i}{\sqrt{2}\sigma}\right), & i = 1 \\
\frac{1}{2}\text{erfc}\left(\frac{g(r_1) - P_{i-1}}{\sqrt{2}\sigma}\right) - \frac{1}{2}\text{erfc}\left(\frac{g(r_1) - P_i}{\sqrt{2}\sigma}\right), & i \geq 2 
\end{cases} \]  
(5)

The PHY throughput \( f(r_1) \) defined by (4) is a monotonically decreasing function of distance \( r_1 \).

Equation (5) gives the probability of existence of each data rate as a function of the distance between the transmitter and receiver. Consequently the outage rate \( P_{\text{out}} \) can be defined as

\[ P_{\text{out}} = 1 - \sum_i P(R_i[r]) \]  
(6)

For a given outage rate, we can determine the maximum coverage distance \( R_{\text{max}} \) by solving the nonlinear relation in (6). Fig. 2 shows the normalized PHY throughput \( f(r_1) \) and the outage rate for a single vehicle using the manufacturer data sheet values from Table I.

### 5.3.2 Throughput for RTS/CTS

In wireless data networks, the MAC protocol is responsible for the coordination of multiple terminals over a shared channel. To extend the coverage of the access point and to increase the throughput to a target vehicle \( T \), relays are deployed within a network. Due to the nature of road traffic, VANETs can be modeled using a linear network topology[30]. Fig. 3 shows a general model of the linear network topology. In [38] the optimal number of relays for IEEE802.11 RTC/CTS MAC to maximize the throughput and minimize the energy consumption was investigated. The throughput-energy relationship included the effects of the PHY and MAC layers. The optimization goal was to select a number of hops to minimize the energy consumption and maximize the throughput for a given data rate. The assumption of free space propagation was assumed to simplify the analysis. Let \( D \) be the distance between the transmitting vehicle and the target vehicle \( T \). For a deployed single relay network, the throughput-distance relation between MAC throughput \( S(X) \) for deployment distance \( X \) is shown in Fig. 4. As an example, consider the receiving vehicle located at a distance \( D = 200m \). The addition of a relaying vehicle to the network at distance \( r_1 \),
increased the throughput to the receiving vehicle located at 145\(m\) and increased the coverage range of the transmitting vehicle to a range of 300\(m\).

![Normalized throughput distance relation for 802.11 using probabilistic model](image)

Fig. 2. General curve shape for PHY throughput-distance relationship using numerical data from manufacturer datasheet shown in Table II.

The tradeoff is that the addition of the relay affects the overall MAC throughput. Let \(S(D)\) be the MAC throughput at the target vehicle located at a distance \(D\) from the transmitting vehicle. Since the introduction of 802.11n, there has been a significant push towards using RTS/CTS protocol in the MAC layer to better utilize the available channels. In our analysis we assume the MAC layer uses RTS/CTS with an information payload of \(I\) bits and a packet size of length \(M\) bits, the physical transmission medium will be shared between the transmitting vehicle and relaying vehicle. At the relaying vehicle, the packet is encapsulated and forwarded to the target T. From [41], The MAC throughput \(S\) is defined as
\[ S = \frac{\text{Payload}}{\text{SlotTime}} = \frac{1}{\tau_{\text{total}}} \]  

(7)

The slot time, \( \tau_{\text{total}} \), is composed of a MAC layer component and a PHY layer component. The PHY component \( \tau_{\text{PHY}} \) arises from the propagation delays of the transmission. For a packet size of \( M \) bits, \( \tau_{\text{PHY}} \) is given by

\[ \tau_{\text{PHY}} = \frac{M}{f} \]  

(8)

where \( f \) is the supported PHY data rate in \( \text{Mbps} \). The MAC layer is the delay associated with the processing time of the control frames for channel access and is given by [41]

\[ \tau_{\text{MAC}} = E[L] \cdot E[X] + T_{\text{RTS}} + T_{\text{CTS}} + T_{\text{ACK}} \]  

(9)

where \( E[L] \) and \( E[X] \) are average length of time for decrease of back off count and the average number of back-off counts, respectively. The window contention size between competing nodes, the probability of successful transmission and the probability of collisions all affect both expectations. \( T_{\text{RTS}} \), \( T_{\text{CTS}} \), and \( T_{\text{ACK}} \) are the time durations to transmit the RTS, CTS, and ACK packets for the last successful RTS/CTS handshake. The time durations are defined as the control packet size divided by the transmission rate similar to the relation shown in (7). From (4), the data rate \( f \) is a function of distance \( r_1 \). For the relay network shown in Fig. 3, assuming a single relaying vehicle, the MAC throughput \( S(D) \) will be bounded by

\[ S_2(D) \leq \frac{1}{2 \left( \frac{M}{f(r_1)} + \tau_{\text{MAC}} \right)}, \text{ where } D = r_1 + r_2 \]  

(10)

Fig. 3. General Scenario for a Multi-Relay Network
To increase the coverage of the Tx to greater distances, the deployment of additional relays is required. Each additional relay, will encapsulate the received packet resulting in a larger packet size and a longer slot duration effectively reducing the MAC throughput even further. In Fig 3, the leftmost vehicle is treated as the first transmitting vehicle followed by $N$ hops, $H_i; i = 1,2,…,N-1$, where $N-1$ is the number of relaying vehicles. The target vehicle $T$ is located at the other end of the network. We assume an information payload of $I$ bits and packet size of $M$ bits resulting in a frame overhead of $M-I$ bits. For a network with $N-1$ relays using the RTS/CTS protocol, the MAC throughput $S_N(D)$ is bounded by

$$S_N(D) \leq \frac{I}{N\left(\frac{M}{f(r_1)} + t_{MAC}\right)}, \quad \text{with} \quad D = \sum_{i=1}^{N} r_i$$

(11)

where $D$ is the distance from the transmit vehicle to target vehicle $T$. Equation (11) indicates that the MAC throughput at distance $D$ is a function of the PHY throughput of the first link $f(r_1)$ and the number of hops $N$. 

Fig. 4. Effect of relay addition on deployment throughput-distance relation.
The definition in (11) is used to estimate the throughput between a transmitter and receiver as function of the links between them. For broadcasting applications, we are interested in maximizing the spatial throughput. For the linear network in Fig. 3. The average spatial throughput for this network can be formulated as the average of the MAC throughput within the desired distance D. That is

\[
\overline{S_N(D)} = \frac{1}{D} \int_0^D S_N(D) dD
\]

(12)
5.4 Optimal solution and number of relays

We start with optimization process by considering the dedicated link scenario from Fig. 1. The first step in optimization is to select a minimum outage rate $P_{out}$. Using this outage rate we can determine the maximum coverage distance $R_{max}$ by solving the non-linear equation in (6). Essentially, we must satisfy the condition $r_i ; 1 \leq i \leq N$, such that $r_i \leq R_{max}$. If this condition does not hold, the communication link will not satisfy the minimum acceptable outage rate. Additionally, relays are added to extend the coverage. Therefore, if we add $N$ relays, we wish to extend the coverage to a bounded distance $NR_{max}$. The second step in optimization is to ensure that all the links on the path can support the PHY throughput in the first hop, $f(r_i) \geq f(r_1)$, and to ensure that we can achieve the maximum MAC throughput of $S_N(D)$. From this condition we can determine a relationship for distance between the relays. This implies that $r_1 \geq r_i$. The third step in optimization is that, since $r_1 \leq D$, for up to certain distance values, if we add a relay at some distance $Z_i$, the initial MAC throughput with no relays ($N=1$) will be greater than throughput with a relay at a distance $D \leq Z_i$. Stated differently, the MAC throughput $S_N(D)$ is less than or equal to the PHY throughput. Adding any number of hops between the transmitter and the receiver will reduce the MAC throughput below the maximum PHY throughput. From (11) we also see that $S(D,N)$ is a function of the throughput of the first hop. Therefore, it depends on the distance of the first relay. When additional relays are added, the goal is increase the MAC throughput to the target vehicle T. Considering (11),(12) and the above discussion, we can describe the optimization problem as:

$$\text{Max: } S_N(D) \leq \frac{I}{N \left( \frac{M}{f(r_1)} + \tau_{MAC} \right)} \text{ where } D = \sum_{i=1}^{N} r_i \text{ subject to }$$

$$S_i(D) \geq S_{i-1}(D) \quad 2 \leq i \leq N$$
For broadcasting applications, we require the minimum number of relays in order to maximize the spatial throughput. In this situation we require that the relay should be chosen such that \( r_i + r_{i+1} \geq R_{\text{max}} \). This gives the added constraint that the first relay should be located after a distance of \( R_{\text{max}}/2 \). The mathematical program can written as

\[
\text{Max: } S_N(D) = \frac{1}{D} \int_0^D S_N(D) \, dD \text{ where } D = \sum_{i=1}^{N} r_i \text{ subject to}
\]

\[
S_i(D) \geq S_{i-1}(D) \quad 2 \leq i \leq N
\]

\[
r_i \leq R_{\text{max}} \quad 2 \leq i \leq N
\]

\[
r_1 \geq r_i \quad 2 \leq i \leq N
\]

\[
r_i = r_{i+1} \quad 2 \leq i \leq N - 1
\]

\[
D \leq N \times R_{\text{max}}
\]

\[
D \leq N \times r_1
\]

\[
r_1 \geq \frac{R_{\text{max}}}{2}
\]

Using the probabilistic throughput-distance relationship in (4), the objective function and set of constraints in (13) and (14) reduce to a classical mathematical programming problem. To
solve this mathematical programming problem, we start with the no relay scenario where \( N=1 \). If we have no relay the distance \( r_1 \) will be the same as \( D \) and therefore the MAC throughput will be the same as the PHY throughput. From (1), the MAC throughput is given by \( S(D) = f(r_1) | r_1 = D \). The solution can be generalized to \( N=k \) hops. The solution is recursive and \( R_{\text{max}} \) is the maximum distance allowed for a hop. In all cases, the values of \( r_i \) depend on the previous \( r_{i-1} \) value. The key is to select an optimum \( r_1 \) and adjust the rest of the distances so they can carry the load. The general solution for the two mathematical programs in (13) and (14) is shown in Fig 5. We require the minimum relay location \( r_1 \) that maximizes the distance \( D \). For the vehicle-to-vehicle scenario, the maximzation objective can be satisfied by selecting \( r_1 = D/N \). The performance bounds for each
case is shown in Fig. 6. Fig. 6 provides the bounds that can be used to select the required number of relays to reach the maximum MAC throughput $S$ for V2V application. For broadcasting applications, we revisit Fig. 5. The number of relays should be selected such that $N = \text{ceil}(2D/N)$. The ceil operator is used to ensure an integer number of relays is selected for practical realization. In otherwords, to select the optimal number of relays to broadcast to a desired coverage distance $D$, we select the rounded up value of half the number of relays from the bounds in Fig. 6.

5.5 Performance Analysis for Emergency Broadcast for Moving Vehicles

Consider the emergency broadcast scenario at the bottom of Fig. 7. We would like to optimize the deployment to deliver the maximum achievable throughput for a target located at a specific distance $D$. We assume a frame header overhead of $H = 400$ bits and the data payload $I = 8184$ bits. In this situation we use the mathematical program in (14). Assuming the target is located at a distance $D = 2900m$. Given the pathloss model described by (3) (4) and (5), as well as the manufacturer data sheet in Table I, we obtain the PHY throughput distance relationship shown in Fig. 1. We consider the first scenario to maximize the spatial throughput up to $D$. Adding more relays will increase the packet size due to the encapsulation process. This in turn, will increase the slot duration and will reduce the MAC throughput. Therefore to maximize the spatial throughput we must use the minimum number of relays. The performance bounds shown in Fig. 5 is used as our relay selection criterion. The bound in Fig 5 is a function of the number of relays and the distance $D$. For each $S_M(D)$, after certain distance, the available data rate will be fully occupied by the header and therefore no data can be transmitted. Therefore we need to add a relay to increase the coverage beyond that threshold distance. From Fig. 5, the curve that satisfies this minimum criterion is the single relay ($N = 2$) curve. The channel access overhead from the MAC layer processing based on the calculation provided in [42] is $T_{\text{MAC}} = 109 \text{ usec}$. To carry the traffic, the
relays must be equally spaced. For this single relay case, the optimal relay selection has to be at a distance of 1450m. Fig.7 shows the optimal deployment MAC throughput-distance relation for this single relay network case.

5.6 Performance Analysis for Vehicle-to-Vehicle Transmission

In this example, we would like to optimize the deployment to deliver the maximum achievable throughput for a vehicle located at a specific distance $D$ from the primary transmitter. Here, we use the same parameters in the emergency broadcast scenario. Using the bounds in Fig. 6, the optimal number of relays is 2 ($N = 3$) for a maximum throughput of 3.485Mbps. Using the last constraint in (13), the relays are selected such that they are 966.6m apart. The two relay deployment throughput-distance relation is shown in Fig. 7. The results in Fig. 7 reveal that the two relay network delivers higher throughput than the single relay network at distances greater than 2900m. This holds for the desired distance at 3000m.

5.7 Summary

In this chapter, we presented a novel approach to obtain the optimal performance bounds for a multi-hop multi-rate Vehicular Ad hoc Network. Using a general description of the PHY model we show the optimal performance bounds for the VANET and the MAC throughput distance relation for a deployment optimized for emergency broadcasting or dedicated V2V link. Second, we determined the optimal relay selection for two scenarios. The first scenario is for emergency broadcast for highways. The second scenario is for a target vehicle located at a distance $D$ away from the primary transmitting vehicle for data sharing in a V2V communication link. In the analysis, we considered the IEEE 802.11 RTS/CTS MAC protocol.
Fig. 7: Comparison of the deployment Throughput-distance relation for emergency broadcasting and V2V link scenarios
Chapter 6

Comparison of TDMA and RTS/CTS Under Interference for Dense Deployments of Small Cells

In this chapter we explored the effect of interference and fading in the physical layer on the performance of the MAC layer throughput for femto cells and Wi-Fi. Under saturation conditions a densely deployed network, the throughput of decentralized networks such as WiFi outperforms cellular networks. Using the throughput-distance relationship, we showed that WLANs employing RTS/CTS in the MAC layer deliver higher throughput than Femto cells in the range of 10m-160m. Additionally, the inclusion of a relay into the network increases the average spatial throughput for certain interference distances.

6.1 Nature of Wireless Transmission in Ultra-dense Networks

Next generation wireless networks are expected to support a diverse range of communication scenarios. Small cells are playing an integral part in the development of the next generation wireless networks. The addition of small cells increases the density of the wireless network. This in turn increases the available network capacity and coverage. However, it comes with its own set of challenges. In the physical layer, performance issues arise from the effects of interference and fading. The physical layer performance will in turn impact medium access control performance, effectively reducing the network throughput. Even with advancements made in the speed and ease of implementation of Wi-Fi networks, the basic nature of wireless transmission is generally unchanged. Ultra-dense networks offer unprecedented capacity by bringing the network closer to mobile users. Theoretically, the overall capacity scales with the number of
small cells deployed [50], [51]. However, in an ultra-dense deployment, not only desired signal strength but also interference from other cells increase due to the reuse of spectrum. Thus, interference has been considered as the key limiting factor for the system capacity in ultra-dense small cell networks. Moreover, small cells are usually opportunistically and irregularly deployed in the hotspots [52] [79] [83]. The ultra-dense and unplanned deployment of small cells has imposed a great challenge on interference management and radio resource optimization [53][54] [81]. The analysis is considered extremely challenging resulting on reliance on model simulations [21][61]. Currently, two trends in dense deployments are emerging in Wireless networks [55]. The first is a linear one-dimensional network and the second is for a two-dimensional network. The first type is popular in transportation and vehicular networks. The latter is more commonly used for wireless access in densely populated areas. They physical layer (PHY) of both methods is very similar. They both use multi-rate modems that drops the data rate as the user gets further away from modem. The main difference is in the medium access control (MAC) in which femto cell deployments use cellular time division multiple access whereas WLANs rely on carrier sensing. Ultimately the degradation of signal to noise ratio (SNR) through fading and interference is going to significantly affect client devices [62][63]. As the network utilization increases, the interference effects will become more destructive. On the other hand, if interference is present and the network is lightly utilized, the effects of interference may not even be noticeable [77]. However, if the Wi-Fi network is highly utilized, then even a small amount of interference can have a large and noticeable effect.

In [56] MAC protocol performance in MANETs was carried out under fading conditions. A comparison was made using the standard distributed coordination function (DCF) in the 802.11 standard and the enhanced DCF (EDCF) introduced in the 802.11e standard. However,
the analysis did not take into consideration interference effects. [57] presents a framework for analyzing the performance in terms of downlink SINR, coverage, and throughput of wireless network densification and for identifying its potential and challenge. They considered a channel model that captures multi-slope pathloss and general channel power distributions, including transmit power, shadowing, fast fading, as well as associated gains such as antenna pattern and beamforming gain. However, both models did not take MAC into consideration. In new MAC protocols for TDMA and CSMA/CA [9] proposed. The main focus was to minimize collisions under poor channel conditions. However, those models do not take multi-rate transmissions into account.

In this chapter we analyze the performance of dense small cells on the performance of the medium access control data rate for both femto cells and wireless local area networks (WLAN) for data applications. We assume the network is fully saturated. We develop a model to evaluate the medium access control throughput for femto cells and WLANs. We use the throughput-distance relation and take into consideration the effects of interference and fading as well as the medium access control overheads in multi-rate modems.

The rest of the chapter is organized as follows: Section II contains the general network model. Section III is the physical layer description and interference model. Section IV describes the medium access control for femto cells and WLANs. Performance analysis is shown in section V. Finally, section VI provides a summary of our contributions.

6.2 General Network Model

In our analysis, we assume the simplified network model in Fig. 1. We assume each transmitting node or access point (AP) has a packet that it wants to send to its receiver. The two
APs are located at a distance $D$ apart from one another. The target receiver $T$ is connected to $AP_1$. The distance from the target $T$ to $AP_1$ is $r_1$. $AP_2$ is the interference source.

![Network model](image)

**Fig. 1.** Network model used to evaluate the performance of dense wireless networks

### 6.3 Data Rate, Distance and Outage

The physical (PHY) layer throughput is a monotonic function of the signal-to-noise ratio in wireless transmissions. Typical manufacturer data sheets specify the data rate as a discrete function of the receiver sensitivity. Table I shows manufacturer parameters for the discrete relationship between the receiver sensitivity and the PHY throughput. Assuming the target receiver $T$ is located at a distance $r_1$ within its access point coverage range, then the general expression for the received signal power $P_r$ can be expressed as [10]

$$P_r(r_1) = h(r_1) + \eta$$

(1)

where $h(r_1)$ is the propagation path loss model used for the transmission medium and $\eta$ is a Gaussian random variable with variance $\sigma$ representing shadow fading. The function $h(r_1)$ has different representations. It is sometimes partitioned into multiple segments. In 802.11 and the Joint Technical communities (JTC), the distance partitioned pathloss models are the most popular. These models can generally be represented by [17]

$$h(r_1) = \begin{cases} P_t - L_0 - 10\alpha_1 \log(r_1), & r \leq r_{bp} \\ P_t - L_0 - 10\alpha_1 \log(r_{bp}) - 10\alpha_2 \log\left(\frac{r_1}{r_{bp}}\right), & r > r_{bp} \end{cases}$$

(2)
where $P_t$ is the transmission power, $L_0$ is the pathloss in the first meter, $\alpha_1$ and $\alpha_2$ are the distance power gradients, and $r_{bp}$ is the break point distance. The most recent pathloss model that is popular in the literature is given by \[57\]

\[
  h(r_1) = \begin{cases} 
  P_t - L_0 - 10\alpha_1 \log\left(\frac{r}{r_0}\right), & r_0 \leq r \leq r_{bp} \\
  P_t - L_0 - 10\alpha_1 \log\left(\frac{r_{bp}}{r_0}\right) - 10\alpha_2 \log\left(\frac{r_{bp}}{r_{bp}}\right), & r > r_{bp}
  \end{cases}
\]  

(3)

where $r_0$ is the reference distance used in the measurement data [4]. Equations (1) and (3) show that $P_t(r_1)$ is a Gaussian random variable with mean $h(r_1)$ and variance $\sigma$. We note that this model is very similar to the 802.11 model.

As the distance between the transmitting node and receiving node changes, the receiver experiences varying data rates at each location due to the random fluctuations in the received signal. Therefore the average spatial PHY throughput $f(r_1)$ can be calculated as

\[f(r_1) = \sum_i R_i P\{R_i(r_1)\}\]  

(4)

<table>
<thead>
<tr>
<th>$P_t$(dBm)</th>
<th>RSS (dBm)</th>
<th>PHY Data Rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>-72</td>
<td>27</td>
</tr>
<tr>
<td>19</td>
<td>-73</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>-78</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>-81</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>-84</td>
<td>6</td>
</tr>
<tr>
<td>20</td>
<td>-85</td>
<td>4.5</td>
</tr>
<tr>
<td>20</td>
<td>-87</td>
<td>3</td>
</tr>
</tbody>
</table>

where $P\{R_i(r_1)\}$ is the probability of getting throughput $R_i$ at distance $r_1$ and is given by [10][13]

\[
P(R_i[r]) = \begin{cases} 
  1 - \text{erfc}\left(\frac{h(r_1) - P_i}{\sqrt{2}\sigma}\right), & i = 1 \\
  \frac{1}{2}\text{erfc}\left(\frac{h(r_1) - P_{i-1}}{\sqrt{2}\sigma}\right) - \frac{1}{2}\text{erfc}\left(\frac{h(r_1) - P_i}{\sqrt{2}\sigma}\right), & i \geq 2
  \end{cases}
\]  

(5)
The PHY throughput $f(r_1)$ defined by (4) is a monotonically decreasing function of distance $r_1$. Equation (5) gives the probability of existence of each data rate as a function of the distance between the transmitter and receiver. Consequently the outage rate $P_{out}$ can be defined as

$$P_{out} = 1 - \sum_i P(R_i[r])$$  \hspace{1cm} (6)

For a given outage rate, we can determine the maximum coverage distance $R_{max}$ by solving the nonlinear relation in (6). Fig. 2 shows the normalized PHY throughput $f(r_1)$ and the outage rate for a single transmitter using the manufacturer data sheet values from Table I.

### 6.4 Interference Model

The probability of two APs sensing each other is related to the outage rate $P_{out}$. When two APs are far apart, the probability that the two APs sense each other decreases. This leads to interference at the target receiver $T$. To correctly receive the information, the received signal to interference ratio has to be above a certain threshold. Practically, this requires an adjustment of the receiver sensitivity. The original receiver sensitivity in Table 1 is obtained in a laboratory environment where interference does not exist [60]. It is defined as

$$SNR = \frac{P_s}{kT_kB}$$  \hspace{1cm} (7)

Where $P_s$ is the receiver sensitivity, $k$ is the Boltzmann constant, $T_k$ is the operating temperature in Kelvin and $B$ is the transmission bandwidth. The denominator in (7) is commonly known as thermal noise. Interference raises the noise floor. As such to account for the interference $I$, an adjustment is required. This can be shown as

$$SNR = \frac{P_s}{(1-P_{out})I+kT_kB}$$  \hspace{1cm} (8)
Where the additional term at the denominator is average interference. $P_{\text{out}}$ is a function of distance. The receiver sensitivity, $P_{S}$, can be adjusted using (8) to account for the interference. This can be expressed as

$$P_{S_{\text{adj}}} = \frac{P_{\text{old}}}{kT_B} [1 - P_{\text{out}}(D)] I + P_{S_{\text{old}}}$$

(9)

Using the adjustment in (9) we can take into account the effects of interference on the throughput-distance relationship of (4).

6.5 Medium Access Control for Femto Cells

Let $S(D)$ be the MAC throughput at the location of the target terminal separated by a distance $r_1$ from the AP. In general, cellular has minimal overhead and as such, the PHY and MAC layer throughputs are nearly identical. In our analysis we assume $S(r_1) = f(r_1)$ for cellular networks.

6.6 Medium Access Control for WLANs

In WLANs, the MAC protocol is responsible for the coordination of multiple terminals over a shared channel. Let $r_1$ be the distance between the primary transmitter and the target receiver $T$. Since the introduction of 802.11n, there has been a significant push towards using the RTS/CTS protocol in the MAC layer of WLANs to better utilize the available channels. In our analysis, we adopt the approach presented in [11], [12],[18]. Assuming the MAC layer uses RTS/CTS with a packet size of length $M$ bits and a data payload of $I$ bits, the physical transmission medium will be shared between the two transmitting APs. In general, the MAC throughput $S$ is defined as

$$S = \frac{\text{Payload}}{\text{SlotTime}} = \frac{I}{\tau_{\text{total}}}$$

(10)
The slot time, $\tau_{total}$ is composed of a PHY layer component and a MAC layer component. The PHY component $\tau_{PHY}$ arises from the propagation delays of the transmission. For a packet size of $M$ bits, $\tau_{PHY}$ is given by

$$\tau_{PHY} = \frac{M}{f}$$  \hfill (11)

where $f$ is the supported PHY data rate in $Mbps$. The MAC layer is the delay associated with the processing time of the control frames for channel access and is given by

$$\tau_{MAC} = E[X] \cdot E[L] + T_{RTS} + T_{CTS} + T_{ACK}$$  \hfill (12)
where $E[X]$ and $E[L]$ are the average number of back-off counts and average length of time for decrease of back off count respectively. Both of these expectations are affected by the window contention size $W$, between competing nodes, the probability of successful transmission and the probability of collisions. $T_{RTS}$, $T_{CTS}$ and $T_{ACK}$ are the time durations to transmit the RTS, CTS, and ACK packets for the last successful RTS/CTS handshake. The time durations are defined as the control packet size divided by the transmission rate similar to the relation shown in (10).

To obtain $E[X]$ and $E[L]$, we use the results in [11]. Bianchi in his famous paper [11] defined the collision probability $p$ using Markov chain analysis as

$$p = 1 - (1 - \tau)^{n-1} \tag{13}$$

Where $\tau$ is the probability that a station tranmits in a genuine slot time and is defined as

$$\tau = \frac{2(1-2p)}{(1-2p)(W+1)+pW(1-[2p]^{m})} \tag{14}$$

where $n$ is the number of stations and $m$ is the maximum back-off stage. The two nonlinear equations can be solved numerically. Let be $P_{tr}$ be the probability that at least one transmission occurs. Then $P_{tr}$ is defined as

$$P_{tr} = 1 - (1 - \tau)^{n} \tag{15}$$

Successful transmission occurs when only a single node transmits over the channel. Then the probability of successful transmission is

$$p_s = \frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^{n-1}} \tag{16}$$

Given (15),(16) we can analyze $E[L]$ the average time to decrease the backoff counter by one.

We have three situations that decrease the backoff count. The first case is when no transmission occurs in the network. Then after some slot time $\sigma$ the backoff counter decreases by one. On average the delay in this situation is $(1 - P_{tr})\sigma$. The second situation is successful transmission when one other node is transmitting. In this situation the source node freezes its back off count
while the other node is transmitting. Let us define $T_H$ and $T_D$ to be the total time to transmit the header and data payload respectively. The time duration for each packet transmission can be calculated by dividing the respective frame size by the rate supported by the PHY layer. The constant time durations of DIFS, SIFS and propagation delays can be represented as $T_{DIFS}$, $T_{SIFS}$ and $\delta$. To successfully transmit a packet the transmitting node sends an RTS packet to the receiving node. This time duration is composed of sending the RTS frame and accounting for the propagation delay. This can be written as $T_{RTS} + \delta$. The receiving node sends back a CTS packet to the transmitting node when it correctly detects an RTS packet. This packet is sent after waiting for a SIFS duration. The delay here, including the propagation delay is $T_{SIFS} + T_{CTS} + \delta$. When the transmitter receives the CTS packet, it starts sending a data packet after waiting a SIFS duration. The data packet contains a header and the information payload. This delay is $T_{SIFS} + T_H + T_D + \delta$. The receiving node waits for SIFS duration and sends ACK packet to the transmitter after the data packet is received. The time duration here is $T_{SIFS} + T_{ACK} + \delta$. After the ACK packet transmission is finished, the remaining nodes in the network sense the channel for a $T_{DIFS}$ duration and decrease the backoff counter by one. Therefore, the total time to decrease the backoff counter for successful transmission is

$$T_s = T_{RTS} + 3\delta + 3T_{SIFS} + T_{CTS} + (T_H + T_D) + T_{ACK} + T_{DIFS}$$

(17)
Fig. 3. Comparison of MAC throughput-distance relationship for RTS/CTS and TDMA under fading and interference

The probability of being in this case is $p_{tr}.p_s$ The third situation is when multiple collisions occur in the networks. This situation occurs when multiple nodes simultaneously transmit packets at the same time. Here, the transmitting node sends an RTS packet. Accounting for the propagation delay, the duration is $T_{RTS} + \delta$. However, the receiving node will not send a CTS packet since it cannot correctly detect the RTS. The transmitting node will wait for a DIFS duration and decide there is a collision. To decrease the backoff count in this situation, the delay is

$$T_C = T_{RTS} + \delta + T_{DIFS}$$

(18)

With a probability of $p_{tr}(1 - p_s)$. Given these probabilities and time durations, the average time to decrease the backoff counter by one is
\[ E[L] = (1 - P_{tr})\sigma + p_{tr}p \tau_s + p_{tr}(1 - p_s)\tau_c \quad (19) \]

In [23],[24] the MAC layer supported a fixed rate. However, due to the throughput-distance model in section III, the variable rate at the physical layer affects the MAC layer delay. Then the average number of backoff counts for one successful transmission is

\[ E[X] = \frac{p^m}{1-p} \frac{W_{max}+1}{2} + \sum_{k=0}^{m} p^k \frac{W_k+1}{2} \quad (20) \]

From (4), the physical layer data rate \( f \) is a function of distance \( r_1 \). Assuming a two AP network, the channel will be shared by the by the two APs. In this case the MAC throughput \( S(r_1) \) will be bounded by

\[ S(r_1) = \frac{l}{\frac{1}{f(r_1) + \tau_{MAC}}} \quad (21) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload (I)/ Header (H)</td>
<td>2250 Bytes/228 Bits</td>
</tr>
<tr>
<td>RTS/ CTS/ ACK</td>
<td>288 / 256 / 228 usec</td>
</tr>
<tr>
<td>Slot time (( \sigma ))/ DIFS/ SIFS</td>
<td>20 / 128 / 28 usec</td>
</tr>
<tr>
<td>( CW_{min}/CW_{max} )</td>
<td>64/1024</td>
</tr>
<tr>
<td>Pathloss exponents ( \alpha_1, \alpha_2 )</td>
<td>2.4, 3.5</td>
</tr>
</tbody>
</table>
Fig. 4. Breakpoint distance in relation to interference distance D for RTS/CTS and TDMA under fading and interference

Where $f(r_1)$ uses the parameters in table 1. As the AP separation distance $D$ gets larger, the APs do not sense each other. This results in interference at the receiver $T$ since the two access points cannot coordinate the channel access. In this case, the throughput

$$S(r_1) = \frac{I}{f_{adj}(r_1)^{TMAC}}$$

(22)

Where $f_{adj}(r_1)$ is the adjusted physical layer model using (8) and (9). When the two APs are close to each other, the outage rate decreases and the medium is shared. That is, the throughput would resemble that of (21), when they are further apart, the performance is similar to that of ALOHA [84] resulting in (22). Then the average throughput for RTS/CTS can be defined as
\[ S(D) = (1 - P_{out}(r)) \frac{I}{(r_1^2 + \tau_{MAC})} + P_{out}(r) \frac{I}{(r_1^2 + \tau_{MAC})} \] (23)

For some applications, there is interest in the performance the spatial throughput. For the linear network in Fig. 1. The average spatial throughput for this network can be formulated as the average of the MAC throughput within the desired distance coverage distance. The network shown in Fig.1, the distance is \( D/2 \). The average spatial MAC throughput can be defined as

\[
\overline{S}(D) = \frac{2}{D} \int_{0}^{D/2} S(D) dD
\] (24)

### 6.7 Performance Analysis for Access Points

We start comparing the performance of femto cells and WLANs under interference by first defining the transmission parameters. We assume TDMA has no overheads and as such the femto cell will be transmitting at the raw physical layer data rate. The throughput-distance relationship for femto cells is determined using (4). To account for interference, we apply the adjustment using (9) to the modem values in Table I. For Wi-Fi, the assumed MAC parameters are shown in Table II. These parameters are used to determine the delay associated with the MAC, \( \tau_{MAC} \), in (12). For the PHY layer delay, \( \tau_{PHY} \), we use (11) while noting that \( f \) is a function of distance. Using (23) we can determine the throughput distance relationship for Wi-Fi. Fig. 3 shows an example for the throughput distance relation for femto cells and WLANs for \( D = 120m \) with and without interference. It can be observed, that up to a certain distance TDMA delivers higher data rates that RTS/CTS. After that distance, RTS/CTS delivers higher throughput than TDMA under fading and interference conditions. Let us define the breakpoint distance as \( r_b \). Fig. 4. Shows the relationship between the access points interference distance \( D \) and the breakpoint.
distance $r_b$. After the distance $r_b$, Wi-Fi delivers higher throughput than femto cells. Next, we vary the access point separation distance $D$ to numerically calculate the average spatial throughput using (24). We use the distance range of $5m$ to $160m$. Fig. 5 shows the average spatial throughput for TDMA and RTS/CTS. When a target receiver is located at distance less than $9m$ from it’s associated access point, TDMA delivers higher MAC throughput than RTS/CTS. When the target receiver is in the distance range of $10 m$ to $160 m$. RTS/CTS delivers higher spatial data rates than TDMA.

Fig. 5. Average spatial throughput for TDMA and RTS/CTS for different interference distances
6.8 Performance Analysis with Relays

To extend the coverage and increase the spatial throughput relays can be deployed. For the analysis of this chapter, we assume a single relay is added. This relay is connected to the primary access point. Whenever a relay is added the channel is shared with the primary access point. The throughout in this case is reduced due to hopping (chapter 3). With the addition of a single relay, the new network model is shown in fig. 6. We assume the relay is located at distance $r_1 = D_{int}/4$. That is, the relay is placed at the midpoint before handoff. The same interreference conditions apply to relays. For TDMA, the relay is closer to the interference source. This will degrade the throughput distance relationship considerably. RTS/CTS is inherently resistant to interference. With the addition of a relay the deployed network will have one of two possible conditions. Those conditions are dependent on the probability of outage. When the probability outage is small, the two access points and relay will detect each other. This will result in 3 nodes sharing the network and changing the MAC delay $\tau_{MAC}$ to that of $n = 3$. When the relay does not detect the interference access point, the situation is the same as that of isolated access points in section 6.5 with one exception. The relay throughput will be halved due to hopping. Performing the same analysis, we can calculate the average spatial throughput $S(D)$ for relay networks. The result is shown in fig. 7. For TDMA, the addition of a relay increased the average spatial throughput substantially up to interference separation distance of 80 m. For RTS/CTS, the throughput increased at smaller interference distances below 40 m. Around 40 m, the average spatial throughput for the no relay case is slightly higher. After interference distance of 122 m, the relay network throughput is the highest of the available test cases. It is important to note that RTS/CTS with and without a relay provides higher average spatial throughput in these dense networks.
Fig. 6. Network model for a single relay network under interference

Fig. 7. Average spatial throughput for TDMA and RTS/CTS for different interference distances with a single relay
Chapter 7

Thesis Conclusion and Suggested Future Work

In this dissertation, we present a novel approach to obtain the optimal performance bounds for a multi-hop multi-rate WLAN as a function of distance between the relays. The optimal distance is derived from the throughput-distance relationship of the access point and the relays subject to a set of constraints on the PHY and MAC throughputs. Furthermore, we show that the general solution to achieve the optimal throughput at distance $D$ depends on two parameters. These parameters are the coverage radius of the access point and relays, $R_{\text{max}}$, and the distance of first relay from the access point. Additionally, we determined the optimal relay selection for two scenarios. The first scenario is for emergency broadcast for highways. The second scenario is for a target vehicle located at a distance $D$ away from the primary transmitting vehicle for data sharing in a V2V communication link. In the analysis, we considered the IEEE 802.11 RTS/CTS MAC protocol. we also explored of the effect of interference and fading in the physical layer on the performance of the MAC layer throughput for femto cells and Wi-Fi. Under saturation conditions a densely deployed network, the throughput of decentralized networks such as WiFi outperforms cellular networks. Using the throughput-distance relationship, we showed that WLANs employing RTS/CTS in the MAC layer deliver higher throughput than Femto cells in the range of 10m-160m. We also extended this analysis to include the effects of relays in densely deployed networks.

Many different adaptations are still left open for future work. The first suggestion is to extend the approach to that of two-dimensional networks. It is recommended to start with topologies that
have equal distances such as triangles. Another suggestion is to use the approach presented in this dissertation for localization applications. In localization applications, throughput is not an issue but minimizing location error is. In that regard, the use of relays can potentially help with the analysis of performance bounds. An alternate suggestion is going for a measurement campaign. Throughout this dissertation, finding suitable outdoor WiFi models was very tedious and cumbersome outside of a few applications. Carrying out a measurement campaign to determine a detailed empirical model for WiFi propagation with the inclusion of parameters such as antenna heights is very lucrative. Such model would need to be comparable to the Okamura-Hata model in previous generational cellular networks.
References


[77] IEEE 802.11p Standard documentation


Appendix

Supporting MATLAB Scripts

This appendix provides supporting MATLAB script used to support the analysis provided in this dissertation as well as plot generation.

The following script is used to generate the MAC delay parameters for CSMA/CA with RTS/CTS for \( n \) number of nodes

```
% Author: Bader Alkandari, Kaveh Pahlavan
% ECE 506 Fundemental of Wide and Local Area Networks
% Worcester Polytechnic Institute Nov. 2015
%
% Description:
% The script generates Fig. 15 of Bianchi's famous paper. The generated
% plot shows that for large data packets, RTS/CTS outperforms CSMA/CA
% "--" lines are for CSMA/CA
% "solid" lines are for RTS/CTS

function bianchi_main
    clear all;
    close all;
   clc;

    n_values = [10, 50];        % number of contending nodes
    n_values = 2; % number of stations
    figure;                     % create and hold semi-log figure
    grid
    set(gca,'xscale','log')
    xlabel('payload I (bits)')
    ylabel('Throughput')
    hold on

    for i = 1:length(n_values)
        W = 16;                 % window size CWmin or CWmax
        m = 5;                  % typical values are 5 or 6
        n = n_values(i);        % number of stations
        calculate_RTS_CTS_FHSS  % calculate p and tau given n
        sig = 50;               % slot duration (usec). depends on PHY
        Ptr = 1 - (1-tau)^n;
        num_Ps = n*tau*(1-tau)^(n-1);
        den_Ps = 1 - (1-tau)^n;
        Ps = num_Ps/den_Ps;

        % assume 1 Mbit rate (all units get converted to usec)
        H = 400;                % header size (bits)
        I = 100:1:20000;        % information payload I (bits)
    
    end
```

\[
\begin{align*}
d &= 1; \quad \text{% propagation delay "delta"}
SIFS &= 28; \quad \text{% usec}
DIFS &= 128; \quad \text{% usec}
H_{\text{phy}} &= 128; \quad \text{% PHY header}
RTS &= 160 + H_{\text{phy}}; \quad \text{% usec}
CTS &= 112 + H_{\text{phy}}; \quad \text{% usec}
ACK &= 112 + H_{\text{phy}}; \quad \text{% usec}
\end{align*}
\]

% Ts and TC for RTS/CTS. Change this for CSMA/CA
\[
Ts = RTS + SIFS + d + CTS + SIFS + d + H + I + SIFS + d + ACK + DIFS + d;
\]
\[
Tc = RTS + DIFS + d;
\]
% Calculate throughput S
\[
\text{num}_S = Ps*Ptr*I;
\]
\[
\text{den}_{S_1} = (1-Ptr)*\text{sig};
\]
\[
\text{den}_{S_2} = \text{Ptr}*Ps*Ts;
\]
\[
\text{den}_{S_3} = \text{Ptr}*(1-Ps)*Tc;
\]
\[
\text{den}_S = \text{den}_{S_1} + \text{den}_{S_2} + \text{den}_{S_3};
\]
\[
S = \frac{\text{num}_S}{\text{den}_S};
\]
% plot throughput vs payload
semilogx(I,S)
end
%----------------
function calculate_RTS_CTS_FHSS
fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);
\[
\tau = x(1);
\]
\[
p = x(2);
\]
%-------------------
function fcnseqns2(z)
\[
t = z(1);
p = z(2);
\]
\[
\text{num} = 2*(1-2*p);
\]
\[
\text{k1} = (1 - 2*p)*(W+1);
\]
\[
\text{k2} = p*W*(1-(2*p)^m);
\]
\[
\text{den} = \text{k1} + \text{k2};
\]
\[
\text{fcns}(1)= t - \text{num}/\text{den};
\]
\[
\text{fcns}(2)= p - 1 + (1 - t)^{(n-1)};
\]
end
\]
% RTS/CTS Throughput vs payload (DSSS)%
for i = 1:length(n_values)
\[
W = 32; \quad \text{% window size CWmin or CWmax}
m = 5; \quad \text{% typical values are 5 or 6}
n = n_values(i);        % number of stations
calculate_RTS_CTS_DSSS  % calculate p and tau given n
sig = 20;               % slot duration (usec). depends on PHY
Ptr = 1 - (1-tau)^n;
num_Ps = n*tau*(1-tau)^(n-1);
den_Ps = 1 - (1-tau)^n;
Ps = num_Ps/den_Ps;

% assume 1 Mbit rate (all units get converted to usec)
H = 400;                % header size (bits)
I = 100:1:20000;        % information payload I (bits)
d = 1;                  % propagation delay "delta"
SIFS = 28;               % usec
DIFS = 128;              % usec
H_phy = 128;             % PHY header
RTS = 160 + H_phy;      % usec
CTS = 112 + H Phy;      % usec
ACK = 112 + H_phy;      % usec

% Ts and TC for RTS/CTS. Change this for CSMA/CA
Ts = RTS + SIFS + d + CTS + SIFS + d + H + I + SIFS + d + ACK + DIFS + d;
Tc = RTS + DIFS + d;

% Calculate throughput S
num_S = Ps*Ptr*I;
den_S_1 = (1-Pt)*sig;
den_S_2 = Ptr*Ps*Ts;
den_S_3 = Ptr*(1-Ps)Tc;
den_S = den_S_1 + den_S_2 + den_S_3;
S = num_S/den_S;

% plot throughput vs payload
semilogx(I,S)
end

%--------------
function calculate_RTS_CTS_DSSS

fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);

tau = x(1);
p = x(2);

%-----------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);
k1 = (1 - 2*p)*(w+1);
k2 = p*w*(1-(2*p)^m);
den = k1 + k2;
fcns(1) = t - num/den;
fcns(2)= p - 1 + (1 - t)^(n-1) ;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RTS/CTS Throughput vs payload (IR)%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)
    W = 64;                % window size CWmin or CWmax
    m = 5;                 % typical values are 5 or 6
    n = n_values(i);       % number of stations
    calculate_RTS_CTS_IR   % calculate p and tau given n
    sig = 8;               % slot duration (usec). depends on PHY
    Ptr = 1 - (1-tau)^n;
    num_Ps = n*tau*(1-tau)^(n-1);
    den_Ps = 1 - (1-tau)^n;
    Ps = num_Ps/den_Ps;

    % assume 1 Mbit rate (all units get converted to usec)
    H = 400;                % header size (bits)
    I = 100:1:20000;        % information payload I (bits)
    d = 1;                  % propagation delay "delta"
    SIFS = 28;              % usec
    DIFS = 128;             % usec
    H_phy = 128;            % PHY header
    RTS = 160 + H_phy;      % usec
    CTS = 112 + H_phy;      % usec
    ACK = 112 + H_phy;      % usec

    % Ts and TC for RTS/CTS. Change this for CSMA/CA
    Ts = RTS + SIFS + d + CTS + SIFS + d + H + I + SIFS + d + ACK + DIFS +

    Tc = RTS + DIFS + d;

    % Calculate throughput S
    num_S = Ps*Ptr*I;
    den_S_1 = (1-Ptr)*sig;
    den_S_2 = Ptr*Ps*Ts;
    den_S_3 = Ptr*(1-Ps)*Tc;
    den_S = num_S + den_S_2 + den_S_3;
    S = num_S/den_S;

    % plot throughput vs payload
    semilogx(I,S)
end

%-----------------------
function calculate_RTS_CTS_IR
    fun = @eqns2;
    x0 = [0 0];
    x = fsolve(fun,x0);
    tau = x(1);
    p = x(2);
%-----------------------
function fcns=eqns2(z)
    t = z(1);
    p = z(2);
    num = 2*(1-2*p);
    k1 = (1-2*p)*(W+1);
    k2 = p*W*(1-(2*p)^m);
    den = k1 + k2;
    fcns(1)= t - num/den;
    fcns(2)= p - 1 + (1 - t)^(n-1);
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CSMA/CA Throughput vs payload (FHSS)%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)
    W = 16;                 % window size CWmin or CWmax
    m = 5;                  % typical values are 5 or 6
    n = n_values(i);        % number of stations
    calculate_CSMA_CA_FHSS   % calculate p and tau given n
    sig = 50;               % slot duration (usec). depends on PHY
    Ptr = 1 - (1-tau)^n;
    num_Ps = n*tau*(1-tau)^(n-1);
    den_Ps = 1 - (1-tau)^n;
    Ps = num_Ps/den_Ps;
    % assume 1 Mbit rate (all units get converted to usec)
    H = 400;                % header size (bits)
    I = 100:1:20000;        % information payload I (bits)
    d = 1;                  % propagation delay "delta"
    SIFS = 28;              % usec
    DIFS = 128;             % usec
    H_phy = 128;            % PHY header
    RTS = 160 + H_phy;      % usec
    CTS = 112 + H_phy;      % usec
    ACK = 112 + H_phy;      % usec
    % Ts and TC for CSMA/CA. Change this for CSMA/CA
    Ts = H + I + SIFS + d + ACK + DIFS + d;
    Tc = H + I + DIFS + d;
    % Calculate throughput S
    num_S = Ps*Ptr*I;
    den_S_1 = (1-Ps)*sig;
    den_S_2 = Ps*Ts;
    den_S_3 = Ps*(1-Ps)*Tc;
    den_S = den_S_1 + den_S_2 + den_S_3;
    S = num_S./den_S;
    % plot throughput vs payload
    semilogx(I,S,'-.')
end

%----------------
function calculate_CSMA_CA_PHSS

fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);

tau = x(1);
p = x(2);

%-----------------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);
k1 = (1 - 2*p)*(W+1);
k2 = p*W*(1-(2*p)^m);
den = k1 + k2;

fcns(1)= t - num/den;
fcns(2)= p - 1 + (1 - t)^(n-1) ;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CSMA/CA Throughput vs payload (DSSS)%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)
    W = 32;                 % window size CWmin or CWmax
    m = 5;                  % typical values are 5 or 6
    n = n_values(i);        % number of stations
    calculate_CSMA_CA_DSSS   % calculate p and tau given n
    sig = 20;               % slot duration (usec). depends on PHY
    Ptr = 1 - (1-tau)^n;
    num_Ps = n*tau*(1-tau)^(n-1);
    den_Ps = 1 - (1-tau)^n;
    Ps = num_Ps/den_Ps;

    % assume 1 Mbit rate (all units get converted to usec)
    H = 400;                % header size (bits)
    I = 100:1:20000;        % information payload I (bits)
    d = 1;                  % propagation delay "delta"
    SIFS = 28;              % usec
    DIFS = 128;             % usec
    H_phy = 128;            % PHY header
    RTS = 160 + H_phy;      % usec
    CTS = 112 + H_phy;      % usec
    ACK = 112 + H_phy;      % usec

    % Ts and TC for CSMA/CA. Change this for CSMA/CA
    Ts = H + I + SIFS + d + ACK + DIFS + d;
    Tc = H + I + DIFS + d;

    % Calculate throughput S
    num_S = Ps*Ptr*I;
end
den_S_1 = (1-Ptr)*sig;
den_S_2 = Ptr*Ps*Ts;
den_S_3 = Ptr*(1-Ps)*Tc;
den_S = den_S_1 + den_S_2 + den_S_3;
S = num_S./den_S;

% plot throughput vs payload
semilogx(I,S,'-.'
end
%--------------------------
function calculate_CSMA_CA_DSSS
fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);

tau = x(1);
p = x(2);
%--------------------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);
k1 = (1-2*p)*(W+1);
k2 = p*W*(1-(2*p)^m);
den = k1 + k2;
fcns(1)= t - num/den;
fcns(2)= p - 1 + (1-t)^(n-1) ;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CSMA/CA Throughput vs payload (IR)%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)
W = 64;                     % window size CWmin or CWmax
m = 5;                      % typical values are 5 or 6
n = n_values(i);           % number of stations
calculate_CSMA_CA_IR        % calculate p and tau given n
sig = 8;                    % slot duration (usec). depends on PHY
Ptr = 1 - (1-tau)^n;
um_Ps = n*tau*(1-tau)^n;
den_Ps = 1 - (1-tau)^n;
Ps = num_Ps/den_Ps;

% assume 1 Mbit rate (all units get converted to usec)
H = 400;                    % header size (bits)
I = 100:1:20000;            % information payload I (bits)
d = 1;                      % propagation delay "delta"
SIFS = 28;                  % usec
DIFS = 128;                 % usec
H_phy = 128;                % PHY header
% RTS = 160 + H_phy; % usec
% CTS = 112 + H_phy; % usec
% ACK = 112 + H_phy; % usec

% Ts and TC for CSMA/CA. Change this for CSMA/CA
Ts = H + I + SIFS + d + ACK + DIFS + d;
Tc = H + I + DIFS + d;

% Calculate throughput S
num_S = Ps* Ptr*I;
den_S_1 = (1-Ptr)*sig;
den_S_2 = Ptr*Ps*Ts;
den_S_3 = Ptr*(1-Ps)*Tc;
den_S = den_S_1 + den_S_2 + den_S_3;
S = num_S/den_S;

% plot throughput vs payload
semilogx(I,S,'.-.');
end

%-----------------
function calculate_CSMA_CA_IR
fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);

tau = x(1);
p = x(2);

%-----------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);
k1 = (1 - 2*p)*(W+1);
k2 = p*W*(1-(2*p)^m);
den = k1 + k2;

fcns(1)= t - num/den;
fcns(2)= p - 1 + (1 - t)^(n-1); end
end
end
The following script is a support script that generates throughput versus payload for FHSS CSMA/CA.

```matlab
function bianchi_compare_CSMA_CW

clear all;
close all;
clc;

n_values = [5,10,20,50];   % number of contending nodes
figure;               % create and hold semi-log figure
m_global = 5;
W_global = [8, 16, 32, 64, 128, 256, 512, 1024];
grid
xlabel('Window Size')
ylabel('Throughput')
hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CSMA/CA Throughput vs payload (FHSS) using Bianchi%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)
    for j = 1: length(W_global)
        m = m_global;        % typical values are 5 or 6
        W = W_global(j);                 % window size CWmin or CWmax
        n = n_values(i);        % number of stations
        calculate_CSMA_CA_FHSS  % calculate p and tau given n
        sig = 50;               % slot duration (usec). depends on PHY
       Ptr = 1 - (1-tau)^n;
       num_Ps = n*tau*(1-tau)^(n-1);
       den_Ps = 1 - (1-tau)^n;
       Ps = num_Ps/den_Ps;

        % assume 1 Mbit rate (all units get converted to usec)
        H = 400;       % header size (bits)
        I = 8184;     % information payload I (bits)
        d = 1;        % propagation delay "delta"
       SIFS = 28;        % usec
       DIFS = 128;    % usec
       H_phy = 128;  % PHY header
       RTS = 160 + H_phy;  % usec
       CTS = 112 + H_phy;  % usec
       ACK = 112 + H_phy;  % usec

        % Ts and TC for CSMA/CA. Change this for CSMA/CA
        Ts = H + I + SIFS + d + ACK + DIFS + d;
        Tc = H + I + DIFS + d;

        % Calculate throughput S
        num_S = Ps*Ptr*I;
        den_S_1 = (1-Ptr)*sig;
        den_S_2 = Ptr*Ps*Ts;
    end
end
```

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\[ \text{den}_S_3 = \text{Ptr} \cdot (1 - \text{Ps}) \cdot \text{Tc}; \]
\[ \text{den}_S = \text{den}_S_1 + \text{den}_S_2 + \text{den}_S_3; \]
\[ \text{S(j)} = \frac{\text{num}_S}{\text{den}_S}; \]

end

% plot throughput vs payload
plot(W_global, S)
plot(W_global, S, 'r*')

end

%-------------------
function calculate_CSMA_CA_FHSS

fun = @eqns2;
x0 = [0 0];
x = fsolve(fun, x0);

tau = x(1);
p = x(2);

%-------------------
function fcns = eqns2(z)
t = z(1);
p = z(2);

num = 2 \cdot (1 - 2 \cdot p);
k1 = (1 - 2 \cdot p) \cdot (W + 1);
k2 = p \cdot W \cdot (1 - (2 \cdot p)^m);
den = k1 + k2;

fcns(1) = t - num/den;
fcns(2) = p - 1 + (1 - t)^{(n-1)};
end

end

end
The following script generates throughput versus payload for CSMA/CA for different contention window sizes

```matlab
function bianchi_compare_RTS_CTS_CW
clear all;
close all;
clc;

n_values = [5,10,20,50];  % number of contending nodes
figure;  % create and hold semi-log figure
m_global = 5;
W_global = [8, 16, 32, 64, 128, 256, 512, 1024];
grid
% set(gca,'xscale','log')
xlabel('Window Size')
ylabel('Throughput')
hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
% CSMA/CA Throughput vs payload (FHSS) using Bianchi%  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
for i = 1:length(n_values)
    for j = 1:length(W_global)
        m = m_global;  % typical values are 5 or 6
        W = W_global(j);  % window size CWmin or CWmax
        n = n_values(i);  % number of stations
        calculate_CSMA_CA_FHSS  % calculate p and tau given n
        sig = 50;  % slot duration (usec). depends on PHY
        Ptr = 1 - (1-tau)^n;
        num_Ps = n*tau*(1-tau)^(n-1);
        den_Ps = 1 - (1-tau)^n;
        Ps = num_Ps/den_Ps;

        % assume 1 Mbit rate (all units get converted to usec)
        H = 400;  % header size (bits)
        I = 8184;  % information payload I (bits)
        d = 1;  % propagation delay "delta"
        SIFS = 28;  % usec
        DIFS = 128;  % usec
        H_phy = 128;  % PHY header
        RTS = 160 + H_phy;  % usec
        CTS = 112 + H_phy;  % usec
        ACK = 112 + H_phy;  % usec

        % Ts and TC for CSMA/CA. Change this for CSMA/CA
        Ts = RTS + SIFS + d + CTS + SIFS + d + H + I + SIFS + d + ACK + DIFS + d;
        Tc = RTS + DIFS + d;

        % Calculate throughput S
        num_S = Ps*Ptr*I;
        den_S = (1-Ptr)*sig;
    end
end
```

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den_S_2 = Ptr*Ps*Ts;
den_S_3 = Ptr*(1-Ps)*Tc;
den_S = den_S_1 + den_S_2 + den_S_3;
S(j) = num_S./den_S;
end
% plot throughput vs payload
plot(W_global,S)
plot(W_global,S,'*')
end
%-----------------------
function calculate_CSMA_CA_FHSS

fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);

tau = x(1);
p = x(2);
%-----------------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);

k1 = (1 - 2*p)*(W+1);
k2 = p*W*(1-(2*p)^m);
den = k1 + k2;

fcns(1)= t - num/den;
fcns(2)= p - 1 + (1 - t)^(n-1) ;
end
end
The following script compares the approximation of MAC throughput of [84] to that of Bianchi’s model

```matlab
function bianchi_compare_with_approx
    clear all;
    close all;
    clc;

    n_values = [5];         % number of contending nodes
    figure;                 % create and hold semi-log figure
    grid
    set(gca,'xscale','log')
    xlabel('payload I (bits)')
    ylabel('Throughput')
    hold on

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % RTS/CTS Throughput vs payload (FHSS)%
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    for i = 1:length(n_values)
        W = 16;             % window size CWmin or CWmax
        m = 0;              % typical values are 5 or 6
        n = n_values(i);    % number of stations
        calculate_RTS_CTS_FHSS % calculate p and tau given n
        sig = 50;           % slot duration (usec). depends on PHY
        Ptr = 1 - (1-tau)^n;
        num_Ps = n*tau*(1-tau)^(n-1);
        den_Ps = 1 - (1-tau)^n;
        Ps = num_Ps/den_Ps;
        % assume 1 Mbit rate (all units get converted to usec)
        H = 400;            % header size (bits)
        I = 100:1:20000;    % information payload I (bits)
        d = 1;              % propagation delay "delta"
        SIFS = 28;          % usec
        DIFS = 128;         % usec
        H_phy = 128;        % PHY header
        RTS = 160 + H_phy;  % usec
        CTS = 112 + H_phy;  % usec
        ACK = 112 + H_phy;  % usec
        % Ts and TC for RTS/CTS. Change this for CSMA/CA
        Ts = RTS + SIFS + d + CTS + SIFS + d + H + I + SIFS + d + ACK + DIFS + d;
        Tc = RTS + DIFS + d;
        % Calculate throughput S
        num_S = Ps*Ptr*I;
        den_S_1 = (1-Ptr)*sig;
        den_S_2 = Ptr*Ps*Ts;
        den_S_3 = Ptr*(1-Ps)*Tc;
        den_S = den_S_1 + den_S_2 + den_S_3;
```
S = num_S./den_S;
%
% plot throughput vs payload
semilogx(I,S,'-.k')
end
%
----------------------
function calculate_RTS_CTS_FHSS
fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);

tau = x(1);
p = x(2);

%----------------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);
k1 = (1-2*p)*(W+1);
k2 = p*W*(1-(2*p)^m);
den = k1 + k2;

fcns(1)= t - num/den;
fcns(2)= p - 1 + (1 - t)^(n-1) ;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CSMA/CA Throughput vs payload (FHSS) using Bianchi%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)

W = 16;                 % window size CWmin or CWmax
m = 0;                  % typical values are 5 or 6
n = n_values(i);        % number of stations
calculate_CSMA_CA_FHSS  % calculate p and tau given n
sig = 50;               % slot duration (usec). depends on PHY
Ptr = 1 - (1-tau)^n;
num_Ps = n*tau*(1-tau)^(n-1);
den_Ps = 1 - (1-tau)^n;
Ps = num_Ps/den_Ps;

% assume 1 Mbit rate (all units get converted to usec)
H = 400;                % header size (bits)
I = 100:1:20000;        % information payload I (bits)
d = 1;                  % propagation delay "delta"
SIFS = 28;              % usec
DIFS = 128;             % usec
H_phy = 128;            % PHY header
RTS = 160 + H_phy;      % usec
CTS = 112 + H_phy;      % usec
ACK = 112 + H_phy;      % usec

end
% Ts and TC for CSMA/CA. Change this for CSMA/CA
Ts = H + I + SIFS + d + ACK + DIFS + d;
Tc = H + I + DIFS + d;

% Calculate throughput S
num_S = Ps*Ptr*I;
den_S_1 = (1-Ptr)*sig;
den_S_2 = Ptr*Ps*Ts;
den_S_3 = Ptr*(1-Ps)*Tc;
den_S = den_S_1 + den_S_2 + den_S_3;
S = num_S./den_S;

% plot throughput vs payload
semilogx(I,S,'-.b')
end

%-----------------------
function calculate_CSMA_CA_FHSS
fun = @eqns2;
x0 = [0 0];
x = fsolve(fun,x0);
tau = x(1);
p = x(2);
%-----------------------
function fcns=eqns2(z)
t = z(1);
p = z(2);

num = 2*(1-2*p);
k1 = (1 - 2*p)*(W+1);
k2 = p*W*(1-(2*p)^m);
den = k1 + k2;
fcns(1)= t - num/den;
fcsn(2)= p - 1 + (1 - t)^(n-1) ;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% 
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)

% assume 1 Mbit rate (all units get converted to usec)
H = 400;                % header size (bits)
I = 100:1:20000;        % information payload I (bits)
d = 1;                  % propagation delay "delta"
SIFS = 28;              % usec
DIFS = 128;             % usec
H_phy = 128;            % PHY header
RTS = 160 + H_phy;      % usec
CTS = 112 + H_phy; % usec
ACK = 112 + H_phy; % usec

CWmin = 16;
sig = 50;

T_data = I;
T_ack = ACK;

T_cw_min = sig*(CWmin/2);
T_cw = T_cw_min

R_data = 1;
R_header = 1;
L_IP = 20;
L_UDP = 0;
L_SAP = 0; % ignore
L_LLCC = 0; % ignore
L_MAC = 30;
L_PLCP = 0; % ignore

T_oh = 8*(L_IP + L_UDP + L_SAP + L_LLCC + L_MAC)/R_data +
8*L_PLCP/R_header

num = T_data;
den = T_cw + T_data + T_oh + DIFS + SIFS + T_ack;
S = (num./den);
semilogx(T_data,S,'.-r')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CSMA Throughput vs payload using other paper approx %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)

% assume 1 Mbit rate (all units get converted to usec)
H = 400; % header size (bits)
I = 100:1:20000; % information payload I (bits)
d = 1; % propagation delay "delta"
SIFS = 28; % usec
DIFS = 128; % usec
H_phy = 128; % PHY header
H_mac = 272;
RTS = 160 + H_phy; % usec
CTS = 112 + H_phy; % usec
ACK = 112 + H_phy; % usec

CWmin = 16;
sig = 50;

T_data = I;
T_ack = ACK;

T_slot = sig;
CWbar = T_slot*(CWmin/2);
T_D_ack = ACK;
T_DIFS = DIFS;
T_SIFS = SIFS;
t = d;
T_p = H_phy;
T_phy = 1;     % ignore
term1 = H_mac + I;
T_D_data = T_p + T_phy + term1;
% T_D_data = T_phy + term1;
L_data = I;

num = 1*L_data;
den = CWbar + T_D_data + T_D_ack + 2*t + T_DIFS + T_SIFS;
S = (num./den);
semilogx(L_data,S,'-.g')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RTS/CTS Throughput vs payload using other paper approx  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:length(n_values)
    % assume 1 Mbit rate (all units get converted to usec)
    H = 400;        % header size (bits)
    I = 100:1:20000; % information payload I (bits)
    d = 1;          % propagation delay "delta"
    SIFS = 28;      % usec
    DIFS = 128;     % usec
    H_phy = 128;    % PHY header
    H_mac = 272;
    RTS = 160 + H_phy; % usec
    CTS = 112 + H_phy; % usec
    ACK = 112 + H_phy; % usec
    CWmin = 16;
    sig = 50;
    T_data = I;
    T_ack = ACK;
    T_slot = sig;
    CWbar = T_slot*(CWmin/2);
    T_D_ack = ACK;
    T_DIFS = DIFS;
    T_SIFS = SIFS;
    t = d;
    T_p = H_phy;
    T_phy = 1;     % ignore
term1 = H_mac + I;
T_D_data = T_p + T_phy + term1;
% T_D_data = T_phy + term1;
L_data = I;

    num = 1*L_data;
den = RTS + CTS + CWbar + T_D_data + T_D_ack + 4*t + T_DIFS + 3*T_SIFS;
S = (num./den);
    semilogx(L_data,S,'-.g')
end
end
The following script generate the throughput versus distance relationship for 802.11p for VANET applications

clear all;
close all;
c1c;

% calculate SNR from RSSI
%RSSI = [-84 -87 -88 -91]
RSSI = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);
RSSI_W = 10.^((RSSI - 30)/10)
k = 1.3806488*10^(-23)%physconst('Boltzmann')
T = 273.18 + 17
B = 20*10^6
N = k*T*B
SNR = RSSI_W / N
Pt = 10.^((20-30)/10)
c = 299792458%physconst('LightSpeed')
f = 2.4*10^9
L = c/f;
a = 3.5
P0 = L^a
d = 80
I = Pt*P0*(1/d)^a;

% new RSSI
RSSI_adjust = SNR * (I + N);

P = 10*log10(RSSI_adjust/(10^-3)); % RSS (dBm)
R = [ 11, 5.5, 2, 1]; % Data rate (Mbps)
R = R(end:-1:1);
ap = 2.4; % alpha 1
a2 = 3.5; % alpha 2
c = 3*10^8; % speed of light
f = 5.6*10^9; % operating center frequency
lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
Pt = [12,13,14,16,18,19,20,20,20]; % dBm
dbp = 104; % 802.11 breakpoint distance
var = sqrt(2)*8; % shadow fading variance
N = 1; % # of hops
H = N-1; % number of relays

db
hold
grid
xlabel('r')
ylabel('S(r)')
title('throughput distance relation for 802.11p using probabiltic model')
linespec = {'b-', 'r-', 'g', 'k','b-'};

for i =1:N
    for j = 1:(length(R))
        % distance range
        d = 0:1:(2*N*100);
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(dbp) - 10*a2*log10(d(k)/(dbp*i));
            end
            % calulate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob(k) = 1 - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
            else
                Prob(k) = (1/2)*erfc((f(k) - P(j-1))/(var*sqrt(2))) ... - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
            end
            Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
    % S(D) (i.e R(i))
    Rd = sum(Rbar);
    end
end
plot(d,Rd),linespec{randi(5,1,1)}) %linespec{randi(5,1,1)} %plot(d,Rbar)
end
end
end
end
axis([0 150 0 (max(R)+0.1*max(R))])

%P = [-82, -87, -91, -94];  % RSS (dBm)  % RSS (dBm)
P = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
P = P(end:-1:1);
%R = [ 11, 5.5, 2, 1];  % Data rate (Mbps)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);

a1 = 2.4;  % alpha 1
a2 = 3.5;  % alpha 2

c = 3*10^8;  % speed of light
f = 5.6*10^9;  % operating center frequency
lambda = c/f;  % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);
Pt = 20; % dBm
Pt = [12,13,14,16,18,19,20,20,20]; % dBm
dbp = 104; % 802.11 breakpoint distance

var = 8; % shadow fading variance
N = 1; % # of hops
H = N-1; % number of relays

figure;
hold
grid
xlabel('D')
ylabel('S(D)')
title('throughput distance relation for 802.11b using probabilistic model')
linespec = {'b-', 'r-', 'g', 'k','b-'};

for i =1:N
for j = 1:(length(R))

% distance range
d = 0:1:(2*N*100);
for k = 1:length(d)

% f(k) is RSS based on 802.11 model
if(d(k) <= dbp)
f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
else
f(k) = Pt - L0 - 10*a1*log10(dbp) -
10*a2*log10(d(k)/(dbp*i));
end
% calulate Prob{Ri(x)} where x is the distance. See reference
if (j == 1)
Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
else
Prob(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ...
- (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
end
Rbar(j,k) = (1/i)*R(j)*Prob(k);
end

% S(D) (i.e R(d))
Rd = sum(Rbar);
end
plot(d,Rd,'-.'%linespec{randi(5,1,1)}) %linespec{randi(5,1,1)}
%plot(d,Rbar)
end
axis([0 150 0 (max(R)+0.1*max(R))])

% Receiver Sensitivity
% P = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
% P = P(end:-1:1);
%
% PHY Layer Rate
% R = [40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
% R = R(end:-1:1);
% a1 = 2; % alpha 1
% a2 = 3.5; % alpha 2
%
% c = 3*10^8; % speed of light
% f = 5*10^9; % operating center frequency
% lambda = c/f; % wavelength
%
% % path loss in 1st meter
% L0 = 20*log10(4*pi/lambda);
%
% Pt = [20,20,20,20,20,20,20,20,20,20]; % dBm
%
% dbp = 5; % 802.11 breakpoint distance
%
% var = 5; % shadow fading variance
%
% N = 1; % # of hops
% H = N-1; % number of relays
% I = 8184; % payload (bits)
% H = 400; % header length (bits)
% M = I + H; % packet length (bits)
% t_MAC = 110; % assume fixed (usec) (old was 24)
%
% % figure;
% % hold
% % grid
% % xlabel('D')
% % ylabel('S(D)')
% % title('throughput distance relation for 802.11a using probabilistic model')
% % linespec = {'b-', 'r-', 'm', 'k', 'c', 'g', 'r', 'k'};
%
% % NOTE: use i=(N-1):N to determine Zi
% for i =1:N
%    for j = 1: (length(R))
%        % distance range
%        d = 0:1:(2*N*100);
%    for k = 1:length(d)
%        % f(k) is RSS based on 802.11 model
%        if(d(k) <= dbp)
%            f(k) = Pt(i) - L0 - 10*a1*log10(d(k)/i);
%        else
%            f(k) = Pt(i) - L0 - 10*a1*log10(dbp) -
%            10*a2*log10(d(k)/(i*dbp));
%        end
%    % calulate Prob{Ri(x)} where x is the distance. See reference
%    if (j == 1)
%        Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
%    else
%        Prob(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2)))
%               - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
%    end
%    %Rbar(j,k) = (1/i)*R(j)*Prob(k);
%    Rbar(j,k) = (1/1)*R(j)*Prob(k);
end

% S(D) (i.e R(d))
% temp = sum(Rbar);
% t_PHY = M./temp;
% Rd = (1/i)*I./(t_PHY + t_MAC);
%
% end
SD(i,:) = Rd;
plot(d, Rd, 'r')

end
end

% for seq_z=1:N-1
% for seq_d = 1: (length(d)-1)
% if SD(seq_z,seq_d) >= SD(seq_z+1,seq_d) && SD(seq_z,seq_d+1) < SD(seq_z+1,seq_d+1)
% z(seq_z) = seq_d;
% z(seq_z);
% end
% end
Zi = z

axis([0 240 0 0.5*(max(R)+0.1*max(R))])
The following script generates the throughput distance relationship for 802.11 under interference for both cellular and WiFi

```matlab
%% TDMA with int
clear all;
close all;
clc;

% calculate SNR from RSSI
RSSI = [-85, -85, -85, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);

k = 1.3806488*10^(-23); % physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k*T*B;
SNR = RSSI_W / N;

Pt = 10.^((20-30)/10);
c = 299792458; % physconst('LightSpeed')
f = 2.4*10^9;
L = c/f;
a = 3.5;
P0 = L^a;
rel_loc = 0;
d_int = 160 - rel_loc % Interference distance
I = Pt*P0*(1/d_int)^a;

% new RSSI
RSSI_adjust = SNR * (I + N);

P = 10*log10(RSSI_adjust/(10^-3)); % RSS (dBm)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540]; % Mbps
R = R(end:-1:1);

a1 = 2.4; % alpha 1
a2 = 3.5; % alpha 2 was 3.5

c = 3*10^8; % speed of light
f = 5.6*10^9; % operating center frequency

lambda = c/f; % wavelength

% path loss in 1st meter
LO = 20*log10(4*pi/lambda);

Pt = 20; % dBm
dbp = 10; % 802.11 breakpoint distance

var = sqrt(2)*8; % shadow fading variance

N = 1; % # of hops
H = N-1; % number of relays
```
figure;
hold
grid
xlabel('normalized distance r')
ylabel('Mac throughput S(r) (Mbps)')
%title('throughput distance relation for 802.11p using probabilistic model')
%linespec = {'b-', 'r-', 'g', 'k','b-'};

for i =1:N
  for j = 1:(length(R))
    d = 0:1:(d_int/2);
    for k = 1:length(d)
      % f(k) is RSS based on 802.11 model
      if(d(k) <= dbp)
        f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
      else
        f(k) = Pt - L0 - 10*a1*log10(dbp) -
           10*a2*log10(d(k)/(dbp*i));
      end
      % calculate Prob{Ri(x)} where x is the distance. See reference
      if (j == 1)
        Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
      else
        Prob(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ...
               - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
      end
      Rbar(j,k) = (1/i)*R(j)*Prob(k);
    end
    Rd = sum(Rbar); % S(D) (i.e R(d))
  end
  plot(d/(d_int/2),Rd,'b-')%,linespec{randi(5,1,1)})
end
%linespec{randi(5,1,1)}
%plot(d,Rbar)
ed
TDMA_av = sum(Rd)
%axis([0 80 0 30])

P = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];       % RSS (dBm)
P = P(end:-1:1);
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);
a1 = 2.4;       % alpha 1
a2 = 3.5;       % alpha 2
c = 3*10^8;     % speed of light
f = 5.6*10^9;   % operating center frequency
lambda = c/f;   % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);
Pt = 20;       % dBm
dbp = 10;      % 802.11 breakpoint distance
var = sqrt(2)*8; % shadow fading variance
N = 1;       % # of hops
H = N-1;     % number of relays

figure;
hold
grid
xlabel('D')
ylabel('S(D)')
title('throughput distance relation for 802.11 using probabilistic model')
linespec = {'b-', 'r-', 'g', 'k','b-'};

for i = 1:N
    for j = 1:length(R)
        % distance range
        d = 0:1:(d_int/2);
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(dbp) -
                      10*a2*log10(d(k)/(dbp*i));
            end
            % calculate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob(k) = 1 - (1/2)*erfc(((f(k)-P(j))/(var*sqrt(2))));
            else
                Prob(k) = (1/2)*erfc(((f(k)-P(j-1))/(var*sqrt(2))) ...
                               - (1/2)*erfc(((f(k)-P(j))/(var*sqrt(2)))));
            end
            Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
        % S(D) (i.e R(d))
        Rd = 1*sum(Rbar);
    end
    plot(d/(d_int/2),Rd,'b-')%linespec(randi(5,1,1))
    %plot(d,Rbar)
end
% axis([0 150 0 (max(R)+0.1*max(R))])

% RTS/CTS no int
P = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62]; % RSS (dBm)
P = P(end:-1:1);
% Data rate (Mbps)
R = 0.71*(27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);

a1 = 2.4;       % alpha 1
a2 = 3.5;       % alpha 2
c = 3*10^8;    % speed of light
f = 5.6*10^9; % operating center frequency

lambda = c/f;  % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20;       % dBm
dbp = 10;       % 802.11 breakpoint distance
var = sqrt(2)*8; % shadow fading variance

N = 1;         % # of hops
H = N-1;       % number of relays

figure;
hold
grid
xlabel('D')
ylabel('S(D)')
title('throughput distance relation for 802.11 using probabilistic model')
linespec = {'b-', 'r-', 'g', 'k','b-'};

for i =1:N
    for j = 1:length(R)
        % distance range
        d = 0:1:(d_int/2);
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(dbp) -
                      10*a2*log10(d(k)/(dbp*i)) ;
            end
            % Calculate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            else
                Prob(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ... 
                          - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2))) ;
            end
            Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
    end
    % S(D) (i.e R(d))
    Rd = sum(Rbar);
end
```matlab
plot(d/(d_int/2),Rd,'r--'); %linespec(randi(5,1,1))
%linespec(randi(5,1,1))
plot(d,Rbar)
end
% axis([0 150 0 (max(R)+0.1*max(R))])

%% RTS/CTS with int
clear all;
close all;
clc;

% calculate SNR from RSSI
RSSI = [-85, -85, -85, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);
RSSI_W = 10.^(RSSI-30)/10;
k = 1.3806488*10^(-23); %physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k*T*B;
SNR = RSSI_W / N;
Pt = 10.^(20-30)/10;
c = 299792458; %physconst('LightSpeed')
f = 2.4*10^9;
L = c/f;
a = 3.5;
P0 = L^a;
%d_int = 200 % meters
I = Pt*P0*(1/d_int)^a;

% new RSSI
RSSI_adjust = SNR * (I + N);

P = 10*log10(RSSI_adjust/(10^(-3))); % RSS (dBm)
%R = [ 11, 5.5, 2, 1]; % Data rate (Mbps)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);
I = 20000; % payload (bits)
H = 228; % header length (bits)
M = I + H; % packet length (bits)
t_MAC = 678; % (usec)
a1 = 2.4; % alpha 1
a2 = 3.5; % alpha 2
c = 3*10^8; % speed of light
f = 5.6*10^9; % operating center frequency 2.4

lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
%Pt = [12,13,14,16,18,19,19,20,20,20]; % dBm
dbp = 10; % 802.11 breakpoint distance
```

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var = sqrt(2)*8; % shadow fading variance

N = 1; % # of hops
H = N-1; % number of relays

% figure;
% hold
% grid
% xlabel('r')
% ylabel('S(r)')
% title('throughput distance relation for 802.11b using probabilistic model')
% linespec = {'b-', 'r-', 'g', 'k', 'b-'};

for i = (1):N
    for j = 1:(length(R))
        % distance range
        d = 1:1:(d_int/2);
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(dbp) - 10*a2*log10(d(k)/(dbp*i));
            end
            % calulate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob_int(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            else
                Prob_int(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ...
                - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            end
            t_PHY = M/R(j);
            Rbar_int(j,k) = (1/i)*(I/(t_PHY + t_MAC))*Prob_int(k);
            %Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
        % S(D) (i.e R(d))
        Rd_int = sum(Rbar_int);
    end
end
% plot(d,Rd_int),linespec{randi(5,1,1)} %linespec{randi(5,1,1)}
%plot(d,Rbar)
end
% axis([0 150 0 (max(R)+0.1*max(R))])

% RTS/CTS no int
% clear all;
% close all;
% clc;

% calculate SNR from RSSI
RSSI = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);
RSSI_W = 10.^[((RSSI-30)/10)];
k = 1.3806488*10^(-23);%physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k*T*B;
SNR = RSSI_W / N;
Pt = 10.^[((20-30)/10)];
c = 299792458;%physconst('LightSpeed')
f = 2.4*10^9;
L = c/f;
a = 3.5;
P0 = L^a;
d = 80;
I = 0;

RSSI_adjust = SNR * (I + N); % new RSSI
P = 10*log10(RSSI_adjust/(10^-3)); % RSS (dBm)

% Data rate (Mbps)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);
I = 20000; % payload (bits) was 8184
H = 228; % header length (bits)
M = I + H; % packet length (bits)
t_MAC = 1258 % (usec)
a1 = 2.4; % alpha 1
a2 = 3.5; % alpha 2
c = 3*10^8; % speed of light
f = 5.6*10^9; % operating center frequency

lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
dbp = 10; % 802.11 breakpoint distance

var = sqrt(2)*8;% shadow fading variance

N = 1; % # of hops
H = N-1; % number of relays

% figure;
% hold
% grid
% xlabel('r')
% ylabel('S(r)')
% title('throughput distance relation for 802.11 using probabiltic model')
% linespec = {'b-', 'r-', 'g', 'k','b-'};

for i =1:N
    for j = 1:length(R)
        % distance range
\begin{verbatim}
d = 1:1:(d_int/2);
for k = 1:length(d)
   
   % f(k) is RSS based on 802.11 model
   if(d(k) <= dbp)
      f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
   else
      f(k) = Pt - L0 - 10*a1*log10(dbp) -
            10*a2*log10(d(k)/(dbp*i));
   end

   % calculate Prob{R_i(x)} where x is the distance. See reference
   if (j == 1)
      Prob_no_int(k) = 1 - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
   else
      Prob_no_int(k) = (1/2)*erfc((f(k) - P(j-1))/(var*sqrt(2))) ...
                        - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
   end

   t_PHY = M/R(j);
   Rbar_no_int(j,k) = (1/2)*(I/(t_PHY + t_MAC))*Prob_no_int(k);
   %Rbar(j,k) = (1/i)*R(j)*Prob(k);
end

% S(D) (i.e R(d))
Rd_no_int = sum(Rbar_no_int);
end
% plot(d,Rd_no_int)
% %linespec{randi(5,1,1)}

% RTS/CTS with int
for w = 1:length(Prob_int)
   Rd_avg(w) = (0.71*(1-Prob_no_int(w))*Rd_no_int(w) + (1-
                 Prob_int(w))*Rd_int(w));
   %Rd_avg(w) = 1*((1-Prob_no_int(w))*Rd_no_int(w) + (1-
                 Prob_no_int(w))*Rd_int(w));
end

plot(d/(d_int/2),Rd_avg,'r-')
RTS_av = sum(Rd_avg)

legend('TDMA_{int}','TDMA_{no int}', 'RTS/CTS_{no int}', 'RTS/CTS_{int}')
\end{verbatim}
The following script generates the probabilistic MAC throughput $S(D)$ versus the distance $D$ for 802.11a

```matlab
% Bader Alkandari
% 802.11a probabilistic model
% $S(D)$ versus $D$ and $f(r)$
clc;
clear all;
close all;
format long;

P = [-72-3, -73-3, -78-3, -81-3, -84-3, -85-3, -87-3]; % RSS (dBm)
R = 0.5*[ 54, 48, 36, 24, 18, 12, 9, 6]; % Data rate (Mbps)

a1 = 2; % alpha 1
a2 = 3.5; % alpha 2

c = 3*10^8; % speed of light
f = 5*10^9; % operating center frequency
lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = [20,20,20,20,20,20,20,20]; % dBm

dbp = 5; % 802.11 breakpoint distance
var = 5;
N = 3; % # of hops
H = N-1; % number of relays

figure;
hold
grid
xlabel('D')
ylabel('S(D)')
title('throughput distance relation for 802.11p using probabilistic model')
linespec = {'b-', 'r-', 'm', 'k', 'c', 'g','r' , 'k'};

% NOTE: use i=(N-1):N to determine Zi
for i =1:N
  for j = 1: (length(R))

    % distance range
    d = 0:1:(2*N*100);
    for k = 1:length(d)

      % $f(k)$ is RSS based on 802.11 model
      if(d(k) <= dbp)
        f(k) = Pt(i) - L0 - 10*a1*log10(d(k)/(2*i));
      else
        f(k) = Pt(i) - L0 - 10*a1*log10(dbp) -
        10*a2*log10(d(k)/(dbp*2*i));
      end

      % add shadow fading
      f(k) = f(k) + var*randn;

    end

  end

  % add Rayleigh fading
  f(i) = f(i) + var*randn;

end
```

% calculate Prob(Ri(x)) where x is the distance. See reference
if (j == 1)
    Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
else
    Prob(k) = (1/2)*erfc(((f(k)-P(j-1))/(var*sqrt(2))) ... 
        - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2))));
end
Rbar(j,k) = (1/i)*R(j)*Prob(k);
end

% S(D) (i.e R(d))
Rd = sum(Rbar);

end
SD(i,:)=Rd;
plot(d,Rd,linespec{i})%{randi(6,1,1)}
end
% for seq_z=1:1:N-1
%     for seq_d= 1: (length(d)-1)
%         if
SD(seq_z,seq_d)>=SD(seq_z+1,seq_d)&&SD(seq_z,seq_d+1)<SD(seq_z+1,seq_d+1)
        z(seq_z)=seq_d;
        z(seq_z);
    end
% end
% end
% Zi=z
axis([0 450 0 (max(R)+0.1*max(R))])
%legend('N=1 (No relay)','N=2 (1 relay)','N=3 (2 relays)','N=3 (3 relays)',...
    'Location','NorthEast')

%%
% plot S(D) = f(r)/N with relay locations
% figure;
% hold
% grid
% xlabel('Distance X (m)')
% ylabel('MAC Throughput S(X)')
% title('Deployment Throughput Distance Relation')
% relay location
% L = 400/N;
% rbp = 5;
% linespec = {'b-','r-','m','k','c','g','r','k'};
% for i =1:N
%     for j = 1: (length(R))
%         % distance range
%         r = 0:0.1:2*N*100;
%         for k = 1:length(r)
%             % f(k) is RSS based on 802.11 model
if(r(k) <= rbp)
  g(k) = Pt(i) - L0 - 10*a1*log10(r(k));
else
  g(k) = Pt(i) - L0 - 10*a1*log10(rbp) -
10*a2*log10(r(k)/rbp);
end

% calulate Prob{Ri(x)} where x is the distance. See reference
if (j == 1)
  Prob(k) = 1 - (1/2)*erfc((g(k)-P(j))/(var*sqrt(2)));
else
  Prob(k) = (1/2)*erfc((g(k)-P(j-1))/(var*sqrt(2))) ...
  - (1/2)*erfc((g(k)-P(j))/(var*sqrt(2)));
end
Rbarp(j,k) = (1/i)*R(j)*Prob(k);

% S(D) (i.e R(d))
Rd = sum(Rbarp);

% plot(r+(i-1)*L,Rd,linespec{i})
% plot(-r+(i-1)*L,Rd,linespec{i})
end

% axis([0 500 0 (max(R)+0.3*max(R))])
The following script generates the performance bounds for 802.11b as well as the deployed relay network throughput:

clear all;
close all;
c1c;

% calculate SNR from RSSI
cl = 10; % distance from interferer
a1 = 2; % alpha 1
a2 = 3.5; % alpha 2
c = 3*10^8; % speed of light
f = 2.4*10^9; % operating center frequency
lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
dbp = 10; % 802.11 breakpoint distance
var = 8; % shadow fading variance

% determine Pout for specific interferer I at d
f_p = Pt - L0 - 10*a1*log10(d/bp) - 10*a2*log10(d/dbp)
P_coverage = 1 - 0.5*erfc((f_p - (-85))/(sqrt(2)*var))

RSSI = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);
RSSI_W = 10.^(RSSI - 30)/10
k = 1.3806488*10^(-23)% physconst('Boltzmann')
T = 273.15 + 17
B = 20*10^6
N = k*T*B
SNR = RSSI_W / N

Pt = 10.^(20-30)/10

c = 299792458% physconst('LightSpeed')
f = 2.4*10^9
L = c/f;
a = 3.5
P0 = L^a

Pout = 1 - P_coverage
I = (Pout)*Pt*P0*(1/d)^a;

% new RSSI
RSSI_adjust = SNR * (I + N);

P = 10*log10(RSSI_adjust/(10^-3)); % RSS (dBm)
R = [40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);

N = 1; % # of hops
H = N-1; % number of relays

figure; hold
grid
xlabel('r')
ylabel('S(r)')
% title('throughput distance relation for 802.11b using probabilistic model')
% linespec = {'b-', 'r-', 'g', 'k','b-'};

for i = 1:N
    for j = 1:(length(R))
        % distance range
        d = 0:1:2*N*100;
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(dbp) - 10*a2*log10(d(k)/(dbp*i));
            end
            % calculate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob(k) = 1 - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
            else
                Prob(k) = (1/2)*erfc((f(k) - P(j-1))/(var*sqrt(2))) ... - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
            end
            Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
        % S(D) (i.e R(d))
        Rd = sum(Rbar);
    end
end

plot(d,Rd), linespec{randi(5,1,1)}) %linespec{randi(5,1,1)} %plot(d,Rbar)
end

% axis([0 150 0 (max(R)+0.1*max(R))])
clear;

P = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
P = P(end:-1:1);

R = 1*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);

a1 = 2; % alpha 1
a2 = 3.5; % alpha 2

c = 3*10^8; % speed of light
f = 2.4*10^9; % operating center frequency

lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
dbp = 10; % 802.11 breakpoint distance
var = 8;  \% shadow fading variance

N = 1;  \% # of hops
H = N-1;  \% number of relays

\% figure;
\% hold
\% grid
\% xlabel('D')
\% ylabel('S(D)')
\% title('throughput distance relation for 802.11b using probabilistic model')
\% linespec = {'b-', 'r-', 'g', 'k', 'b-'};

for i = (1):N
    for j = 1:(length(R))
        \% distance range
        d = 0:1:(2*N*100);
        for k = 1:length(d)
            \% f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(d(k)/(dbp*i)) - 10*a2*log10(d(k)/(dbp*i));
            end
            \% calculate Prob{R_i(x)} where x is the distance. See reference
            if (j == 1)
                Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            else
                Prob(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ...
                - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            end
            Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
    \% S(D) (i.e R(d))
    Rd = sum(Rbar);
    plot(d,Rd,'-'); linespec{randi(5,1,1)} %linespec{randi(5,1,1)}
    plot(d,Rbar)
end
\% axis([0 150 0 (max(R)+0.1*max(R))])

\% \% Receiver Sensitivity
\% clear;
\% \% P = [-85, -85, -85, -77, -77, -70, -68, -63, -62];
\% P = P(end:-1:1);
\% \% PHY Layer Rate
\% R = 0.5*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
\% R = R(end:-1:1);
% a1 = 2; % alpha 1
% a2 = 3.5; % alpha 2
%
% c = 3*10^8; % speed of light
% f = 2.4*10^9; % operating center frequency
% lambda = c/f; % wavelength
%
% path loss in 1st meter
% L0 = 20*log10(4*pi/lambda);
%
% Pt = [20,20,20,20,20,20,20,20,20,20]; % dBm
%
% dbp = 10; % 802.11 breakpoint distance
%
% var = 8; % shadow fading variance
%
% N = 1; % # of hops
% H = N-1; % number of relays
% I = 8184; % payload (bits)
% H = 400; % header length (bits) was 400
% M = I + H; % packet length (bits)
% t_MAC = 110; % assume fixed (usec) (old was 110)
%
figure;
hold
grid
xlabel('D')
ylabel('S(D)')
title('throughput distance relation for 802.11a using probabilistic
model')
linespec = {'b-', 'r-', 'm', 'k', 'c', 'g','r','k'};

% NOTE: use i=(N-1):N to determine Zi
for i =1:N
    for j = 1: (length(R))
        % distance range
        d = 0:1:(2*N*100);
        for k = 1:length(d)

            % f(k) is RSS based on 802.11 model
            if(d(k) <= dbp)
                f(k) = Pt(i) - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt(i) - L0 - 10*a1*log10(dbp) -
                10*a2*log10(d(k)/(i*dbp));
            end
            % calulate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            else
                Prob(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ...
                - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            end
            Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
    end
Rbar(j,k) = (1/1)*R(j)*Prob(k);
end
\% S(D) (i.e R(d))
\% temp = sum(Rbar);
\% t PHY = M./temp;
\% Rd = (l/i)*I./(t PHY + t MAC);
\%
\% end
\% SD(i,:) = Rd;
\% plot(d,Rd,'*r')%. linespec{i})%{rand(6,1,1)}
\% end
\% for seq z = 1:1:N-1
\% for seq d = 1: (length(d)-1)
\% if
SD(seq z, seq d) >= SD(seq z+1, seq d) && SD(seq z, seq d+1) < SD(seq z+1, seq d+1)
\% z(seq z) = seq d;
\% z(seq z);
\% end
\% end
\% Zi = z
\% axis([0 240 0.5*(max(R)+0.1*max(R))])
The following script generates the throughput distance relationship for CSMA/CA with RTS/CTS under the effect of interference for variable interference distance $D_{int}$

```matlab
%% RTS/CTS with int
clear all;
close all;
clc;

for d_int = 80:20:160;                    % meters

% for AP
% calculate SNR from RSSI
RSSI = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);
RSSI_W = 10.^(RSSI-30)/10;
k = 1.3806488*10^(-23);         % physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k*T*B;
SNR = RSSI_W / N;
Pt = 10.^(20-30)/10;
c = 299792458;                  % physconst('LightSpeed')
f = 2.4*10^9;
L = c/f;
a = 3.5;
P0 = L^a;
rel_loc = 0;
I = Pt*P0*(1/d_int)^a;

% new RSSI
RSSI_adjust = SNR * (I + N);

P = 10*log10(RSSI_adjust/(10^(-3)));  % RSS (dBm)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];  
% Mbps
R = R(end:-1:1);
I = 20000;                          % payload (bits)
H = 228;                            % header length (bits)
M = I + H;                          % packet length (bits)
t_MAC = 678;                         % (usec)
a1 = 2.4;                           % alpha 1
a2 = 3.5;                           % alpha 2

% a2 = 3.5;                           % alpha 2
% a1 = a2;
c = 3*10^8;                         % speed of light
f = 5.6*10^9;                       % operating center frequency

lambda = c/f;                       % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20;                           % dBm
```

dbp = 10;       % 802.11 breakpoint distance

var = sqrt(2)*8;        % shadow fading variance

N = 1;          % # of hops
H = N-1;        % number of relays

figure;
hold
grid
xlabel('distance')
ylabel('MAC throughput S(r)')
% title('throughput distance relation for 802.11b using probabilistic model')
% linespec = { 'b-', 'r-', 'g', 'k', 'b-' }

for i =N
    for j = 1:(length(R))
        % distance range
        d = 0:1:((d_int+rel_loc)/2);
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
            if (d(k) <= dbp)
                f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
            else
                f(k) = Pt - L0 - 10*a1*log10(dbp) - 10*a2*log10(d(k)/(dbp*i));
            end
            % calculate Prob{Ri(x)} where x is the distance. See reference
            if (j == 1)
                Prob_int(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            else
                Prob_int(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ...
                - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
            end
            t_PHY = M/R(j);
            Rbar_int(j,k) = (1/i)*(I/(t_PHY + t_MAC))*Prob_int(k);
            % Rbar(j,k) = (1/i)*R(j)*Prob(k);
        end
        % S(D) (i.e R(d))
        Rd_int = sum(Rbar_int);
    end
    Rd = Rd_int(1:length(Rd_int)-1);
    %plot(d(1:length(Rd)),Rd,'b-')
    %plot(d,Rd_int,'%linespec(randi(5,1,1))) %linespec(randi(5,1,1))
    %plot(d,Rbar)
end
% axis([0 150 0 (max(R)+0.1*max(R))])

% RTS/CTS no int
% For AP

% clear all;
% close all;
% clc;
% calculate SNR from RSSI
RSSI = [-85, -85, -85, 77, -77, -71, -70, -68, -63, -62];
RSSI = RSSI(end:-1:1);
RSSI_W = 10.^((RSSI-30)/10);
k = 1.3806488*10^(-23);%physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k*T*B;
SNR = RSSI_W / N;
Pt = 10.^((20-30)/10);
c = 299792458;%physconst('LightSpeed')
f = 2.4*10^9;
L = c/f;
a = 3.5;
P0 = L^a;
d = 80;
I = 0;

RSSI_adjust = SNR * (I + N); % new RSSI
P = 10*log10(RSSI_adjust/(10^(-3))); % RSS (dBm)

% Data rate (Mbps)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);
I = 20000; % payload (bits) was 8184
H = 228; % header length (bits)
M = I + H; % packet length (bits)
t_MAC = 1258; % (usec)
a1 = 2.4; % alpha 1
a2 = 3.5; % alpha 2

a2 = 3.5; % alpha 2
a1 = a2;

c = 3*10^8; % speed of light
f = 5.6*10^9; % operating center frequency

lambda = c/f; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
dbp = 10; % 802.11 breakpoint distance

var = sqrt(2)*8;% shadow fading variance

N = 2; % # of hops
H = N-1; % number of relays

for i = N
    for j = 1:(length(R))
        % distance range
        d = 0:1:(d_int/2);
for k = 1:length(d)
    % f(k) is RSS based on 802.11 model
    if(d(k) <= dbp)
        f(k) = Pt - L0 - 10*a1*log10(d(k)/i);
    else
        f(k) = Pt - L0 - 10*a1*log10(dbp) - 10*a2*log10(d(k)/(dbp*i));
    end

    % calculate Prob{Ri(x)} where x is the distance. See reference
    if (j == 1)
        Prob_no_int(k) = 1 - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
    else
        Prob_no_int(k) = (1/2)*erfc((f(k)-P(j-1))/(var*sqrt(2))) ... - (1/2)*erfc((f(k)-P(j))/(var*sqrt(2)));
    end

    t_PHY = M/R(j);
    Rbar_no_int(j,k) = (1/2)*(I/(t_PHY + t_MAC))*Prob_no_int(k);
    %Rbar(j,k) = (1/i)*R(j)*Prob(k);
end

% S(D) (i.e R(d))
Rd_no_int = sum(Rbar_no_int);
end
Rd=Rd_no_int(1:length(Rd_no_int)-1);
%plot(d(1:length(Rd)),Rd,'r-')
%plot(d,Rd_no_int)%linespec(randi(5,1,1))) %linespec(randi(5,1,1))
%plot(d,Rbär)
end

% axis([0 150 0 (max(R)+0.1*max(R))])

%% RTS/CTS int and no int combined

for w = 1:(length(Prob_int))
    Rd_avg(w) = (0.71*(1-Prob_no_int(w))*Rd_no_int(w) + (1-Prob_int(w))*Rd_int(w));
    %Rd_avg(w) = 1*((1-Prob_no_int(w))*Rd_no_int(w) + (1-Prob_no_int(w))*Rd_int(w));
end
%plot(d/(d_int/2),Rd_avg,'g-')
plot(d,Rd_avg,'g-')
RTS_av = sum(Rd_avg);

%% For relay. with int
% clear all;
% close all;
% clc;
% d_int = 100

% calculate SNR from RSSI
RSSI_r = [-85, -85, -85, -77, -77, -71, -70, -63, -63, -62];
RSSI_r = RSSI_r(end:-1:1);
RSSI_W_r = 10.^(RSSI_r-30)/10;
k2 = 1.3806488*10^(-23); %physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k2*T*B;
SNR = RSSI_W_r / N;
Pt = 10.^(20-(30)/10);
c = 299792458;                    %physconst('LightSpeed')
f2 = 2.4*10^9;
L = c/f2;
a = 3.5;
P0 = L^a;
rel_loc = d_int/4;
I = \text{Pt}\*P0\*(1/d_int)^a;

\% new RSSI
RSSI_adjust = SNR * (I + N);

P = 10*\text{log10}(RSSI_adjust/(10^-3));   \% RSS (dBm)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];  \%Mbps
R = R(end:-1:1);
I = 20000;           \% payload (bits)
H = 228;            \% header length (bits)
M = I + H;           \% packet length (bits)
t_MAC = 1257;        \% (usec) 678

a1 = 2.4;           \% alpha 1
a2 = 3.5;           \% alpha 2

a1 = a2;

c = 3*10^8;        \% speed of light
f2 = 5.6*10^9;   \% operating center frequency

lambda = c/f2;     \% wavelength

\% path loss in 1st meter
L0 = 20*\text{log10}(4*\pi/\lambda);

Pt = 20;       \% dBm
dbp = 10;       \% 802.11 breakpoint distance

var = \text{sqrt}(2)*8; \% shadow fading variance

N = 2;        \% # of hops
H = N-1;     \% number of relays

\% figure;
\% hold
\% grid
\% xlabel('distance')
\% ylabel('MAC throughput S(r)')
\% title('throughput distance relation for 802.11b using probabilistic model')
\% linespec = {'b-', 'r-', 'g', 'k','b-'};

for i = N
    for j = 1:(length(R))
% distance range
d2 = 0:1:(d_int/4);
%d2 = 0:1:((d_int-rel_loc)/3);
for k2 = 1:length(d2)

    % f(k) is RSS based on 802.11 model
    if(d2(k2) <= dbp)
        f2(k2) = Pt - L0 - 10*a1*log10(d2(k2)/i);
    else
        f2(k2) = Pt - L0 - 10*a1*log10(dbp) -
                 10*a2*log10((d2(k2))/(dbp*i));
    end

    % calulate Prob{Ri(x)} where x is the distance. See reference
    if (j == 1)
        Prob_int2(k2) = 1 - (1/2)*erfc((f2(k2)-P(j))/(var*sqrt(2))));
    else
        Prob_int2(k2) = (1/2)*erfc((f2(k2)-P(j-1))/(var*sqrt(2)))
                        - (1/2)*erfc((f2(k2)-P(j))/(var*sqrt(2))));
    end

    t_PHY = M/R(j);
    Rbar_int2(j,k2) = (1/i)*(I/(t_PHY +t_MAC))*Prob_int2(k2);
    %Rbar(j,k) = (1/i)*R(j)*Prob(k);
end

% S(D) (i.e R(d))
Rd_int2 = sum(Rbar_int2);
end

plot(d_int/4+d(1:length(Rd)),Rd,'b-')
plot(d,Rd_int) %,linespec{randi(5,1,1)}) %linespec{randi(5,1,1)}
plot(d,Rbar)
axis([0 (d_int/2) 0 (max(R)+0.1*max(R))])

% RTS/CTS relay no int
% clear all;
% close all;
% clc;
%
% calculate SNR from RSSI
RSSI_r = [-85, -85, -85, -77, -77, -71, -70, -68, -63, -62];
RSSI_r = RSSI_r(1:end-1:1);
RSSI_W_r = 10.^((RSSI_r-30)/10);
k2 = 1.380648*10^(-23);%physconst('Boltzmann')
T = 273.18 + 17;
B = 20*10^6;
N = k2*T*B;
SNR = RSSI_W_r / N;
Pt = 10.^((20-30)/10);
c = 299792458;%physconst('LightSpeed')
f2 = 2.4*10^9;
L = c/f2;
a = 3.5;
P0 = L^a;
d2 = 80;
I = 0;

RSSI_adjust = SNR * (I + N); % new RSSI
P = 10*log10(RSSI_adjust/(10^-3)); % RSS (dBm)

% Data rate (Mbps)
R = (27/540)*[40.5, 81, 121.5, 162, 243, 324, 364.5, 405, 486, 540];
R = R(end:-1:1);
I = 20000; % payload (bits) was 8184
H = 228; % header length (bits)
M = I + H; % packet length (bits)
t_MAC = 1771; % (usec) 1257
a1 = 2.4; % alpha 1
a2 = 3.5; % alpha 2

% a2 = 3.5; % alpha 2
% a1 = a2;

c = 3*10^8; % speed of light
f2 = 5.6*10^9; % operating center frequency
lambda = c/f2; % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = 20; % dBm
dbp = 10; % 802.11 breakpoint distance

var = sqrt(2)*8; % shadow fading variance

N = 2; % # of hops
H = N-1; % number of relays

for i = N
for j = 1:(length(R))

% distance range
d2 = 0:1:(d_int/4);
% d2 = 0:1:((d_int-rel_loc)/3);
for k2 = 1:length(d2)

% f(k) is RSS based on 802.11 model
if(d2(k2) <= dbp)
    f2(k2) = Pt - L0 - 10*a1*log10(d2(k2)/i);
else
    f2(k2) = Pt - L0 - 10*a1*log10(dbp) -
            10*a2*log10(d2(k2)/(dbp*i));
end

% calculate Prob(Ri(x)) where x is the distance. See reference
if (j == 1)
    Prob_no_int2(k2) = 1 - (1/2)*erfc((f2(k2) -
P(j))/(var*sqrt(2)));
end

end
end
else
    Prob_no_int2(k2) = (1/2)*erfc((f2(k2)-P(j-1))/(var*sqrt(2)));

end

t_PHY = M/R(j);
Rbar_no_int2(j,k2) = (1/2)*(1/(t_PHY + t_MAC))*Prob_no_int2(k2);

end

% S(D) (i.e R(d))
Rd_no_int2 = sum(Rbar_no_int2);
end

Rd=Rd_no_int2(1:length(Rd_no_int2));
end

Rd=Rd_no_int2(1:length(Rd_no_int2));
end

% RTS/CTS int and no int combined
for w2 = 1:1:(length(Prob_int2))
    Rd_avg2(w2) = (0.71*(1-Prob_no_int2(w2))*Rd_no_int2(w2) + (1-Prob_int2(w2))*Rd_int2(w2));
    Rd_avg(w) = Rd_int(w) = (1-Prob_no_int(w))*Rd_no_int(w) + (1-Prob_no_int(w))*Rd_int(w));
end

% form relay Sr
temp = fliplr(Rd_avg2);
temp2 = temp(2:end);
d3=d2(1:(length(d2)-1));
plot(d3, temp2,'r-.')
Rd_avg;
% Rd_avg2=Rd_avg2(2:end);
rel_Sr = [temp2, Rd_avg2];

% if d is 10,30,50 etc use temp. otherwise use temp2
%d_temp = d2(2:end)+length(d2);
%new_d = [d2(2:end), d_temp];

plot(new_d-d_int/4, rel_Sr)
max_Sr = max(Rd_avg,rel_Sr);
RTS_relay_av = sum(max_Sr)
end
The following script generates the performance bounds for a relay network using the empirical approximation for 802.11g with second degree polynomial curve fit.

```matlab
% Bader Alkandari
% Throughput Analysis problem for 802.11g
% polynomial: f(r1) = a(r-Rmax)^2
% 2013 12 28

% N = k solution. Note there are multiple N in the code. So change things
% accordingly
clc;
clear all;
close all;
format long;

syms D; % distance D
R = 1; % normalized Rmax
Smax = 1;
a = Smax/R;
r1 = 0:0.01:1; % r1 normalized to Rmax
N = 2;

for i=1:length(r1)
    % (1) solve S(D) <= fr(1)/N
    x=double(solve((a/N)*(r1(i)-R)^2-(a/(N-1))*((D/(N-1))-R)^2 == 0));
    % Two solutions due to parabaloic shape/ Quadratic model.
    % Select D s.t. D < N*Rmax and ignore other solution
    for j=1:length(x)
        y(i,j)=x(j);
    end
end

figure;
plot(r1,y(:,2)) % accepted solution from for loop

% adjust display region
axis([-0.2 (R+.3) -0.2 (N*R+0.3)]),grid

xlabel('r_1')
ylabel('D')
title('solution region for N = k')
hold;
stem(1,N*R,'r') % (2) r1 = Rmax
plot(r1,N*R,'r') % (3) D  = N*Rmax
c=N*r1; % (4) D <= 2*r1 constraint
plot(r1,c,'r')

%%
% % compute Z1 directly from S[k](D) = S[k+1](D)
syms D R N
Zi=solve((a/(N-1))*((D/(N-1))-R)^2 -(a/(N))*((D/(N))-R)^2, D);
```
Zi=Zi(2) % assign accepted solution
pretty(Zi)
R = 1; % normalized Rmax
N = 2; % number of hops
Zi=subs(Zi) % convert from symbolic to numeric

%%
% Sketch S[k](D)
figure;
xlabel('D')
ylabel('S(D)')
grid
title('bounds for throughput distance relation')
hold;
R = 1;
N = 2;
for k=1:N
    D = 0:0.01:k*R;
    SD = (a/k)*(D/k-R).^2;
    plot(D,SD)
end
The following script generates the plot for throughput for CSMA/CA with RTS/CTS using both a probabilistic model and a curve fit

```matlab
% RTS Analytical vs Curve fit
clear all;
close all;
format long;

P = [-72, -73, -78, -81, -84, -85, -87, -87];   % RSS (dBm)
R = [ 54,  48,  36,  24,  18,  12,   9,   6];   % Data rate (Mbps)
a1 = 2;         % alpha 1
a2 = 3.5;       % alpha 2
c = 3*10^8;     % speed of light
f = 2.4*10^9;   % operating center frequency
lambda = c/f;   % wavelength

% path loss in 1st meter
L0 = 20*log10(4*pi/lambda);

Pt = [17,19,20,20,20,20,20,20];   % dBm
dbp = 5;       % 802.11 breakpoint distance
var = 5;        % shadow fading variance
N = 1;          % # of hops
H = N-1;        % number of relays
I = 8184;       % payload (bits)
H = 400;        % header length (bits)
M = I + H;      % packet length (bits)
t_MAC = 220;    % assume fixed (usec)

figure;
hold
grid
xlabel('r_1')
ylabel('f(r_1)')
title('throughput distance relation for 802.11a with RTS/CTS MAC')
linespec = {'b-', 'r-', 'm', 'k', 'c', 'g','r','k'};

% NOTE: use i=(N-1):N to determine Zi
for i =1:N
    for j = 1: (length(R)-1)
        % distance range
d = 0:1:(2*N*150);
        for k = 1:length(d)
            % f(k) is RSS based on 802.11 model
```

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if(d(k) <= dbp)
    f(k) = Pt(i) - L0 - 10*a1*log10(d(k)/i);
else
    f(k) = Pt(i) - L0 - 10*a1*log10(dbp) - 10*a2*log10(d(k)/(dbp*i));
end

% calculate Prob{Ri(x)} where x is the distance. See reference
if (j == 1)
    Prob(k) = 1 - (1/2)*erfc((f(k) - P(j))/(var*sqrt(2)));
else
    Prob(k) = (1/2)*erfc((f(k) - P(j))/(var*sqrt(2))) ... - (1/2)*erfc((f(k) - P(j+1))/(var*sqrt(2)));
end
t_PHY = M/R(j);
Rbar(j,k) = (1/i)*(I/(t_PHY + t_MAC)) * Prob(k);
end

% S(D) (i.e R(d))
Rd = sum(Rbar);

end
SD(i,:)=Rd;
plot(d,Rd,linespec{i})%{randi(6,1,1)}
end

% for seq_z=1:1:N-1
%   for seq_d= 1: (length(d)-1)
%     if
SD(seq_z,seq_d)>=SD(seq_z+1,seq_d)&&(SD(seq_z,seq_d+1)<SD(seq_z+1,seq_d+1))
%       z(seq_z)=seq_d;
%     end
%   end
% end
% 2i=z
%axis([0 450 0 1*(max(R)+0.1*max(R))])

% clear;
% MAC throughput bounds (best fit)
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
I = 8184; % payload (bits)
H = 400; % header length (bits)
M = I + H; % packet length (bits)
c = 3e8; % speed of light (m/s)
Smax = 54;
d_bp = 25;
R = 220;
N= 1;
a = Smax/((R-d_bp)^2);
t_MAC = 220; % need to work on this parameter

for k =1:N
    D = 0:0.1:k*R;
    for j=1:length(D)
        if(D(j) <= k*d_bp)
            t_PHY = M/Smax;
        end
    end
end
SD(j) = \frac{I}{k*(t_{PHY}+t_{MAC})};
else
\quad t_{PHY} = M/(a*(D(j)/k-R)^2);
\quad SD(j) = \frac{I}{(k*(t_{PHY}+t_{MAC}))};
end
end
plot(D,SD,'-.r')
end
legend('Analytical Model','Curve Fit','Location','NorthEast')