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Risk Management Project

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Abstract

In order to evaluate and manage portfolio risk, we separated this project into three sections. In the first section we constructed a portfolio with 15 different stocks and six options with different strategies. The portfolio was implemented in Interactive Brokers[1] and rebalanced weekly through five holding periods. In the second section we modeled the loss distribution of the whole portfolio with normal and student-t distributions, we computed the Value-at-Risk and expected shortfall in detail for the portfolio loss in each holding week, and then we evaluated differences between the normal and student-t distributions. In the third section we applied the ARMA(1,1)-GARCH(1,1) model to simulate our assets and compared the polynomial tails with Gaussian and t-distribution innovations.

Key Words: Risk Management ; Value-at-Risk; Expected Shortfall; ARMA-GARCH; x^2 test; AIC; BIC; Portfolio Optimization

[1] Provided by Interactive Brokers Group, Inc. (IB), which is an online discount brokerage firm in the United States.

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Chapter 1 Introduction

Modeling losses of a portfolio is a central issue in modern risk management. How much will an investor lose in a portfolio in a bad scenario? This is a question not only risk mangers but also researchers care about.[1] A good model for modeling losses will provide guidance for investors; conversely, a bad model will mislead investors.

An estimation of a portfolio loss distribution provides the basic precondition for computing risk measures such as value-at-risk and expected shortfall. Value-at-risk, abbreviated as VaR, attempts to explain the potential loss of a risky asset or a portfolio over a defined period under certain confidence level. It became popular when JP Morgan started a VaR system as an internal system of risk disclosure. This system was published in 1994.[2] VaR is a widely used financial tool for risk assessment currently. Expected shortfall is an alternative risk measure to VaR. It is the average of value-at-risk beyond a certain threshold, and is also called conditional value at risk.

The main goal of this project is loss distribution modeling with 5 different methods. Then risk measures could be computed and analyzed among these models. There are 4 components of the project. First we formed a risky portfolio with stocks and options worth $500,000. $420,000 is distributed to stocks part and the other $80,000 is for options. The allocation for each stock is chosen according to the Efficient Frontier Strategy. We computed the optimal weights from this theory and rebalanced these stocks each holding week. As for option portion, we purchase these options according to option strategies such as long straddle, long strangle, and protective puts. We traded these stocks and options in Interactive Brokers[2] paper trading account to get actual investing value for profit or loss.

Second, we model the portfolio loss distribution with the assumption of stock loss stationarity. Normal and student-t distribution are the two types of distributions we are considering. The unconditional mean and standard deviation can be estimated through historical stock data. The VaR and ES can then be calculated. We compare and analyze these risk measures for normal and student-t distributions.

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[2] Provided by Interactive Brokers Group, Inc. (IB), which is an online discount brokerage firm in the United States.

Third, we assume the stock returns distribution has a polynomial left tail. With historical data of underlying assets, the tail index can be estimated. A semi-parametric method is applied for computing VaR and ES. This gives another model for the loss distribution.

Fourth, we assume the portfolio loss follows an ARMA(1,1)-GARCH(1,1) with Gaussian and student-t innovations. We model both the stock portfolio portion and the entire portfolio. The difference between this model and previous models is that we can obtain both the conditional mean and the conditional standard deviation. VaR and ES for the conditional loss distribution then can be estimated.

Finally, we make a comparison of these models.

In the last part we implement a risk reduction for our portfolio. The basic idea is to add negatively correlated assets. We short a call to hedge the risk of holding long stocks, and similarly we short a put to manage the risk of shorting stocks. Then we recalculate the VaR in different models and compare the values of new portfolio with the previous one.

Chapter 2 Portfolio Development Strategies

In this project we formed a portfolio containing \$500,000 of risky assets and \$500,000 of risk-free assets using an Interactive Brokers paper trading account. We bought \$500,000 Treasury Bills as risk free assets, and our risky assets consisted of 15 positions in stocks and 10 positions in options. The underlying assets were from different sectors so that we could maintain a diversified portfolio. We rebalanced these assets weekly, then modeled the log-returns of the underlying assets using a normal distribution and estimated the linearized loss distribution.

2.1 Stock Selection

In order to make our investment, we chose 15 stocks from American stock exchanges. These stocks were selected from different economic sectors to obtain the benefit of diversification. Since we intended to form a portfolio with option strategies, underlying assets with a wide range of option strike prices and large trading volumes were primarily taken into consideration. We detail the information for purchased stocks here.

Name	Symbol	Sector
Apple Inc.	AAPL	Technology
Dell Inc.	DELL	Technology
Google Inc.	GOOG	Technology
Hewlett-Packard Company	HPQ	Technology
Microsoft Corporation	MSFT	Technology
Best Buy Co., Inc.	BBY	Service
Wal-Mart Stores, Inc.	WMT	Services
American Eagle Outfitters, Inc.	AEO	Service
McDonald's Corporation	MCD	Service
Nike, Inc.	NKE	Consumer Goods
Sony Corporation	SNE	Consumer Goods
Coca-Cola Company	KO	Consumer Goods
General Electric Company	GE	Industrial Goods
Citigroup, Inc.	C	Financial
HSBC Holdings, plc.	HBC	Financial

Table 2.1: Stock Selection

2.2 Portfolio Investment Strategies

Once we chose our underlying assets, we could determine our portfolio investment strategies. Our initial investment amount was $1,000,000 in our Interactive Brokers paper trading account. $500,000 was invested in risk-free assets, and the other $500,000 was our initial capital for risky assets with stock and option positions. Since stock prices are much higher than option prices, we purchased $420,000 worth of stocks (Table 2.1) and $80,000 worth of options.

2.2.1 Optimal Strategy for Underlying Assets

Efficient Frontier Theory, which was creatively defined by Harry Markowitz in 1952[3], is the most influential component in modern portfolio theory. The theory explores what the most optimal portfolio is for a given level of risk. Thus we decided to apply this theory in investing in our underlying assets. The stocks were bought based on weights calculated using the Efficient Frontier Theory and rebalanced weekly.

There are several factors that were considered in the process of implementing the theory. First, we considered an interval of time to evaluate a representative historical stock price. Stocks' expected returns and volatility could be derived from this period price dataset with the assumption of stocks' returns being stationary. In order to include a representative amount of changes in stock prices, a proper length of looking back period had to be selected. Since our holding period of the stocks was one week, we decided to look back six months. Second, we chose the BofA Merrill Lynch US Corporate AAA Effective Yield as the risk-free interest rate (2.09% for March 16 2012). This rate is different from the US Three-Month Treasury Bill managed by The Federal Reserve. We choose this as our risk-free rate because we believe this interest rate reflects more realistic economic conditions. Third, to avoid margin violations, we set our weight boundary between -0.3 to 0.4, which means the largest weight is 30% for any one short position and 40% for any one long position.

The green star on the curve in Figure 2.1 is the optimal portfolio we developed using the Efficient Frontier Theory. With these optimal weights we created a portfolio with our Interactive Brokers account on March 16 2012. The trading details are shown in Table 2.2:

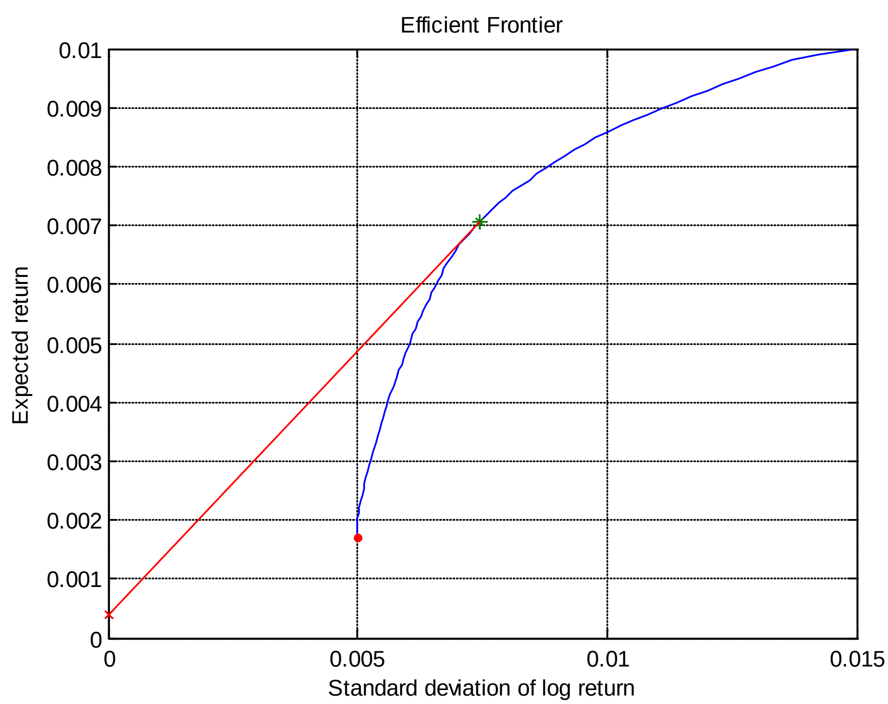
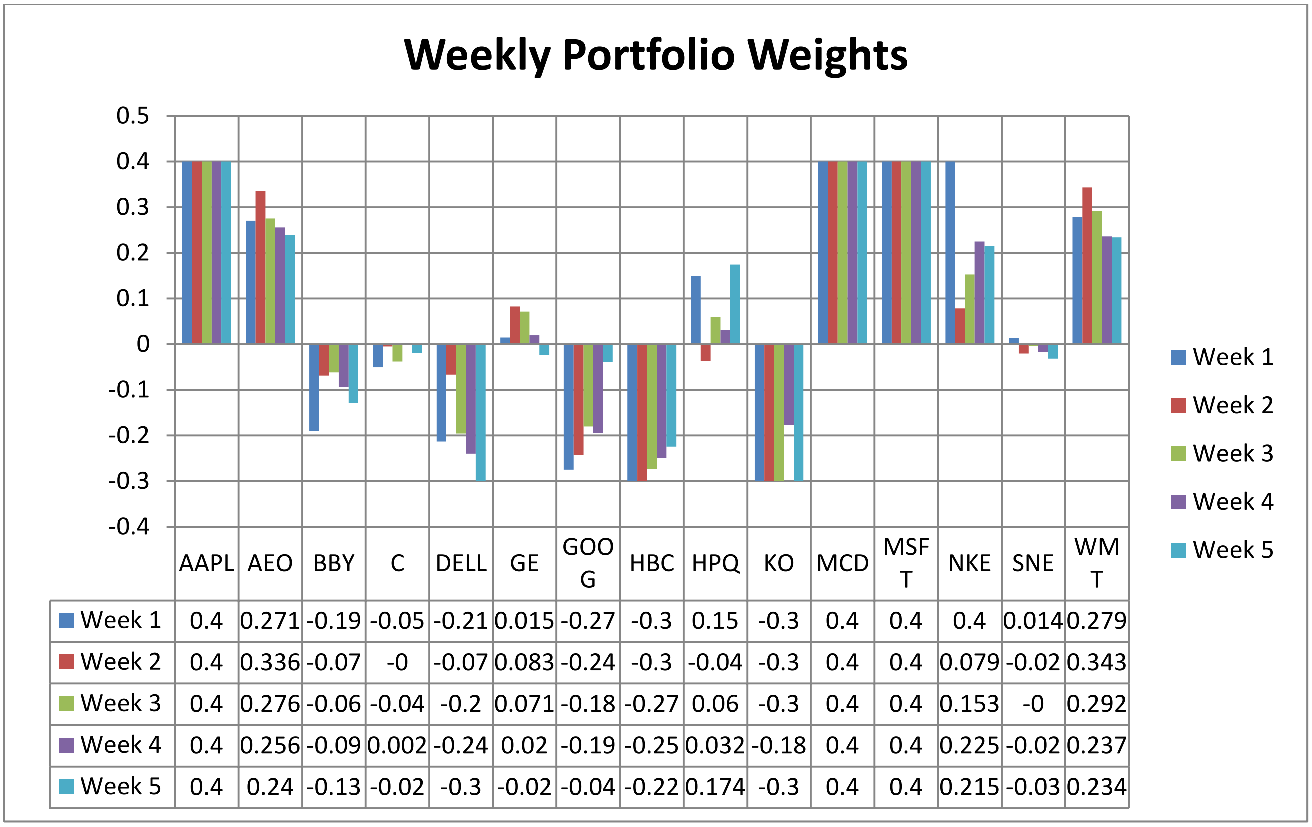


Figure 2.1: Efficient Frontier of Underlying Assets (1st week)

Stocks	Weights	Corresponding Amount($)	Unit Price($)	Quantity	Position
AAPL	0.4	168000	588	285.7143	285
AEO	0.270776	113725.93	16.47	6905.036	6905
BBY	-0.18991	-79761.36	25.13	-3173.95	-3173
C	-0.05024	-21100.25	36.58	-576.825	-576
DELL	-0.21334	-89603.87	17.23	-5200.46	-5200
GE	0.014766	6201.68	20.1	308.5387	308
GOOG	-0.27442	-115256.96	623	-185.003	-185
HBC	-0.3	-126000	46.01	-2738.54	-2738
HPQ	0.149597	62830.92	24.42	2572.929	2572
KO	-0.3	-126000	70.12	-1796.92	-1796
MCD	0.4	168000	98.21	1710.62	1710
MSFT	0.4	168000	32.81	5120.39	5120
NKE	0.4	168000	111.2	1510.791	1510
SNE	0.014138	5937.87	22	269.9031	269
WMT	0.278634	117026.07	61.09	1915.634	1915
Initial Amount=$42,000					

Table 2.2: 1st Portfolio Maintenance Detail

We rebalance the underlying assets according to optimal weights from Efficient Frontier Theory each week. At the beginning of the next week, we buy or sell the underlying assets to make sure they worth the corresponding weights. The following clustered column chart gives the shifted weights among 5 weeks:



	AAPL	AEO	BBY	C	DELL	GE	GOOG	HBC	HPQ	KO	MCD	MSFT	NKE	SNE	WMT
Week 1	0.4	0.271	-0.19	-0.05	-0.21	0.015	-0.27	-0.3	0.15	-0.3	0.4	0.4	0.4	0.014	0.279
Week 2	0.4	0.336	-0.07	-0	-0.07	0.083	-0.24	-0.3	-0.04	-0.3	0.4	0.4	0.079	-0.02	0.343
Week 3	0.4	0.276	-0.06	-0.04	-0.2	0.071	-0.18	-0.27	0.06	-0.3	0.4	0.4	0.153	-0	0.292
Week 4	0.4	0.256	-0.09	0.002	-0.24	0.02	-0.19	-0.25	0.032	-0.18	0.4	0.4	0.225	-0.02	0.237
Week 5	0.4	0.24	-0.13	-0.02	-0.3	-0.02	-0.04	-0.22	0.174	-0.3	0.4	0.4	0.215	-0.03	0.234

Figure 2.2: Weekly Portfolio Weights

2.2.2 Option Strategies

According to our investment capital allocation, $80,000 is the initial capital we invest in options. We decided to use four kinds of option strategies: long straddles, long strangles, a covered call and a protective put. Before determining which options could be used on these strategies, we estimated the weekly expected returns and volatilities of the underlying assets using six months (15th Sep. 2011--15th Mar. 2012) of stock prices. This provided a basis for our choices.

Stocks	Expected return	Volatility
AAPL	**0.006345**	**0.152172**
AEO	**0.006574**	**0.118474**
BBY	7.31E-05	0.056142

C	**0.003754**	**0.117066**
DELL	0.00221	0.077434
GE	**0.003829**	**0.104547**
GOOG	0.002133	0.066071
HBC	0.001822	0.074739
HPQ	0.000763	0.081484
KO	3.09E-05	0.021339
MCD	0.001992	0.05736
MSFT	**0.003437**	**0.098534**
NKE	0.003623	0.080008
SNE	0.000989	0.081925
WMT	0.002736	0.06132

Table 2.3: Expected Return and Volatility of Underlying Assets

From the table we find that AAPL, AEO, C, GE and MSFT have relatively higher volatilities, so we decided to purchase options based on these underlying assets.

- Long Straddles

If you purchase both the at-the-money call and put you are using the long straddle strategy[4], which means the trader will purchase a long call and a long put with the same underlying asset, expiration date and strike price. The strategy will make a profit when the price of the underlying asset moves up or down from its present level.

The diagram of pay-off for long straddle is as following:

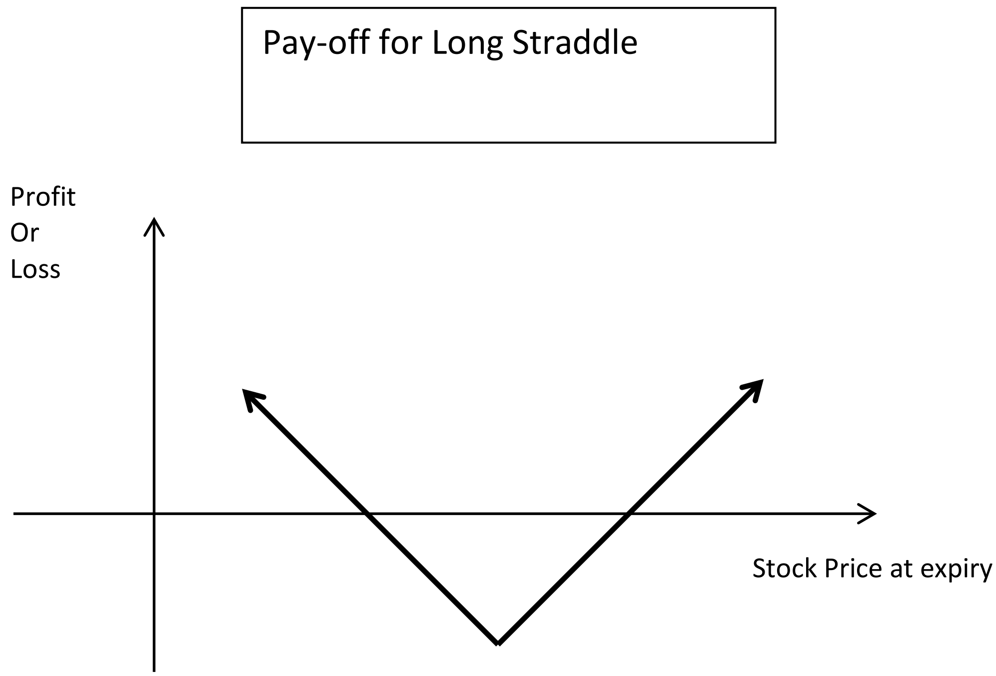


Figure 2.3: Pay-Off for Long Straddle

From Table 2.3, we find AAPL and AEO have higher volatilities than the other underlying assets. Apple recently released the new iPad, which may give a boost to Apple's stock, but its stock might fall if there is a poor performance of the new iPad. Thus we decided to use a long straddle strategy for AAPL. Also cotton prices are much cheaper in 2012. This could give American Eagle a chance to make profit, but recently AEO has found a replacement for its retiring CEO, which means the company is much less likely to be supported by private investors. We believe the stock price of AEO will fluctuate, but we are not sure which direction, so we invested in AEO options with long straddle strategy.

- Long Strangles

A long strangle option strategy has similar characteristics to the long straddle except that the options purchased are of different exercise prices and are out-of-the-money[5]. This strategy involves buying a put option at a strike price and a call option with a higher strike price for the same underlying asset and expiry date.

The pay-off of a long strangle strategy is:

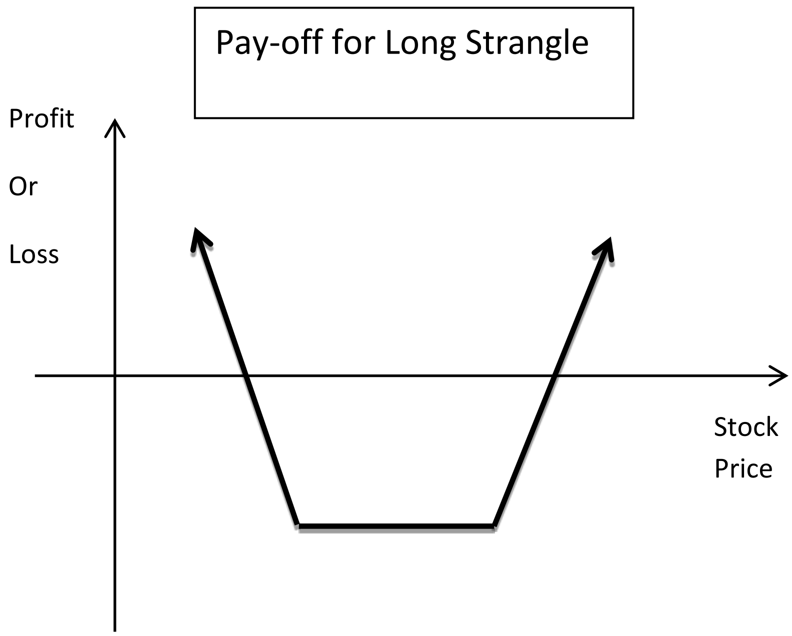


Figure 2.4: Pay-Off for Long Strangle

We chose the call and put strike price close to the present stock price when trading options. With this strategy, the profit potential is unlimited in either direction.

This is also a bet on the volatility of stock price just like long straddles, Table 3 describes that C (CitiGroup) and MSFT (Microsoft) are volatile stocks. For Citigroup, "The treasury recently announced that it completed the sale of the rest of the mortgage-backed securities that it bought as part of the Fannie and Freddie bailouts."[6] Citigroup (C) benefitted from this move, its stock price boosted drastically, and we chose to buy a long strangle strategy in the hope that Citigroup's stock price would continue appreciating. The reason why we chose Microsoft was that it recently released its Windows Phone to the Chinese smartphone market officially, with the intension of beating AAPL's iPhone in China's market. It was good news for the Chinese Andriod phone users, and we are interested in Windows Phone's performance in the future.

- Covered Call

If we sell a call option with the same amount of underlying shares we hold, the call option is "covered". Since we pay premiums for the right to acquire the shares at a slightly higher price than the current price, we need to choose this strategy when underlying assets are flat or rising slowly - as this allows us to receive income from option premiums and keep hold of modest gains [7].

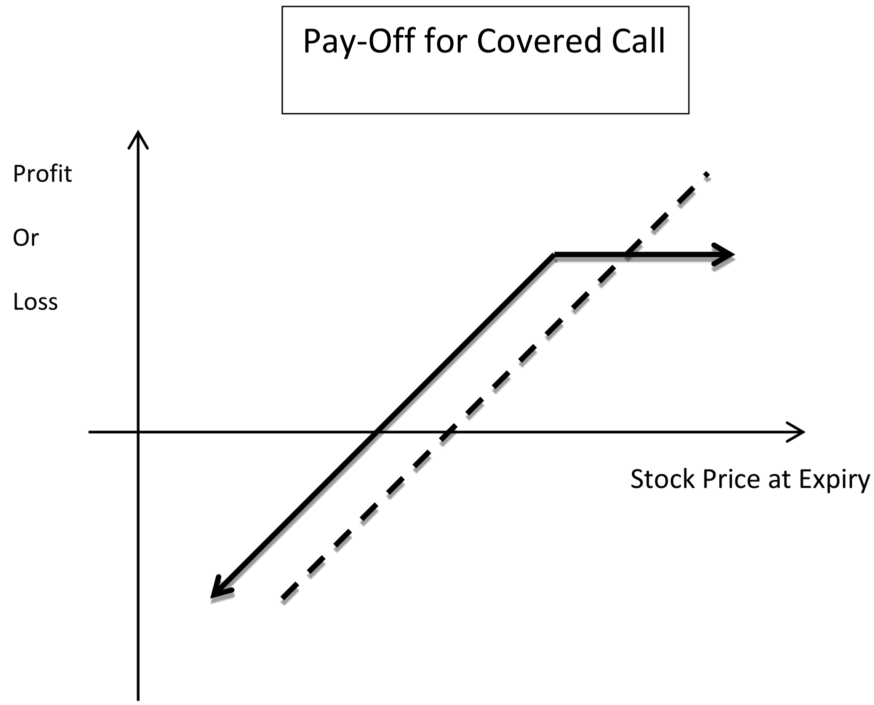


Figure 2.5: Pay-Off for Covered Call

From the market observations in March, 2012, most Asian markets dropped because China's economic growth was slowing down. We reckoned SNE would remain flat since Japan's market was influence by China. The Nikkei Index fell 0.6% on March 21. Moreover, from our estimated returns of 15 stock prices, Sony (SNE) has relatively lower log returns, thus we decided to use a covered call with this stock. We hold a long position in SNE stock and also write a call option at the same time.

- Protective Put

A well-known strategy to protect loss is one of a "protective put [8]," or buying a put option to protect a long stock position. A protective put would limit possible losses regardless of how far stock prices dropped while allowing further profits to accrue as long as the market kept going up.

The payoff graph of protective put is:

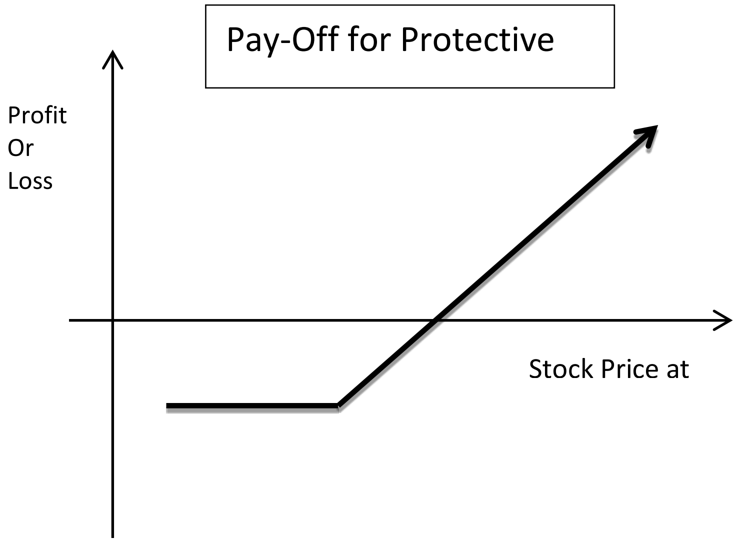


Figure 2.6 Pay-off for Protective Put

We invested in a protective put strategy to avoid large losses if GE's stock went down, owning a long position in GE's stock and purchasing the same number of shares of put options as well. Details of our option strategies and positions are listed in the table below.

Strategy	Options	Style	Expiry	Strike Price	Unit Price	Position
Long Straddle	AAPL	CALL	18th May	590	35.75	4
	AAPL	PUT	18th May	590	38.85	4
	AEO	CALL	18th May	16	1.1	100
	AEO	PUT	18th May	16	0.8	100
Long Strangle	C	CALL	18th May	34	3.5	33
	C	PUT	18th May	35	1.32	33
	MSFT	CALL	18th May	31	2.24	50
	MSFT	PUT	18th May	32	0.82	50
Protective Put	GE	PUT	18th May	21	1.19	3
Covered Call	SNE	CALL	20th July	22	1.55	-3
	Initial Amount=$8,000					

Table 2.4: Option Strategies and Positions

Chapter 3 Estimation of Loss Distribution

A central issue in modern risk management is the measurement of risk. We manage our risky assets by working with the loss distribution. Particular attention will be given to Value-at-Risk (VaR) and the related notion of expected shortfall. Both of these concepts are widely used risk measurements for the loss distribution. In this chapter we estimated the loss distribution of our underlying assets and options separately at first, then combined them together to get the entire portfolio value-at-Risk and expected shortfall. The estimation of the loss distribution is a basic input for value-at-risk and expected shortfall calculation.

Definition[9]: (Loss Distribution)

For a given time horizon, such as 1 or 10 days, the loss of the portfolio over the period $[s, s+\Delta]$ is given by

$$L_{[s,s+\Delta]} := -(V(s+\Delta) - V(s))$$

While $L_{[s,s+\Delta]}$ is assumed to be observable at time $s+\Delta$, it is typically random from the viewpoint of time s. The distribution of $L_{[s,s+\Delta]}$ is termed the loss distribution, denoted as F_L.

3.1 Methods for Linearized Loss Distribution

3.1.1 Linearized Loss Distribution for Underlying Assets

For each stock we have a separate log return risk factor, and we use the following formulas to deal with the linearized stock loss distribution,

$$L_{t+1}^{\Delta} = -V_t \sum_{i=1}^{N} W_{t,i} X_{t+1,i} \,, \tag{3.3}$$

where $X_{t+1,i} = \ln S_{t+1,i} - \ln S_{t,i}$, and $W_{t,i}$ is the weight of the i^{th} stock in our portfolio.

The mean of linearized loss distribution can be calculated through a linearized loss operator

$$\ell_{[t]}^{\Delta}(\underline{X}) = -V_t \sum_{i=1}^{N} W_{t,i} X_{t,i} = -V_t \underline{W_t^T}\, \underline{X} \tag{3.4}$$

Suppose $\underline{X}$ follows a distribution with mean $\underline{\mu}$ and covariance matrix Σ. Then

$$E\left[\ell_{[t]}^{\Delta}(\underline{X})\right] = E\left[-V_t \underline{W_t^T X}\right] = -V_t \underline{W_t^T \mu} \text{ , and} \tag{3.5}$$

$$Var\left[\ell_{[t]}^{\Delta}(\underline{X})\right] = Var\left[-V_t \underline{W_t^T X}\right] = V_t^2 \, \underline{W_t^T} \, \Sigma \, \underline{W_t} \, . \tag{3.6}$$

In our case, $\underline{X}$ is the log-returns of our 15 stocks, and it follows $N(\mu, \Sigma)$[3] distribution.

3.1.2 Linearized Loss Estimation of Options

Since we have six options positions, we simulate the linearized loss for the options separately from what we had done with stocks.

Suppose a European call option on a non-dividend paying stock with maturity T and strike price K has time-t value $V_t = C^{BS}(s, S; r, \sigma, K, T)$. Similarly $P^{BS}(s, S; r, \sigma, K, T)$ is the value of a European put option on day t.

The risk factors are

$$\underline{Z_t} = [lnS_t,\ \ r_t,\ \ \sigma_t] \tag{3.7}$$

where S_t is the stock price at time t, r_t is the interest rate, σ_t is volatility.

The change in the risk factors is thus

$$\underline{X_{t+1}} = [lnS_{t+1} - lnS_t, r_{t+1} - r_t, \sigma_{t+1} - \sigma_t] \; . \tag{3.8}$$

The loss for call option is given by

$$L_{t+1} = -[V_{t+1} - V_t] = [C^{BS}([t+1]\Delta, Z_t + X_{t+1}) - C^{BS}(t\Delta, Z_t)] \tag{3.9}$$

The linearized loss is

$$L_{t+1}^{\Delta} = -(C_t^{BS}\Delta + C_S^{BS} S_t X_{t+1.1} + C_r^{BS} X_{t+1.2} + C_\sigma^{BS} X_{t+1.3}) \tag{3.10}$$

where C_t^{BS} is the partial derivative with respect to the calendar time t, C_S^{BS} is the partial derivative with respect to the stock price S, C_r^{BS} is the partial derivative with respect to the interest rate r, and C_σ^{BS} is the partial derivative with respect to volatility σ.

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[3] N(μ,Σ) means a multivariate normal distribution with meanμ, and covariance matrixΣ

In case of the linearized loss distribution for the put option,

$$L_{t+1}^{\Delta} = -(P_t^{BS}\Delta + P_S^{BS}S_t X_{t+1.1} + P_r^{BS}X_{t+1.2} + P_\sigma^{BS}X_{t+1.3}) \quad (3.11)$$

where P_t^{BS} is the partial derivative with respect to the calendar time t, P_S^{BS} is the partial derivative with respect to the stock price S, P_r^{BS} is the partial derivative with respect to the interest rate r, and P_σ^{BS} is the partial derivative with respect to volatility σ.

3.1.3 Portfolio Mean and Variance Estimation

After separately modeling stocks and options, we combined stocks and options together to construct our entire portfolio. Suppose we invest in 15 stocks, n_1 positions of call option on the ith stock, n_2 positions of put option on the j^{th} stock.

Denote the risk factor as $$\underline{X_{t+1}} = \begin{pmatrix} lnS_{t+1} - lnS_t \\ r_{t+1} - r_t \\ \sigma_{t+1} - \sigma_t \end{pmatrix} \quad (3.12)$$

In our model, the risk factor for 15 stocks is

$$\underline{X_{t+1}} = \begin{pmatrix} lnS_{t+1} - lnS_t \\ r_{t+1} - r_t \\ \sigma_{t+1} - \sigma_t \end{pmatrix} = \begin{pmatrix} X_{t+1,1} \\ X_{t+1,2} \\ \vdots \\ X_{t+1,15} \\ Y_{t+1} \\ Z_{t+1} \end{pmatrix} \quad (3.13)$$

where $X_{t+1,i} = lnS_{t+1,i} - lnS_{t,i}$ is the log return of the i^{th} stock, and

$$Y_{t+1} = r_{t+1} - r_t \ , Z_{t+1} = \sigma_{t+1} - \sigma_t.$$

The linearized loss of the portfolio including both stocks and options is

$$L_{t+1}^{\Delta} = -V_t \sum_{s=1}^{15} W_{t,s} X_{t+1,s} + n_1\left(-(C_t^{BS}\Delta + C_S^{BS}S_t X_{t+1.i} + C_r^{BS}Y_{t+1} + C_\sigma^{BS}Z_{t+1})\right) + n_2(-(P_t^{BS}\Delta + P_S^{BS}S_t X_{t+1.j} + P_r^{BS}Y_{t+1} + P_\sigma^{BS}Z_{t+1})) . \quad (3.14)$$

Suppose the risk factors are all normally distributed with mean

$$\underline{\mu}=E\underline{(X_{t+1})} = \begin{pmatrix} \mu 1 \\ \mu 2 \\ \vdots \\ \mu 15 \\ \mu 16 \\ \mu 17 \end{pmatrix} \tag{3.15}$$

and covariance matrix Σ.

The expected linearized loss of the whole portfolio is

$$E(L_{t+1}^{\Delta}) = E\{-V_t \sum_{s=1}^{15} W_{t,s}X_{t+1,s} + n_1\left(-(C_t^{BS}\Delta + C_S^{BS}S_tX_{t+1.i} + C_r^{BS}Y_{t+1} + C_\sigma^{BS}Z_{t+1})\right) + n_2(-(P_t^{BS}\Delta + P_S^{BS}S_tX_{t+1.j} + P_r^{BS}Y_{t+1} + P_\sigma^{BS}Z_{t+1}))\}$$

$$= -V_t \sum_{s=1}^{15} W_{t,s}\,\mu_s + n_1\left(-(C_t^{BS}\Delta + C_S^{BS}S_t\mu_i + C_r^{BS}\mu_{16} + C_\sigma^{BS}\mu_{17})\right) + n_2(-(P_t^{BS}\Delta + P_S^{BS}S_tX_{t+1.j} + P_r^{BS}Y_{t+1} + P_\sigma^{BS}Z_{t+1})) \tag{3.16}$$

Thus we have the expected loss of the combined portfolio of stocks and options.

For variance of our portfolio, we can combine these items with the same risk factors,

$$L_{t+1}^{\Delta} = -V_t \sum_{s=1}^{n} W_{t,s}X_{t+1,s} + n_1\left(-(C_t^{BS}\Delta + C_S^{BS}S_tX_{t+1.i} + C_r^{BS}Y_{t+1} + C_\sigma^{BS}Z_{t+1})\right)$$

$$+ n_2(-(P_t^{BS}\Delta + P_S^{BS}S_tX_{t+1.j} + P_r^{BS}Y_{t+1} + P_\sigma^{BS}Z_{t+1}))$$

$$= -V_t \sum_{\substack{s=1 \\ s\neq i,j}}^{15} W_{t,s}X_{t+1,s} + \left[-V_tW_{t,i}X_{t+1,i} - n_1(C_S^{BS}S_t)\right]X_{t+1.i}$$

$$+\left[-V_tW_{t,i}X_{t+1,j} - n_2(P_S^{BS}S_t)\right]X_{t+1.j} + (-n_1C_r^{BS} - n_2P_r^{BS})Y_{t+1}$$

$$+(-n_1C_\sigma^{BS} - n_2P_\sigma^{BS})\, Z_{t+1}$$

$$= \underline{W_{new}} * [X_{t+1,1},\ X_{t+1,2}, \cdots, X_{t+1,15}, Y_{t+1}, Z_{t+1}]^T \tag{3.17}$$

where $\underline{W_{new}}$ is the new coefficients vector (1×17) of these risk factors.

Thus we can calculate the variance of the whole portfolio,

$$Var(L_{t+1}^{\Delta}) = \underline{W_{new}} * \Sigma * \underline{W_{new}}^T \tag{3.18}$$

where Σ is the covariance matrix of the risk factors.

3.2 Basic Concepts in Risk Management

Definition [10]**: (Value-at-Risk).** The Value-at-Risk (which is often abbreviated VaR) of our portfolio at the confidence level $\alpha \in (0,1)$ is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-\alpha)$. Formally,

$$VaR_\alpha = \inf\{l \in \mathbb{R}: P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R}: F_L(l) \geq \alpha\} \tag{3.19}$$

There are two parameters when we use VaR, the horizon T and the confidence level $1-\alpha$. One example that can explain VaR is, if the time horizon is one week, the confidence parameter is 5%, and VaR is $5000, then there is a 5% chance of the loss exceeding $5000 in the next week [11].

Definition [12]: **(Expected Shortfall).** For a loss L with $E(|L|) < \infty$ and df F_L , the expected shortfall at confidence level $\alpha \in (0,1)$ is defined as

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_u(F_L)du \ ,$$

where $q_u(F_L) = F^{\leftarrow}(u)$ is the quantile function of F_L.

Thus expected shortfall is related to VaR by

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 VaR_u(L)du. \tag{3.20}$$

Expected shortfall can be interpreted as the expected loss that is incurred in the event that VaR is exceeded.

If we have $\boldsymbol{F_L} \sim N(\mu, \sigma^2)$, then

Value-at-Risk is: $\mathrm{VaR}_\alpha = \mu + \sigma N^{-1}(\alpha)$ (3.21)

Expected short fall is : $\mathrm{ES}_\alpha = \mu + \frac{\sigma\varphi(N^{-1}(\alpha)}{1-\alpha}$ (3.22) where φ is the probability density function (PDF) and N is the cumulative density function (CDF) of the standard normal distribution.

For student-t distributions, if $\tilde{L} = \frac{L-\mu}{\sigma}$ has a standard t distribution with ν degrees of freedom, $\nu > 1$

Thus we have

$$ES_\alpha(\tilde{L}) = \frac{g_\nu[t_\nu^{-1}(\alpha)]}{1-\alpha}\left(\frac{\nu+[t_\nu^{-1}(\alpha)]^2}{\nu-1}\right) \quad (3.23)$$

Where t_ν is the C.D.F and g_ν is the P.D.F of the standard t distribution.

We obtain $ES_\alpha = \mu + \sigma ES_\alpha(\tilde{L})$,where $\tilde{L}$ has standard t-distribution. [13]

We compute the VaR and expected shortfall of our portfolio by the method of covariance estimation.

3.3 Fitting the Loss Distribution

3.3.1 Normal Distribution Estimation

Suppose the risk factors $\underline{X_{t+1}} = \begin{pmatrix} lnS_{t+1} - lnS_t \\ r_{t+1} - r_t \\ \sigma_{t+1} - \sigma_t \end{pmatrix}$ follows a multivariate normal distribution with mean $\underline{\mu}$ and covariance matrix Σ, so the whole portfolio loss is univariate normally distributed.

The mean and covariance matrix of risk factor can be estimated from the historical data, then we obtain the estimated mean and variance of the linearized loss.

1) 1st Week (16 - 23 March 2012)

Expected Loss $E(L^{\Delta}_{t+1})$	Expected Variance $Var(L^{\Delta}_{t+1})$
1931.563	1.25E+08

Table 3.1: 1st Week Estimated Loss and Variance

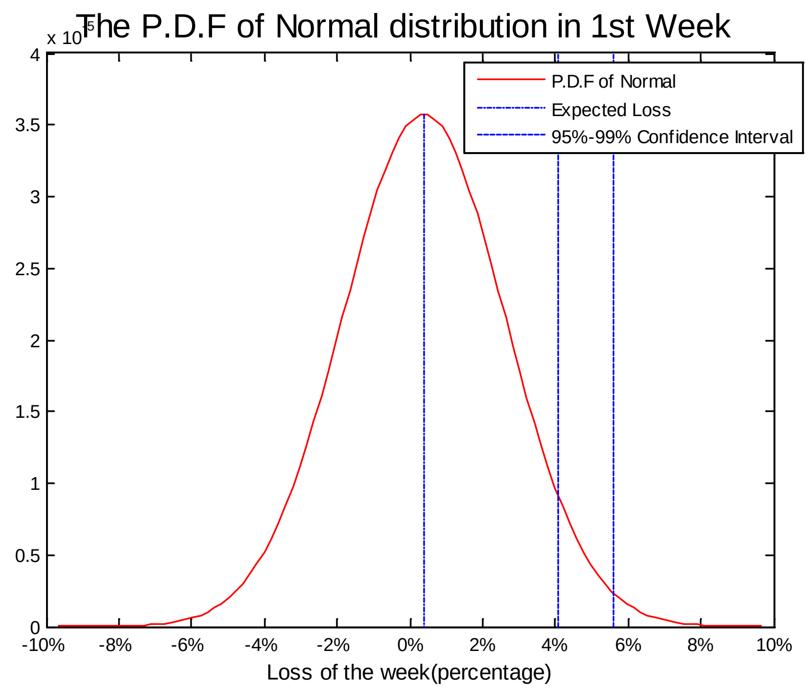

Figure 3.1: 1st Week P.D.F of Normal Distribution

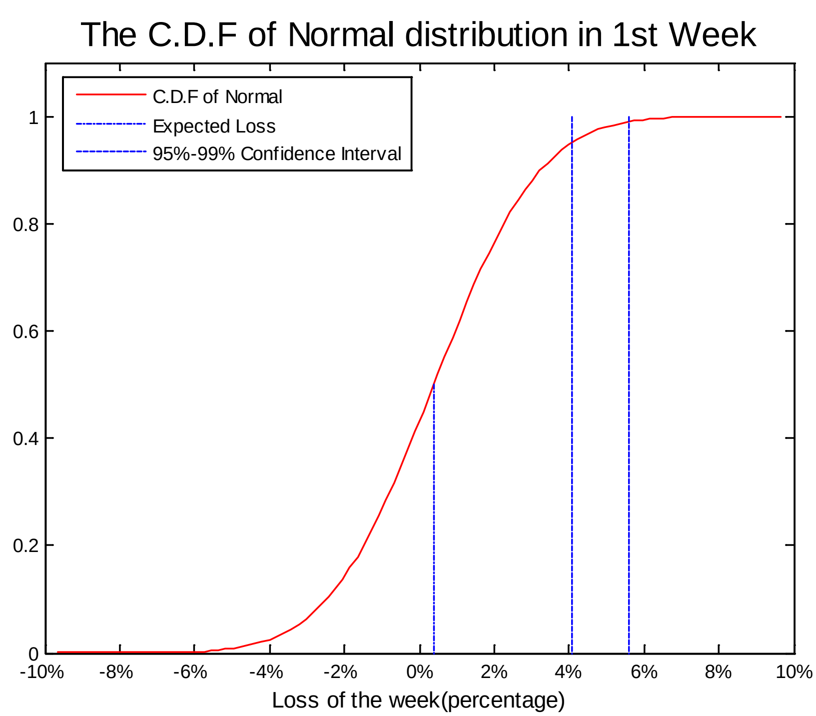

Figure 3.2: 1st Week C.D.F. of Normal Distribution

Value-at-risk and Expected shortfall of Normal distribution

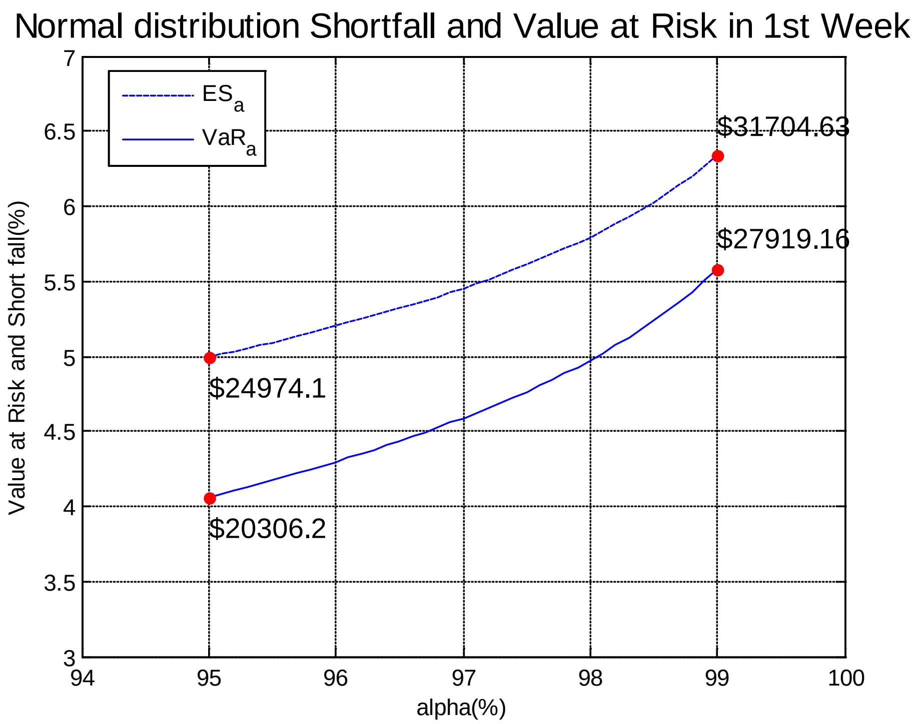

Figure 3.3 : 1st Week VaR and ES of Normal Distribution

3.3.2 Student t-distribution Estimation

We applied a similar method to estimate the linearized loss distribution using a student-t distribution. In this part the risk factors $\underline{X_{t+1}} = \begin{pmatrix} \ln S_{t+1} - \ln S_t \\ r_{t+1} - r_t \\ \sigma_{t+1} - \sigma_t \end{pmatrix}$ follow multivariate student-t distribution with mean $\underline{\mu}$ and covariance matrix Σ, rather than a normal distribution. In this case the loss follows a univariate student-t distribution. We plot the graphs of PDF and CDF here.

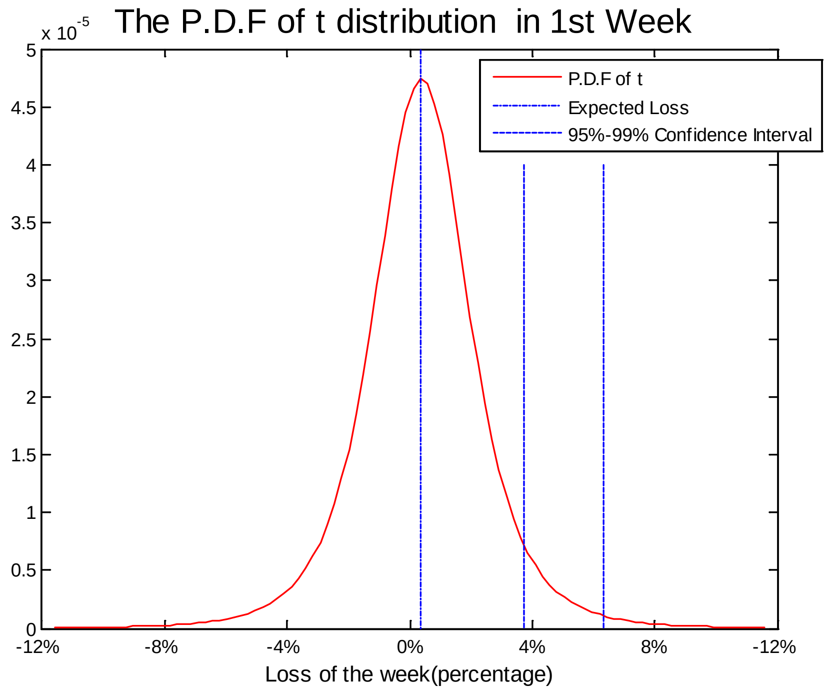

Figure 3.4: 1st Week P.D.F. of t Distribution

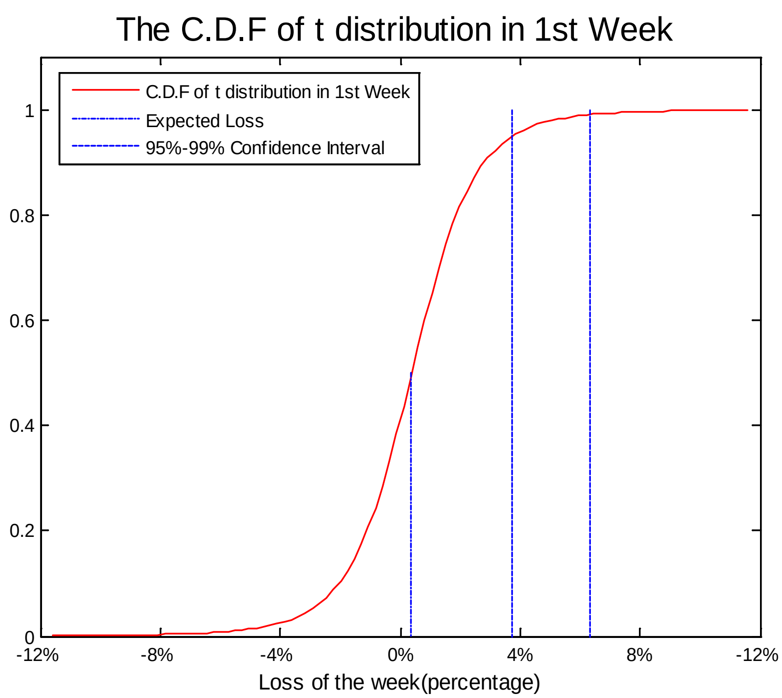

Figure 3.5: 1st Week C.D.F. of t Distribution

Value-at-risk and Expected shortfall for t distribution

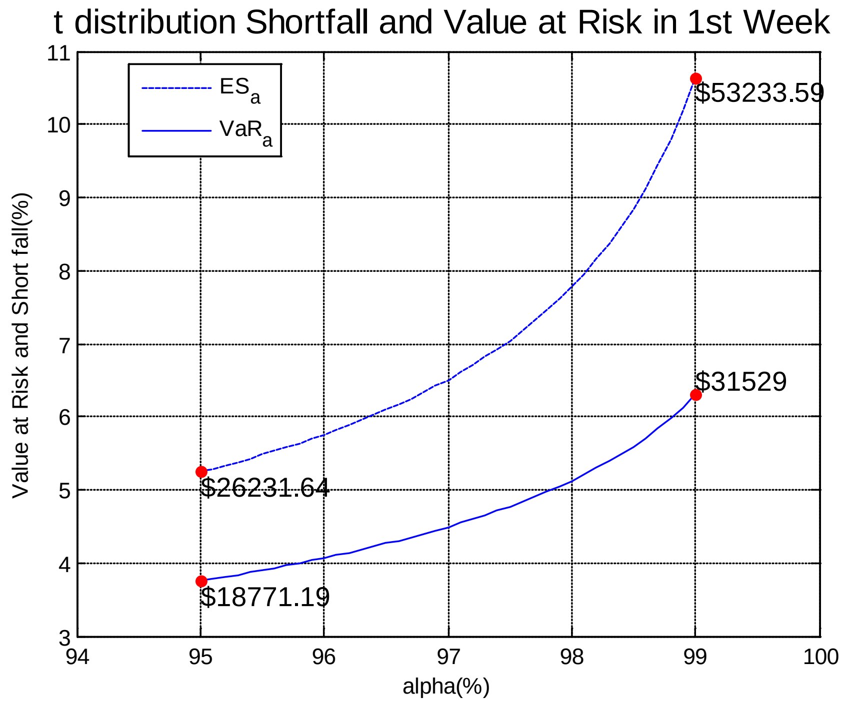

Figure 3.6: 1st Week VaR and ES of t Distribution

3.3.3 Comparison and Analysis between Two Distributions

We plot the PDFs of the two distributions in one graph so that we can make a comparison and analysis. The dashed line is the PDF of the linearized loss under the t distribution and the line is PDF of the linearized loss under the normal distribution. Comparing the PDF of the t with that of the normal, we have found it has a heavier tail The t distribution is more prone to having values far away from its mean value. Therefore a loss happening with a slim probability from the normal distribution could be a common event from the student-t distribution. The blue spot describes the first week's actual loss ($12,576.23, 2.7% in percentage), which is greater than the expected loss.

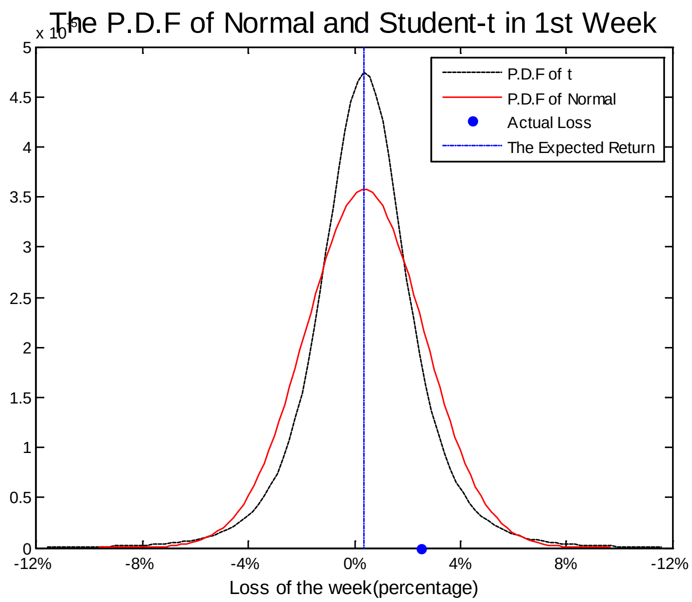

Figure 3.7: 1st Week P.D.F. Comparison between Normal and t Distribution

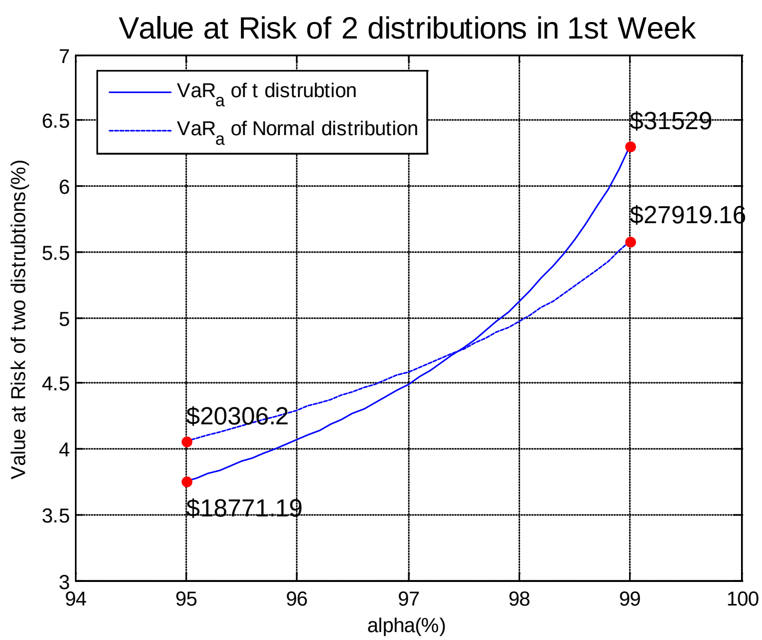

Figure 3.8: 1st Week VaR Comparison between Normal and t Distribution

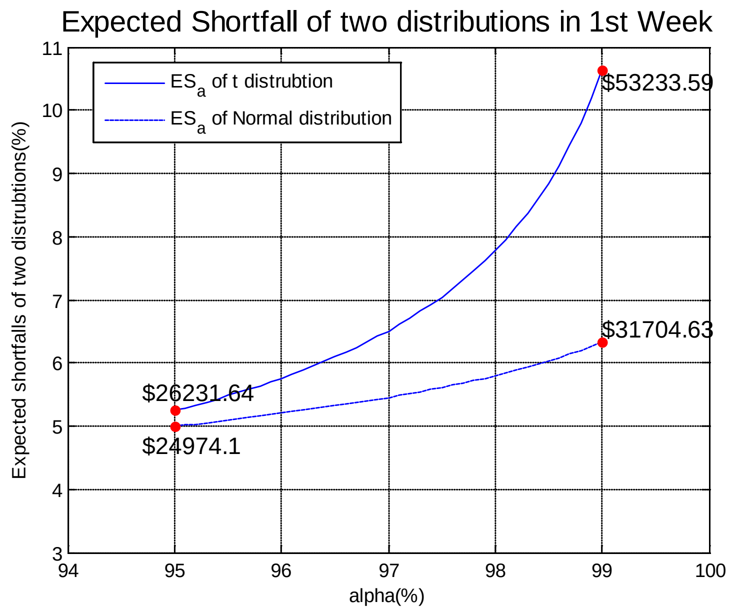

Figure 3.9: 1st week ES Comparison between Normal and t Distribution

Now we would like to compare and analyze the Value-at-Risk of the two distributions.

1) 1st week (Mar.16 –Mar.23, 2012)

Distribution	α(%)	VaR ($)	VaR (%)	ES ($)	ES (%)
Normal	95	20306.2	4.06	24974.1	4.99
T	95	18771.19	3.75	26231.64	5.25
Normal	99	27919.16	5.58	31704.63	6.34
T	99	31529	6.31	53233.59	1.06

Table 3.2: 1st Week VaR and ES Comparison between Normal and t Distribution

We check the confidence interval between α = 0.95 and 0.99. Value-at-Risk using the normal distribution in higher in value than that of the t distribution when the confidence level is low; however, when the confidence level increases above 0.975, VaR using the normal distribution has relatively lower value than that of t distribution.

Next we take expected shortfall as a measurement of risk. Expected shortfall is an overview risk measure of the loss distribution tail, and it is the average level of the tail of the distribution. From the graph below, we see that the t distribution reflects a higher expected shortfall than the normal distribution. The first week's actual loss was $12,576.23, 2.7%, lower than each of our risk measures.

We ran this model for the next four weeks. During each week we updated the stock prices and calculated optimal weights using Markowitz's Efficient Frontier. The results showed a similar trend, with only slight changes in stock prices from week to week.

2) 2nd Week (Mar.23 –Mar.30, 2012)

We did the same steps in the second week. Figure shows the actual loss ($-3,603.79) of the second week is below the expected loss ($2,177.28).

Expected Loss $E(L^{\Delta}_{t+1})$	Expected Variance $Var(L^{\Delta}_{t+1})$
2177.28	3.88E+08

Table 3.3: 2nd Week Estimated Loss and Variance

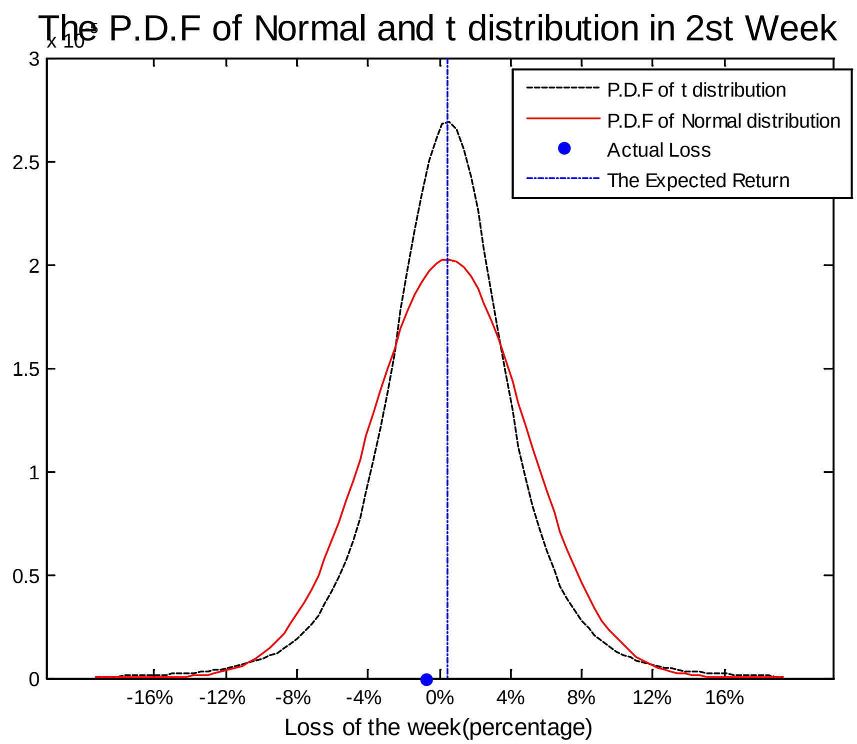

Figure 3.10: 2nd week P.D.F. comparison

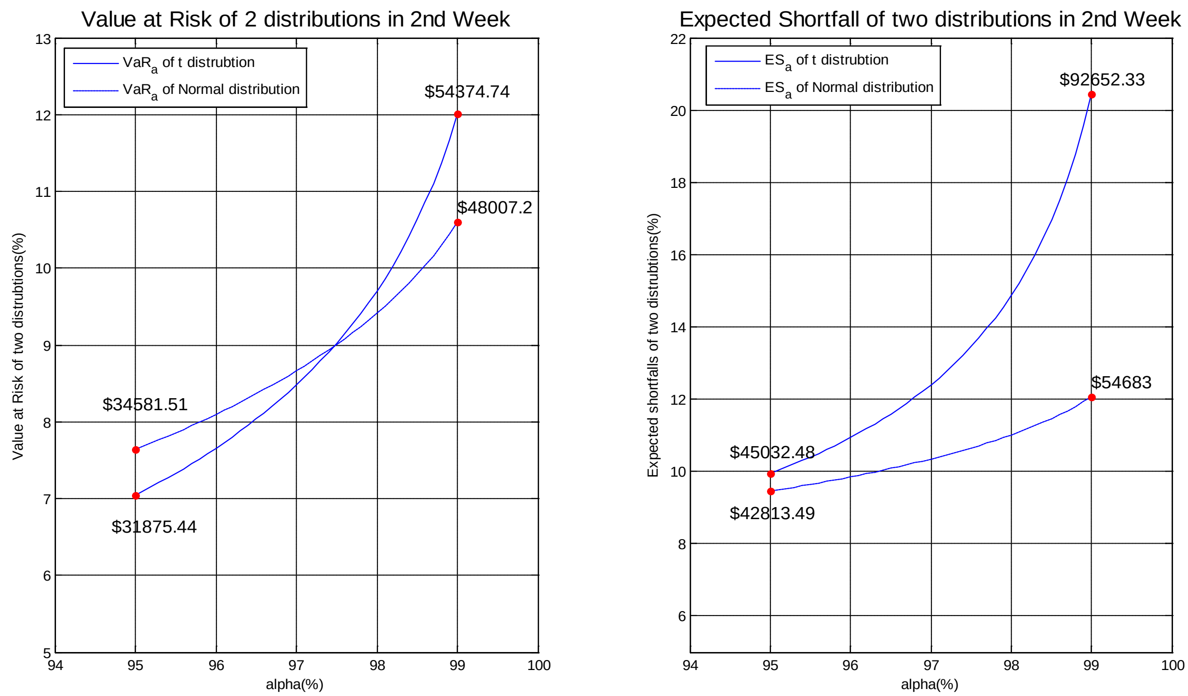

Figure 3.11: 2nd Week VaR and ES Comparison between Normal and t Distribution

2nd week comparison between normal distribution and t-distribution is as following:

Distribution	α(%)	VaR ($)	VaR (%)	ES ($)	ES (%)
Normal	95	34581.51	7.64	42813.49	9.45
T	95	31875.44	7.04	45032.49	9.94
Normal	99	48007.2	10.6	54693	12.07
T	99	54374.74	12.01	93652	20.46

Table 3.4: 2nd Week VaR and ES Comparison between Normal and t Distribution

3) 3rd week

Expected Loss $E(L^{\Delta}_{t+1})$	Expected Variance $Var(L^{\Delta}_{t+1})$
2344.49	1.1E+08

Table 3.5: 3rd week estimated loss and variance

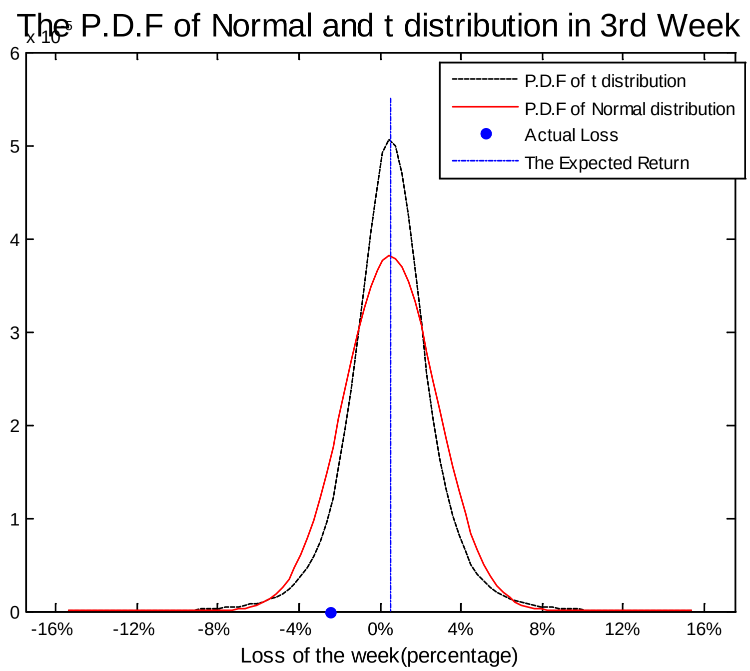

Figure 3.12: 3rd week P.D.F. Comparison between Normal and t Distribution

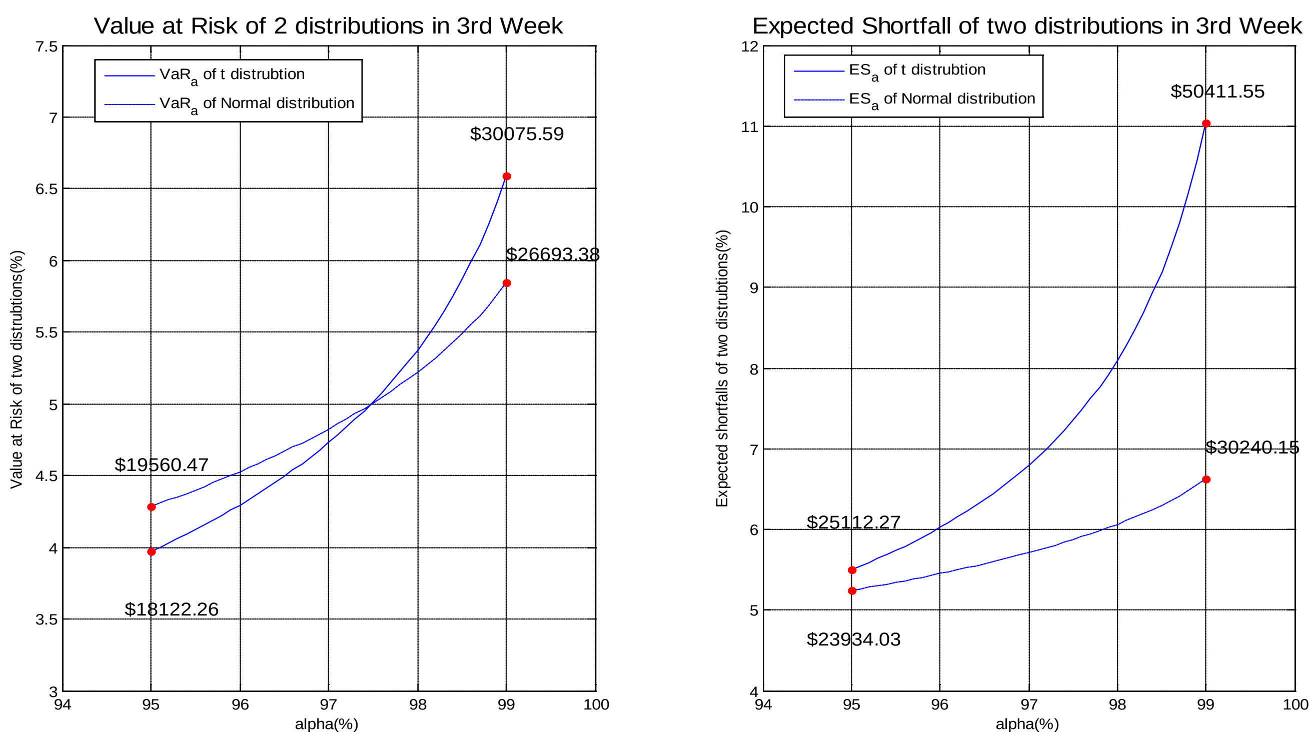

Figure 3.13: 3rd Week VaR and ES Comparison between Normal and t Distribution

Distribution	α(%)	VaR ($)	VaR (%)	ES ($)	ES (%)
Normal	95	19560.47	4.28	23934.03	5.24
T	95	18122.26	3.97	25112.27	5.55
Normal	99	26693.38	5.85	30240.15	6.62
T	99	30075.59	6.59	50411.55	11.04

Table 3.6: 3rd Week Comparison of VaR and ES between Normal Distribution and t Distribution

4) 4th Week

Expected Loss $E(L^{\Delta}_{t+1})$	Expected Variance $Var(L^{\Delta}_{t+1})$
2361.07479540231	8.4E+07

Table 3.7: 4th Week Estimated Loss and Variance

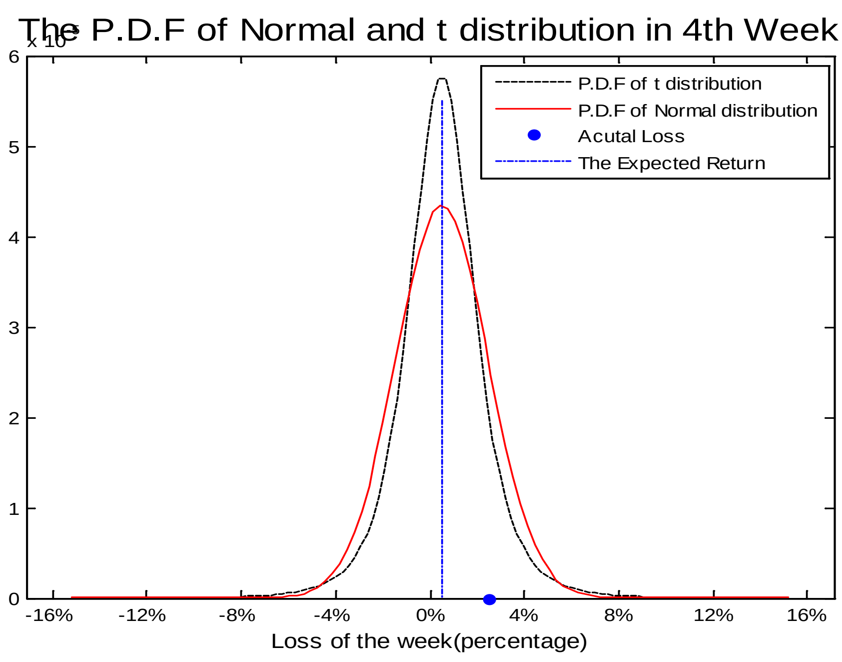

Figure 3.14: 4th week P.D.F. Comparison between Normal and t Distribution

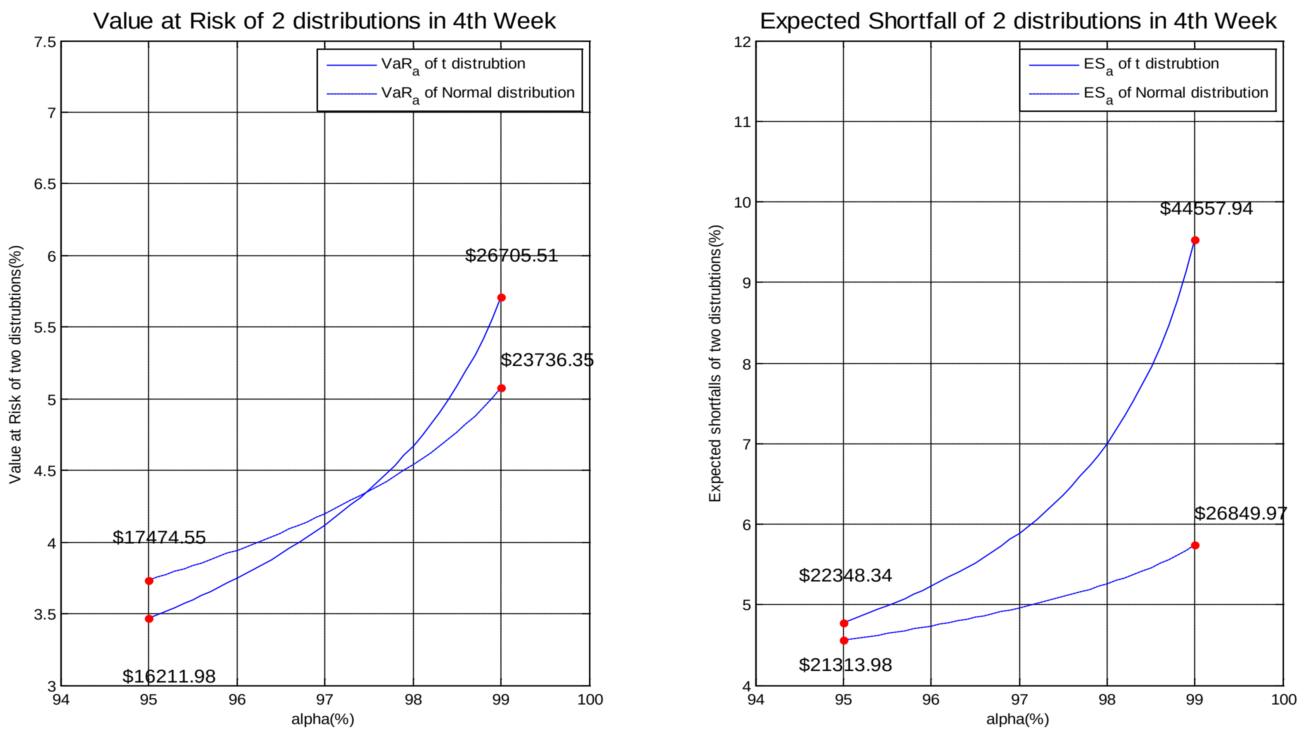

Figure 3.15: 4th Week VaR and ES Comparison between Normal and t Distribution

Distribution	α(%)	VaR ($)	VaR (%)	ES ($)	ES (%)
Normal	95	17474.55	3.74	21313.98	4.56
T	95	16211.98	3.47	22348.34	4.78
Normal	99	23736.35	5.08	26849.97	5.74
T	99	26705.51	5.71	44557.94	9.52

Table 3.8: 4th Week Comparison of VaR and ES between Normal Distribution and t Distribution

5) 5th Week

Expected Loss $E(L^{\Delta}_{t+1})$	Expected Variance $Var(L^{\Delta}_{t+1})$
3856.86	7.58E+7

Table 3.9: 5th Week Estimated Loss and Variance

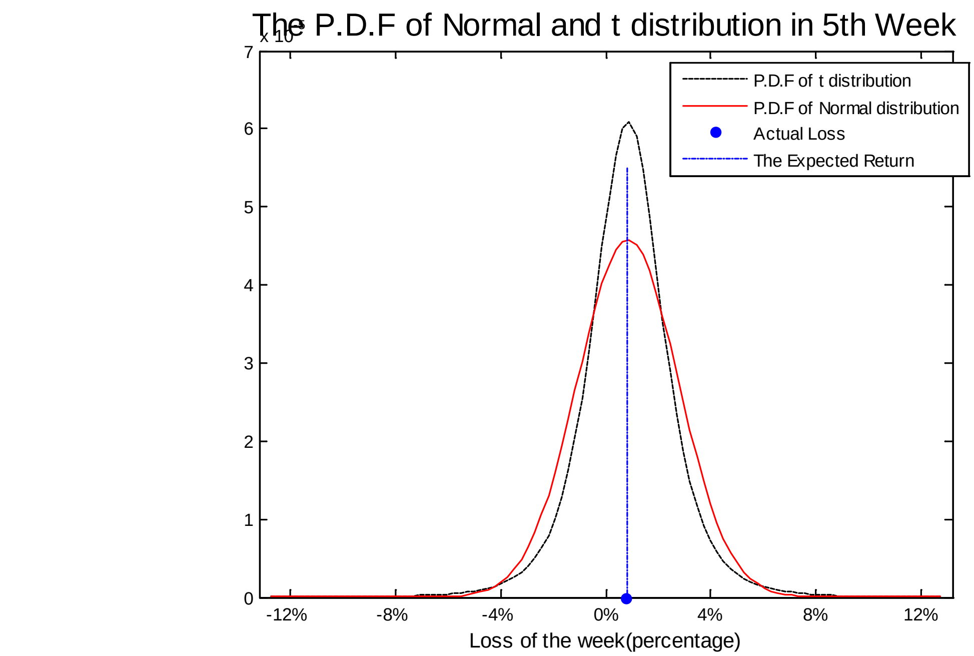

Figure 3.16: 5th Week P.D.F. Comparison between Normal and t Distribution

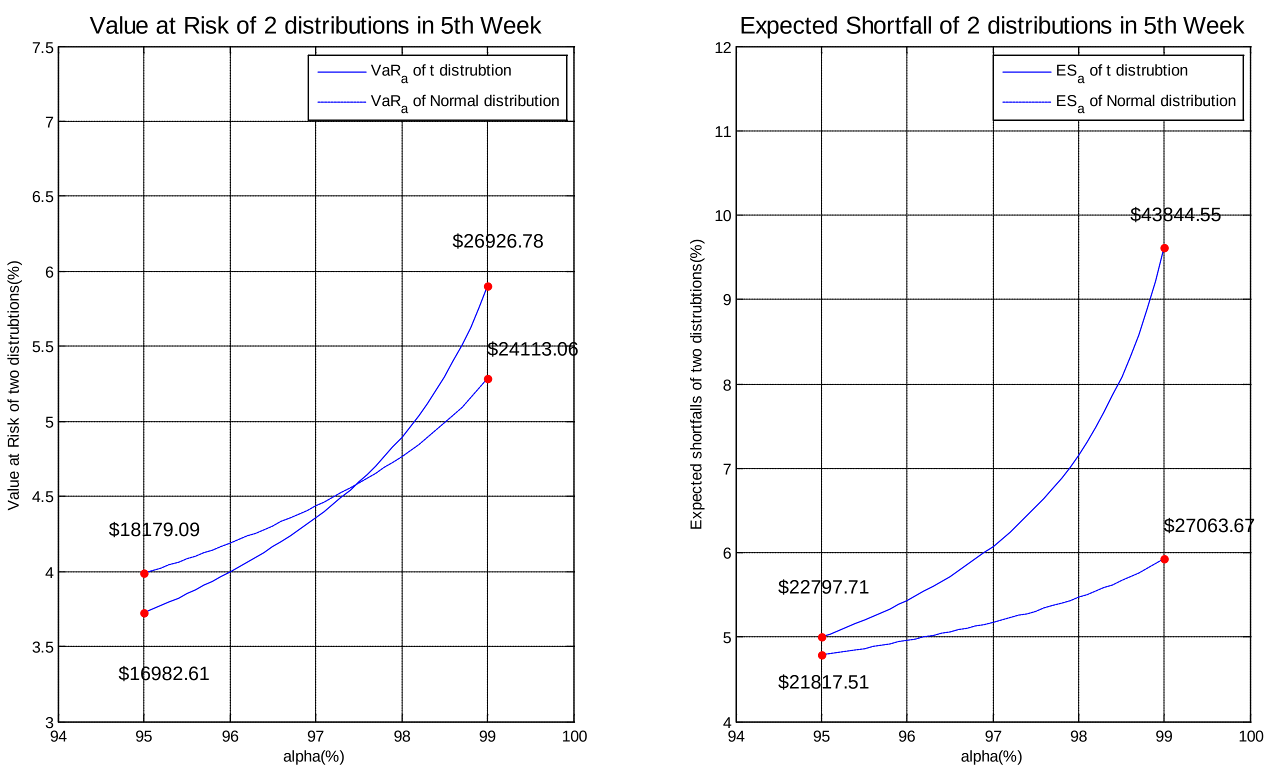

Figure 3.17: 5th Week VaR and ES Comparison between Normal and t Distribution

Distribution	α(%)	VaR ($)	VaR (%)	ES ($)	ES (%)
Normal	95	18179.09	3.99	21817.51	4.78
T	95	16982.61	3.72	22797.71	4.99
Normal	99	24113.06	5.29	27063.67	5.93
T	99	26926.77	5.90	43844.55	9.61

Table 3.10: 5^{th} Week Comparison of VaR and ES between Normal and t Distribution

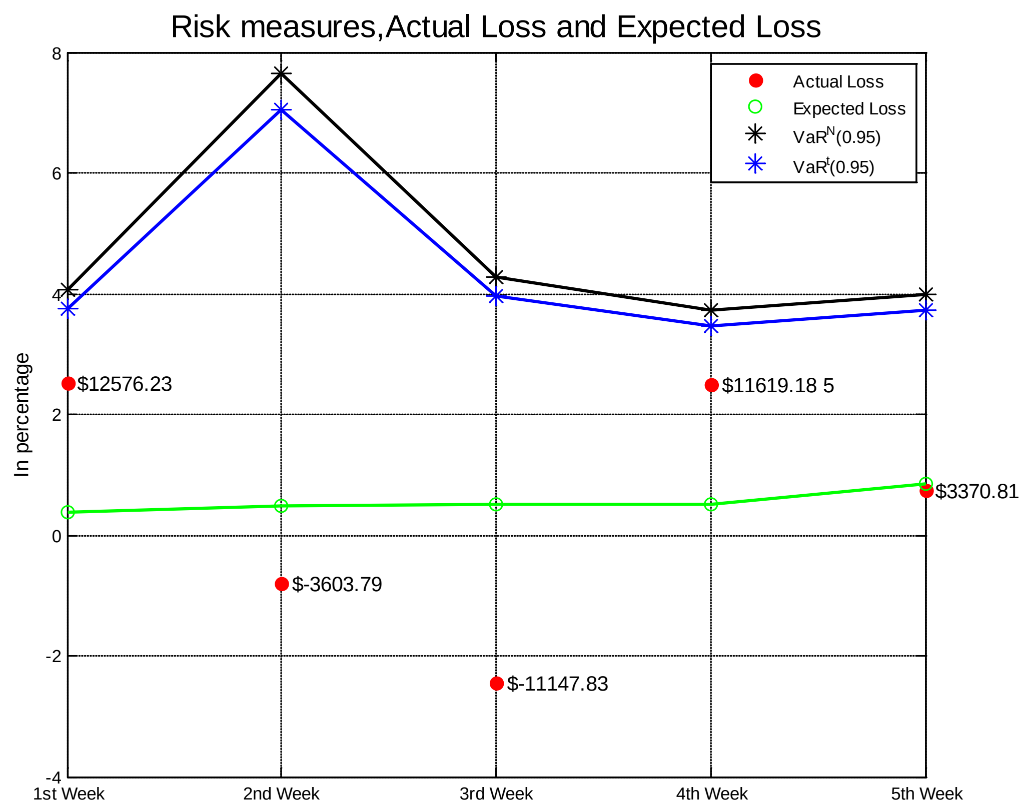

Figure 3.18: Comprehensive Comparison through Five weeks

Figure 2.18 depicts the expected loss, actual loss and VaR(0.95) for both normal and t distributions. The actual loss of five weeks were all below VaR(0.95) for both normal and t cases; namely, the probability of the losses less than VaR(0.95) is 5%. The expected loss displays a stable trend over the 5 week period; however, the actual loss fluctuates dramatically.

We made an assumption of stationarity of stock losses and returns for each model we have used; however, in realistic financial markets, stock returns and losses have loss distributions that are significantly changing [14].

3.4 Goodness of Fit

We use a Chi-squared hypothesis test to check the goodness of fit for the normal and t-distributions. The results are in the chart below:

	Normal distribution		t-distribution	
Stocks	h	p	h	p
AAPL	0	0.560968	0	0.545434
AEO	0	0.208693	0	0.115148
BBY	0	0.093645	0	0.067239
C	0	0.129941	0	0.328431
DELL	0	0.577239	0	0.582663
GE	0	0.510744	0	0.43868
GOOG	1	0.032212	1	0.015746
HBC	0	0.163755	0	0.137313
HPQ	1	0.018183	1	0.015211
KO	0	0.275655	0	0.609468
MCD	0	0.3588	0	0.331533
MSFT	0	0.84079	0	0.917548
NKE	0	0.310658	0	0.272692
SNE	0	0.748495	0	0.357017
WMT	0	0.485201	0	0.29862
Stock Portfolio	0	0.055010	1	4.6595e-05

Table 3.11: Chi-squared Test for Underlying Assets

We check the historical weekly returns of individual underlying asset with Chi-squared test. The decision rule is: If h=0, we fail to reject null hypothesis (which says that the specific stock returns are from the population of the distribution) under confidence level $\alpha = 5\%$.

Otherwise, if h=1, we reject the null hypothesis under confidence level $\alpha = 5\%$. P-values in the chart indicate the probability of observed results, or the chance of the hypothesis is true. So if h=0, the bigger p-value give the better goodness of fit.

In our underlying assets, MSFT (Microsoft) is a good example. In this case h=0 and the p-value is 0.84079 for the normal distribution, which means we could believe that the weekly return of MSFT is from a of normally-distributed population. Rather, h=0 and p=0.917548 for the t-distribution, we could accept that the weekly return of MSFT is from t-distribution with probability of 90%. The results show that we have to reject that the stock returns are from a student-t distribution, but fail to reject that the stock returns are from normal distribution.

Chapter 4 Polynomial Tail and ARMA-GARCH Model

In this chapter we model the portfolio loss with two different methods. Instead of estimating the entire loss distribution, we only consider the tail part of the loss density, which we model as a polynomial tail. The ARMA(1,1)-GARCH(1,1) model provides us a new way of exploring the conditional loss distribution rather than unconditional loss distribution as we did previously. Two styles of innovations (Gaussian and student-t) were used in the model. Then we calculated Value-at-Risk and expected shortfalls to check the effects of using different innovations.

4.1 Polynomial tail estimation

We assume that the loss density has a polynomial tail. Let f be the return density function It has a polynomial left tail of the form

$$f(y) \sim A|y|^{-(a+1)} \quad \text{as} \quad y \rightarrow -\infty \qquad (4.1)$$

where $A > 0$ is a constant and $a > 0$ is the tail index.

1) Construction of historical time series

Since 16th March when we formed our portfolio, we accumulated five weekly returns on our stock portfolio. We take the 4th week as the representative example describes here. To estimate the tail index, we assume R is the weekly return of the portfolio, and

$$P(R < -y) = \frac{A}{a}|y|^{-a} = \frac{A}{a}y^{-a} \quad y > 0\,, \qquad ln[P(R < -y)] = ln\left(\frac{A}{a}\right) - a\;ln(y)$$

Let $R_{(1)},\ R_{(2)}, \cdots, R_{(n)}$ be the order statistics of a sample of returns. This sample is what we obtained from the last step-historical weekly returns of our stock portfolio. We assume that the number of returns smaller than or equal to $R_{(k)}$ is k.[15]

Thus with the estimation of $P\left(R < R_{(k)}\right) \approx k/n$

$$ln(-R_{(k)}) = \frac{1}{a}ln\left(\frac{A}{a}\right) - \frac{1}{a}l\,n\left(\frac{k}{n}\right). \tag{4.2}$$

We perform linear regression on $ln(k/n)$ and $ln(-R_{(k)})$, then we plot

$\left\{ln(k/n),\ \ ln(-R_{(k)})\right\}_{k=1}^{n}$, where k is a small percentage of n ($\leq 20\%$).

In this case the regression slope $\hat{\beta}$ is an estimation of $-1/$ a. So $\hat{a} = -\frac{1}{\hat{\beta}}$ is the our estimator of the tail index, $\hat{A}$ can also be calculated from the intercept of the linear regression model.

We took *k*=30 ≈17.6%, which picked up 30 samples to apply in the regression. The estimated â and Â are as listed below for 5 weeks:

Parameters	$\hat{a}$	$\hat{A}$
1st Week	5.78	5.23E-05
2nd Week	5.78	5.20E-05
3rd Week	5.78	5.17E-05
4th Week	5.78	5.14E-05
5th Week	5.78	5.11E-05

Table 4.1: Regression Parameters

Note that $\hat{a}$ is the same and $\hat{A}$ varies slightly over the five week period. Because we are using 30 smallest returns in our regression, if the newest return of next week is not small enough to get into the top 30, we still use the same 30 points to do regression for the next week. The same results with the prior week would be obtained. Figure 4.1 demonstrates the regression lines for the 4th week.

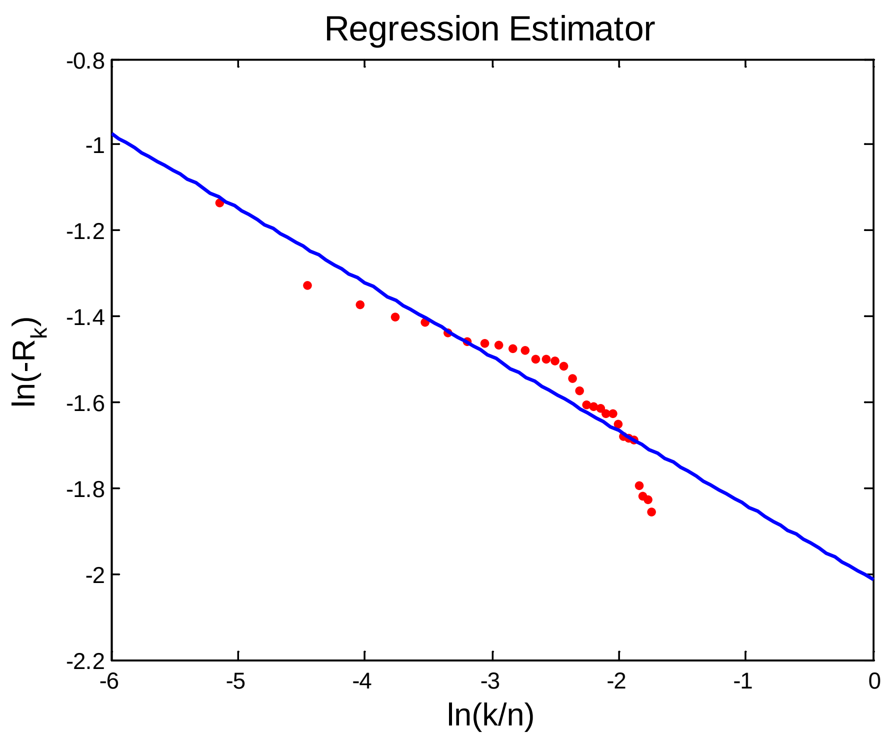

Figure 4.1: Regression Estimators

2) Value-at-Risk and Expected shortfall of polynomial tail

The method of computing value-at-risk and expected shortfalls for a polynomial tail is different from the previous approaches. We are going to use a semi-parametric method, which combines parametric and nonparametric components[16]. We perform a non-parametric estimation on $\mathrm{VaR}_{\alpha 0}$ for a small α_0, and a parametric estimation on $\mathrm{VaR}_{\alpha 1}$ for $\alpha_1 > \alpha_0$, with a formula for the value-at-risk

$$\frac{\mathrm{VaR}_{\alpha 1}}{\mathrm{VaR}_{\alpha 0}} = \left(\frac{1-\alpha_1}{1-\alpha_0}\right)^{-1/a} \quad . \tag{4.3}$$

The non-parametric estimation of $VaR(\alpha_0)$ is one of finding the K^{th} (K is the nearest integer to $(1-\alpha_0) \times n$) smallest return.

Suppose S is the initial investment, then $VaR(\alpha_0) = -S \times R_{(K)}$.

We choose a small $\alpha_0 = 0.9$, and the $1 - \alpha_0 = 0.1$ quantile of the sample is

$$n \times (1 - \alpha_0) = 170 \times 0.1 = 17,$$

so we take the 17th return which is -0.02684.

The stock portfolio value is $395,652.24 on 6th April, thus

$VaR(0.9) = -S \times R_{(K)} = -\$395{,}652.24 \times (-0.02684) = \ \$10617.81$ (2.68% in percentage).

The expected shortfall is calculated as

$$ES_{\alpha_0} = \frac{a}{a-1} VaR_\alpha = \frac{5.78}{5.78-1} \times \$10617.81 = \$12839.36 \ \text{(3.25\% in percentage).}$$

After calculating $VaR_{\alpha 0}$ and ES_{α_0}, we plug them into formula (4.3) to obtain $\mathrm{VaR}_{\alpha 1}$ and ES_{α_1}(in percentage) where $\alpha_1 \in [0.9, 1)$.

A graph of VaR_α and ES_α for confidence levels of $\alpha \geq 0.9$ is given below.

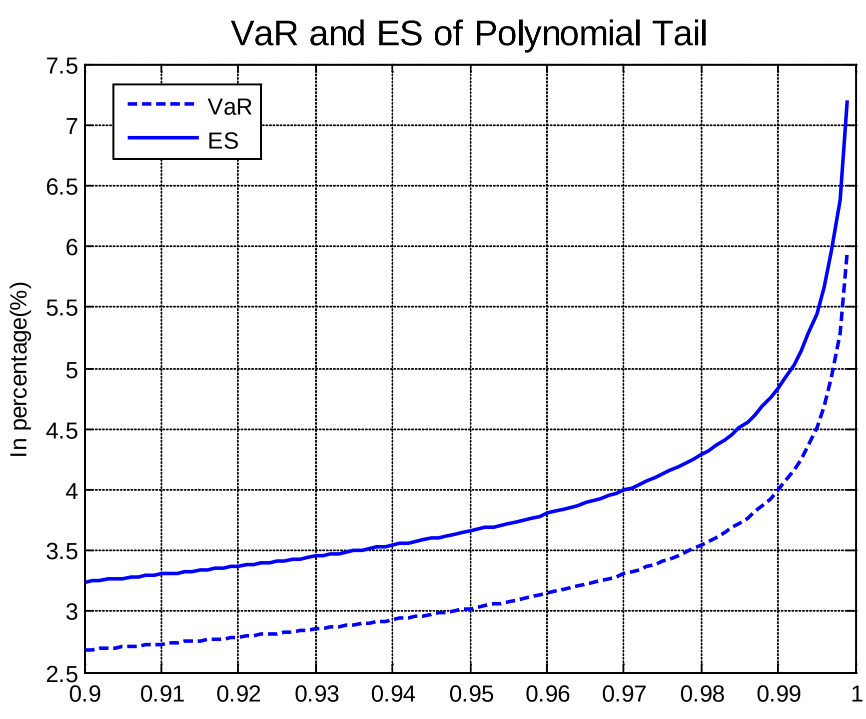

Figure 4.2: VaR and ES for Semi-parameter Estimation

Figure 4.2 depicts VaR and ES for a polynomial tail estimation of the 4th week. The trends of both risk measures are the same with previous plots of the normal and t distributions.

Date	VaR(0.95)	ES(0.95)	VaR(0.99)	ES(0.99)
1st Week	11706.58	14155.93	15465.72	18701.6
2nd Week	11435.51	13828.15	15107.62	18268.57
3rd Week	11667.24	14108.36	15413.75	18638.75
4th Week	11970.74	14475.37	15814.72	19123.61
5th Week	11726.85	14180.44	15492.51	18733.99
In percentage	3.03%	3.66%	4.0%	4.83%

Table 4.2: Value-at-Risk for Polynomial Tail Estimation

We list the VaR and ES values at confidence level of 0.95 and 0.99 over five weeks. The absolute values of VaR and ES vary slightly and percentages of those hardly change. Since no large enough loss happened over the five weeks, VaR(0.9) computed from the 17th lowest returns remained stable. Therefore these risk measures are not changing much.

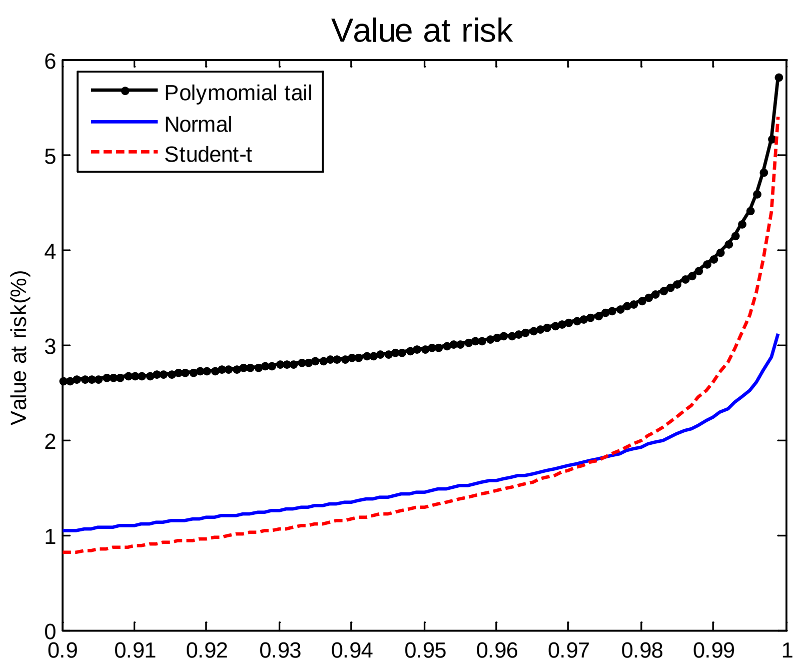

Figure 4.3: Comparison of VaR among Polynomial Tail, Normal and t Distribution

We compared the value-at-risk estimated from this semi-parametric method with our prior parametric methods. Figure 3.3 displays the value-at-risk of our polynomial tail, normal and student-t distribution for our portfolio of underlying assets over confidence level (0.95, 0.1). The value-at-risk of our polynomial index is much higher than the estimated value from the normal and t distributions. It is considered to be riskier than the previous two loss distribution estimation methods. This difference might be due to the heavier tail estimation; also it could be brought on by the difference between parametric (normal and t estimation) and semi-parametric methods. We use a small size of historical data to estimate normal and t parameters, however a large sample size is used for estimating VaR(α_0) .

4.2 Application of ARMA(1,1)-GARCH(1,1) Model

In previous modeling, we assumed the stock losses are stationary, at least over the historic period we consider [17]. However, in real financial world, it these distributions can vary dramatically over time. We simulated conditional mean and variance of portfolio loss with an ARMA (1, 1)-GARCH (1, 1) model, which is popularly used to estimate the process of stock returns and volatility [18].

4.2.1 Model construction

We considered our portfolio of underlying asset positions independently of the option positions.

Assume weekly stock portfolio loss follows an ARMA (1, 1)-GARCH (1, 1) model

$$L_t = \mu_t + \sigma_t Z_t \tag{4.4}$$

Where $\mu_t = \mu + \varphi(L_{t-1} - \mu) + \theta(\varepsilon_{t-1})$ is an ARMA (1, 1) model, $\varepsilon_t = \sigma_t Z_t$ is a GARCH (1, 1) model with White Noise term Z_t.[19]

With the stock portfolio historical time series, we form a series of losses. In the Matlab Economic ToolBox[4], “garchset”[5] is the function we first use to create a structure of our ARMA

[4] Matlab is a numerical computing software, MatlabEconometrics Toolbox™ provides functions for modeling economic data.

[5] Garchset, garchfit, garchinfter and garchdisp are all Maltab command.

(1, 1)/GARCH (1, 1) model with different innovations (Gaussian or student-t). Then “garchfit” fits our data into this structure. We then apply “garchdisp” and “garchinfer” to get coefficients of the model. We obtained the graphs of conditional means and conditional standard deviations from the two versions of innovations (Gaussian and student-t). In comparing the data and graphs of the two innovations, there were little differences between conditional means and standard deviations. Gaussian and student-t innovations affect only on the residuals of the models.

4.2.2 Fitting with Underlying Assets Portfolio

We consider our stock positions independently of our options positions. In this session, we model our stocks using ARMA-GARCH. The data was fitted for each holding week. With implementation in Matlab software, we obtained the following time series parameters (conditional means μ_t, standard deviations σ_t and standardized residuals Z_t) from our ARMA-GARCH model . Figure 3.4 shows the time series parameters’ plots for the last weeks.

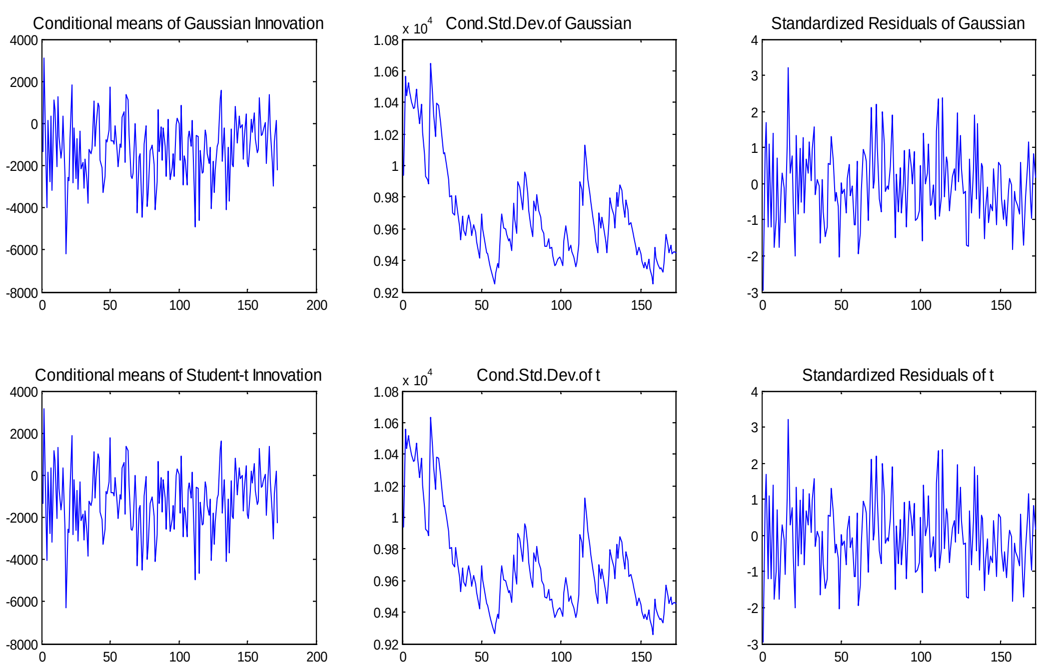

Figure 4.4: Conditional Characteristics of Gaussian and Student-t Innovations

With these conditional means and standard deviations (μ_t and σ_t) that we have at time t, we can estimate the conditional mean and variance of the next time t+1 using the coefficients of our model

$$\begin{cases} \mu_{t+1} = \mu + \varphi(L_t - \mu) + \theta(L_t - \mu_t) \\ \sigma_{t+1}^2 = \alpha_0 + \alpha_1(L_t - \mu_t)^2 + \beta\sigma_t^2. \end{cases} \quad (4.5)$$

Then we get the estimated conditional means and standard deviations of each week we hold the stock portfolio.

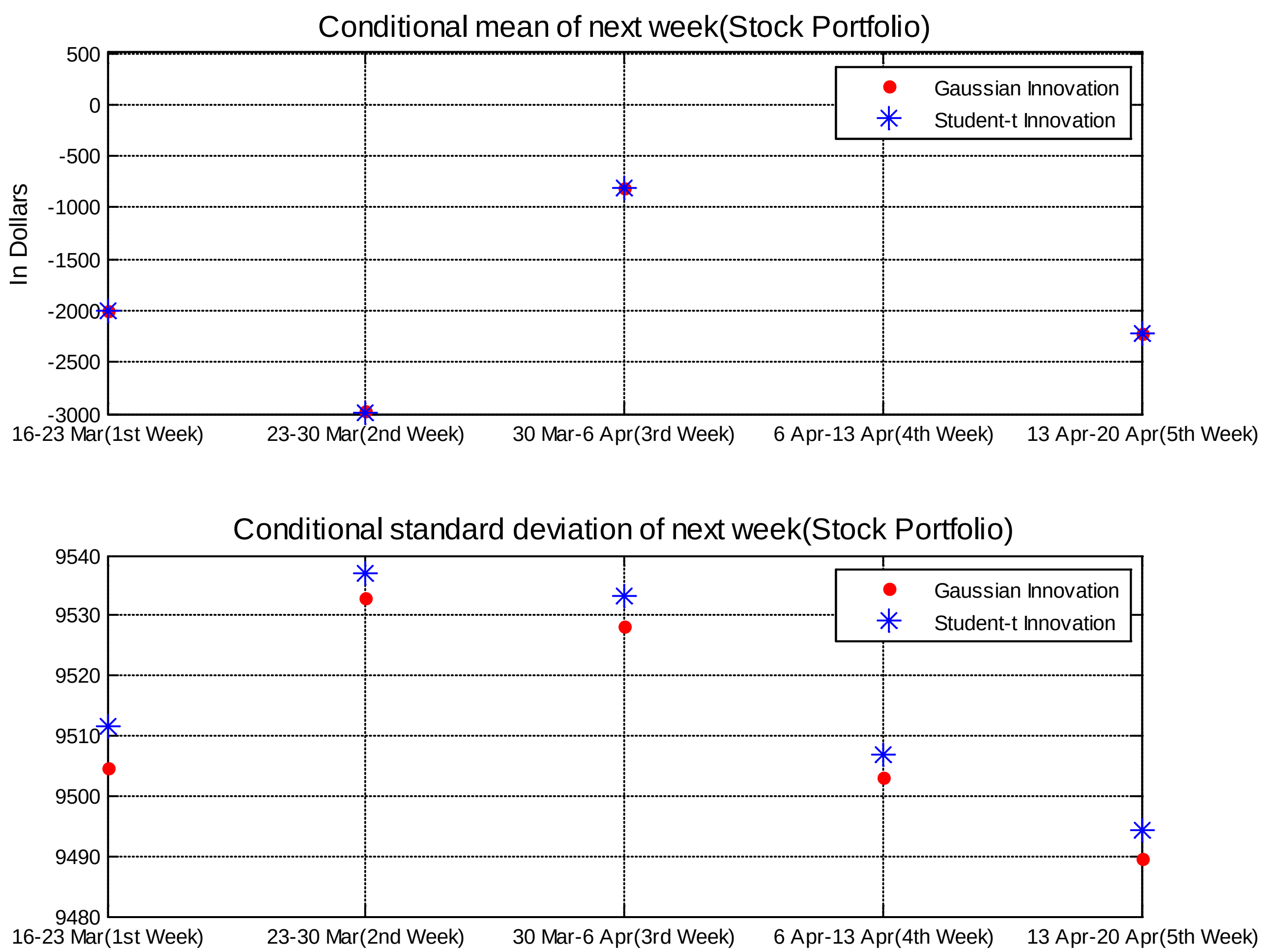

Figure 4.5: Conditional Means and Standard Deviations through Five Weeks

Figure 4.5 describes the forecasted conditional mean and standard deviation of the next time step based on the historical data in a rolling style. The two models predict almost the same conditional mean. The t innovation model gives a slightly higher conditional standard deviation.

- Value-at-risk and Expected shortfall calculation

We obtained the value-at-risk and expected shortfalls of each week holding the portfolio using the two formulas below:

$$\begin{cases} VaR_\alpha^t = \mu_{t+1} + \sigma_{t+1} q_\alpha(Z) \\ ES_\alpha^t = \mu_{t+1} + \sigma_{t+1} ES_\alpha(Z) \end{cases} \tag{4.6}$$

where Z is a random variable with cumulative distribution function G.

The following two graphs are VaR and ES values with Gaussian and student-t innovations for the 2nd week (23rd-30th March) and 4th week (6th-13th April). According to these graphs, there is no obvious difference between Gaussian innovations and t innovations with confidence levels between 0.9 and 0.99.

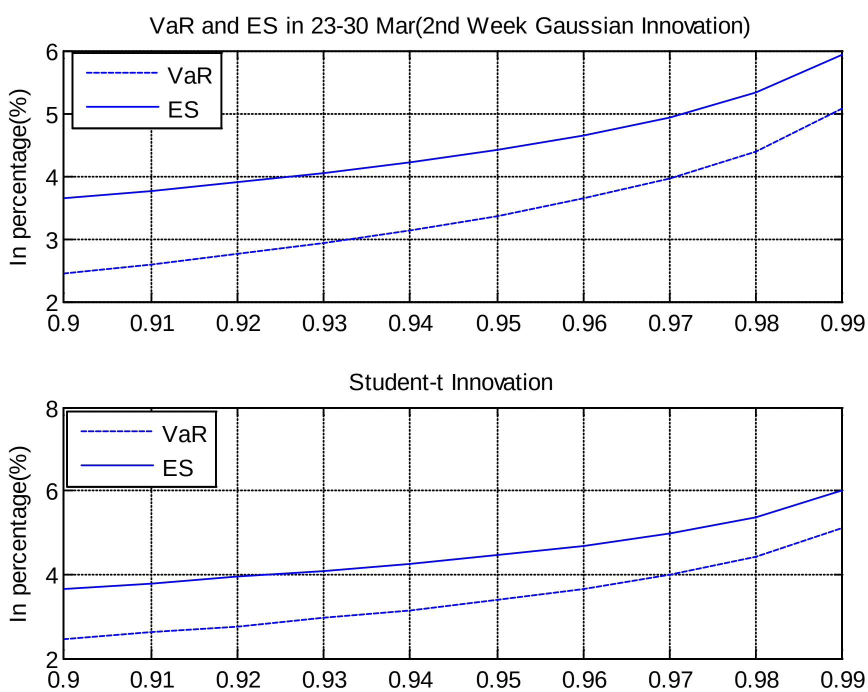

Figure 4.6: 2nd Week VaR and ES of Gaussian and student-t Innovations

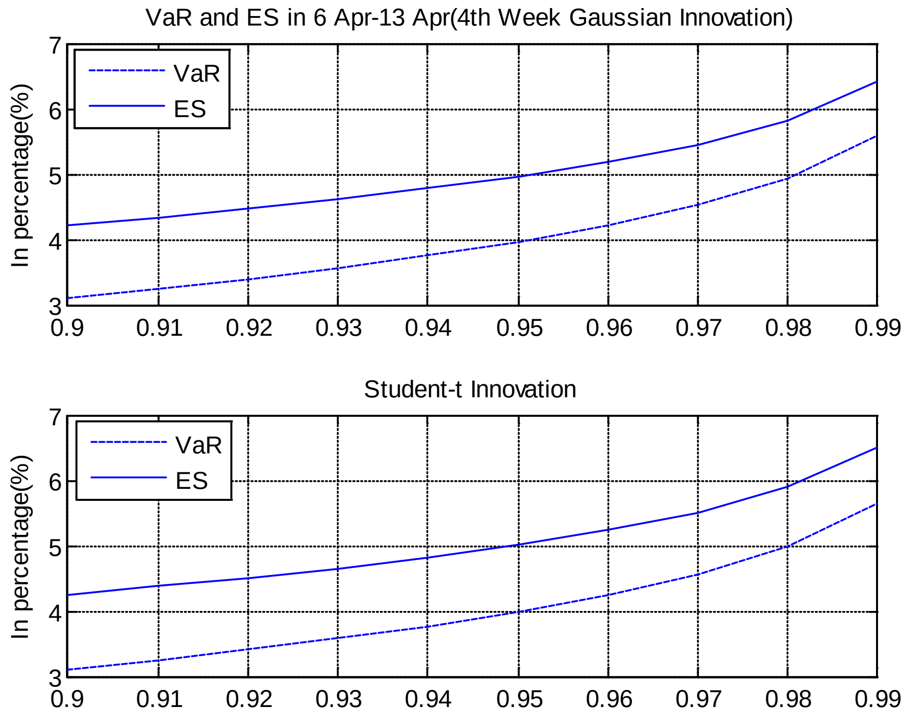

Figure 4.7: 4th Week VaR and ES of Gaussian and student-t Innovations

Figure3.7 displays Value-at-Risks and expected shortfalls of the two innovations over the five-week period at confidence level α =0.9. Both risk measures show the similar trends for the two types of innovations, and they fluctuate slightly over the time interval. Even though the actual losses do not come stably, they are all under the VaR(0.9) value.

Below is a graph of five-week Value-at-risk and expected shortfall values with confidence level α=0.9. VaR(0.9) and ES(0.9) are shown in lines in the graph, they have the similar trends for Gaussian innovations and Student-t innovations, and they fluctuate slightly over the weeks. While our actual loss of each week is shown as dots, they are all below VaR (0.9).

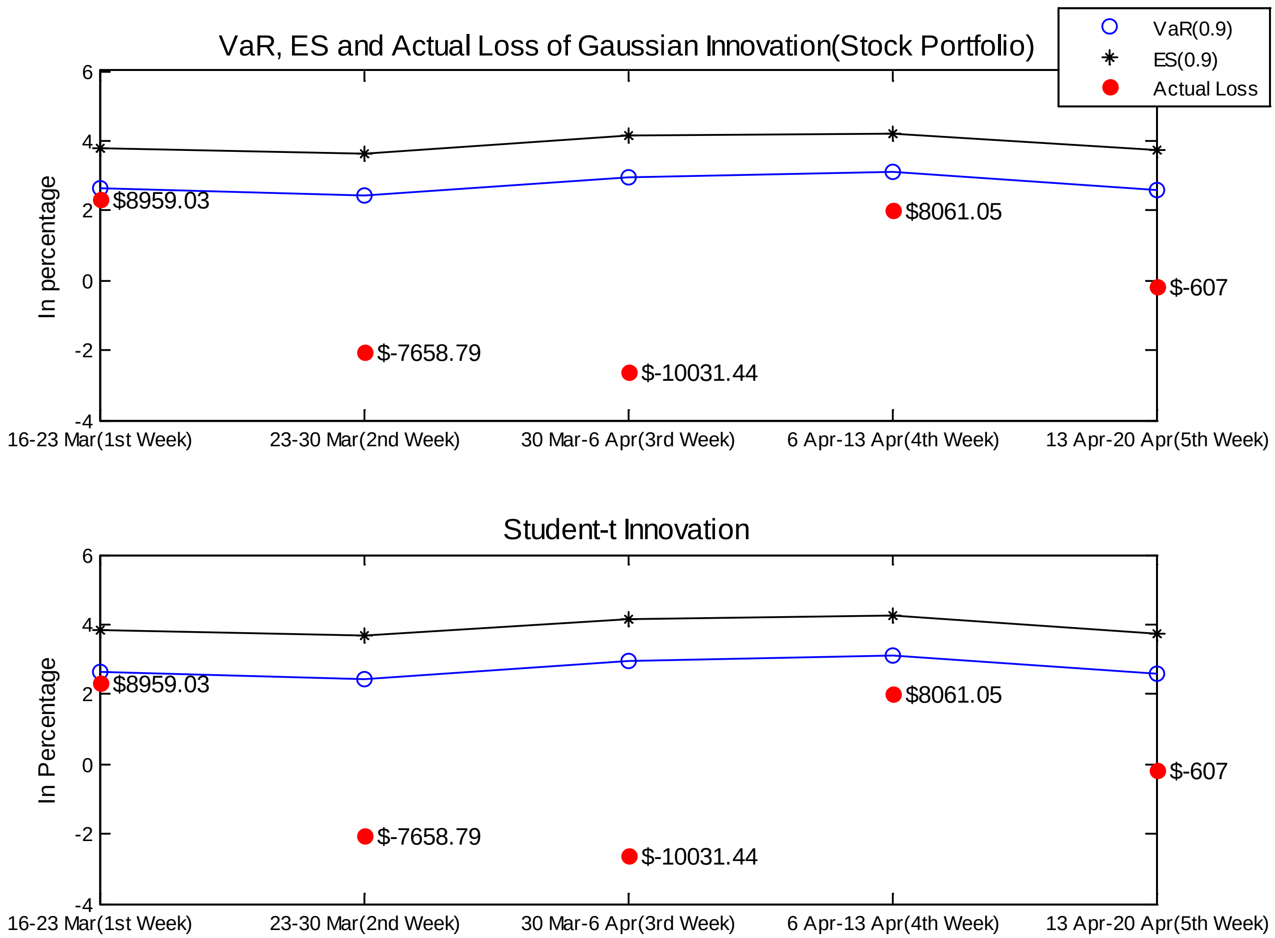

Figure 4.8: Five Weeks VaR, ES and Actual Loss Comparison between Two Innovations

AIC BIC test

We did an AIC and BIC test for the two fitting models. AIC (Akaike's information criterion) and BIC (Bayesian information criterion) are measures of good fit test. A goal of a good model is to minimize the values defined by these criteria [20]. The results suggest that the model with normal innovations is a better fit than student-t innovation model according to both criteria.

Test	AIC		BIC	
	Normal	Student-t	Normal	Student-t
1st Week	3562.46	3564.44	3581.17	3586.26
2nd Week	3584.01	3586	3602.75	3607.87
3rd Week	3604.41	3606.39	3623.19	3628.3
4th Week	3625.49	3627.48	3644.31	3649.44
5th Week	3646.49	3648.49	3665.34	3670.48

Table 4.3: AIC and BIC Test of Stock Portfolio

4.2.3 Fitting with Entire Portfolio

We do the same fitting with the entire portfolio as we did in the last section using ARMA(1,1)-GARCH(1,1) with different innovations. The only change for the entire portfolio is that we have 16 daily losses to fit into the ARMA(1,1)/GARCH(1,1) model. With much fewer losses, the conditional means and standard deviations of the two models for the entire portfolio are quite different. We computed $\sigma_t, \mu_t, VaR_\alpha$ and ES_α of Gaussian innovations and student-t innovations for the conditional loss distribution of L on a rolling daily basis. Then we plotted them for each innovation, and analyzed the results.

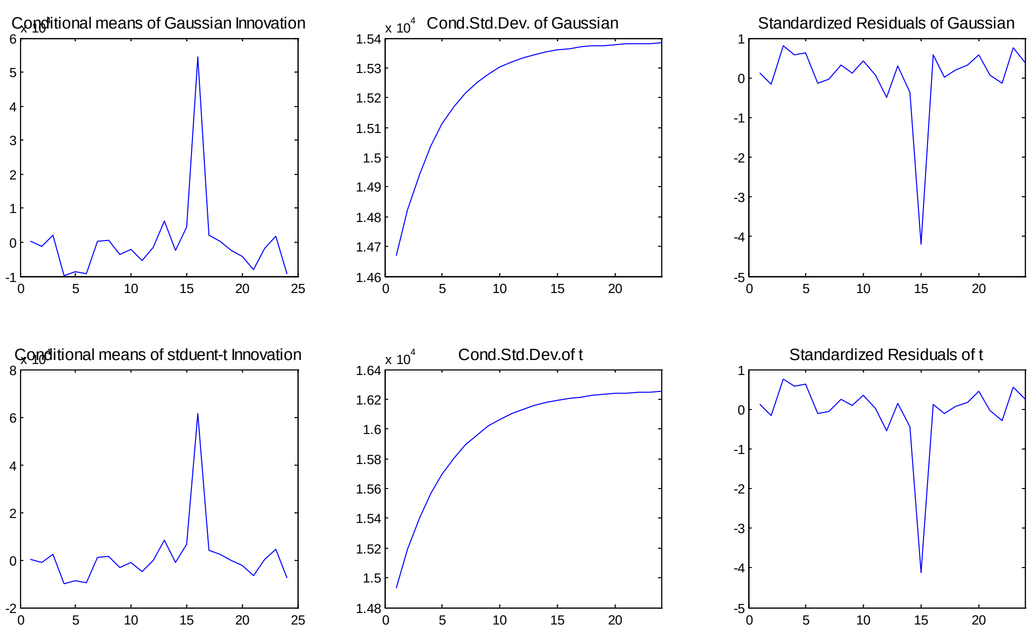

Figure 4.9: Conditional Characteristics of Gaussian and student-t Innovations

Figure 4.9 depicts the time series of parameters (conditional means μ_t, standard deviations σ_t and standardized residuals Z_t) generated by the two versions of ARMA(1,1)-GARCH(1,1) model on the last day's data fitting.

With the sequence of parameters, we predicted conditional mean μ_{t+1} and conditional standard deviation σ_{t+1}. According to Figure 4.10, the standard deviation of Gaussian innovations shows an upward trend while student-t innovations changes up and down. For both graphs, the 15th daily loss standardized residual's absolute value is larger than three, which implied this point is an "outlier" for our portfolio.

1) Comparison of conditional mean of Gaussian innovations and student-t innovations:

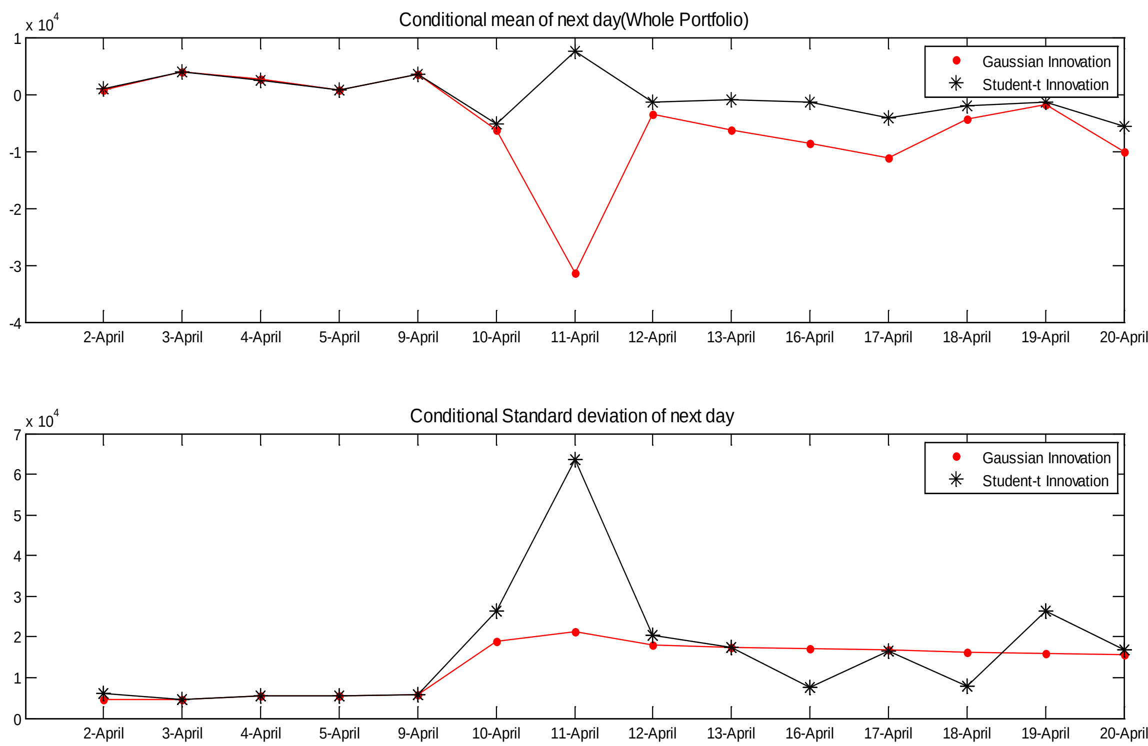

Figure 4.10: Rolling Conditional Mean and Standard Deviations

As the Figure 4.10 shows above, there is a dividing point at 9th April (Monday after Easter). Before 9th April, the conditional mean and conditional standard deviation for the two innovations seem like almost the same, but after 9th April, student-t innovations' conditional mean and conditional standard deviation is larger than that of the Gaussian innovations.

2) Value-at-risk and Expected Shortfall of Gaussian innovations and student-t innovations:

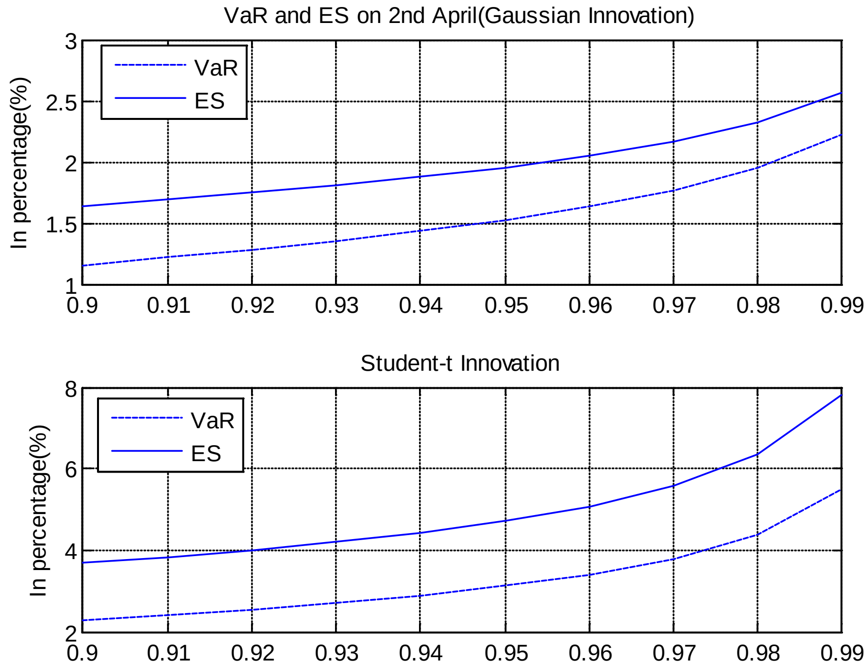

Figure 4.11: 2nd week VaR and ES of Gaussian and student-t Innovations

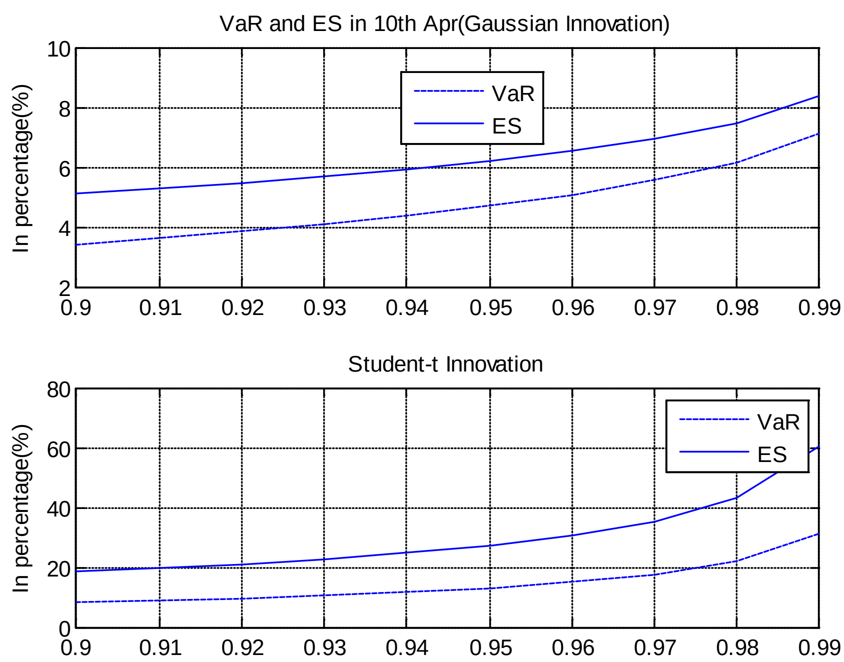

Figure 4.12: 3rd week VaR and ES of Gaussian and student-t innovations

3) Actual loss, VaR and ES of Gaussian Innovations and Student-t Innovations:

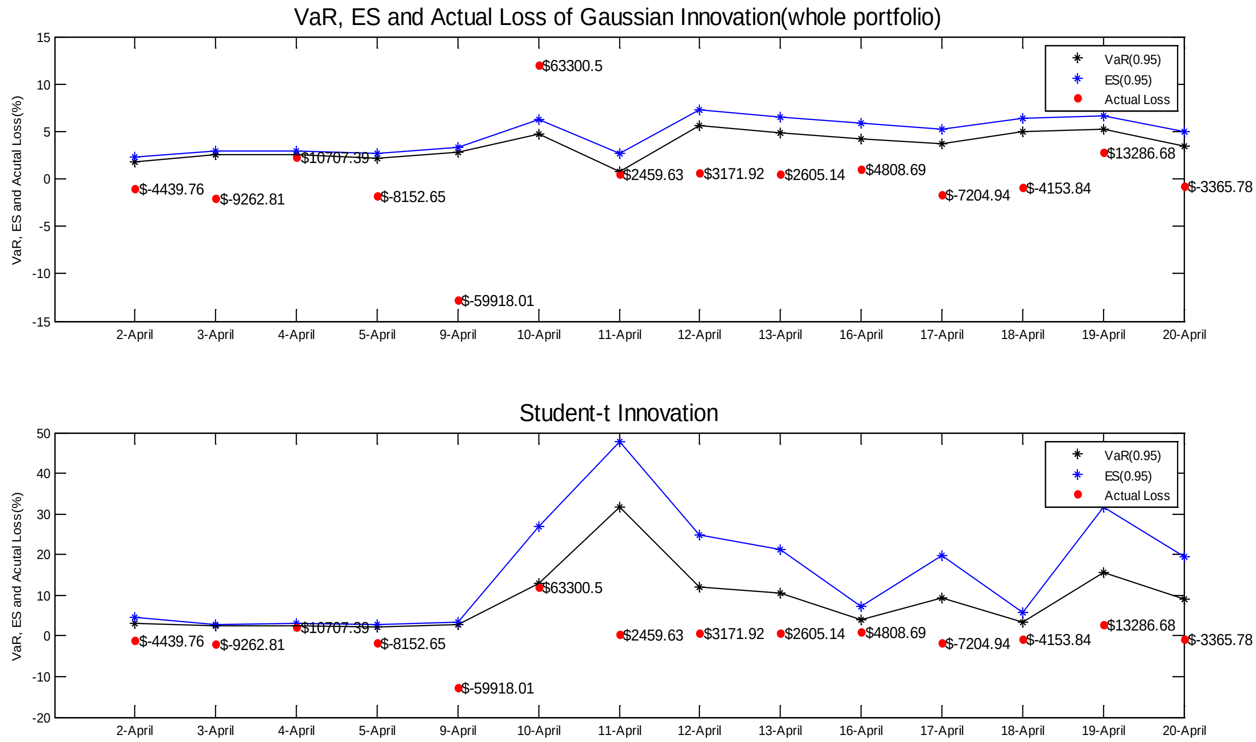

Figure 4.13: Rolling VaR, ES and Actual Loss of Gaussian and student-t Innovations

Figure 4.13 displays the risk measures for the two innovation models and the actual loss over time. From the two VaR(0.95) plots, we can observe that the VaR(0.95) curve has much more dramatic fluctuation as the actual loss is changing.

It is interesting to see that before 10th April the two VaR(0.95) values have almost the same stable trend and actual losses are all below the VaR(0.95) lines. However, when a big loss ($63300.5) happened on 10th April, the outcome occured with a 5% chance of loss exceeding VaR(0.95) for the Gaussian model but not for student-t model. Therefore we believe the t innovation model is better than Gaussian model in this case.

AIC&BIC Test

	AIC		BIC	
Date	**Normal**	**Student-t**	**Normal**	**Student-t**
2nd April	211.07	215.46	212.89	217.57
3rd April	230.96	232.96	233.35	235.75
4th April	254.45	256.45	257.36	259.85
5th April	273.51	275.52	276.9	279.47
9th April	295.2	297.22	299.04	301.7
10th April	343.24	331.38	347.49	336.34
11th April	370.42	365.67	375.05	371.08
12th April	389.8	371.47	394.8	377.31
13th April	411.18	390	416.52	396.23
16th April	432.68	410.61	438.34	417.22
17th April	454.5	429.8	460.47	436.77
18th April	475.7	458.22	481.96	465.53
19th April	496.82	479.37	503.37	487.
20th April	518.61	491.47	525.42	499.41

Table 4.4: AIC and BIC Test of The Entire Portfolio

We performed AIC and BIC tests of our ARMA(1,1)-GARCH(1,1) model on a daily basis. According to Table 3.4, 10th April is the watershed for the test. Before 10th April, normal innovations perform better than t innovations, and conversely, t has a better goodness of fit after 10th April since both of the criteria have smaller values for t. This fact verifies what we concluded from Figure 3.13. The loss on 10th April exceeds the predicted VaR(0.95) of the normal but not of the t. Thus t innovations seem more effective for fitting losses.

Chapter 5 Risk Reduction

We have been estimating loss distributions during our previous work. In this chapter we added new positions into our portfolio in order to reduce the risk of the investment.

There are many ways of decreasing the portfolio risk, such as diversification and option strategies. When we are constructing our portfolio, option strategies are been used to manage risk. We could also incorporate a new component, such as VXZ, which is negative correlated with the stock prices of the underlying assets in our portfolio. We tried to add VXZ into our portfolio at first; however, the risk reduction was achieved. Then we decided to add options on the underlying assets that we held. Table 4.1 displays the current portfolio in 5th week we held.

Stocks	Position	Unit Price($)	Corresponding Amount($)
AAPL	250	610.03	152507.5
AEO	6201	16.85	104486.9
BBY	-1627	22.11	-35972.97
C	23	33.45	769.35
DELL	-5828	16.16	-94180.48
GE	401	18.95	7598.95
GOOG	-122	629.52	-76801.44
HBC	-2252	42.92	-96655.84
HPQ	544	24.56	13360.64
KO	-952	72.1	-68639.2
MCD	1605	97.47	156439.4
MSFT	5021	31	155651
NKE	807	109.48	88350.36
SNE	-349	17.4	-6072.6
WMT	1562	60.09	93860.58

Table 5.1: 5th Week Stock Portfolio Details

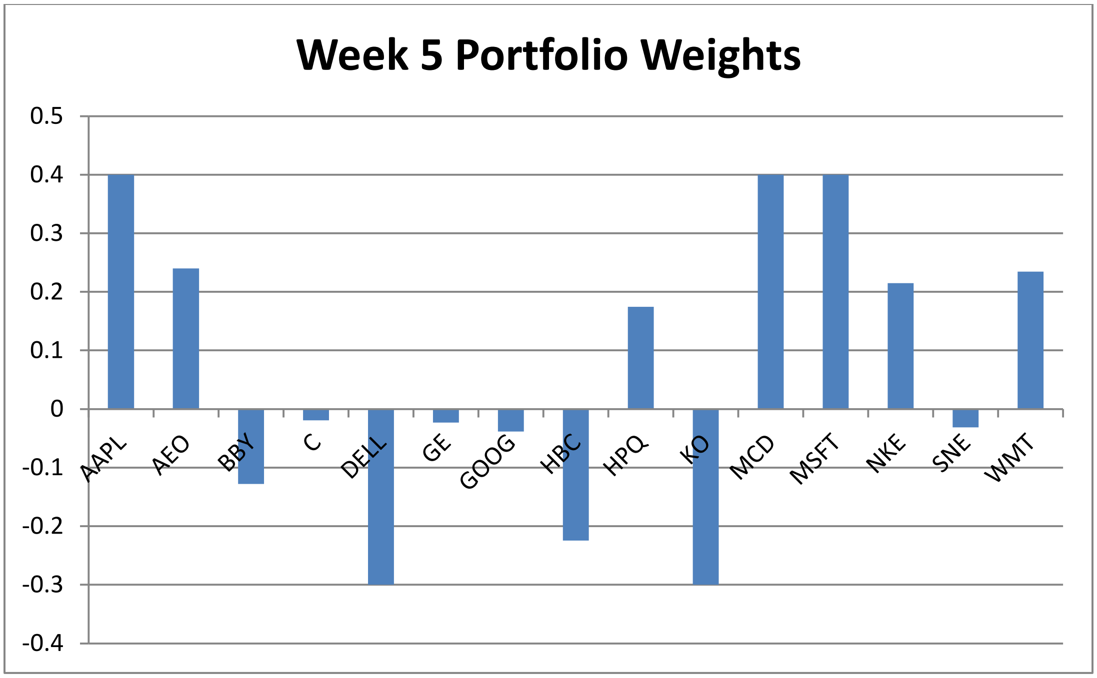

Figure 5.1: 5th Week Stock Portfolio Weights

To lower the risk of the portfolio, we are going to add options that negatively affect the stocks with large weights. Figure 4.1 illustrates the weights of the 15 underlying assets. AAPL, MCD and MSFT take up the biggest positive weight (0.4), on the other hand, DELL and KO are the biggest negative weight proportion (-0.3). So we short call option on MCD and a put option on DELL to reduce our portfolio risk.

Options	Style	Strike Price	Unit Price	Position(1=100shares)
MCD	CALL	95	95.94	-20
DELL	PUT	16	16.16	-5

Table 5.2: Risk Reduction Options

From the figure below, the VaR and ES of our portfolio after this modification are lower than in the original portfolio, under significance levels between 95% and 99%, which means our risk reduction succeeded.

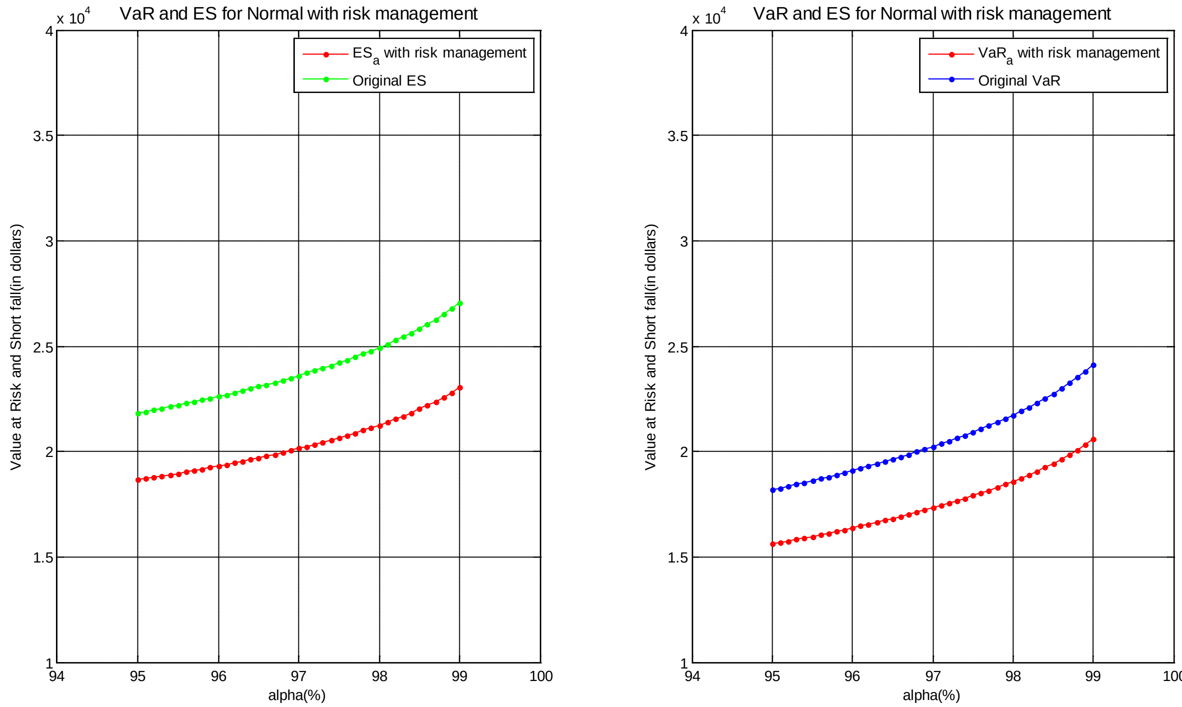

Figure5.2 : Comparison of VaR and ES between original portfolio and risk reduced portfolio

In detail, the VaR and expected shortfall before and after our risk reduction are listed below:

	α(%)	VaR ($)	VaR (%)	ES ($)	ES (%)
Original	95	18179.1	3.99	21817.5	4.78
After reduction	95	15605.7	3.42	18649.4	4.08
Original	99	24113.1	5.29	27063.7	5.93
After reduction	99	20569.8	4.51	23038.1	5.05

Table 5.3: VaR and ES of Original Portfolio and Risk Reduced Portfolio

The red line represents our portfolio after risk reduction, the VaR reduced by 0.57% under significant level $\alpha = 0.95$ and 0.78% under significant level $\alpha = 0.99$, the expected shortfall reduced by 0.70% under significant level $\alpha = 0.95$ and 0.88% under significant level $\alpha = 0.99$.

Chapter 6 Conclusion

For the three models we used with the assumption of stationarity, we make some comparisons and analysis. The student-t distribution has a heavier tail than the normal distribution. Under a confidence level around 0.95, VaR of the student-t for the portfolio loss is under that of normal; however, when the confidence level increases to 0.99, VaR of the student-t exceeds that of normal. The actual losses of our five weeks do not exceed VaR(0.95) of the two models. According to the AIC and BIC test, normal distribution fits better than student-t. The chi-square test also displays the stock portfolio part is prone to follow a normal rather than t. The two models fit equally well.

In the second part, we perform a polynomial tail estimation to stock portfolio portion. The expected polynomial tail indices for each holding week are the same. It depends on the length of sample for historical stock returns. In our model the actual losses never go to the top 20 percentage of the smallest returns, thus the VaR and ES under each confidence level from the semi-parametric method we calculate stay the same over the five-week period. Thus the tail index is very sensitive to the sample we choose. If we choose too long a period, historical returns and actual losses never exceed the smallest top 20 percent, and the unchanged predicted VaR and ES are not good risk assessment for our portfolio. So a polynomial tail is hard to manage and estimate.

We consider independently the stock part and then the entire portfolio. We fit both the stock portfolio and the entire portfolio to an ARMA(1,1)-GARCH(1,1) model with Gaussian and student-t innovations. For the stock portfolio part, both VaR and ES under the 2 innovations are very close to each other. Actual losses for the 5 weeks are under VaR(0.90) for both innovations. AIC and BIC tests show that the Gaussian innovations have a better goodness of fit. For the entire portfolio fitting, we have a different scenario. The entire portfolio fitting is based on a daily basis and it is total of 14 days from April 2nd to April 20th. At the first 6 days, VaR and ES are very close and predicted well; however, 10th April is a watershed for the 2 models. VaR of Gaussian is quite small and the VaR of the student-t model has a dramatic jump. The actual loss that happened on April 10th was quite large. The actual loss on April 10th happened under the potential loss predicted by the VaR at the 95% confidence level from the student-t innovations, but it happened in the 5% “outliers” of the VaR derived from Gaussian innovations. AIC and BIC tests describe that after the big loss happened, the t innovation model fits better than the Gaussian.

In conclusion, for unconditional estimation, the normal and student-t distributions have equally goodness of fit for our portfolio. Polynomial tail index estimation is not a good way of fitting since it mostly depends on the sample we choose. For conditional GARCH modeling, the historical time series of loss we are using determines the goodness of fit. Adding one big loss into the data can cause a big change to the fitting results.

Appendix

Matlab Code

Project1.m

```
%% Stock Porfolio
Wt= xlsread('stock rebalace.xlsx','Sheet2','B56:P56')'; %Stock weights for
Nmu=xlsread('Stock Log reutn(1 Year).xlsx','Sheet2','B56:P56')'; % Expected
return of N stocks
Ncov=xlsread('Stock Log reutn(1 Year).xlsx','Sheet2','B60:P74');
Vt=420000; %Initial Capital for stock
ExpLoss_S=-Vt*Wt'*Nmu;

%%  Option Portfolio
%Risk Factor:1 Log return of stock price
callSlogreturn=[0.004772627,0.001180538,-0.001791964,0.002573041,-
0.003103893]/50; % call: AAPL AEO C MSFT SNE 1 year of stock price return

putSlogreturn=[0.004772627,0.001180538,-
0.001791964,0.000686127,0.002573041]/50; % put: AAPL AEO C GE MSFT 1  Log
return of stock price

%2  Interest rate changes of 1 year weekly
InterChange=-0.0186538461538462/50;

%3  Volatility changes: Use VIX index change of 1 year weekly as our
volatility change
VIXchange=-0.173269231*0.01/50;

%Call option
callS=[580.56,16.69,36.27,32.85,21.75]; % call: AAPL AEO C MSFT SNE stock
price when trading
callK=[590,16,34,31,22]; % call: AAPL AEO C MSFT SNE
callPrice=[34.85,1.1,3.55,2.22,1.45];
putS=[585.56,16.69,36.27,20.16,32.85]; %put: AAPL AEO C GE MSFT stock price
when trading put
putK=[590,16,35,21,32]; %
putPrice=[39.45,0.8,1.33,1.21,0.85];
r=1.98/100;  callTau=[1/6,1/6,1/6,1/6,1/3]; putTau=1/6; delta=1/52;

for i=1:5 % 5 calls and 5 puts
callVol(i) = blsimpv(callS(i), callK(i), r, callTau(i), callPrice(i), [],0,
[], {'call'});
putVol(i)  = blsimpv(putS(i), putK(i), r, putTau, putPrice(i), [],0, [],
{'put'});

[CallTheta(i), b] = blstheta(callS(i), callK(i), r, callTau(i), callVol(i));
[a,  PutTheta(i)] = blstheta(putS(i), putK(i), r, putTau, putVol(i));

[CallDelta(i),b] = blsdelta(callS(i), callK(i), r, callTau(i), callVol(i));
[a, PutDelta(i)] = blsdelta(putS(i), putK(i), r, putTau, putVol(i));
```

```
[CallRho(i),b] = blsrho(callS(i), callK(i), r, callTau(i), callVol(i));
[a, PutRho(i)] = blsrho(putS(i), putK(i), r, putTau, putVol(i));

CallVega(i) = blsvega(callS(i), callK(i), r, callTau(i), callVol(i));
PutVega(i)  = blsvega(putS(i), putK(i), r, putTau, putVol(i));

callExpLoss(i)=-
[CallTheta(i)*delta+CallDelta(i)*callS(i)*callSlogreturn(i)+CallRho(i)*InterC
hange+CallVega(i)*VIXchange];
putExpLoss(i)=-
[PutTheta(i)*delta+PutDelta(i)*putS(i)*putSlogreturn(i)+PutRho(i)*InterChange
+PutVega(i)*VIXchange];
end

%% Position for call: AAPL AEO C MSFT SNE and put: AAPL AEO C GE MSFT
callN=[4,100,33,50,-3]*100; putN=[4,100,33,3,50]*100;
ExpLoss_all=ExpLoss_S+callN*callExpLoss'+putN*putExpLoss';

WT=zeros(17,1);
WT(1)=Vt*Wt(1)+400*CallDelta(1)*callS(1)+400*PutDelta(1)*putS(1);
WT(2)=Vt*Wt(2)+10000*CallDelta(2)*callS(2)+10000*PutDelta(2)*putS(2);
WT(3)=Vt*Wt(3);
WT(4)=Vt*Wt(2)+3300*CallDelta(3)*callS(3)+3300*PutDelta(3)*putS(3);
WT(5)=Vt*Wt(5);
WT(6)=Vt*Wt(6)+300*PutDelta(4)*putS(4);
for i=7:11
    WT(i)=Vt*Wt(i);
end
WT(12)=Vt*Wt(12)+5000*CallDelta(4)*callS(4)+5000*PutDelta(5)*putS(5);
WT(13)=Vt*Wt(13);
WT(14)=Vt*Wt(14)-300*CallDelta(5)*callS(5);
WT(15)=Vt*Wt(15);
WT(16)=callN*CallRho'+putN*PutRho';
WT(17)=callN*CallVega'+putN*PutVega';
Factor_cov=xlsread('e:\MA575\Project 1\Stock Log reutn(1
Year).xlsx','Sheet1','B58:R74');
ExpVar_all=WT'*Factor_cov*WT;

%% Value at Risk and Expected Shortfall
alpha=[0.95:0.001:0.99]; n=length(alpha);
VaR_alpha=ExpLoss_all+sqrt(ExpVar_all)*norminv(alpha);
ES_alpha=ExpLoss_all+sqrt(ExpVar_all)*normpdf(norminv(alpha))./(1-alpha);

%Plot the pdf and cdf of the expected loss distribution
figure(1)
xx=linspace(-25*ExpLoss_all,25*ExpLoss_all);
yy=normpdf(xx,ExpLoss_all,sqrt(ExpVar_all));
zz=normcdf(xx,ExpLoss_all,sqrt(ExpVar_all));
plot(xx,yy,'r');
hold on
V=normpdf(ExpLoss_all,ExpLoss_all,sqrt(ExpVar_all));
plot([ExpLoss_all,ExpLoss_all],[V,0],'-.'); % Expected Loss
hold on
plot([VaR_alpha(1),VaR_alpha(1)],[4*10^(-5),0],'--');% 95% confidence
```

```
interval
hold on
plot([VaR_alpha(n),VaR_alpha(n)],[4*10^(-5),0],'--');% 99% condidence
interval
 title('The P.D.F of Normal distribution in 1st Week ','FontSize',15);
 legend('P.D.F of Normal','Expected Loss','95%-99% Confidence Interval');
 xlabel('Loss of the week(percentage)');
 set(gca,'xtick',[-5*10^4:10000:5*10^4]);
set(gca,'xtick',[-5*10^4:10000:5*10^4],'xticklabel',{ '-10%','-8%','-6%','-
4%','-2%', '0%' ,'2%','4%','6%','8%','10%'},'FontSize',8);


 figure(2)
plot(xx,zz,'r');
hold on
plot([ExpLoss_all,ExpLoss_all],[1/2,0],'-.');
hold on
plot([VaR_alpha(1),VaR_alpha(1)],[1,0],'--');% 95% confidence interval
hold on
plot([VaR_alpha(n),VaR_alpha(n)],[1,0],'--');% 99% condidence interval
title('The C.D.F of Normal distribution in 1st Week','FontSize',15);
 legend('C.D.F of Normal','Expected Loss','95%-99% Confidence Interval');
 xlabel('Loss of the week(percentage)');
 set(gca,'xtick',[-5*10^4:10000:5*10^4]);
set(gca,'xtick',[-5*10^4:10000:5*10^4],'xticklabel',{ '-10%','-8%','-6%','-
4%','-2%', '0%' ,'2%','4%','6%','8%','10%'},'FontSize',8)
axis([-5*10^4 5*10^4 0 1.1]);

figure(3)
plot(alpha*100,ES_alpha/500000*100,'--');
hold on
plot(alpha*100,VaR_alpha/500000*100);
hold on
legend('ES_a','VaR_a');
title('Normal distribution Shortfall and Value at Risk in 1st
Week','FontSize',15);
xlabel('alpha(%)'); ylabel('Value at Risk and Short fall(%)');
text(alpha(1)*100,VaR_alpha(1)/500000*100-.2, '$20306.2','FontSize',12);
text(alpha(41)*100,VaR_alpha(41)/500000*100+.2, '$27919.16','FontSize',12);
text(alpha(1)*100,ES_alpha(1)/500000*100-0.2, '$24974.1','FontSize',12);
text(alpha(41)*100,ES_alpha(41)/500000*100+.2, '$31704.63','FontSize',12);
plot(alpha(41)*100,VaR_alpha(41)/500000*100,'.r','MarkerSize',15);
plot(alpha(1)*100,VaR_alpha(1)/500000*100,'.r','MarkerSize',15);
plot(alpha(1)*100,ES_alpha(1)/500000*100,'.r','MarkerSize',15);
plot(alpha(41)*100,ES_alpha(41)/500000*100,'.r','MarkerSize',15);
axis([94,100,3,7]);
grid on


figure(4)
plot(alpha*100,ES_alpha,'--');
hold on
plot(alpha*100,VaR_alpha);
hold on
legend('ES_a','VaR_a');
title('Normal distribution Shortfall and Value at Risk in absolute value in
```

```
1st Week');
xlabel('alpha(%)'); ylabel('Value at Risk and Short fall(in dollars)');
grid on
```

PolymonialTail.m

```
%% Polynomial Tail 1st Week
n=168; k=30; % k is 20 percentage of n
for i=1:k
X(i)=log(i/n); % Take Observation data in a
end
Y= xlsread('stock rebalace.xlsx','Sheet2','B56:P56')';

X_matrix=[ones(k,1),X'];
B=zeros(k,1);
B=inv((X_matrix)'*X_matrix)*X_matrix'*Y;
a=-1/B(2);
A=a*exp(B(1)*a);
%% Value at risk and Expected Shortfall
alpha0=0.90; VaR_alpha0=10383.49764; Vt=386921.04;
alpha1=[0.9:0.001:0.999];
VaR_alpha1=((1-alpha1)./(1-alpha0)).^(-1/a)*VaR_alpha0;

ES_alpha0=a/(a-1)*VaR_alpha0;
ES_alpha1=a/(a-1)*VaR_alpha1;


%% Plot 7 observation points and Estimation line
figure(1)
for i=1:k
plot(X(i),Y(i),'r.','MarkerSize',10);
hold on
end
hold on
%Plot estimation line
xx1=linspace(-6,0);
xx=[ones(100,1),xx1'];
yy=xx*B;
plot(xx1,yy);
title('Regression Estimator','FontSize',15);
xlabel('ln(k/n)','FontSize',13);
ylabel('ln(-R_k)','FontSize',13);

%% Plot Value at risk and ES
figure(2)
x=[0.9:0.001:0.999];
y=[VaR_alpha1];
z=[ES_alpha1];
plot(x,y/Vt*100,'--b','LineWidth',2);
hold on
plot(x,z/Vt*100,'LineWidth',2);
title('VaR and ES of Polynomial Tail','FontSize',15);
legend('VaR','ES');
ylabel('In percentage(%)');
grid on
```

ARMA_GARCH.m

```
Lt=xlsread('Data.xlsx', 'Stock Portfolio Loss','C3:C173')     % The entire
portfolio daily returns
n=length(Lt);

%% Gaussian Innovations
spec1 = garchset('R',1,'M',1,'P',1,'Q',1);
spec1 = garchset(spec1,'Display','off');
[coeff1,error1] = garchfit(spec1,Lt);
garchdisp(coeff1,error1)
[res1,sig1,LogL1] = garchinfer(coeff1,Lt); % sigma1 is the conditional
standarized deviation
mu1=Lt-res1; % Conditional mean of Gaussion Innovations
C1=coeff1.C; AR1=coeff1.AR; MA1=coeff1.MA; K1=coeff1.K; ARCH1=coeff1.ARCH;
GARCH1=coeff1.GARCH;
Con_mu1=C1 +AR1*Lt(n)+MA1*res1(n);
Con_std1=sqrt(K1+ARCH1*Lt(n)^2+GARCH1*sig1(n)^2);

%% Student-t innovations
spec2 = garchset('R',1,'M',1,'P',1,'Q',1,'Dist','t');
spec2 = garchset(spec2,'Display','off');
[coeff2,error2,LogL2] = garchfit(spec2,Lt);
garchdisp(coeff2,error2)
[res2,sig2,LogL2] = garchinfer(coeff2,Lt);
mu2=Lt-res2; %Conditional mean of student-t innovations
C2=coeff2.C; AR2=coeff2.AR; MA2=coeff2.MA; K2=coeff2.K; ARCH2=coeff2.ARCH;
GARCH2=coeff2.GARCH; DoF=coeff2.DoF;
Con_mu2=C2 +AR2*Lt(n)+MA2*res2(n);
Con_std2=sqrt(K2+ARCH2*Lt(n)^2+GARCH2*sig2(n)^2);

[aic,bic] = aicbic([LogL1,LogL2],[6,7],length(Lt))

figure(1)
subplot(2,3,1)
plot(mu1);
title('Conditional means of Gaussian Innovation','FontSize',12);

subplot(2,3,2);
plot(sig1);
xlim([0,length(Lt)]);
title('Cond.Std.Dev. of Gaussian','FontSize',12);
subplot(2,3,3)
plot(res1./sig1)
xlim([0,length(Lt)])
title('Standardized Residuals of Gaussian','FontSize',12)
% Conditional mean of Gaussian Innovations

subplot(2,3,4)
plot(mu2);
title('Conditional means of stduent-t Innovation','FontSize',12);
subplot(2,3,5);
plot(sig2);
xlim([0,length(Lt)]);
```

```
title('Cond.Std.Dev.of t ','FontSize',12);
subplot(2,3,6)
plot(res2./sig2);
xlim([0,length(Lt)]);
title('Standardized Residuals of t','FontSize',12);



%% Value at risk and Expected Shortfall
alpha=[0.9:0.01:0.99]';  Vt=398763.56;
```

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