2012-05-02

Investment Analysis: Evaluating the Loss and Risk of a Stocks and Options Portfolio

Azuri Shah
Worcester Polytechnic Institute

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Investment Analysis:

Evaluating the Loss and Risk of a Stocks and Options Portfolio

by

Azuri Shah

A Project Report
Submitted to the Faculty
of the
WORCESTER POLYTECHNIC INSTITUTE
In partial fulfillment of the requirements for the
Degree of Master of Science
in
Financial Mathematics

May 2012

APPROVED:

______________________________
Professor Marcel Y. Blais, Capstone Advisor

______________________________
Professor Bogdan Vernescu, Head of Department
Abstract

With the ripples in the financial markets and economic stresses that occur around the world today, it would be beneficial to have some insight into the tools that help investors learn about the riskiness of their portfolios. At what value is one’s portfolio in danger of being completely wiped out? We aim to further the understanding of values such as these and give an assessment of some risk measures by investing in an interactive portfolio, as well as estimating the values at risk and expected shortfalls of this portfolio.
Acknowledgements

I, Azuri Shah, would like to formally acknowledge those who have contributed to the successful completion of this capstone project. First, I would like to sincerely thank my project partner, Shanna Infantino, for her participation, diligence, and focus throughout the semester. Her dedication and hard work is greatly appreciated. Next, I would like to thank my classmates, who have graciously answered questions in my time of need even when they themselves were under a time crunch. Thirdly, I would to thank the institution of Worcester Polytechnic Institute, for the quality education and teachers that have been provided for me to further my education. Lastly, and most importantly, I would genuinely like to thank Professor Marcel Blais, for his encouragement, patience, guidance, and support not only throughout this project, but throughout the Financial Mathematics program.
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Introduction

Investing in any type of security will result in a return, but whether or not the return is positive or negative is the question. While one would rejoice in the fact that he or she received a positive return, it is the negative returns that most individuals would prefer to keep hidden from public knowledge. No one will ever complain about a portfolio that is performing well in the market, so when analyzing the expected loss of a portfolio and the risk of a portfolio, it is those large losses that investors want to study in order to avoid being hit with a big negative return. This is where the idea of loss distributions comes in when assessing the risk of a portfolio.

Loss distributions are a risk management technique used to explore the right tail of a distribution of losses. Furthermore, an examination of loss distributions are combined with an analysis of value at risk and expected shortfall in order to obtain a greater understanding of the potential loss of an investment portfolio. In order to make such assessments, a number of assets were chosen and joined into a portfolio to evaluate.

1. Stock and Option Investment Plan

The stocks and underlying assets chosen for the portfolio span six different sectors and categories of the economy: technology, services, financial, basic material, commodities precious metals, and commodities energy. These assets are displayed below.

<table>
<thead>
<tr>
<th>Underlying Asset</th>
<th>Ticker Symbol</th>
<th>Sector/Category</th>
<th>Industry</th>
<th>Option Trading Strategy</th>
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<td>AAPL</td>
<td>Technology</td>
<td>Personal Computers</td>
<td>Straddle</td>
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<tr>
<td>Bank of America</td>
<td>BAC</td>
<td>Financial</td>
<td>Regional-Mid-Atlantic Banks</td>
<td>Short Call Ladder</td>
</tr>
<tr>
<td>Dell</td>
<td>DELL</td>
<td>Technology</td>
<td>Personal Computers</td>
<td></td>
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<tr>
<td>Disney</td>
<td>DIS</td>
<td>Services</td>
<td>Entertainment Diversified</td>
<td></td>
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<tr>
<td>EMC Corporation</td>
<td>EMC</td>
<td>Technology</td>
<td>Data Storage Services</td>
<td>Bull Call Spread</td>
</tr>
<tr>
<td>Exxon-Mobil</td>
<td>XOM</td>
<td>Basic Material</td>
<td>Major Integrated Oil &amp; Gas</td>
<td></td>
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<tr>
<td>SPDR Gold Shares*</td>
<td>GLD</td>
<td>Commodities Precious Metals</td>
<td>Bull Put Spread</td>
<td></td>
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<tr>
<td>Google</td>
<td>GOOG</td>
<td>Technology</td>
<td>Internet Information Providers</td>
<td></td>
</tr>
<tr>
<td>International Business Machines (IBM)</td>
<td>IBM</td>
<td>Technology</td>
<td>Diversified Computer Systems</td>
<td>Bear Put Spread</td>
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<td>JPMorgan Chase &amp; Co.</td>
<td>JPM</td>
<td>Financial</td>
<td>Money Center Banks</td>
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<tr>
<td>Microsoft</td>
<td>MSFT</td>
<td>Technology</td>
<td>Application Software</td>
<td></td>
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<td>Netflix</td>
<td>NFLX</td>
<td>Services</td>
<td>Music &amp; Video Stores</td>
<td>Strip Straddle</td>
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<tr>
<td>Nokia</td>
<td>NOK</td>
<td>Technology</td>
<td>Communication Equipment</td>
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<tr>
<td>iPath S&amp;P GSCI Crude Oil TR Index**</td>
<td>OIL</td>
<td>Commodities Energy</td>
<td>Bull Call Spread</td>
<td></td>
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<tr>
<td>Visa</td>
<td>V</td>
<td>Services</td>
<td>Business Services</td>
<td></td>
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Table 1: List of Stocks and Underlying Assets

*indicates a Exchange Traded Fund
**indicates a Exchange Traded Note

1 All information was obtained through Yahoo! Finance.
2 There are two breakeven points: 1. Strike Price+ sum of call and put premiums;
The stocks and underlying assets were chosen based on either personal interest, interrelations between companies (i.e. collaborations, competitors, etc.), or current events going on in the economy involving the selected companies. In addition to the symbol, sector, and industry, we have also indicated which assets will serve as the underlying for option strategies implemented in this investment.

The investment plan is broken down into three security categories: stocks, options, and risk-free assets. As seen in Table 2, there are also three stocks that we chose to invest in both a stock position and an option position – Apple, SPDR Gold, and IBM.

<table>
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<th>Stocks</th>
<th>Risk-free Assets</th>
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<td>AAPL</td>
<td>U.S. Treasury Bills</td>
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<td>EMC</td>
<td>DIS</td>
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<td>GLD</td>
<td>XOM</td>
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<td>IBM</td>
<td>GLD</td>
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<td>NFLX</td>
<td>GOOG</td>
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<tr>
<td>NOK</td>
<td>IBM</td>
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<th>15%</th>
<th>35%</th>
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<td>MSFT</td>
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<td>OIL</td>
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Table 2: Overview of Portfolio Investment

1.1 Technology Sector

The technology sector of the market is always a developing and improving sector, leaving a wide expanse of possibility to branch into. When it comes to the computer world, the integral question is “Are you a Mac or PC?” Thus Apple was chosen to be included into the portfolio. Aside from their computers, the company is always expanding their products, such as the iPod, iPhone, and iPad, coming up with innovative ways to keep the technology loving consumers entertained. It is a strong branded company and has a history of success with their products. With the current release of the new iPad, one can only assume the market will respond positively. International Business Machines Corp. (IBM) and Dell were chosen to be included as competitors for Apple.

Google was chosen due to its domination in the search engine field. The trademark line “Google It!” is often the response when someone asks a person a question that he or she does not know the answer to. The information is easy to navigate and appears to give better compatible results to what one is searching for.

Microsoft Corporation, the world’s largest independent computer software company, was chosen due to its widely used products. Microsoft Office itself is a staple resource in day-to-day work activities with programs such as Word, Excel, and PowerPoint. These products are highly standard tools in the business and educational world.

As one of the largest smartphone makers in the world, Nokia seems to have some great potential coming in its horizon. Although the company is going up against other companies, such as Apple, in the highly competitive smartphone market, Nokia’s new Lumia 900 Windows smartphone has been gaining interest. The phone, set to be release
later this year, is said to be more affordable, putting the product in a better position when going up against other Android phones and the iPhone.[33] Additionally, Nokia may be launching a new Windows 8 tablet later this year.

Lastly from this sector, EMC Corporation was chosen due to the visible leadership in storage solutions. The company develops, delivers, and supports information infrastructure and virtual infrastructure technologies and solutions.[19]

1.2 Services Sector

The services sector is one where most people have interaction with in one way or another, whether it is from entertainment or consumer service companies. Netflix was chosen due to the damaging effect the company’s decision had on the stock and value of the company when it split into Qwikster and Netflix in 2011. By forcing their consumers to pay two separate fees in order to instantly stream and get DVDs sent through the mail to them, the company lost a lot of its popularity and the value of the company plummeted as a result of this decision. Currently the company’s stock value is in recovery and based on the trend since the beginning of 2012, the value of the company seems to be improving. Netflix CEO Reed Hastings says that he hopes the company can phase out the DVD rental service, which would be an interesting development in the company when this plan is put into play. The company has seen a sharp increase in the number of new subscribers streaming online, as well as an increase in the amount of streaming that old subscribers have been doing.[31]

With dozens of companies and subsidiaries ranging from movies and music, to television and parks, the Walt Disney Company is seen as a “right of passage” for kids and young adults alike. The company prides itself on keeping the magic alive and is equally as strict on its employees to provide the best experience to all whenever they encounter anything Disney related. Additionally, the company will soon gain even more appeal once the expansion of FantasyLand in Walt Disney World’s Magic Kingdom opens in late fall 2012, as well as other expansions to portions of their parks throughout the year. Thus, through personal interest, as well as the stability of the company, Disney was chosen to be included in the portfolio.

Lastly, Visa was chosen as a collaborator to both Disney and JPMorgan Chase & Co.. The company is widely popular when it comes to credit cards and electronic payment networks.

1.3 Financial Sector

Though the economy is seen as in recovery from the market crash in 2008, we thought it would be interesting to include a few companies from the financial district. JPMorgan Chase & Co. has an association with Disney, so the company was added to our portfolio. It also collaborates with Visa in relation to its credit card services. Bank of America was included as a competitor for JPMorgan, as well as to look at the company’s stock market behavior during the current foreign economic crisis.

1.4 Basic Materials Sector

Exxon-Mobil was chosen due to its natural demand in the economy. In 2011, the United States’ oil imports were reported as approximately 8.6 million barrels per day, showcasing the importance of oil sales, as well as the country’s foreign dependency.[22] Exxon participates in exploration, production, transportation, and sale of crude oil and
natural gas.[62] Moreover, the European debt crisis has been making its presence known in the rising oil and gas prices, which will be discuss next.

1.5 Commodities (Precious Metals and Energy) Category

Since the United States has such a foreign dependency on certain commodities, events taking place in the economies of these foreign countries are sure to have an effect on how these commodities perform in the U.S. market. In particular, fuel prices have been increasing these past few months in response to the foreign debt crises in Europe, and with a rise in prices come a decline in demand. While the prices are said to eventually stabilize, we wanted to see how oil is affected in the markets.[24] Thus we decided to include the exchange-traded note (ETN) in relation to the Goldman Sachs Crude Oil Return index (OIL) in our portfolio.

The value of currency is a big problem in the current economy. With the current debt crisis, we thought it would be interesting to look at other currencies, specifically gold. Since gold is reported to be outperforming the market in terms of returns, we choose to add Standard and Poor’s Depositary Receipts (SPDR) Gold Shares to our portfolio. SPDR Gold Shares is the largest backed gold exchange traded fund (ETF) in the world.[52] Whether in bullion or ETF form, owning gold in some form seems to be a decent investment. It has achieved an inflation-adjusted annualized return of 16.3% in the past 10-year period.[18] While the size of that return normally would raise suspicions because of the exceptionally high return, we are only looking at a short-term investment and wanted to see the response from currency in the market as the crisis plays out.

1.6 Option Strategies

The seven option strategies that make up 15% of our portfolio span five different types: straddle, short call ladder, bull call spread, bear put spread, and strip straddle. Table 3 below displays the information of what underlying assets were used, whether to buy or sell, type of option, strike, and maturity of each option.

<table>
<thead>
<tr>
<th>Option Strategy</th>
<th>Underlying Asset Symbol</th>
<th>Buy/Sell</th>
<th>Put/Call</th>
<th>Strike ($ U.S.D.)</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straddle</td>
<td>AAPL</td>
<td>Buy</td>
<td>Call</td>
<td>560</td>
<td>July 2012</td>
</tr>
<tr>
<td>Short Call Ladder</td>
<td>BAC</td>
<td>Sell</td>
<td>Call</td>
<td>8</td>
<td>August 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Buy</td>
<td></td>
<td>9</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Buy</td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Bull Call Spread</td>
<td>EMC</td>
<td>Buy</td>
<td>Call</td>
<td>28</td>
<td>October 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell</td>
<td></td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Bear Put Spread</td>
<td>GLD</td>
<td>Buy</td>
<td>Put</td>
<td>167</td>
<td>September 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell</td>
<td></td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>Bear Put Spread</td>
<td>IBM</td>
<td>Buy</td>
<td>Put</td>
<td>190</td>
<td>July 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell</td>
<td></td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>Strip Straddle</td>
<td>NFLX</td>
<td>Buy</td>
<td>1 Call</td>
<td>105</td>
<td>June 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 Puts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bull Call Spread</td>
<td>NOK</td>
<td>Buy</td>
<td>Call</td>
<td>6</td>
<td>October 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell</td>
<td></td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Option Strategies

The option strategy explanations are going to be in terms of making one contract of the strategy. However, options are bought in lots of 100 shares/contracts. The numbers of lots
should be multiplied by the profit/loss reported in this section in order to get the total potential profit/loss of the entire option position.

### 2.1.1 Apple Straddle

A straddle is defined as an option strategy where the investor holds a position in both a call and put with the same strike price and expiration date. As a risky strategy overall, it is a good position to engage in if there is a belief that the stock’s price will move significantly, either up or down. In order to make a profit, the stock’s price needs to make a considerable move, with the potential to make an unlimited profit if the price goes up and a limited profit if the price goes to zero.[42] Relative to the breakeven points (BEP)\(^2\), we have the following:

\[
\begin{align*}
\text{Upside Profit at Expiration} &= (S_T - \text{total premium paid}) - K, \\
given that S_T \text{ is above the BEP} \\
\text{Downside Profit at Expiration} &= K - (S_T + \text{total premium paid}), \\
given that S_T \text{ is below the BEP},
\end{align*}
\]

where

\[ S_T \] is the stock's price at maturity, and \\
\[ K \] is the strike price.

**Equation 1: Upside and Downside Profit at Maturity for a Straddle Position**

A small change in price, whether increasing or decreasing, results in a loss, but the potential loss is limited at the sum of the call and put premiums paid.[42]

Since a straddle is risky by nature, we chose Apple as the underlying asset for this strategy. Apple is an American multinational corporation that is involved in designing and selling consumer electronics, computer software, and personal computers. The company has a wide range of best selling products, such as Mac computers, iPhone, iPod, and iPad, popular operating systems, such as MAC OS X and iOS, and creative software, such as iTunes and the App Store. Overall the company is a model for substantial success, outputting statistics like the following:

- App Store breaks the 500,000 apps barrier in only 4 years.[61]
- App Store hits 25 billion app downloads barrier.[15]
- The most recent quarter reports that iPhone sales rose 128%, iPad sales rose 111%, and Mac sales rose 26% compared to the previous year.[27]
- Since the beginning of 2012, Apple shares have risen approximately 30%.[29]

While the company has been exceeding expectations, we do not know how much more of an increase the price will make. However, there is still plenty of opportunity for growth to happen that will only benefit the company. With the recent launch of the awaited new iPad and Apple TV, the market value of Apple is approximately $505.5 billion.[50] Only five other companies in history have ever crossed this $500 billion threshold, them being CISCO, Microsoft, General Electric, Intel, and Exxon.[29] That is not to say that Apple completely overpowers their competition, since there are new tablets, ultra books, Windows 8, smartphones, and other products that prove to be running in this technological race. For the time being, we feel that investing in a straddle

---

\(^2\) There are two breakeven points: 1. Strike Price+ sum of call and put premiums; 2. Strike Price – sum of call and put premiums.
strategy with Apple is a wise position to pursue, especially since the 52-week range of the stock price is $310.50 – $548.21. The last year has definitely seen a significant change in price. Thus, the straddle strategy taken with Apple per contract is the following:

- Long 1 Apple July 2012 call option with a strike price of $560 at $31.25
- Long 1 Apple July 2012 put option with a strike price of $560 at $45.70

So, for every option position bought, an investment of $76.95 is taken. At expiry, if Apple’s stock price is outside of the range $483.05 to $636.95, we will make money, and if it is exactly $560, we will lose $76.95.

![Total payoff profile](image)

Figure 1: Total Payoff Profile for Apple Long Straddle when $S_T = K$

The number of lots bought for Apple will be discussed later when determining the weights that each option has on our overall portfolio.

### 2.1.2 Bank of America Short Call Ladder

A short call ladder is defined as an option strategy where the investor buys an additional highest strike price call option on a bear call spread. A bear call spread is an option strategy where the investor sells a call option at a specified strike price and buys the same number of call options at a higher strike price.[4] The short call ladder overall includes longing an at-the-money call option while at the same time buying two out-of-the-money call options, both with higher strike prices than the call option originally sold. Typically a strategy such as this is used with an underlying stock that is either expected to stay stationary, decrease slightly, or perform badly. The maximum profit is unlimited and the smallest profit that can be made is the net credit of making the position. The maximum loss is limited, equaling the following:

\[
\text{Maximum Loss} = \text{Lower Long Call Strike} - \text{Short Call Strike} - \text{Net Credit},
\]

Equation 2: Maximum Loss for a Short Call Ladder

where the net credit is equal to the amount gained by entering the position.[51]
Since a short call ladder is meant for underperforming stocks, we chose Bank of America for the underlying asset of the position. Bank of America is an American multinational banking and financial service corporation that consists of six major business segments. These segments are Deposits, Global Card Services, Home Loans and Insurance, Global Commercial Banking and Markets, and Global Wealth and Investment Management.[3] With the 2008 Credit Crunch and default on mortgage-backed securities came a less than desirable hit to Bank of America as a company. The company’s share price reached its lowest of $2.53 on February 20, 2009 and has been slow to recover as time progresses. In attempt to drive the stock price up, Warren Buffet purchased $5 billion preferred shares in August 2011, but instead shares further dropped by 28%.[23] The price seemed to start to recuperate in October 2011, but then again declined and fell below $5 on December 19, 2011. At the moment Bank of America has performed relatively well compared to the previous year, currently staying above $7.50 for the past few weeks. The company has recently reduced principal for up to 200,000 homeowners, which could potentially help the company recover.[47] However, at the same time the company has had a $1.4 billion setback when the advising team from Merrill Lynch left in early March.[59] Furthermore, Bank of America faces an additional $2.34 billion in litigation expenses after taxes, equating 10.9% of the $21.455 billion expected earnings for the 2012-2013 year.[2] At this point in time, Bank of America has been facing many charges and penalties, leading us to believe that the stock price of the company will either have a low performance or remain the same during this year. Thus the short call ladder strategy taken with Bank of America per contract is the following:

- Short 1 Bank of America August 2012 call option with a strike price of $8 at $0.88
- Long 1 Bank of America August 2012 call option with a strike price of $9 at $0.47
- Long 1 Bank of America August 2012 call option with a strike price of $10 at $0.24

So, for every option position bought, an investment of –$0.18 is taken. Therefore at expiry the minimum profit is $0.18 and the maximum loss is $0.82 [9-$8-$0.18]. The option payoff diagrams shown next are the payoffs if the underlying stock price is equal to each of the strike prices of the options.

![Total payoff profile](image)

**Figure 2: Total Payoff for Bank of America Short Call Ladder when $S_T = K = 9**
The number of lots bought for Bank of America will be discussed later when determining the weights that each option has on our overall portfolio.

2.1.3 EMC Corporation Bull Call Spread

A bull call spread is defined as an option strategy where the investor buys a call option at a specified strike price and sells a call option with the same maturity at a higher strike price.[13] Typically this strategy is used when the investor expects there to be a reasonable increase in the stock price of the underlying asset. The maximum profit is the following:

\[
\text{Maximum Profit} = (\text{Strike Price of Short Call} - \text{Strike Price of Long Call}) - \text{Net Debit},
\]

Equation 3: Maximum Profit for a Bull Call Spread

where the net debit is equal to the amount invested by entering into the position. There is a limited loss in this strategy equal to the net debit.[14]

Since a bull call spread is to be used for stocks that are expected to rise, but not significantly, we chose EMC Corporation as the underlying asset of this position. As previously stated, EMC Corporation develops, delivers, and supports the information and virtual infrastructure technologies and solutions. The company has a big presence in enterprise storage systems and software, as well as information security solutions in various areas.[19] In a world where a lot of data is created, EMC has given these creators a place to store it. Over the years EMC has proved to be a successful and strong company, progressing ahead with its innovative solutions. Its most current fastest growing products are FAST and Cloud Computing, a storage product and a data management product, respectively. Some noteworthy achievements that EMC has had are the following:

- Quarterly revenue in 2011 increased 18% to $5 billion and earnings per share (EPS) jumped 23% to $0.36.
- Revenue from VMware, majorly owned by EMC, increased by 32%.[39]
- EMC had a record-stomping fourth-quarter earnings and another EPS jump of 17% to $0.49.
- The close of the 2011 fiscal year reported a $20 billion figure for total sales, which is 18% more than the previous year.[38]

EMC Corporation is forecasted to have sales around the $22 billion mark this 2012 year, which is a steady amount of growth from one year to the next.[38] The company has already started the year with a 33% gain so far and we think that it will have a small pull back followed by a steady, slow growth stream. Thus the bull call spread strategy taken with EMC Corporation per contract is the following:

- Long 1 EMC Corporation October 2012 call option with a strike price of $28 at $2.99
- Short 1 EMC Corporation October 2012 call option with a strike price of $33 at $0.87

Thus for every option position bought, an investment of $2.12 is taken. Therefore at expiry the maximum profit $2.88 [($33-$28)-2.12] and the maximum loss is $2.12.
The number of lots bought for EMC Corporation will be discussed later when determining the weights that each option has in our overall portfolio.

2.1.4 SPDR Gold Shares Bear Put Spread

A bear put spread is defined as an option strategy where the investor buys a put at a specified strike price and sells a put at the same maturity with a lower strike price than the one bought.[5] Typically this strategy is used when the investor expects that the underlying stock price will decrease moderately in the near future. The maximum profit for this position is the following:

$$\text{Maximum Profit} = (\text{Strike Price of Long Put} - \text{Strike Price of Short Put}) - \text{Net Debit},$$

Equation 4: Maximum Profit for a Bear Put Spread.

where the net debit is equal to the amount invested by entering into the position. There is a maximum loss equal to the net debit.[6]

Since a bear put spread is used for stocks that are expected to slightly decline, we chose to have SPDR Gold Shares as the underlying asset for this position. SPDR Gold Shares, an investment trust, is an exchange-traded fund (ETF) that tracks the gold commodity like an index, but trades like a stock on an exchange.[21] The main objective of the trust is for the performance of the gold bullion price to be reflected by the shares.[46] Many investors in 2010 rushed to use gold as a hedge due to the financial disaster in Europe, specifically European Union nations Greece and Ireland, who were in danger of defaulting on their debt. Most governments responded to the crises by printing money, which forced the paper currencies to decline in value, causing gold to increase in value. In the past decade gold prices increased 400% overall, with a record-breaking 26% increase, as well as an intraday high of $1,637.50 an ounce, in 2010. The price of gold had been unpredictable for the 2011 year; however, with the commodity being hit with
double-digit sell-offs and rallies. Investors were jumping back and forth with selling their ownership in gold to earn a profit, but then coming back to gold when the political issues in the Middle East and North Africa arose, as well as the environmental disasters that Japan faced.[53] As for 2012, gold started to increase in value again, making an 11% increase in January alone. This particular increase was mainly the result of the announcement by the Federal Reserve to keep the interest rates low until late 2014.[54] Despite the current rise in gold, there is a great deal of skepticism about economic recovery, though the President and his advisors are trying to convey a robust, recovery economy, and gold has been experiencing a volatile market. In the event that the market continues to improve, there is a good chance that money will flow into markets and bring down the value of gold. Hence gold looks highly vulnerable to some sort of price correction in 2012, leading us to think that gold is entering a bearish trend for 2012. Thus the bear put spread strategy taken with SPDR Gold *per contract* is the following:

- **Long 1 SPDR Gold September 2012 put option** with a strike price of $167 at $9.55
- **Short 1 SPDR Gold September 2012 put option** with a strike price of $166 at $9.20

So, for every option position bought, an investment of $0.35 is taken. Therefore, at expiry, the maximum profit will be $0.65 [($167-$166)-$0.35] and the maximum loss is $0.35.

![Total Payoff Profile](image)

*Figure 4: Total Payoff for SPDR Gold Shares Bear Put Spread with S_T = K = 166*

The number of lots bought for SPDR Gold will be discussed later when determining the weights that each option has on our overall portfolio.

### 2.1.5 International Business Machines Bear Put Spread

As previously stated in 2.1.4, a bear put spread is defined as an option strategy where the investor buys a put at a specified strike price and sells a put at the same maturity with a lower strike price than the one bought.[5] Since a bear put spread is used for stocks that are expected to slightly decline, we chose to have International Business...
Machines (IBM) as the underlying asset for this position. IBM provides information technology products and services worldwide and consists of five major segments. These segments are Global Technology Services, Global Business Services, Software, Systems and Technology, and Global Financing.[44] Overall we think that IBM has proved many times that it has a very smart business model and is a strong company. The company made some smart business moves, especially with the merger between Netezza Corporation, a leading provider of high-performance analytics in data warehousing. Since the merger with IBM, Netezza has seen a 40% growth, while IBM benefits with helping its clients gain a more comprehensive sight into their business information.[34] IBM’s share price has also gained 22% over the past year and 122% over a two-year prior.[30] Additionally, not only has IBM climbed 9% in 2012, but also for the first time its 100-year history, it has reached a closing price of over $200 per share.[28] IBM’s super computer WATSON, used for analyzing customer, financial, and economic data, has already made its mark in the healthcare industry and it is predicted to add a few billion dollars of revenue and have a significant positive impact on the EPS by 2015.[60] Despite these favorable statistics, while the price of IBM stock is generally increasing, it has been experiencing periods of consistent up and down movement in the last 6 to 8 months, leading us to believe that with the current upward trend occurring in these first weeks of March, a slight decrease is anticipated in a few months. Thus the bear put spread strategy taken with IBM per contract is the following:

- Long 1 IBM July 2012 put option with a strike price of $190 at $5.60
- Short 1 IBM July 2012 put option with a strike price of $180 at $3.55

Thus for every option position bought, an investment of $2.05 is made. Therefore at expiry, the maximum profit will be $7.95 [($190-$180)-$2.05] and the maximum loss is $2.05.

![Total Payoff Profile](image)

**Figure 5: Total Payoff for IBM Bear Put Spread when \( S_T = K = 180 \)**

The number of lots bought for IBM will be discussed later when determining the weights that each option has on our overall portfolio.
2.1.6 Netflix Strip Straddle

A strip straddle is defined as an option strategy where the investor buys more put options than call options at the same strike price. Typically this strategy is used for an underlying asset whose price change is uncertain, but thought to be bearish. The main difference between a strip straddle and a long straddle is that a strip straddle has a higher upside profit and a lower downside profit compared to a regular straddle, due to the extra put option bought.[56] In order to make a profit the stock’s price needs to make a move relative to its two breakeven points, much like with a regular straddle. In this case the lower breakeven point is closer to the strike price than the upper breakeven point, which is not the case with a regular straddle, thus the different between the potential profits that can be made. In this case the breakeven points are the following:

\[
\begin{align*}
\text{Upper Breakeven Point} &= K + \text{Net Debit} \\
\text{Lower Breakeven Point} &= K - \frac{\text{Net Debit}}{\left(\frac{\#\text{ of Put Options}}{\#\text{ of Call Options}}\right)},
\end{align*}
\]

where

\[K\] is the strike price of the options,

Equation 5: Upper and Lower Breakeven Points for a Strip Straddle.

and net debit is equal to the amount invested by taking the position. The maximum profit is unlimited as long as the stock price continues to move in one direction. The downside profit then follows as:

\[
\text{Downside Profit} = \begin{cases} 
|K - S_T| * \#\text{ of Call Options} - \text{Net Debit}, & \text{if } S_T > K \\
|K - S_T| * \#\text{ of Put Options} - \text{Net Debit}, & \text{if } S_T < K'
\end{cases}
\]

where

\[S_T\] is the stock’s price at maturity,

\[K\] is the strike price, and

\[|K - S_T|\] indicates the difference between the stock price and the strike price.

Equation 6: Downside Profit for a Strip Straddle.

Similar to a regular straddle, the maximum loss is equal to the net debit.[56]

Since a strip straddle gives the versatility for both an upward or downward price movement, we chose to use Netflix as the underlying asset of this position. Netflix is an Internet subscription service consisting of three major segments, which are Domestic Streaming, International Streaming, and Domestic DVD. The service allows its subscribers to watch unlimited television shows and movies over the Internet.[45] The 2011 downfall of Netflix was caused by the CEO’s decision to rebrand Netflix’s DVDs service as Qwikster, charging twice as much for those who wanted the option to both receive DVDs by mail and through streaming.[36] It goes without saying that this decision did not sit well with their subscribers. Netflix lost about 800,000 subscribers in September 2011, followed by the stock price plummeting by 27% in a day.[37] The stock price tumbled down over $200 in the months that followed, closing at prices under $100 when it was formerly around $300. Despite the serious damage this decision has done to the company, Netflix was able to come out with a strong fourth quarter, and has been performing better than expected. By the end of January, shares increased 22% compared to the beginning of the year, a likely result from the increase of subscribers to the
streaming portion of the company.[26] Netflix is taking aggressive moves to make a comeback in the market, such as making deals with Apple TV, but its expenses for content, international expansion, and brand marketing continue to extensively rise.[43] The 52-week range of the stock price is $62.37 – $304.79 and is currently trading around $110. While we feel that the Netflix is more bearish, as well as has a cash flow problem, it could really move in either direction. Thus the strip straddle strategy taken with Netflix per contract is the following:

- Long 1 Netflix June 2012 call option with a strike price of $105 at $16.50
- Long 2 Netflix June 2012 put options with a strike price of $105 at $12.40 each

Thus for every option position bought, an investment of $41.30 is made. At expiry if Netflix’s stock price is outside of the range $84.35 to $146.30, we will make an unlimited profit. We will, in fact, make an even larger profit if the price goes below $84.35 than if we used a long straddle strategy instead of a strip straddle. If the price is exactly $560, we will lose $41.30.

![Total payoff profile](image)

Figure 6: Total Payoff for Netflix Strip Straddle when $S_T = K$

The number of lots bought for Netflix will be discussed later when determining the weights that each option has on our overall portfolio.

2.1.7 Nokia Bull Call Spread

As previously stated in 2.1.3, a bull call spread is defined as an option strategy where the investor buys a call option at a specified strike price and sells a call option with the same maturity at a higher strike price.[13] Since a bull call spread is used for stocks that are expected to have a reasonable increase in the stock price, we chose to use Nokia for the underlying asset for this position. Nokia provides telecommunications infrastructure hardware, software, and services worldwide. It manufactures mobile electronic devices, mostly cell phones and other devices related to uniting the communication and Internet industries.[41] In the current technological race, Nokia is facing a bit of trouble lately due to the competitive market in smartphones and communication devices. Nokia holds 38% of the world market for handheld mobile devices.
devices, making it the world’s largest distributor with respect to this category. After being recently downgraded to underperform from perform by Oppenheimer, a financial services company, Nokia has started to make moves to place itself in a better position for the future.[35] The launch of Nokia’s new smartphone, Lumia, offers several different models of the phone at various price points to give consumers the opportunity to still buy the product, but at a price range they are comfortable with. Additionally, the company is working on adapting or re-writing ClearView software to be compatible with Windows phones. Nokia is said to have the best smartphone cameras and already has a presence in the Apps market. The possibility of integrating Skype, now owned by Microsoft, in Windows phones also gives an additional leg up in the phone market. When Nokia comes out with a phone that incorporates all these types of improvements and features, such as Windows 8, Skype, a great processor, etc., it will be one of best phones on the market that spans several categories.[32] The potential that Nokia has in the market and their aggressive moves to improving their technology lead us to believe that it will be a good long-term investment. Though we are making a short-term investment, we think that the stock price will increase over the period of our investment. Thus the bull call spread strategy taken with Nokia per contract is the following:

- **Long** 1 Nokia October 2012 call option with a strike price of $6 at $0.41
- **Short** 1 Nokia October 2012 call option with a strike price of $7 at $0.24

Thus for every option position bought, an investment of $0.17 is made. Therefore at expiry the maximum profit is $0.83 [($7-$6)-$0.17] and the maximum loss is $0.17. By October we do not think that the stock price will reach $7, but even if it crosses $6, we would still make a profit by exercising the long call options and buying back the shorted options.

![Figure 7: Total Payoff for Nokia Bull Call Spread when \( S_T = K = 6 \)](image)

The number of lots bought for Nokia will be discussed later when determining the weights that each option has on our overall portfolio.
NOTE: All of these profit and loss calculations are based on the option data taken on Friday, March 9, 2012. When the options are bought when the market opens Monday morning, there will possibly be a slight change in the price at which the options are purchased.

NOTE: The option payoff diagrams were constructed from an EXCEL download obtained online and created by Andreas Emmert. It is stated as being free to use for academic purposes. [20] The EXCEL file is also located in the folder submitted with this project and is titled “optionpayoff.xls”.

1.7 Stocks and Risk-free assets

There are eleven stocks to which we decided to invest 35% of our total portfolio and ten risk-free assets to which we decided to invest 50% of our total portfolio. Recall from Table 2 the chosen stocks and risk-free assets. The reasons for choosing these stocks in general were discussed in Section 1.1 through Section 1.5, but there was a different process in how we chose the risk-free assets. We invested in U.S. Treasury bills and notes, which are considered virtually risk-free since it is improbable that the government will default and not be able to pay back the debt when the assets reach expiry. In other words, 50% of our portfolio is essentially invested in cash. Bills and notes were specifically chosen since they are more compatible for short-term investments compared to long-term investments where we would more likely invest in a bond. The bills and notes we invested in were chosen such that there maturity dates coincide with the maturity dates of our options. The specific U.S. Treasuries are the following:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Type</th>
<th>Underlying</th>
<th>Issue Date</th>
<th>Maturity Date</th>
<th>CUISP</th>
<th>Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States Treasury</td>
<td>Bill</td>
<td>US-T</td>
<td>June 30, 2011</td>
<td>June 2012</td>
<td>IBCID90377392</td>
<td>No Coupon</td>
</tr>
<tr>
<td>United States Treasury</td>
<td>Bill</td>
<td>US-T</td>
<td>September 22, 2011</td>
<td>September 2012</td>
<td>IBCID94979476</td>
<td>No Coupon</td>
</tr>
<tr>
<td>United States Treasury</td>
<td>Bill</td>
<td>US-T</td>
<td>October 20, 2011</td>
<td>October 2012</td>
<td>IBCID96069228</td>
<td>No Coupon</td>
</tr>
<tr>
<td>United States Treasury</td>
<td>Note</td>
<td>US-T</td>
<td>June 15, 2009</td>
<td>June 2012</td>
<td>IBCID60826954</td>
<td>1.875</td>
</tr>
<tr>
<td>United States Treasury</td>
<td>Note</td>
<td>US-T</td>
<td>July 15, 2009</td>
<td>July 2012</td>
<td>IBCID61610100</td>
<td>1.5</td>
</tr>
<tr>
<td>United States Treasury</td>
<td>Note</td>
<td>US-T</td>
<td>September 15, 2009</td>
<td>September 2012</td>
<td>IBCID68471564</td>
<td>1.375</td>
</tr>
<tr>
<td>United States Treasury</td>
<td>Note</td>
<td>US-T</td>
<td>October 15, 2009</td>
<td>October 2012</td>
<td>IBCID69322894</td>
<td>1.375</td>
</tr>
</tbody>
</table>

Table 5: U.S. Treasury Bills and Notes
In the event that our option positions are exercised against us, we will be able to pay the payoff of the options with the money invested in the treasuries that expire that month. Each month of expiration for an option – June, July, August, September, and October – has a Treasury bill and a Treasury note that each reach expiry in the same month. The weights for the stocks and treasuries, as well as the option positions, will be discussed next.

1.8 Determining the Weights of the Portfolio

The performance of our portfolio was not only dependent on the types of stocks and options we invested in, but how heavily each stock and underlying asset of the options contributed to the portfolio as a whole. Out of the allotted $1,000,000 given to invest, $500,000 was distributed to risk-free assets and $500,000 was distributed to stocks and options. After much consideration, the stock and option portion of our investment consisted of 70% stocks and 30% options. In other words, 50% of our total portfolio consisted of risk-free assets, 35% of our total portfolio consisted of stocks, and 15% of our total portfolio consisted of option positions. The reason for the 70%-30% breakdown was that we suspected it would be easier to track trends of a stock on a day-to-day basis than it would be to determine if an option would be exercised at maturity. The 15% of our total portfolio going to options was redistributed over seven strategy positions, so if options are exercised against us, it would pose a smaller threat to the overall portfolio performance. The overall weights were distributed over all of the securities in each group.

1.8.1 Stock Portfolio

Markowitz portfolio theory essentially revolves around maximizing the expected return while at the same time minimizing the amount of risk taken by making such investments. There are times when these two concepts conflict, since generally the riskier the asset, the higher the expected returns. Thus investors rely on the risk premium, which is the difference between the expected return of a risky asset and the risk-free rate of return, to determine the minimum amount of money they are willing to accept for bearing a higher risk on an asset.[49]

The individual weights of the stock portion of our portfolio were determined using Markowitz’s portfolio theory of using portfolio optimization and the efficient frontier. The efficient frontier are points at which, when graphing risk versus reward, are on a location on the curve that have an expected return that is at least as large as the minimum variance portfolio.3 Those points that are on this efficient frontier are called the efficient portfolios. Efficient portfolios mix the tangency portfolio of n number of risky assets with the risk-free rate, and have the following properties:

- A higher expected return than any other portfolio with the same risk
- A smaller risk than any other portfolio with the same expected return

A line drawn from the risk free asset to the tangency portfolio on the efficient frontier indicates the possible values of risk and rate of return that can occur with an overall portfolio consisting of these both. The slope of this line is called Sharpe’s ratio, or the “reward to risk” ratio, and is calculated in the following way:

---

[49] The minimum variance portfolio is the leftmost point of the curve and achieves the minimum value of the risk.
\[
\frac{\mu_M - \mu_f}{\sigma_M},
\]

where
\(\mu_M\) is the expected portfolio return, 
\(\mu_f\) is the risk-free rate, and 
\(\sigma_M\) is the portfolio volatility.  

**Equation 7: Sharpe’s Ratio**

The larger Sharpe’s ratio is, the higher the expected return for a given level of risk. The portfolios that mix the tangency portfolio with the risk-free asset have to maximal Sharpe’s ratio. \[49\]

The return for an optimal portfolio combining the tangency portfolio and the risk-free asset is as follows:

\[
R = \omega R_T + (1 - \omega) \mu_f,
\]

where
\(\omega\) is the weight allocated to the tangency portfolio,
\((1 - \omega)\) is the weight allocated to the risk-free asset,
\(R_T\) is the return of the tangency portfolio, and
\(\mu_f\) is the risk-free rate of return.  

**Equation 8: Optimal Portfolio Return Formula**

Since we want to manage the risk of the portfolio, this optimal portfolio concept was used to calculate the optimal weights for each stock. All returns that were calculated in relation to the portfolio optimization are referring to log returns. In other words,

\[
R = \ln \left( \frac{P_t}{P_{t-1}} \right),
\]

where
\(P_t\) is the closing price of the asset at time \(t\).  

**Equation 9: Log Return Formula**

1.8.1.1 Determining the Historical Time Period and Risk-free Rate for Tangency Calculation

Portfolio theory makes the big assumption that asset returns are stationary, which needs to be taken into careful consideration when choosing the historical time period to use to calibrate a portfolio optimization model. Looking back over a short period leaves too few data points. We thought a reasonable period to look over would be 3 months, therefore making the period going from December 5, 2011 to March 5, 2012 of weekly data. The weekly data used for the assets can be found in the Excel document titled “OptimizationData.xlsx,” and all data was downloaded from Yahoo! Finance. \[63\] We felt that this time period was reasonable because it takes into account not only more recent events happening at the moment, such as the economic crisis going on in Europe, but also captures the movements that the market has been experiencing this year. Mid to late 2011 experienced high volatility in the market, but the current market at the time this portfolio was formed had much lower volatility. Looking at Figure 8, we can see through the VIX changes how market volatility had a significant drop from the beginning of
December onward. Going back any further before December 2011 would include data that does not accurately describe the current market fluctuations.

The risk-free interest rate chosen was 0.09%, which is the interest rate on the most recent report at the time for a 3-Month Weekly Treasury Bill.[1] The data for this interest rate was retrieved from the Federal Reserve Bank of St. Louis Website. We chose to use a rate from a Treasury bill since, as stated before, T-bills are considered the least risky asset to invest in. Specifically, a 3-month weekly bill was chosen in order to match up with the historical time period that we chose to run the optimization on.

1.8.1.2 Determining the Efficient Frontier and Tangency Weights

In order to construct the efficient frontier, we need to calculate the optimal weights for the minimal variance for each of the assets. The optimal weights are calculated in the following way:

$$\mu_{mp} = g + \mu_p h,$$

where

$$g = \frac{B\Omega^{-1}1 - A\Omega^{-1}\mu}{D}, \quad h = \frac{C\Omega^{-1}\mu - A\Omega^{-1}1}{D},$$

$\mu_p$ is the target expected return,
$\mu$ is the expected return of the asset $j$, and
$\Omega$ is the covariance matrix between the assets.

The covariance matrix between the assets was calculated using Excel function COVARIANCE.S. Note that the target expected return can be varied over some range of values, enabling a locus $\omega_{\mu_p}$ of efficient portfolios, which, as previously stated, are called the efficient frontier.[49] The above equation is a simplified notation for obtaining the optimal weights from Ruppert’s Statistic and Finance. The coefficients A, B, C, and D are calculated in the following way:

$$A = 1^T\Omega^{-1}\mu, \quad B = \mu^T\Omega^{-1}\mu, \quad C = 1^T\Omega^{-1}1, \quad D = BC - A^2.$$
The efficient frontier is constructed on the constraints that $\omega^T \mathbf{1} = 1$ (a fully invested portfolio), and $\mu_p = E(R) = \omega^T \mu$. Then the tangency portfolio was calculated in the following way:

$$\omega_T = \frac{\vec{\omega}}{1^T \vec{\omega}},$$

where

$$\vec{\omega} = \Omega^{-1}(\mu - \mu_f \mathbf{1}),$$

$\mu$ is the expected returns of the assets.

Equation 12: Optimal Tangency Weights

Using these optimization tools and Matlab, the tangency weights for the stock portion of our portfolio were computed. As you can see in Table 6, the weights were readjusted to represent the allotted 70% of our portfolio. We found it interesting that the optimization reports for shorting Apple stock. If anything, we were thinking it would tell us to long Apple stock since it is more probable that the stock price is going to increase. Despite this prediction, we stayed true to the weights. The efficient frontier for our stock portfolio is displayed in Figure 9.

<table>
<thead>
<tr>
<th>Stock Ticker</th>
<th>Tangency Weights 12/5/11-3/5/12</th>
<th>70% of Portfolio Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>-0.3141</td>
<td>-0.21987</td>
</tr>
<tr>
<td>DELL</td>
<td>-0.0729</td>
<td>-0.05103</td>
</tr>
<tr>
<td>DIS</td>
<td>0.1624</td>
<td>0.11368</td>
</tr>
<tr>
<td>XOM</td>
<td>0.4042</td>
<td>0.28294</td>
</tr>
<tr>
<td>GLD</td>
<td>0.6792</td>
<td>0.47544</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.5733</td>
<td>0.40131</td>
</tr>
<tr>
<td>IBM</td>
<td>0.0878</td>
<td>0.06146</td>
</tr>
<tr>
<td>JPM</td>
<td>-0.1532</td>
<td>-0.10724</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.0353</td>
<td>0.02471</td>
</tr>
<tr>
<td>OIL</td>
<td>-0.4389</td>
<td>-0.30723</td>
</tr>
<tr>
<td>V</td>
<td>0.0369</td>
<td>0.02583</td>
</tr>
</tbody>
</table>

Table 6: Tangency Weights of Stocks from Portfolio Optimization

Figure 9: Efficient Frontier from Stock Portfolio

---

4 Refer to Appendix A for the Matlab code used for this optimization.
Based on the weights outputted, the numbers of shares to short and long, as well as the amount to invest per stock, are displayed in Table 7. Note that 35% of the allotted $1,000,000 is $350,000, which is the same as 70% of the $500,000 that we are investing in risky assets.

<table>
<thead>
<tr>
<th>Stock Ticker</th>
<th>70% of Portfolio Weight</th>
<th>Stock Price (3/9/12)</th>
<th>Amount Invested Based on Weight</th>
<th>Number of Shares</th>
<th>Number of Shares to Purchase</th>
<th>Amount to Invest</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>-0.21987</td>
<td>$545.17</td>
<td>$109,935.00</td>
<td>201.6526955</td>
<td>201</td>
<td>$109,579.17</td>
</tr>
<tr>
<td>DELL</td>
<td>-0.05103</td>
<td>$16.93</td>
<td>$25,515.00</td>
<td>1507.088009</td>
<td>1507</td>
<td>$25,513.51</td>
</tr>
<tr>
<td>DIS</td>
<td>0.11368</td>
<td>$42.24</td>
<td>$56,840.00</td>
<td>1345.643939</td>
<td>1345</td>
<td>$56,812.80</td>
</tr>
<tr>
<td>XOM</td>
<td>0.28294</td>
<td>$84.30</td>
<td>$141,470.00</td>
<td>1678.173191</td>
<td>1678</td>
<td>$141,455.40</td>
</tr>
<tr>
<td>GLD</td>
<td>0.47544</td>
<td>$166.38</td>
<td>$237,720.00</td>
<td>1428.777497</td>
<td>1428</td>
<td>$237,590.64</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.40131</td>
<td>$600.25</td>
<td>$200,655.00</td>
<td>334</td>
<td>334</td>
<td>$200,483.50</td>
</tr>
<tr>
<td>IBM</td>
<td>0.06146</td>
<td>$200.62</td>
<td>$30,730.00</td>
<td>153</td>
<td>153</td>
<td>$30,694.86</td>
</tr>
<tr>
<td>JPM</td>
<td>-0.10724</td>
<td>$41.03</td>
<td>$53,620.00</td>
<td>-1306.848647</td>
<td>-1306</td>
<td>-$53,585.18</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.02471</td>
<td>$31.99</td>
<td>$12,355.00</td>
<td>386</td>
<td>386</td>
<td>$12,348.14</td>
</tr>
<tr>
<td>OIL</td>
<td>-0.30723</td>
<td>$27.26</td>
<td>$153,615.00</td>
<td>-5635.179751</td>
<td>-5635</td>
<td>-$153,610.10</td>
</tr>
<tr>
<td>V</td>
<td>0.02583</td>
<td>$117.17</td>
<td>$12,915.00</td>
<td>110</td>
<td>110</td>
<td>$12,888.70</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td></td>
<td>$350,000.00</td>
<td></td>
<td></td>
<td>$349,986.08</td>
</tr>
</tbody>
</table>

Table 7: Stock Investment According to Optimization Weights

The amounts shown above were based on the closing price on Friday, March 9, 2012, and therefore when the market opened on Monday, March 12, there was a slight difference in the exact prices that the stocks were bought at.

1.8.1.3 Stock Positions Taken in Interactive Brokers

As stated, we formed our positions based on the Friday market close price of each stock, inputting them in our Interactive Brokers paper trading account as market orders for Monday morning.[58] Referring to “Market Portfolio Project 1.xlsx”, the tab labeled “Initial Portfolio Calculations” displays how the tangency weights were used to determine the shorting and longing of each stock. In order to avoid going over the $350,000 allotted to the stock positions, the number of stocks that should have been bought or sold into was rounded down to the next largest whole number (i.e. Instead of shorting 201.6526955 shares of Apple, we shorted 201). This would give us a little cushion to the price of the stock when the market opened Monday morning. The exact prices at which the stocks were bought can be seen in the trade report provided labeled as “Activity Statement” in the folder accompanying this project. The following table is a summary of the trade report for stock positions:

<table>
<thead>
<tr>
<th>Stock Ticker</th>
<th>Shares</th>
<th>Low</th>
<th>High</th>
<th>Difference in Prices</th>
<th>Total Amount Invested</th>
<th>New Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>-201</td>
<td>548.49</td>
<td>548.5</td>
<td>0.01</td>
<td>-110,247.50</td>
<td>-0.225672588</td>
</tr>
<tr>
<td>DELL</td>
<td>1507</td>
<td>16.885</td>
<td>16.915</td>
<td>0.03</td>
<td>-25,471.84</td>
<td>-0.052139922</td>
</tr>
<tr>
<td>DIS</td>
<td>1345</td>
<td>42.465</td>
<td>42.475</td>
<td>0.01</td>
<td>57,123.42</td>
<td>0.116929545</td>
</tr>
<tr>
<td>XOM</td>
<td>1678</td>
<td>84.415</td>
<td>84.45</td>
<td>0.035</td>
<td>141,652.37</td>
<td>0.289957205</td>
</tr>
<tr>
<td>GLD</td>
<td>1428</td>
<td>165.365</td>
<td>165.38</td>
<td>0.015</td>
<td>236,143.22</td>
<td>0.483376509</td>
</tr>
</tbody>
</table>
As you can see above, the range of prices at which the stocks were bought or sold is [0,0.16077]. Even with rounding down the share value that we obtained from the portfolio optimization, we still went over the $350,000 by $1,055.45, making stocks 71.8597% of the stock and option part of the portfolio and 35.4365% of our total portfolio. In the last column you can see the new weights that represent what each stock contributes to the portfolio based on how the market orders were executed. There is a minimal difference between the new weights and the target weights we were aiming for.

NOTE: When re-running the optimization to get the tangency weights, an unknown error occurred that was not caught until all calculations and positions were entered into Interactive Brokers. Table 9 and Figure 10 below display what the tangency weights should have been and the deviation from the tangency weights that we used, as well as what the efficient frontier should have been.
<table>
<thead>
<tr>
<th>Stock Ticker</th>
<th>Correct Tangency Weights</th>
<th>Tangency Weights Used</th>
<th>Deviation</th>
<th>Shares To Invest</th>
<th>Amount Should Have Invested</th>
<th>Amount Invested</th>
<th>Deviation to Correct Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>-0.4061</td>
<td>-0.3141</td>
<td>0.092</td>
<td>-259</td>
<td>-$142,060.21</td>
<td>-$110,247.50</td>
<td>-$31,812.71</td>
</tr>
<tr>
<td>DELL</td>
<td>-0.1934</td>
<td>-0.0729</td>
<td>0.1205</td>
<td>-4004</td>
<td>-$67,677.01</td>
<td>-$25,471.84</td>
<td>-$42,205.17</td>
</tr>
<tr>
<td>DIS</td>
<td>0.1536</td>
<td>0.1624</td>
<td>0.0088</td>
<td>1265</td>
<td>$53,725.74</td>
<td>$57,123.42</td>
<td>$3,397.68</td>
</tr>
<tr>
<td>XOM</td>
<td>0.3162</td>
<td>0.4042</td>
<td>0.088</td>
<td>1310</td>
<td>$110,586.77</td>
<td>$141,652.37</td>
<td>-$31,065.60</td>
</tr>
<tr>
<td>GLD</td>
<td>0.7107</td>
<td>0.6792</td>
<td>0.0315</td>
<td>1504</td>
<td>$248,711.07</td>
<td>$236,143.22</td>
<td>$12,567.85</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.5841</td>
<td>0.5733</td>
<td>0.0108</td>
<td>340</td>
<td>$204,366.90</td>
<td>$200,760.43</td>
<td>$3,606.47</td>
</tr>
<tr>
<td>IBM</td>
<td>0.2008</td>
<td>0.0878</td>
<td>0.113</td>
<td>349</td>
<td>$70,143.23</td>
<td>$30,750.47</td>
<td>$39,392.76</td>
</tr>
<tr>
<td>JPM</td>
<td>-0.0359</td>
<td>-0.1532</td>
<td>0.1173</td>
<td>-307</td>
<td>-$12,560.91</td>
<td>-$53,434.99</td>
<td>$40,874.09</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.0319</td>
<td>0.0353</td>
<td>0.0034</td>
<td>349</td>
<td>$11,155.79</td>
<td>$12,338.49</td>
<td>-$1,182.71</td>
</tr>
<tr>
<td>OIL</td>
<td>-0.4514</td>
<td>-0.4389</td>
<td>0.0125</td>
<td>-5878</td>
<td>-$157,971.24</td>
<td>-$151,440.62</td>
<td>-$6,530.62</td>
</tr>
<tr>
<td>V</td>
<td>0.0895</td>
<td>0.0369</td>
<td>0.0526</td>
<td>267</td>
<td>$31,268.13</td>
<td>$12,882.00</td>
<td>$18,386.13</td>
</tr>
</tbody>
</table>

As seen in Table 9, the tangency weights do not have a large deviation from the ones we used for most of the assets. One of the things that we hope will not penalize us with this mistake is the fact that at least whether we shorted or longed the asset stayed consistent, though the real weights should have been heavier. Contrary to the efficient frontier found before, in actuality, our real efficient frontier does have a small selection of inefficient frontiers that did not show up previously. Overall the amount that was supposed to be invested in assets did not have a huge deviation from the amount we actually invested, though the deviation in the amount of some of the assets was high.

### 1.8.2 Option Portfolio

The option weights were determined differently than the stock weights. We wanted to have the weights in each underlying asset be equal, but since the weights were going to be redistributed over puts and calls of the same underlying asset, the individual weights of the puts and calls were determined by the net debit (or credit) of making one position of each option strategy. The initial weight for each was assigned to 1/7, and then adjusted to represent 30% of our stock and option portfolio. Because the option strategy taken with Bank of America left us with a net credit, we reinvested that money into the other options. Our target investments for the option strategies were the following:
Contrary to determining the number of shares with the stocks, rounding down the shares left us in a position where we were not investing enough of the allotted amount to options, so we rounded the number of shares up. We still were not using enough of the $150,000, so we increased the number of shares by 25%, which brought us up to potentially investing $145,918. The difference in the two numbers we used as a cushion for when the market opened on Monday morning. We then used the number of lots to determine how the weight would be distributed in each of the individual options strategies. The amount purchased for each position in the strategy was computed by multiplying the number of lots by the price of the put or call and then the weight was determined by dividing that number by $500,000. To illustrate this process we use the information from our straddle position in Apple. Recall that our strategy in Apple was the following:

- Long 1 Apple July 2012 call option with a strike price of $560 at $31.25
- Long 1 Apple July 2012 put option with a strike price of $560 at $45.70

The weight for the call and put position in Apple was calculating by the following way:

\[
Weight\ Call = \frac{(4\ \text{lots} \times 100\ \text{shares}) \times $31.25}{$500,000} = \frac{12,500}{500,000} = 0.025
\]

\[
Weight\ Put = \frac{(4\ \text{lots} \times 100\ \text{shares}) \times $45.70}{$500,000} = \frac{18,280}{500,000} = 0.03656
\]

Equation 13: Example of Option Weight Calculation

Combined, the weight of taking the position of 4 lots in Apple is 0.06156, a deviation of about +0.0187 from our target investment weight. The target weights for each of the options strategies are displayed in Table 11.
The underlying assets that are displayed in green represent call positions, whereas the underlying assets displayed in red are put positions. Table 12 displays the new total target weights per underlying asset for our portfolio, as well as the deviation between the target weight and the equal weights we started with. The calculations for these weights can be found in the Excel file titled “Market Portfolio Project 1.xlsx.” The amounts shown above are based on the closing price on Friday, March 9, 2012, and therefore when the market opens on Monday, these may not be the exact prices that the options are bought at.

### Table 11: Target Weights for Option Strategies

<table>
<thead>
<tr>
<th>Underlying Asset</th>
<th>Total Weight in Underlying</th>
<th>Deviation from Equal Weight</th>
<th>Deviation from Equal Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMC</td>
<td>-0.022272</td>
<td>33</td>
<td>$0.87</td>
</tr>
<tr>
<td>GLD</td>
<td>1.52227</td>
<td>167</td>
<td>$9.55</td>
</tr>
<tr>
<td>GLD</td>
<td>-1.46648</td>
<td>166</td>
<td>$9.20</td>
</tr>
<tr>
<td>IBM</td>
<td>0.14784</td>
<td>190</td>
<td>$5.60</td>
</tr>
<tr>
<td>IBM</td>
<td>-0.09372</td>
<td>180</td>
<td>$3.55</td>
</tr>
<tr>
<td>NFLX</td>
<td>0.0264</td>
<td>105</td>
<td>$16.50</td>
</tr>
<tr>
<td>NFLX</td>
<td>0.03968</td>
<td>105</td>
<td>$12.40</td>
</tr>
<tr>
<td>NFLX</td>
<td>0.129314</td>
<td>6</td>
<td>$0.41</td>
</tr>
<tr>
<td>NFLX</td>
<td>-0.075696</td>
<td>7</td>
<td>$0.24</td>
</tr>
</tbody>
</table>

|                            |                             |                             |                             |                             |                            |
|                            |                             |                             |                             |                             |                             |
|                            |                             |                             |                             |                             |                            |

<table>
<thead>
<tr>
<th>Underlying Asset</th>
<th>Total Weight in Underlying</th>
<th>Deviation from Equal Weight</th>
<th>Deviation from Equal Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFLX</td>
<td>0.291836</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 12: Total Target Option Weights for Each Underlying Asset

### 1.8.2.1 Option Positions Taken in Interactive Brokers

When calculating the weights in Table 11, we computed the positions with the intention of placing a limit order on two of the positions we were going to take just to see if it would be picked up. We randomly picked Bank of America and SPDR Gold Shares and calculated the weights and lots according to the bid price on March 9th instead of the ask price. All other positions were placed in as a market order. The bid price for the position on Bank of America was filled when the market opened on Monday, but the position for SPDR Gold Shares was not picked up for the duration of holding our portfolio. We noticed that the ask price for Gold was floating around $0.80, which was well over the $0.35 cost of implementing the position for Gold on Friday. After two days we did increase the limit order to $0.45 and adjusted the lot size to 476, but the position was never picked up. Since the option position for gold was never picked up, we did not have the original target amount of $145,918 invested. The exact prices at which the options were bought can be seen in the trade report provided labeled as “Activity Statement” in the folder accompanying this project. Table 13 displays a summary of the trade report for actual option positions.
The range of prices at which the options were bought or sold is [0,0.25]. Our option positions make up 28.14023% of the stock and option portfolio and 13.87692% of our total portfolio. The new weights are also displayed which represent what each put and call contributed to the portfolio based on how the market orders and limit orders were executed. Table 14 displays the weights per underling asset for our option strategies and the deviation from the target weights we were looking to use. There is a minimal difference between the new weights and the target weights we were aiming for. Note Bank of America has a negative weight compared to the other option positions, which is due to the positive cash flow we obtained from making the option position. Overall, 49.31326% of our total $990,659.54 investment was invested in stock and option positions. Figure 11 on the next page displays the overall leverage for the stock and option positions taken in our portfolio in respect to the $488,528.54 invested. The other 50.6865% of our investment went to risk-free assets, specifically U.S. Treasury bills and notes whose weights are discussed next.
Figure 11: Stock and Option Investment
1.8.3 Risk-free Portfolio

The U.S. Treasuries that we invested in were carefully chosen so that there would be one bill and one note that mature in each of the months where we had options reaching maturity. Ultimately we invested $500,000 in one of the most liquid securities on the market, second to cash. We had $11,471.46 left that was not invested from allotted amount to the stock and option portion of our portfolio, as well as had the bills and notes contributing to any potential exercising of treasuries that may happen. We decided to equally split up the $500,000 over five bills and five notes, so each treasury carried an investment of $50,000. Basically, each bill and note will represent 10% of the risk-free asset section of our portfolio and 5% of our total portfolio.

1.8.3.1 Treasury Positions Taken in Interactive Brokers

U.S. Treasury Investment

![Pie Chart](Figure 12: U.S. Treasury Investment)

When entering the positions for the treasuries into our Interactive Brokers account, we set up the order so that the bills and notes would have a face value of $50,000 each. When the purchase of these treasuries went through, a quantity of 500 of each bill or note was bought with a $100 face value at prices that ranged from $99.94 to $101.88. Table 16 is a summary of the trade report for the treasury positions:

<table>
<thead>
<tr>
<th>Treasury CUSIP</th>
<th>Type</th>
<th>Maturity Date</th>
<th>Number of Treasuries</th>
<th>Price</th>
<th>Total Amount Invested</th>
<th>New Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBCID90377982</td>
<td>Bill</td>
<td>June 2012</td>
<td>500</td>
<td>$100.003</td>
<td>$50,001.50</td>
<td>0.099578596</td>
</tr>
<tr>
<td>IBCID91854016</td>
<td>Bill</td>
<td>July 2012</td>
<td>500</td>
<td>$99.997</td>
<td>$49,998.50</td>
<td>0.099572621</td>
</tr>
<tr>
<td>IBCID93618000</td>
<td>Bill</td>
<td>August 2012</td>
<td>500</td>
<td>$99.974</td>
<td>$49,987.00</td>
<td>0.099549719</td>
</tr>
<tr>
<td>IBCID94979476</td>
<td>Bill</td>
<td>September 2012</td>
<td>500</td>
<td>$99.975</td>
<td>$49,987.50</td>
<td>0.099550715</td>
</tr>
<tr>
<td>IBCID96069228</td>
<td>Bill</td>
<td>October 2012</td>
<td>500</td>
<td>$99.941</td>
<td>$49,970.50</td>
<td>0.099516859</td>
</tr>
<tr>
<td>IBCID60826954</td>
<td>Note</td>
<td>June 2012</td>
<td>500</td>
<td>$100.497</td>
<td>$50,248.50</td>
<td>0.10000705</td>
</tr>
<tr>
<td>IBCID61610100</td>
<td>Note</td>
<td>July 2012</td>
<td>500</td>
<td>$100.513</td>
<td>$50,256.50</td>
<td>0.100086432</td>
</tr>
<tr>
<td>IBCID15960295</td>
<td>Note</td>
<td>August 2012</td>
<td>500</td>
<td>$101.880</td>
<td>$50,940.00</td>
<td>0.10144763</td>
</tr>
<tr>
<td>IBCID68471564</td>
<td>Note</td>
<td>September 2012</td>
<td>500</td>
<td>$100.692</td>
<td>$50,346.00</td>
<td>0.100264672</td>
</tr>
<tr>
<td>IBCID69322894</td>
<td>Note</td>
<td>October 2012</td>
<td>500</td>
<td>$100.790</td>
<td>$50,395.00</td>
<td>0.100362256</td>
</tr>
</tbody>
</table>

$502,131.00

Table 16: Interactive Brokers Treasury Investment

Because the bills and notes can be bought either at a discount or premium price, the total investment for risk-free assets when a little over the allotted amount by $2,131.
Looking at the portfolio as a whole, readjustment needed to be made to reflect the overall weight of the securities. Dividing the weight associated with the security by the total portfolio amount of $990,659.54 did this adjustment. The leverage for each security taken in the portfolio was the following:

![Total Portfolio Leverage](image)

**Note:** The new weights stated for the stocks, options, and risk-free assets are not a reflection of the total portfolio. The stocks and options weights refer to the $488,528.54 amount invested, which is 49.31% of the total portfolio. The risk-free asset weights refer to the $502,131 amount invested. The next section will readjust these weights to reflect the overall portfolio.


2. Loss Distributions

The distribution of losses from time \( t \) to time \( t + \Delta \) is defined as the portfolio’s loss distribution. The actual loss on a portfolio is often denoted as the following:

\[
L_{t+1} = -[V_{t+1} - V_t],
\]

where

\( V_t \) is the value of the portfolio at time \( t \).

**Equation 14: Formula for Actual Loss**

One of the important things to note when dealing with loss distributions is that a negative loss is considered a profit and a positive loss is considered a loss in the sense of losing money.[7] The part of the loss distribution that will be analyzed is the right tail of positive losses. The mean and variance of the linearized loss will be used to estimate the loss distribution and to determine the value at risk and expected shortfall of our stock and option portfolio. The weekly data used for the calculations are described below.

<table>
<thead>
<tr>
<th>Week Number</th>
<th>Data Span (Week Beginning)</th>
<th>Predicts Week Beginning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>3/9/11-3/5/12</td>
<td>3/12/12</td>
</tr>
<tr>
<td>Week 2</td>
<td>3/14/11-3/12/12</td>
<td>3/19/12</td>
</tr>
<tr>
<td>Week 3</td>
<td>3/21/11-3/19/12</td>
<td>3/26/12</td>
</tr>
<tr>
<td>Week 4</td>
<td>3/28/11-3/26/12</td>
<td>4/2/12</td>
</tr>
<tr>
<td>Week 5</td>
<td>4/4/11-4/2/12</td>
<td>4/9/12</td>
</tr>
<tr>
<td>Week 6</td>
<td>4/11/11-4/9/12</td>
<td>4/16/12</td>
</tr>
<tr>
<td>Week 7</td>
<td>4/18/11-4/16/12</td>
<td>4/23/12</td>
</tr>
</tbody>
</table>

**Table 17: Weekly Data Date Intervals**

2.1 Estimating the Weekly Unconditional Loss Distribution

The weekly unconditional loss distribution was estimated using both the normal distribution and student t’s distribution. In doing these calculations we are assuming stationarity of weekly losses for the assets in our portfolio, which is not necessarily true in reality. The mean and variance for the linearized loss distribution are used to calculate probability density values and cumulative distribution values, as well as in the estimation of the parameters when calculating the value at risk and expected shortfall. In order words, to get an overall risk assessment for the risky part of our portfolio, the expected loss and variances for the stocks and options will to be combined in the end. We first look at the stock portfolio individually modeled by the normal distribution since it is so widely assumed in the financial world that returns come from a normal distribution. Then the stock and options portfolio is combined into one and evaluated under both the normal and student’s t distribution.

2.1.1 Modeling the Value of the Portfolio

In essence we are modeling the value of the portfolio as a function of risk factors, \( Z_t \), denoted by

\[
V_t = f(t, Z_t),
\]

where

\( Z_t \) is a vector of risk factors.

**Equation 15: Value of Portfolio**
It follows that the loss, $L_{t+1}$, can be rewritten in terms of a loss operator, $\ell_{[t]}(\cdot)$. A loss operator maps risk factor changes into losses, and thus we have

$$L_{t+1} = \ell_{[t]}(X_{t+1})$$

Equation 16: Formula for Actual Loss in terms of Risk Factor Changes

The next step is to represent the loss as a linear function of the risk factors. This is where the linearized loss operator comes into play. Since we are assuming stationarity of risk factor changes, we will in fact be calculating the unconditional loss distribution of weekly log returns for our portfolio. The linearized loss operator is defined in the following way:

$$\ell_{[t]}^\Delta(X) = -V_t \omega_t^T X,$$

where

- $V_t$ is the value of the portfolio at time $t$,
- $\omega_t$ is the weight assigned to asset $j$ at time $t$, and
- $X$ are the risk factor changes of asset $j$.

Equation 17: Linearized Loss Operator

The following graphs are visual displays of the log returns for the first and last week our portfolio was held. These graphs span one year, where the end of the year is the specified week our portfolio was being held.

![Graphs of Log Returns for Weeks 1 and 7](image)

Figure 15: Stock Log Returns for Weeks 1 and 7

It can be seen how the log returns of stocks were more volatile a year ago than they were at the start of our portfolio. By the last week our portfolio is held, there are a few spikes of volatility but not nearly as much as last year.\(^5\)

We suppose that $X$ follows a distribution with mean $\mu$ and covariance matrix $\Sigma$.

The portfolio value process is then defined in the following way:

$$V_t = \sum_{i=1}^d \lambda_i S_{t,i} = \sum_{i=1}^d \lambda_i e^{Z_{t,i}},$$

where

- $\lambda_i$ is the number of shares of stock $i$,
- $Z_{t,i}$ is the log price of asset $i$ at time $t$.

Equation 18: Portfolio Valuation for Stocks

---

\(^5\) Refer to Appendix C for graphs of the log returns for Weeks 2-6
We modeled the value of our portfolio in the manner described above. Since the stock prices change in real-time, the weights of the stocks will change as well with respect to our stock portfolio. For example, for the prediction of the distribution for the second week, the stock data collected ran from week beginning March 14, 2011 to March 12, 2012. The data from this time series would result in the expected returns and covariance matrix needed to find the mean linearized loss and variance of the linearized loss. The weights used for this time span would be the weights recalculated based on the Friday, March 16th closing price, as opposed to using the initial weights when the portfolio was created. Each week’s data in modeling the value of the portfolio takes into consideration this change in weight.

2.1.2 Calculating the Mean and Variance of the Normal Linearized Loss of the Stock Portfolio

The normal linearized loss for stocks and options is determined in two separate ways. For stocks the parameters are determined by the first and second moment of the linearized loss operator, while for options, the linearized loss is determined using partial derivatives with respect to the stock price, time, the interest rate, and the volatility. Since we assume that the log returns and the options follow a normal distribution, the sum of normal random variables also follows a normal distribution. The procedure on how to combine the formula for stock loss and option loss is later discussed, but at the moment we look at the individual loss of each stock.

In the normal case the risk factors only consist of the log of the stock prices. Using this factor we determine the risk factor changes, which are denoted in the following way:

\[ X_{t,j} = Z_{t,j} - Z_{t-1,j} = \ln S_{t,j} - \ln S_{t-1,j}, \]

where

\[ S_{t,j} \] is the stock price for asset \( j \) at time \( t \).

Equation 19: Formula for Normal Risk Factor Changes
By Equation 17, the mean linearized loss is the first moment of the linearized loss operator and defined as the following:

\[ E\left[\ell_t^\Delta(X)\right] = -V_t^\top \mu, \]

where

\( \mu \) is the expected log returns of each asset.

Equation 20: Expected Linearized Loss for a Normal Distribution

It also follows from the second moment of the loss operator that the variance for the linearized loss is the following:

\[ Var\left[\ell_t^\Delta(X)\right] = -V_t^2 \omega^\top \Sigma \omega, \]

where

\( \Sigma \) is the covariance matrix between the asset returns.

Equation 21: Variance Linearized Loss for a Normal Distribution

Using Matlab, these estimates were calculated every week. The weekly change in the weights for each stock results in a change to the mean and variance, as well as to the value of the modeled portfolio. The following table displays the mean and variance of the normal linearized loss for the weeks the portfolio was held:

<table>
<thead>
<tr>
<th>Week</th>
<th>Expected Loss (in U.S.D.)</th>
<th>Expected Loss (%)</th>
<th>Standard Deviation (in U.S.D.)</th>
<th>Standard Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>$-639.5316</td>
<td>-0.1647%</td>
<td>$10,947</td>
<td>2.8193%</td>
</tr>
<tr>
<td>Week 2</td>
<td>$-436.4713</td>
<td>-0.1206%</td>
<td>$10,562</td>
<td>2.9193%</td>
</tr>
<tr>
<td>Week 3</td>
<td>$-448.6499</td>
<td>-0.1224%</td>
<td>$10,747</td>
<td>2.9328%</td>
</tr>
<tr>
<td>Week 4</td>
<td>$-463.7327</td>
<td>-0.1216%</td>
<td>$11,091</td>
<td>2.9087%</td>
</tr>
<tr>
<td>Week 5</td>
<td>$-13.4311</td>
<td>-0.0039%</td>
<td>$10,670</td>
<td>3.0747%</td>
</tr>
<tr>
<td>Week 6</td>
<td>$-352.8460</td>
<td>-0.0965%</td>
<td>$10,978</td>
<td>3.0030%</td>
</tr>
<tr>
<td>Week 7</td>
<td>$-476.5262</td>
<td>-0.1366%</td>
<td>$10,264</td>
<td>2.9428%</td>
</tr>
</tbody>
</table>

Table 18: Linearized Mean and Variance for Stocks

Looking at the actual values for the expected loss, it can be concluded that the value of the portfolio decreasing will result in a less negative expected loss and the value of the portfolio increase will result in a larger negative expected loss.\(^6\) With these values we can estimate the loss distribution of the stock portfolio, the value at risk, and the expected shortfall. Keep in mind though that we are looking to estimate future behavior of our portfolio, so the expected loss and variance for Week 1 are the values we will use to hypothesize the distribution for the losses in the first week our portfolio is held. The data for this prediction, though, are estimated based on the previous week’s data. In the end, we want to look at our portfolio as a whole, so the risk values for the entire portfolio are what we aim to analyze. The purpose of making the above calculations for just the stock portfolio will give us a little insight on how our options portfolio affects the expected loss and variance with the change in values it has.

---

\(^6\) Refer to Appendix D: Matlab Code – linearLossDistNormal.m for the Matlab Code used for this calculation.
2.1.3 The Linearized Loss of the Options Portfolio

The linearized loss for options relies on the Black-Scholes option pricing formulas for European put and call options. The value of a European put and call option at time $t$ is the following:

$$
C_{BS}(t,S;r,\sigma,K,T) = SN(d_1) - Ke^{-r(T-t)}N(d_2)
$$

$$
p_{BS}(t,S;r,\sigma,K,T) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1)
$$

where

$$
d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}
$$

$N(\cdot)$ is the cumulative distribution function of the standard normal distribution,

$T - t$ is the time until maturity,

$S$ is the price of the underlying asset,

$r$ is the continuously compounded risk-free rate,

$\sigma$ is the volatility of returns of the underlying asset, and

$K$ is the strike price.

Equation 22: Black-Scholes Option Pricing Formulas

The risk factor changes for the options are

$$
X_{t+1} = \left[\ln S_{t+1} - \ln S_t, \frac{r_{t+1} - r_t}{\sigma_{t+1} - \sigma_t}\right],
$$

Equation 23: Risk Factor Changes for Options

where $\sigma_t$ is some measure of volatility at time $t$. In our case, $\sigma_t$ will be the historical volatility for the underlying asset. The actual loss functions for put and call options is the following:

$$
\begin{align*}
L_{t+1} &= -\left[\frac{C_{BS}(t+\Delta T, Z_t + X_{t+1}) - C_{BS}(t, Z_t)}{\Delta T}\right] \\
L_{t+1} &= -\left[\frac{p_{BS}(t+\Delta T, Z_t + X_{t+1}) - p_{BS}(t, Z_t)}{\Delta T}\right].
\end{align*}
$$

Equation 24: Loss Functions for Options

It follows from Equation 24 that the mean linearized loss is defined in the following way:

$$
\begin{align*}
L^\Delta_{t+1} &= -\left[C_{t+\Delta T}^{BS}A + C_S^{BS}S_t X_{t+1,1} + C_r^{BS} X_{t+1,2} + C_{\sigma}^{BS} X_{t+1,3}\right] \\
L^\Delta_{t+1} &= -\left[P_{t+\Delta T}^{BS}A + P_S^{BS}S_t X_{t+1,1} + P_r^{BS} X_{t+1,2} + P_{\sigma}^{BS} X_{t+1,3}\right]
\end{align*}
$$

where

$C_t^{BS}, P_t^{BS}$ are the partial derivatives of the put and call option price with respect to time ($\theta$),

$C_S^{BS}, P_S^{BS}$ are the partial derivatives of the put and call option price with respect to the underlying ($\delta$),

$C_r^{BS}, P_r^{BS}$ are the partial derivatives of the put and call option price with respect to the risk-free rate ($\rho$),

$C_{\sigma}^{BS}, P_{\sigma}^{BS}$ are the partial derivatives of the put and call option price with respect to the volatility ($\nu$),

$\Delta$ is the time horizon $\left(\Delta = \frac{1}{250 \text{ trading days}}\right)$. 

42
$S_t$ is the stock price of the underlying at time $t$

$X_{t+1,i}$ are the risk factor changes.

Equation 25: Linearized Loss Functions for Options

The linearized loss for options is a function of three “Greeks” of the option, which are indicated above. We assume a constant risk-free interest rate so the term $X_{t+1,2}$ was knocked out of the equation. We combine the linearized loss of options with the linearized loss of the stocks in order to compute the expected loss and variance of the loss for the stock and option portfolio. This is done using matrix notation and linear combinations as will be described next.

2.1.4 The Linearized Loss of the Total Portfolio

We suppose that $X$ follows a distribution with mean $\mu$ and covariance matrix $\Sigma$, where these parameters are calculated according to the combined stock and option portfolio. The linearized loss of the portfolio is defined in the following way:

$$L_t^A = z^T X_{t+1} + c,$$

where

- $z$ is a vector of coefficients for the risk factors,
- $X_{t+1}$ is a vector of risk factors, and
- $c$ is a vector of constants.

Equation 26: Portfolio Linearized Loss

The constant term in this linearized loss is the sensitivity to time, $\theta$, and does not get multiplied by any risk factors. For illustration purposes, the vector of coefficients and risk factors are shown in vector notation.
The mean and variance linearized loss of the stock and option

where $a_i$ is the number of shares invested in that particular call, $b_i$ is the number of shares invested in that particular put option, and $\Delta$ is the time horizon. The $V_i w_t$ term indicates the amount invested in the stock of the particular underlying asset. If the option position calls for multiple calls or multiple puts, then the extra number accompanying the sensitivity indicates the strike price of the call or put.

\[ z = \begin{bmatrix} -(a_i C_{iG}^{BS} + b_i P_{iG}^{BS}) \\ -(a_i C_{iG,28}^{BS} + a_i C_{iG,9}^{BS} + a_i C_{iG,10}^{BS}) \\ -(a_i C_{iG,28}^{BS} + a_i C_{iG,33}^{BS}) \\ -(b_i P_{iG,1}^{BS} + b_i P_{iG,166}^{BS}) \\ -(b_i P_{iG,190}^{BS} + b_i P_{iG,180}^{BS}) \\ -(a_i C_{iG}^{BS} + b_i P_{iG}^{BS}) \\ -(a_i C_{iG,166}^{BS} + a_i C_{iG,7}^{BS}) \\ -V_i w_t - S_t (a_i C_{iG,28}^{BS} + b_i P_{iG}^{BS}) \\ -V_i w_t - S_t (a_i C_{iG,33}^{BS}) \\ -V_i w_t - S_t (b_i P_{iG,166}^{BS} + b_i P_{iG,166}^{BS}) \\ -V_i w_t - S_t (b_i P_{iG,190}^{BS} + b_i P_{iG,180}^{BS}) \\ -V_i w_t - S_t (a_i C_{iG,7}^{BS} + b_i P_{iG}^{BS}) \\ -V_i w_t - S_t (a_i C_{iG,166}^{BS} + a_i C_{iG,7}^{BS}) \end{bmatrix} \]

\[ z = \begin{bmatrix} X_{t+1,3, AAPL} \\ X_{t+1,3, BAC} \\ X_{t+1,3, EMC} \\ X_{t+1,3, GLD} \\ X_{t+1,3, I BM} \\ V_{t+1,3, NFLX} \\ X_{t+1,3, NOK} \\ X_{t+1,1, AAPL} \\ X_{t+1,2, BAC} \\ X_{t+1,2, DELL} \\ X_{t+1,2, DIS} \\ X_{t+1,1, EMC} \\ X_{t+1,2, XOM} \\ X_{t+1,1, GLD} \\ X_{t+1,2, GOOG} \\ X_{t+1,1, I BM} \\ X_{t+1,2, PM} \\ X_{t+1,2, M S FT} \\ X_{t+1,1, NFLX} \\ X_{t+1,1, NOK} \\ X_{t+1,2, OIL} \\ X_{t+1,2, V} \end{bmatrix}, \quad \text{and } c = \Delta \]

These are the vectors for portfolio linearized loss.

2.1.4.1 Determining the Portfolio Mean and Variance Using the Normal Distribution

The linearized mean and variance of the total portfolio linearized loss are similar to that of the stock portfolio. The values are calculated in the following way:

\[
E[L_{t+1}^\Delta] = z^T \mu + c \\
Var[L_{t+1}^\Delta] = z^T \Sigma z,
\]

where

\[ \Sigma \] is the covariance matrix associated with the risk factors, and

\[ \mu \] is a vector of the mean of the time series of the risk factors.

Equation 27: Portfolio Mean and Variance Linearized Loss

The mean and variance linearized loss of the stock and option portfolio are displayed in Table 19.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>$16,255</td>
<td>3.3274%</td>
<td>$110,2184</td>
<td>-0.0226%</td>
<td>$18,573</td>
<td>3.8018%</td>
</tr>
<tr>
<td>Week 2</td>
<td>-176,440</td>
<td>-33.5665%</td>
<td>$216,8533</td>
<td>0.0413%</td>
<td>$61,640</td>
<td>11.7263%</td>
</tr>
<tr>
<td>Week 3</td>
<td>-8,816.7</td>
<td>-1.7398%</td>
<td>$1,303.7</td>
<td>0.2573%</td>
<td>$60,469</td>
<td>11.9321%</td>
</tr>
</tbody>
</table>

44
Recall that when looking at only the stock portfolio that the expected loss for all the weeks was reported as being negative losses. With options included in our portfolio, we see that for most of the weeks, the expected loss moved towards the positive direction compared to the stock expected loss. Additionally, the standard deviation of the stock and option portfolio increased dramatically compared to just the stock portfolio. The standard deviation of the stock linearized loss floated around $10,000-$11,000 for all the weeks, whereas the standard deviation for the stock and options linearized loss had a huge jump from week 1 to week 2 and was a lot more volatile, increasing or decreasing by thousands instead of a few hundreds. Essentially we are expected to have more deviation in our loss with the stock and option portfolio than with just the stock portfolio.

We are estimating the loss distribution with these numbers for future weeks. By the results we obtained for the mean and variance, it is estimated that Week 1 will have an expected loss of $-110,218.4$ and standard deviation of $18,573$; Week 2 will have an expected loss of $216,853.3$ and standard deviation of $61,640$, etc. How well these predictions are will be compared with the actual loss in the portfolio when discussing the value at risk in the next section.7

2.1.4.1.1 Value at Risk

Risk is often measured by the volatility of an asset, but there is a building trend in measuring risk by assessing the value at risk.[84] The value at risk, denoted VaR, of the portfolio is given by

\[ \text{VaR}_\alpha = \inf \{ \ell \in \mathbb{R} : P(L > \ell) \leq 1 - \alpha \}. \]

Equation 28: Definition for Value at Risk

Essentially the VaR is the value such that the probability of getting a loss smaller than it has a probability of \( \alpha \).[8] In order to calculate our portfolio’s value at risk for \( \alpha = 0.95 \) to \( \alpha = 0.995 \), we used the following formula:

\[ \text{VaR}_\alpha = \mu + \sigma N^{-1}(\alpha), \]

Equation 29: Value at Risk Formula

where \( N \) is the cumulative distribution function of the standard normal distribution and \( N^{-1}(\alpha) \) is the \( \alpha \)-quantile of \( N \). If follows from our portfolio linearized mean and variance that the value at risk is the following:

<table>
<thead>
<tr>
<th>Week</th>
<th>( \text{VaR}_{\alpha=0.95} )</th>
<th>( \text{VaR}_{\alpha=0.95%} )</th>
<th>( \text{VaR}_{\alpha=0.99} )</th>
<th>( \text{VaR}_{\alpha=0.99%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>$30,440$</td>
<td>6.2309%</td>
<td>$43,097$</td>
<td>8.8218%</td>
</tr>
<tr>
<td>Week 2</td>
<td>$101,610$</td>
<td>19.3293%</td>
<td>$143,610$</td>
<td>27.3207%</td>
</tr>
<tr>
<td>Week 3</td>
<td>$100,770$</td>
<td>19.8837%</td>
<td>$141,980$</td>
<td>28.0153%</td>
</tr>
</tbody>
</table>

---

7 Refer to Appendix E: Matlab Code – portfolioLoss.m for the Matlab Code used for this calculation.
The first thing to note about the value at risk is that there is a linear relationship between the value at risk and the value of our portfolio. The value of our portfolio and the value at risk are positively correlated, meaning the two values rise and fall together. The first week our portfolio was held, we experienced a -7.6% loss, hence a 7.6% profit, on our portfolio. The value at risk increased dramatically because of this going from the first to the second week. Since week 3’s distribution was estimated with the negative loss data from the previous week, it was predicted that the value at risk would increase, hence the value of our portfolio would increase. However, this was not the case. The trend that the
value at risk gives us as the weeks pass allow up to conclude which weeks were the best and which ones were the worse. By this calculations, it is obvious that our worst week overall was the last, but this mostly has to do with the closing of our positions in Interactive Brokers, which we closed on Friday, April 20, 2012. We will address the performance our portfolio in more detail at the end of this paper, but the extreme drop in our portfolio value, and thus value at risk, was due to the option portion of our portfolio when we close our positions. Looking above at Figure 18, we can view the 95% and 99% value at risk against the normal cumulative distribution function (CDF). While this is a good way to view the relationship between each week’s CDF, it is more helpful to view the value at risk values against the probability density function (PDF), which is shown next.

![Percentage VaR Versus Normal PDF](image)

![Close-Up of Expected Loss](image)

![Close-Up of VaR 95% and 99%](image)

**Figure 19: Percentage Value at Risk Versus Normal PDF**

By looking at the normal PDF, it is seen how even the highest probability of our loss distribution, approximately 0.0000215 during the first week, is very small. As the loss moves away from the expected loss and toward either tail, the probability of having such a loss gets progressively smaller. It is highly unlikely, according to the assumption
of normally distributed losses, to get a value that is more than about three standard deviations away from the mean, our a normal distribution. Despite this characteristic of the distribution, we can see that, in fact, our last week is located very far in the right tail. According to the normal distribution, this loss should not have happened. Furthermore, but the seventh week’s loss exceeded the value at risk at alpha 0.99 by more than 5%. This was one of the first indications that the normal distribution was most likely not going to be the best overall fit for our data. Comparing the actual losses to the expected loss, the only weeks that predicted whether a positive or negative loss would occur correctly was weeks 3, 4, and 6, though the expected losses were very small when compared to the real losses. Looking at the probability density, we see that there is a higher probability of getting a loss in the tail for weeks 2, 3, and 4 compared to week 1, but as weeks 5, 6, and 7 approached, the probability of getting a loss in the tails decreased.

The expected loss going from week 1 to week 2 increased a small percentage, an amount of about $106, but the value at risk increased significantly. Both the 95% and 99% value at risk approximately doubled going from week 1 to week 2. Our thought on why the expected loss positively increased is since the stocks had a higher weight in our portfolio, the positive loss we experienced from week 1 to week 2 had more of an effect. However, the positive loss from the stocks did not push the value at risk inward due to the performance of our portfolio, which is where our increase in portfolio value came from going from week 1 to week 2. A final thing to note before looking at the expected shortfall is that the distance between the 95% and 99% value at risk increased for the first few weeks, then started to decrease after week 5 came and past, which is not surprising considering the market took a big decent hit during this week. We now move onto the expected shortfall.

2.1.4.1.2 Expected Shortfall

The expected shortfall, also referred to as the conditional value at risk (CVaR), is derived by taking the weighted average between the value at risk and the losses exceeding the value at risk.[17] The expected shortfall is calculated in the following way:

$$ES_\alpha = \mu + \sigma \frac{\Phi[N^{-1}(\alpha)]}{1 - \alpha},$$

*Equation 30: Expected Shortfall Formula*

where $\Phi$ is the probability density function and $N$ is the cumulative distribution function of the standard normal distribution.

<table>
<thead>
<tr>
<th>Week</th>
<th>$ES_{\alpha=0.95}$</th>
<th>$ES_{\alpha=95%}$</th>
<th>$ES_{\alpha=0.99}$</th>
<th>$ES_{\alpha=99%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>$38,201$</td>
<td>7.8195%</td>
<td>$49,391$</td>
<td>10.1101%</td>
</tr>
<tr>
<td>Week 2</td>
<td>$127,360$</td>
<td>24.2292%</td>
<td>$164,500$</td>
<td>31.2943%</td>
</tr>
<tr>
<td>Week 3</td>
<td>$126,030$</td>
<td>24.8696%</td>
<td>$162,470$</td>
<td>32.0587%</td>
</tr>
<tr>
<td>Week 4</td>
<td>$115,590$</td>
<td>23.3415%</td>
<td>$148,850$</td>
<td>30.0574%</td>
</tr>
<tr>
<td>Week 5</td>
<td>$90,284$</td>
<td>20.5924%</td>
<td>$116,720$</td>
<td>26.6220%</td>
</tr>
<tr>
<td>Week 6</td>
<td>$56,223$</td>
<td>12.3531%</td>
<td>$72,870$</td>
<td>16.0106%</td>
</tr>
<tr>
<td>Week 7</td>
<td>$43,286$</td>
<td>11.8620%</td>
<td>$55,930$</td>
<td>15.3269%</td>
</tr>
</tbody>
</table>

*Table 21: Normal Distribution Portfolio Expected Shortfall for Alpha 0.95 and 0.99*
Similar to the situation with the value at risk, the expected shortfall also increased significantly going from week 1 to week 2. Given that you surpassed the 95% value at risk in the second week, which is 19.3293%, the average loss that can be experienced is $127,360, or 24.2292%. That’s over a 50% increase of average loss than if the 95% value of risk in week 1 is surpassed. As the value at risk gets larger, the difference between the expected shortfall and value at risk gets smaller. In this case, the probability of getting a loss smaller than alpha, given that you already reached alpha, has a higher value, but depending how far away from the expected shortfall that the value at risk is could also indicate that you might face a larger loss. In the case of our portfolio for the last week held, we faced higher than the average loss given that we surpassed the 99% value at risk.

2.1.4.2 Determining the Portfolio Mean and Variance Using Student’s t Distribution

Assuming normality in the markets disregards those values that can exist that are more than the typical 3 standard deviations out from the mean. According to the normal distribution, these extremely high or low values do not happen. The student’s t distribution has fatter tails than the normal distribution and allows larger losses (and gains) to occur with a higher probability. Whereas before we took a look at both the individual stock losses and portfolio loss when discussing the normal, we are only going to look at the values for the overall portfolio for student’s t. The standard student’s t distribution is displayed in Figure 21.

---

8 Refer to Appendix F: Matlab Code – VaRES_Normal.m for the Matlab Code used for calculating both the value at risk and expected shortfall.
In order for the distribution to represent our data, we needed to scale the t distribution using our linearized portfolio parameters. Using $\mu$ as a location parameter, $\sigma$ as the scale parameter, and $\nu$ as the shape parameter, this scaling was done. The following transformation of $x$ was made to the PDF in order for the standard t distribution to reflect our data:

$$\frac{1}{\sigma} f \left( \frac{x - \mu}{\sigma} \right),$$

which has a student’s t distribution with $\nu$ degrees of freedom.[57] Using the maximum likelihood function in Matlab, `mle(risk factors))`, we were able to calculate the parameters for the t distribution. Specifically we used this function to estimate the degrees of freedom that we should use with our calculations. Since the degrees of freedom needed to be greater than 2 in order for the variance to not diverge to infinity or to be undefined, we placed a constraint that if $\nu < 3$, set $\nu = 3$, otherwise, it was set to the degrees of freedom the maximum likelihood function calculated.\(^9\)

Similar to the linearized loss for the normal distribution, the linearized loss for student’s t is quite similar. Primarily the calculation itself follows the same format as the normal, but with the assumption that the risk factors, $X_{t+1}$, are multivariate t, instead of multivariate normal. After making this assumption about $X_{t+1}$, the mean and variance were calculated in the following way:

$$\mu_T = z^T \mu + \zeta \quad \sigma_T^2 = \frac{\nu}{\nu - 2} z^T \Sigma z$$

where

- $z$ is a vector of risk factors,
- $\mu$ is a vector of expected returns for the risk factors,

\(^9\) If the degrees of freedom are between 1 and 2, the variance diverges to infinity, but if the degrees of freedom are less than 1, the variance is undefined. This is due to the $\frac{\nu}{\nu - 2}$ term in the variance calculation.
\( c \) is a vector of constants that are made up of the option sensitivities to time, 
\( \nu \) is the degrees of freedom, and 
\( \Sigma \) is the covariance matrix for the risk factors.

**Equation 31: Linearized Mean and Variance for Student’s t Distribution**

The risk factor vector refers to the same one previously discussed in the beginning of Section 2.1.4. The linearized loss, mean, and variance for our portfolio using student’s \( t \) are the following:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>$16,255</td>
<td>3.3274%</td>
<td>- $110,2184</td>
<td>-0.0226%</td>
<td>$32,169</td>
<td>6.5849%</td>
<td>0.9043 (set to 3)</td>
</tr>
<tr>
<td>Week 2</td>
<td>- $176,440</td>
<td>-33.5665%</td>
<td>$216,853</td>
<td>0.0413%</td>
<td>$106,760</td>
<td>20.3105%</td>
<td>1.7033 (set to 3)</td>
</tr>
<tr>
<td>Week 3</td>
<td>- $8,816.7</td>
<td>-1.7398%</td>
<td>$1,303.7</td>
<td>0.2573%</td>
<td>$104,740</td>
<td>20.6670%</td>
<td>1.6789 (set to 3)</td>
</tr>
<tr>
<td>Week 4</td>
<td>$1,106.9</td>
<td>0.2235%</td>
<td>$1,728.7</td>
<td>0.3491%</td>
<td>$95,609</td>
<td>19.3067%</td>
<td>1.3663 (set to 3)</td>
</tr>
<tr>
<td>Week 5</td>
<td>$34,163</td>
<td>7.7919%</td>
<td>- $221,2526</td>
<td>-0.0505%</td>
<td>$65,809</td>
<td>15.0099%</td>
<td>3.6006</td>
</tr>
<tr>
<td>Week 6</td>
<td>$19,711</td>
<td>4.3307%</td>
<td>- $767,5914</td>
<td>-0.1687%</td>
<td>$47,855</td>
<td>10.5145%</td>
<td>1.4760 (set to 3)</td>
</tr>
<tr>
<td>Week 7</td>
<td>$16,947</td>
<td>4.6441%</td>
<td>- $2,071.1</td>
<td>-0.5675%</td>
<td>$36,349</td>
<td>9.9609%</td>
<td>2.9703 (set to 3)</td>
</tr>
</tbody>
</table>

**Equation 32: Linearized Loss, Mean, and Variance for Student’s t Distribution**

In most of the weeks, the degrees of freedom outputted from the maximum likelihood estimate were less than 3, and where therefore reset to be 3. Since the standard deviation is also used as a risk measure, an initial observation that can be made is that our entire portfolio became riskier during the middle weeks, and then became less risky by the last week. However, remember that we experienced a very big loss in our options portfolio when closing out the positions, which is not reflected in the standard deviation of the estimated distribution for week 7. Since the loss of week 7 is being estimated by week 6, where we had an increase in the portfolio value, we were expected to have the biggest negative loss over all the weeks in week 7. Unfortunately, the last week’s estimated loss distribution was not very accurate since we experienced our biggest positive loss over all the weeks. Comparing the standard deviations to those of the normal distribution, the estimated standard deviations using student’s \( t \) increased more than 40%. Being that the probability density of student’s \( t \) gets pushed down, allowing more probability in the tails, an increase in the variability is expected. The next section will discuss how to calculate the value at risk and expected shortfall for a \( t \) distribution.

2.1.4.2.1 Value at Risk

In order to calculate the weekly values at risk, we used the following formula:

\[
VaR_\alpha = \mu + \sigma t_\nu^{-1}(\alpha),
\]

where

- \( t_\nu \) is the cumulative distribution function for a standard \( t \) distribution with \( \nu \) degrees of freedom,
- \( t_\nu^{-1} \) is the \( \alpha \)-quantile of \( t \).

**Equation 33: Value at Risk Formula for Student’s \( t \)**

The 95% and 99% value at risk are displayed in the table below, as well as in the PDF and CDF plots following the table.
Observe that compared to the normal distribution values at risk, the student’s t values at risk are pushed more into the tail, hence greater than the normal values. Similar to the trend with the normal, the value at risk increases and decreases with the portfolio value.
The value at risk for both the 95% and 99% increased at a larger rate going from week 1 to 2 compared to the normal, more than 50% in fact. During the middle weeks, it is estimated that there is a 95% probability that the loss we experience will be around half of our stock and option portfolio or lower. In the last week it was predicted that we would have a loss that was 22.8742% or lower with a 95% probability. Our actual loss during the last week was, in fact, very close to that number, but by comparison with the normal, student’s t allows this type of losses to more than likely happen, whereas the 19.82% actual loss we had during the last week drastically surpassed the 95% value at risk for the normal distribution. We can see how the probability gets pushed into the tail in the next table.

![Percentage VaR Versus Student's t PDF](image1)

![Close-Up Expected Loss](image2)

![Close-Up VaR 95% and 99%](image3)

Figure 23: Percentage Value at Risk Versus Student’s t PDF

The interval at which the distribution is plotted is from the $[-VaR_{0.05}, VaR_{0.05}]$, so it does appear as if the CDF and PDF do not reach the 99% value at risk, but conceptually it is understood by now that the 99% value at risk is the value at which, according to the student’s t distribution, there is a 99% chance of having a loss at or below this value. Just by looking at the scale from the CDF and PDF, we can see how this distribution allows greater positive and negative losses. In weeks 2, 3, and 4, the 99% value at risk can
essentially wipe out most of our stock and option portfolio, but thankfully our actual losses were nowhere near those. Additionally, the actual losses we had are given a higher probability of occurring. We move on to looking at how the expected shortfall compares to the value at risk for the student’s t distribution.

### 2.1.4.2.2 Expected Shortfall

As previously mentioned, the expected shortfall is derived by taking the weighted average between the value at risk and the losses exceeding the value at risk. The expected shortfall for the t distribution is computed in the following way:

\[
ES_\alpha = \mu + \sigma ES_\alpha(L),
\]

where

\[
ES_\alpha(L) = \frac{g_\nu[t^{-1}(\alpha)]}{1 - \alpha} \left( \frac{\nu + [t^{-1}(\alpha)]^2}{\nu - 1} \right).
\]

Equation 34: Expected Shortfall Formula for Student’s t

The 95% and 99% expected shortfall percentages are displayed in the table below.

<table>
<thead>
<tr>
<th>Week</th>
<th>(ES_\alpha=0.95)</th>
<th>(ES_\alpha=95%)</th>
<th>(ES_\alpha=0.99)</th>
<th>(ES_\alpha=99%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>$115,700</td>
<td>23.6828%</td>
<td>$320,230</td>
<td>65.5493%</td>
</tr>
<tr>
<td>Week 2</td>
<td>$384,550</td>
<td>73.1561%</td>
<td>$1,063,300</td>
<td>202.2855%</td>
</tr>
<tr>
<td>Week 3</td>
<td>$378,370</td>
<td>74.6608%</td>
<td>$1,044,300</td>
<td>206.0660%</td>
</tr>
<tr>
<td>Week 4</td>
<td>$345,920</td>
<td>69.8524%</td>
<td>$953,800</td>
<td>192.6033%</td>
</tr>
<tr>
<td>Week 5</td>
<td>$212,270</td>
<td>48.4164%</td>
<td>$490,270</td>
<td>111.8228%</td>
</tr>
<tr>
<td>Week 6</td>
<td>$171,510</td>
<td>37.6832%</td>
<td>$475,770</td>
<td>104.5339%</td>
</tr>
<tr>
<td>Week 7</td>
<td>$128,780</td>
<td>35.2916%</td>
<td>$359,890</td>
<td>98.6231%</td>
</tr>
</tbody>
</table>

Table 23: Expected Shortfall for t Distribution

As the value at risk increased, the expected shortfall increases significantly. Recall that for the normal distribution, as the value at risk increases, it approaches the expected shortfall. In the case of student’s t, as alpha approaches 1, the value at risk and expected shortfall get further away from each other. In the case of our portfolio, the average loss given that the 99% value at risk has been reached was so high that it can do one of four things: wipe out our entire portfolio, wipe out just our stock portfolio, wipe out just our options portfolio, or wipe out our entire stock and option portfolio, depending on the week you are considering. Remember that we do have about $500,000 invested in risk-free assets, which in the event our 99% value at risk for weeks 2 to 7 were surpassed, would be liquidated in order to meet our margin requirement.
As shown in Figure 24, we can see the clear relationship between the expected shortfall and value at risk. One thing that has not been mentioned yet is how the distance changes from week to week. For most of the middle weeks our portfolio was held, given that we surpassed the value at risk, expected shortfall was substantially higher, especially for weeks 2, 3, and 4. Granted, our portfolio loss would need to surpass a higher threshold, but the damaging it could cause would be huge. By the last week, the distance between the value at risk and expected shortfall decreased. Again, we are not saying that the two values approach each other, but that in the middle weeks the average loss given the threshold has been passed is smaller distance than in the first and last week.

In summary, student’s t distribution gives the estimated distributions a better chance to predict a higher loss, thus of the two distributions, it is the one well feel is better suited when modeling the losses of our portfolio. We conclude this section with a chart depicting the value at risk and expected shortfall at alpha 0.95 as an ending representation of the trend of these risk measures for both distributions.

<table>
<thead>
<tr>
<th>Week</th>
<th>VaR$_{\alpha=95%}$ Normal</th>
<th>ES$_{\alpha=95%}$ Normal</th>
<th>VaR$_{\alpha=95%}$ Student’s t</th>
<th>ES$_{\alpha=95%}$ Student’s t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>6.2309%</td>
<td>7.8195%</td>
<td>15.4740%</td>
<td>23.6828%</td>
</tr>
<tr>
<td>Week 2</td>
<td>19.3293%</td>
<td>24.2292%</td>
<td>47.8377%</td>
<td>73.1561%</td>
</tr>
<tr>
<td>Week 3</td>
<td>19.8837%</td>
<td>24.8696%</td>
<td>48.8961%</td>
<td>74.6608%</td>
</tr>
<tr>
<td>Week 4</td>
<td>18.6838%</td>
<td>23.3415%</td>
<td>45.7846%</td>
<td>69.8524%</td>
</tr>
<tr>
<td>Week 5</td>
<td>16.4106%</td>
<td>20.5924%</td>
<td>32.9961%</td>
<td>48.4164%</td>
</tr>
<tr>
<td>Week 6</td>
<td>9.8165%</td>
<td>12.3531%</td>
<td>24.5747%</td>
<td>37.6832%</td>
</tr>
<tr>
<td>Week 7</td>
<td>9.4589%</td>
<td>11.8620%</td>
<td>22.8742%</td>
<td>35.2916%</td>
</tr>
</tbody>
</table>

Refer to Appendix G: Matlab Code – VaR_ES_T_Dist for the Matlab code used to calculate the value at risk and expected shortfall using Student’s t Distribution.
2.1.4.3 Reducing the Value at Risk

If a reduction in the value at risk were desired, this reduction would result in a greater probability of getting a lower positive loss or potentially a negative loss. For example, if it was desired to reduce the value at risk at 99% from 16% to 9%, this would mean that there is a 99% chance that the loss is 9% or below, instead of 16% of below. Investors usually look to diversification as a way to reduce risk. Stocks that have a negative correlation ensure that while Stock A in your portfolio may decrease, it can be hedged by a long position in Stock B, which responds to the decrease of Stock A with an increase in its own price. In order to reduce our value at risk, we added two additional securities to our stock portfolio - iPath S&P 500 VIX Short-Term Futures ETN (VXX) and iPath S&P 500 VIX Mid-Term Futures ETN (VXZ). Launched in January 2009, these two securities track VIX futures instead of the index itself, allowing investors to speculate on the index. VXX sustains a rolling long position in the first and second month VIX futures contracts, and VXZ sustains a rolling position in the fourth to seventh month contracts. Since market volatility rises with market risk, we added these securities to our portfolio as a way to diversify our portfolio’s exposure to risky assets, therefore providing protection during times of high volatility. There was a high negative correlation between VXX and our portfolio, as well as between VXZ and our portfolio, thus these additions were enough to reduce our value at risk since the securities compensate each other when one has a loss or gain. We had a remaining $8,090 in cash left from our original investment so we split the money evenly and hypothetically invested in both by making the adjustments to the weights and recalculating the risk values.11

<table>
<thead>
<tr>
<th></th>
<th>Week 7</th>
<th>Week 7 with VXX and VXZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearized Loss (Actual)</td>
<td>$16,947</td>
<td>$17,358</td>
</tr>
<tr>
<td>Linearized Loss (%)</td>
<td>4.6441%</td>
<td>4.6540%</td>
</tr>
<tr>
<td>Expected Loss (U.S.D. [%])</td>
<td>$2,071.1 [-0.5675%]</td>
<td>$2,036.2 [-0.5459%]</td>
</tr>
<tr>
<td>Normal $\sigma$ (U.S.D. [%])</td>
<td>$20,986 [5.7509%]</td>
<td>$20,555 [5.5113%]</td>
</tr>
<tr>
<td>Student’s t $\sigma$</td>
<td>$36,349 [9.9609%]</td>
<td>$32,544 [8.7259%]</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>3</td>
<td>3.3274</td>
</tr>
<tr>
<td>Normal $\textit{VaR}_{\alpha=0.95}$</td>
<td>9.4589%</td>
<td>8.5192%</td>
</tr>
<tr>
<td>Normal $\textit{VaR}_{\alpha=0.99}$</td>
<td>13.3781%</td>
<td>12.2751%</td>
</tr>
<tr>
<td>Student’s t $\textit{VaR}_{\alpha=0.95}$</td>
<td>22.8742%</td>
<td>19.1908%</td>
</tr>
<tr>
<td>Student’s t $\textit{VaR}_{\alpha=0.99}$</td>
<td>44.6622%</td>
<td>36.1207%</td>
</tr>
</tbody>
</table>

Table 24: Value at Risk Reduction Comparison

Using the normal distribution, the value at risk at alpha 0.95 and 0.99 was reduced by 9.937% and 8.221%, respectively. Using student’s t distribution, the value at risk at alpha 0.95 and 0.99 was reduced by 16.087% and 19.121%. Additionally, by adding these two ETNs, the value of our stock portfolio went up, thus the value of our entire portfolio went up, since we were investing in the asset. We previously stated that an increase in the value at risk had a positive linear relationship with the value of the portfolio, but with the increase in value of our portfolio, we just stated there was a decrease in the value at risk. This is due to diversification. While it is true that there is a linear relationship between the value at risk and value of the portfolio, diversification allows this to occur.

---

11 Refer to Excel file “Market Portfolio Project 1.xlsx” tab titled “Week 7 Weight – VaR Reduction” to see how the weight were adjusted.
2.2 Estimating the Weekly Loss Using Polynomial Tails

In the prior chapter we assumed that the risk factors in relation to our portfolio came from either a normal or student’s t distribution, and used this assumption to calculate the value at risk and expected shortfall at various alpha values. But what if the loss distributions of our portfolio returns do not follow a parametric family? Since it is not known whether our returns follow either one of those distributions, we used the returns to calculate nonparametric and parametric estimates, which in return were used for calculating a semi-parametric estimate for the value at risk and expected shortfall. This section, however, only refers to the stock portfolio. Whereas before we were modeling the $1 - \alpha$-upper quantile of the loss distribution (right tail), we are now modeling the $1 - \alpha$-quantile of the return distribution (left tail), which in essence is the same values represented in a different way. This quantile is calculated as the $1 - \alpha$-quantile of a sample of historic log returns. The nonparametric estimate of the value at risk is estimated in the following way:

$$\overline{VaR}_{1-\alpha}^{np} = -V_t \hat{q}(1 - \alpha),$$

where

- $V_t$ is the value of the return times initial investment, and
- $\hat{q}(1 - \alpha)$ is the $1 - \alpha$-quantile of a sample of historic log returns.

Equation 35: Non-Parametric VaR Equation

Note that the $-V_t$ converts the revenue to a loss.[48]

We assume that the return density has a polynomial left tail, which is comparable to the loss density having a polynomial right tail. For our purposes, we used the above formula to calculate the nonparametric estimate of $VaR_{1-\alpha}$, a value we need in order to calculate the value at risk and expected shortfall using polynomial tails. By our assumption, the return density $f$ is the following:

$$f(y) \sim A|y|^{-(\alpha+1)}, \quad \forall y \leq c \text{ for some } c < 0 \text{ and } A, \alpha > 0, \quad [12]$$

where

- $A$ is a constant, and
- $\alpha$ is the tail index.

Equation 36: Polynomial Tail Return Density Function

Hence the probability that a return is less than or equal to $y$ is the following:

$$P(R \leq y) = \frac{A}{\alpha} y^{-\alpha}, \quad \text{for } y > 0. \quad [48][11]$$

Equation 37: Polynomial Tail Probability Function

The relationship between polynomial tails and the normal and student’s t distribution is in the heaviness of the tail. The normal distribution has tails in which the probability distributed in those tails is small, hence a smaller probability of high losses or gains to happen. The student’s t distribution has tails that are heavier than the normal distribution. The heaviness of the tails is determined by the degrees of freedom, $\nu$, of student’s t, where the smaller $\nu$ is, the heavier the tails are. Hence, as $\nu \to \infty$, the t

---

12 The notation $\sim$ is equivalent to saying approximately. Hence, we are saying that the polynomial tail density is equivalent to the state formula.
distribution approaches the normal distribution. It follows that the t distribution has polynomial tails with tail index \( a = \nu \).

### 2.2.1 Estimating the Tail Index

We estimate the tail index by using a straight-line regression, so using the model in the form of

\[
Y_i = \beta_0 + \beta_1 X_i + \epsilon_i,
\]

where

- \( \beta_0 \) is the intercept, and
- \( \beta_1 \) is the slope of the linear of the regression, and
- \( \epsilon_i \) is the residual between the \( Y \) value and fitted value.

#### Equation 38: General Straight-Line Regression Model

The regression coefficients are calculated using a least squares approach. Since we are estimating the polynomial tail, the regression coefficients will be calculated with respect to only the portfolio returns in a specified range of the tail, which we have chosen to be 10% of our sample ordered returns. The monetary portfolio returns are calculated and ordered, where \( R(1), R(2), \ldots, R(n) \) are the ordered statistics of the returns. Thus the number of observed values less than or equal \( R(k) \) is \( k \). As stated, we choose \( k \) to be 10% of our sample size of 51 actual returns, where from the 52 weeks of data we get 51 returns, so \( k \) is the first five returns of our ordered statistics. It follows from Equation 37 that by taking the log of the probability, we get the following:

\[
\log\{P\left(R \leq -y\right)\} = \log\left(\frac{A}{a}\right) - a\log(y).
\]

#### Equation 39: Log Probability Formula

From this we estimate that \( \log\{P\left(R \leq -R(k)\right)\} = \log\left(\frac{k}{n}\right) \), which is the \( X \) for the regression. By rearrangement of the above equation with respect to \( y=R(k) \), we have

\[
\log(-R(k)) \approx \frac{1}{a} \log\left(\frac{A}{a}\right) - \frac{1}{a} X
\]

#### Equation 40: Straight-Line Model for Least Squares Regression

This estimation is expected to be accurate for \( k \) values that are small compared to the same size \( n \), hence \( k \) being 10% of our sample size. The coefficients for least squares regression are estimated from fitting Equation 40 to the points \( \{\log\left(\frac{k}{n}\right), \log(-R(k))\} \). Note that since we are fitting the coefficients to these points, we do not regard the residuals as seen in the general straight-line regression model. The least squares coefficients are calculated in the following way,

\[
\beta_1 = \frac{\sum_{k=1}^{5} Y_k (X_k - \bar{X})}{\sum_{k=1}^{5} (X_k - \bar{X})^2} \quad \beta_0 = \bar{Y} - \beta_1 \bar{X}
\]

where

- \( Y_k \) is \( \log(-R(k)) \), and
- \( X_k \) is \( \log\left(\frac{k}{n}\right) \).
From these estimators we can get that the regression estimator for the tail index to be

\[ \hat{\alpha} = -\frac{1}{\hat{\beta}_1}, \]

Equation 42: Polynomial Tail Index Estimation Formula

which can be derived from Equation 40, where the regression slope is \(-\frac{1}{\hat{\alpha}} = \hat{\beta}_1\). The constant \(A\) can also be derived from Equation 40 in the following way:

\[ \hat{\beta}_0 = \frac{1}{\hat{\alpha}} \log \left( \frac{A}{\hat{\alpha}} \right) \]

\[ \hat{\beta}_0 = \frac{1}{\hat{\alpha}} \left[ \log(A) - \log(\hat{\alpha}) \right] \]

\[ \hat{\alpha}\hat{\beta}_0 = \log(A) - \log(\hat{\alpha}) \]

\[ A = e^{\hat{\alpha}\hat{\beta}_0} \log(\hat{\alpha}) \]

Equation 43: Polynomial Tail Constant "A" Estimation Formula

The estimated tail index and constant \(A\) for each week are the following:

<table>
<thead>
<tr>
<th>Week</th>
<th>(A)</th>
<th>(\hat{\alpha})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12,580</td>
<td>1.2609</td>
</tr>
<tr>
<td>2</td>
<td>$79,379</td>
<td>1.4356</td>
</tr>
<tr>
<td>3</td>
<td>$73,861</td>
<td>1.4298</td>
</tr>
<tr>
<td>4</td>
<td>$69,114</td>
<td>1.4217</td>
</tr>
<tr>
<td>5</td>
<td>$588,190</td>
<td>1.6105</td>
</tr>
<tr>
<td>6</td>
<td>$599,420</td>
<td>1.6091</td>
</tr>
<tr>
<td>7</td>
<td>$507,200</td>
<td>1.6024</td>
</tr>
</tbody>
</table>

Table 25: Weekly Values for the Tail Index and Constant \(A\)

The weekly tail index had an overall increase compared to the first week. It is interesting to realize that the tail index had an increase between 0.17-0.19 from the first to second week and from the fourth to fifth week, but the amount that \(A\) increased was dramatic. Even though the fifth week’s tail index only experienced an increase of 0.0141 more than the first to second, the constant \(A\) made more than a half a million dollar jump.

Looking at the value of the stock portfolio each week in Figure 26, it can be seen how a decrease in value caused both factors to increase, and an increase in value resulted in both factors decreasing for the first five weeks. There seems to be an inverse relationship up until we took a big hit in our stock portfolio in week 5, where the relationship between the value of the portfolio and the tail index change to moving in the same direction for the rest of the weeks the portfolio is held. Since there was an increase in portfolio value in weeks 3 and 4, the estimation for week 5 did not
predict such a big loss for our portfolio, considering it is estimated using data up to week 4.

Recall that the smaller the tail index, the heavier the tail is. Looking at the estimated tail indices, the heaviest tail for the distribution was in the first week and the least heavy was in fifth week. With the tail indices as such, we would expect to have a better chance of getting a return in the tail during the first week, and a smaller chance of getting a return in the tail during the fifth week. While we did have a decent increase in portfolio value in the first week, we should not, according to the tail index, have had such a big hit in the fifth and last. This was more of a preliminary observation before looking at the value at risk. With these estimates, we can calculate the probability density of the tail, as well as the value at risk and expected shortfall using polynomial tails for our stock portfolio. In order to visualize the difference the constant and tail index can have on the density function of our loss distribution, the following graph displays the tail of our losses and the actual losses for each week:

The actual losses displayed in the graph above refer to the actual losses of only the stock portfolio, not the stock and option portfolio. Since we are estimating over the largest 10% of our modeled positive losses, it is not surprising that only the week 7’s loss showed up in the tail. The biggest noticeable difference is how fast the curve of the tail of the distribution increases for weeks 5, 6, and 7. The higher the constant and tail index, the steeper the curve from one loss to the other.

---

Refer to Appendix J: Matlab Code - Polydensity.m for the Matlab code used for graphing the density.

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Figure 27: Polynomial Density and Actual Losses

60
2.2.2 Estimating the Value at Risk and Expected Shortfall

After obtaining estimates for the tail index and the constant \( A \), approximating the value at risk and expected shortfall are quite simple. Recall that we assume our tail for the loss distribution has the probability density function as stated in Equation 3. Suppose that \( y_1 = VaR_{1-\alpha_1} \) and \( y_2 = VaR_{1-\alpha_0} \), where \( y_1, y_2 > 0 \) and \( \alpha_0 = 0.9 \). By Equation 3, we get the following ratio:

\[
\frac{P(R < -y_1)}{P(R < -y_2)} \approx \left( \frac{y_1}{y_2} \right)^{-\alpha}
\]

\[
\frac{1 - \alpha_1}{1 - \alpha_0} = \frac{P(R < -VaR_{1-\alpha_1})}{P(R < -VaR_{1-\alpha_0})} \approx \left( \frac{VaR_{1-\alpha_1}}{VaR_{1-\alpha_0}} \right)^{-\alpha}
\]

\[
VaR_{1-\alpha} \approx VaR_{1-\alpha_0} \left( \frac{1 - \alpha_0}{1 - \alpha} \right)^{\frac{1}{\alpha}}
\]

Equation 44: Polynomial Tail Value at Risk Formula

Recall that we are going to use non-parametric estimation in order to calculate \( VaR_{1-\alpha_0} \). The value at risk using a polynomial tail is the following:

<table>
<thead>
<tr>
<th>Week</th>
<th>( VaR_{0.99} )</th>
<th>( VaR_{0.9} )</th>
<th>( VaR_{0.95} )</th>
<th>( VaR_{0.95} )</th>
<th>( VaR_{0.99} )</th>
<th>( VaR_{0.99} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7,913.7</td>
<td>2.2543%</td>
<td>$13,712</td>
<td>3.9061%</td>
<td>$49,141</td>
<td>13.9979%</td>
</tr>
<tr>
<td>2</td>
<td>$9,004.3</td>
<td>2.6164%</td>
<td>$14,593</td>
<td>4.2403%</td>
<td>$44,775</td>
<td>13.0103%</td>
</tr>
<tr>
<td>3</td>
<td>$9,288</td>
<td>2.6568%</td>
<td>$15,082</td>
<td>4.3143%</td>
<td>$46,488</td>
<td>13.2978%</td>
</tr>
<tr>
<td>4</td>
<td>$9,376.5</td>
<td>2.6306%</td>
<td>$15,268</td>
<td>4.2836%</td>
<td>$47,364</td>
<td>13.2882%</td>
</tr>
<tr>
<td>5</td>
<td>$10,034</td>
<td>2.9639%</td>
<td>$15,431</td>
<td>4.5581%</td>
<td>$41,920</td>
<td>12.3825%</td>
</tr>
<tr>
<td>6</td>
<td>$10,039</td>
<td>2.9315%</td>
<td>$15,445</td>
<td>4.5099%</td>
<td>$41,992</td>
<td>12.2619%</td>
</tr>
<tr>
<td>7</td>
<td>$9,889</td>
<td>2.8945%</td>
<td>$15,241</td>
<td>4.4610%</td>
<td>$41,612</td>
<td>12.1796%</td>
</tr>
</tbody>
</table>

Table 26: Value at Risk Using Polynomial Tails

Overall, the value at risk from week to week stays within 1-2% of each other, which is equivalent to about $2,000-$8,000. Note that the value at risk for 90% is at its lowest the first week and at its highest the last week, while the 99% value is at its highest the first week and its lowest the last week. Given that in the last week took such a large loss, it is easy to see that the value at risk would also decline. Over the time we held our portfolio the value at risk increased from week 1 to week 7. We next look at the relationship value at risk has with expected shortfall.

It follows that we can use the value at risk calculation in order to compute the expected shortfall for a specified \( \alpha \). The expected shortfall can be calculated from the expectation of the conditional density of \( R \) given that \( R \leq d \), which is as follows:

\[
E\{\{ |R| |R \leq d \} = a |d|^a \int_{-\infty}^{d} |y|^{-a} dy = \frac{a}{a - 1} |d|.
\]

Equation 45: Expectation of the Conditional Density of \( R \)

Let \( d = -VaR_\alpha \) and we have

\[
ES_1-\alpha = \frac{a}{a - 1} |VaR_{1-\alpha}| = \frac{a}{a - 1} VaR_{1-\alpha}, \quad \text{for } a > 1.
\]

Equation 46: Polynomial Tail Expected Shortfall Formula
Note the \( a \) must be greater than 1 so that the integral in the expectation of the conditional density does not diverge to \( \infty \). The next table displays the expected shortfall using a polynomial tail:

<table>
<thead>
<tr>
<th>Week</th>
<th>( ES_{0.90} )</th>
<th>( ES_{99%} )</th>
<th>( ES_{0.95} )</th>
<th>( ES_{95%} )</th>
<th>( ES_{0.99} )</th>
<th>( ES_{99%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$38,242</td>
<td>10.8933%</td>
<td>$66,263</td>
<td>18.8754%</td>
<td>$237,460</td>
<td>67.6426%</td>
</tr>
<tr>
<td>2</td>
<td>$29,676</td>
<td>8.6231%</td>
<td>$48,095</td>
<td>13.9751%</td>
<td>$147,570</td>
<td>42.8792%</td>
</tr>
<tr>
<td>3</td>
<td>$30,900</td>
<td>8.8389%</td>
<td>$50,177</td>
<td>14.3531%</td>
<td>$154,660</td>
<td>44.2404%</td>
</tr>
<tr>
<td>4</td>
<td>$31,614</td>
<td>8.8695%</td>
<td>$51,479</td>
<td>14.4426%</td>
<td>$159,690</td>
<td>44.8028%</td>
</tr>
<tr>
<td>5</td>
<td>$26,471</td>
<td>7.8191%</td>
<td>$40,709</td>
<td>12.0249%</td>
<td>$110,590</td>
<td>32.663%</td>
</tr>
<tr>
<td>6</td>
<td>$26,521</td>
<td>7.7444%</td>
<td>$40,801</td>
<td>11.9143%</td>
<td>$110,930</td>
<td>32.3935%</td>
</tr>
<tr>
<td>7</td>
<td>$26,305</td>
<td>7.6994%</td>
<td>$40,542</td>
<td>11.8664%</td>
<td>$110,690</td>
<td>32.3982%</td>
</tr>
</tbody>
</table>

Table 27: Expected Shortfall Using Polynomial Tails

Overall the expected shortfall made more of a dramatic increase going from 90% to 95% to 99% compared to the percentage increases for value at risk. In the first week our portfolio could expect to lose $38,242 given that the \( VaR_{0.90} \) has been reached, while by the last week our portfolio was expected to lose more than $10,000 less given that \( VaR_{0.90} \) has been reached. As weeks went by the expected shortfall experienced an overall decrease, but since the 90% and 95% values at risk increased, a higher loss is more probable at these two alpha values. More clearly, even though the expected shortfall decreased, an increased value at risk results in a better chance of experiencing a loss at or below that increased \( VaR_\alpha \) value. There is a huge difference between the first and last week when comparing the value at risk and expected shortfall at 99%. Given that the loss has exceeded \( VaR_{0.99} \) during week 1, the expected loss is a little less than a quarter of a million dollars. This is a huge difference compared to the last week where given that the loss has exceeded \( VaR_{0.99} \), the average loss is less than half of that. Granted there is
about an $8,000 difference between the two $\text{VaR}_{0.99}$ values, with the first week being higher than the last, but that difference between the average losses is a lot larger.\textsuperscript{14}

Figure 28 displays the relationship between the value at risk and expected shortfall for the weeks the stock portfolio was held. It is much easier to see in the graph how the distance between the expected shortfall and value at risk increases as alpha increases. Though difficult to see for weeks 2 to 6 since they were so close to each other, we can see the huge difference between the distance of the value at risk and expected shortfall the first week, and the distance the last week. During the beginning weeks our stock portfolio was held, we were more likely to have a higher average loss compared to the last weeks. Our stock portfolio experienced a negative loss during week 6 and only a small positive loss during the last week. Overall we decided that using only 10% of the data to estimate the tail is not the most favorable way to model our losses because we feel our sample size is not large enough to accurately estimate the data, being that our sample size of 10% is only five values.

2.3 Estimating the Weekly Conditional Loss Distribution Using ARMA(1,1)-GARCH(1,1) Modeling

The portfolio loss was previously modeled using the unconditional loss distribution. We will now look into modeling the portfolio loss using two versions of a conditional loss distribution model, Gaussian innovations (normal) and student’s t innovations using ARMA(1,1)-GARCH(1,1) modeling. For conditional loss distributions we assume that the risk factor changes from $t$ to $t+1$, given information up to time $t$, follow a certain distribution. For example, for Gaussian innovations, we assume that $X_{t+1} | \mathcal{F}_t \sim N(\mu_{t+1}, \Sigma_{t+1})$, where $\mu_{t+1}$ and $\Sigma_{t+1}$ denote the conditional mean and covariance matrix given $\mathcal{F}_t$, and $\mathcal{F}_t$ is the information that is known up to time $t$.\textsuperscript{9}

Autoregressive Moving Average (ARMA) models are used for modeling the conditional expectation of a process given the past, but this model assumes that the conditional variance given the past is constant. Formally, the model is denoted as ARMA(p,q), where the p indicates the order of the autoregressive part of the model and q indicates the order of the moving average part of the model. Since the assumption of constant volatility does not follow a realistic setting according to time-series market data, another model is taken into consideration. Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) models are used for modeling the conditional variance of a process given the past. Formally, the model is denoted as GARCH(p,q), where p indicates the order of the noise in the ARCH term and q is the order of the variances in the GARCH term.\textsuperscript{48}

We assume that our weekly portfolio loss follows an ARMA(1,1)-GARCH(1,1) model of the form

$$L_t = \mu_t + \sigma_t Z_t$$

Equation 47: ARMA(1,1)-GARCH(1,1) Loss Model

where $\mu_t$ and $\sigma_t$ are determined by fitting the portfolio returns to the model using the $\text{garchfit}$ function in Matlab. Let $(Z_t)_{t \in \mathbb{Z}}$ be a strong white noise with mean 0 and variance

\textsuperscript{14} Refer to Appendix I: Matlab Code - polynomialTail.m for the Matlab code used to calculate the value at risk and expected shortfall for polynomial tail modeling.
1. More specifically, the process \((L_t)_{t \in \mathbb{Z}}\) is called an ARMA(1,1) process with GARCH(1,1) errors if it is covariance stationary and satisfies Equation 13. We have the following relations for the residuals, mean and variance according to this model:

\[
X_t - \mu_t = \epsilon_t, \quad \text{where} \quad \epsilon_t = \sigma_t Z_t
\]

\[
\mu_t = \mu + \phi (X_{t-1} - \mu) + \theta (X_{t-1} - \mu_{t-1}), \quad \text{and}
\]

\[
\sigma_t^2 = \alpha_0 + \alpha_1 (X_{t-1} - \mu_{t-1})^2 + \beta \sigma_{t-1}^2
\]

Equation 48: Residuals, Mean, and Variance Formulas for ARMA(1,1)-GARCH(1,1) Model

By modeling the portfolio loss with ARMA-GARCH, the value at risk and expected shortfall are modeled in a way where they can adjust to periods of high and low volatility.[48] The `garchfit` function outputs the parameter estimates, the standard errors of the parameter estimates, the optimized log likelihood function value associated with the parameter estimates, a vector of fit residuals, and a vector of conditional standard deviations of the fit residuals. From these outputs we can calculate both the Gaussian and student’s t mean and strong white noise terms with the residuals, sigma, and portfolio log returns. The following graphs display the model losses versus the actual losses of our modeled value of our portfolio for the first and last week our portfolio was held.

![Model Losses Versus Actual Losses Week 1](image1)

![Model Losses Versus Actual Losses Week 7](image2)

Figure 29: ARMA-GARCH Modeled Losses Versus Our Modeled Portfolio Actual Losses

The modeled losses in the form of Equation 47 are quite close to the actual losses that occurred during the 52-week span of our own time series model. For the most part, even though the Gaussian and student’s t models output a different mean and sigma for the calculation of the loss, they are quite close, so close that you can barely notice the difference between the two. For the most part the actual losses follow pretty closely the modeled losses, with a few of the bigger losses being underestimated by the ARMA-GARCH model. The relationship between the means and variances for each week can be seen next.

![muN for Normal Distribution](image3)

![muT for T Distribution](image4)

Figure 30: ARMA-GARCH Expected Returns for Gaussian and Student's t Innovations
where for n shortfall were calculated in Chapter 1, when the log returns were modeled using the exp standard deviation outputted for each distribution to compute the value at risk and innovations and Student’s t Innovations for the Stock Portfolio.

2.3.1 Calculating the Weekly Value at Risk and Expected Shortfall Using Gaussian Innovations and Student’s t Innovations for the Stock Portfolio

Once the parameters for the GARCH model are fitted, we use the mean and standard deviation outputted for each distribution to compute the value at risk and expected shortfall. The computation is identical to how the value at risk and expected shortfall were calculated in Chapter 1, when the log returns were modeled using the normal distribution and student’s t distribution. Recall the following value at risk formulas for the Gaussian (left) and student’s t (right) innovations:

\[ VaR_\alpha = \mu + \sigma N^{-1}(\alpha) \quad \text{VaR}_\alpha = \mu + \sigma t^{-1}_\nu(\alpha), \]

where

\( N \) is the cumulative distribution function of the standard normal distribution,

\( N^{-1}(\alpha) \) is the \( \alpha \)-quantile of \( N \),

\( t_\nu \) is the cumulative distribution function for a standard \( t \) distribution with \( \nu \) degrees of freedom, and

\( t^{-1}_\nu(\alpha) \) is the \( \alpha \)-quantile of the standard \( t \) distribution.

Equation 49: Normal and Student’s t Value at Risk Formulas
The weekly value at risk for $\alpha = [0.90:0.001:1]$ was calculated at every time step of the series from the specified week our portfolio was held back 52 weeks, though only the 90%, 95%, and 99% will be indicated. The change in the value at risk for the year ending with our invested weeks will be displayed in graphical form over this interval. As before, week 1 denotes the week we are currently analyzing in our portfolio, for instance if we are looking at the value at risk graph for week 7, week 1 on the y-axis of the graph is referring to week 7 of our portfolio being held. Week 52 on the y-axis denotes the first week of the time series, so if we were looking at the seventh week of the portfolio being held, week 52 refers to the week one-year prior. Another example of this is for the first week the portfolio was invested in, our time series data for our estimations spanned from March 9, 2011 to March 5, 2012. In the graphs for week 1, week 1 is March 5, 2012 and week 52 is March 9, 2011. We compare the Gaussian and student’s t value at risk values.

<table>
<thead>
<tr>
<th>Week</th>
<th>$VaR_{90% G}$</th>
<th>$VaR_{90% t}$</th>
<th>$VaR_{95% G}$</th>
<th>$VaR_{95% t}$</th>
<th>$VaR_{99% G}$</th>
<th>$VaR_{99% t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>3.6713%</td>
<td>5.0754%</td>
<td>4.8311%</td>
<td>7.6831%</td>
<td>7.0067%</td>
<td>16.4057%</td>
</tr>
<tr>
<td>Week 2</td>
<td>3.0874%</td>
<td>4.4336%</td>
<td>4.2333%</td>
<td>6.9590%</td>
<td>6.3829%</td>
<td>15.2929%</td>
</tr>
<tr>
<td>Week 3</td>
<td>3.6930%</td>
<td>5.0995%</td>
<td>4.8676%</td>
<td>7.7211%</td>
<td>7.0710%</td>
<td>16.4469%</td>
</tr>
<tr>
<td>Week 4</td>
<td>3.8288%</td>
<td>5.2696%</td>
<td>5.1890%</td>
<td>7.7573%</td>
<td>7.4405%</td>
<td>15.6312%</td>
</tr>
<tr>
<td>Week 5</td>
<td>3.8297%</td>
<td>5.1176%</td>
<td>5.0952%</td>
<td>7.6595%</td>
<td>7.4632%</td>
<td>15.5475%</td>
</tr>
<tr>
<td>Week 6</td>
<td>4.2491%</td>
<td>5.6199%</td>
<td>5.5635%</td>
<td>8.3059%</td>
<td>8.0291%</td>
<td>16.7259%</td>
</tr>
<tr>
<td>Week 7</td>
<td>3.5388%</td>
<td>4.8831%</td>
<td>4.6811%</td>
<td>7.4033%</td>
<td>6.8239%</td>
<td>15.7266%</td>
</tr>
</tbody>
</table>

Figure 32: ARMA-GARCH Modeling Value at Risk

Figure 33: Weekly Value at Risk Time Series for Gaussian and Student’s t Innovations
From week to week, value at risk did not vary much percentage wise. The biggest difference that can be seen is that the student’s t innovation outputs a larger value at risk than the Gaussian innovation. Additionally, the 99% value at risk for student’s t is more than twice the 99% value at risk for Gaussian. Looking at the time series graphs for the first week, we see that the value at risk is more volatile when calculated using the normal distribution compared to using student’s t, which has a more subtle change. We can also see how the market was very erratic. A similarity between both distributions is that once the beginning of 2012 came, the value at risk became a lot smoother increasing curve as a function of alpha. The increase in the curve indicates the steady increase of the market. Looking at the last week, we are shown the progression of the value at risk during the weeks we held our portfolio. It is easy to see where week 5 occurred on the graph, which corresponds to the week when we had our biggest stock portfolio loss. This decrease is more pronounced for the normal distribution. If you look at the value at risk graphs for both Gaussian and student’s t for week 7, we can see how during the weeks we held our portfolio, more specifically week 5, the market was more volatile compared to the beginning of the 2012 year, which can be seen through the dip in the value at risk.

One other noteworthy piece of information is that the degrees of freedom calculated is significantly larger than degrees of freedom calculated with both the polynomial tail model and the maximum likelihood estimation we used in the first section. Recall that the tail index of a polynomial tail is the degrees of freedom for the t distribution. All of those estimates for the tail index were between 1.2 and 1.61, where all of the degrees of freedom outputted in this model were above 2.5 and had an increasing pattern moving from week to week. The degrees of freedom outputted from the maximum likelihood estimation were almost all fewer than 2, the exception being week 5 and 7.

We now look at the expected shortfall for the ARMA-GARCH model. Recall the following expected shortfall formulas for the Gaussian (right) and student’s t (left) innovations:

\[
ES_\alpha = \mu + \sigma \Phi[N^{-1}(\alpha)] \\
ES_\alpha = \mu + \sigma ES_\alpha(L),
\]

where

- \( \Phi \) is the probability density function,
- \( N \) is the cumulative distribution function of the standard normal distribution, and

\[
ES_\alpha(L) = \frac{g_\nu[t^{-1}(\alpha)]}{1-\alpha} \left( \frac{\nu + [t^{-1}(\alpha)]^2}{\nu - 1} \right).
\]

Equation 50: Normal and Student’s t Expected Shortfall Formulas

The weekly expected shortfall for \( \alpha = [0.90: 0.001: 1] \) was calculated at every time step of the one-year series ending with the specified week our portfolio was held. The graphs displayed below display how the expected shortfall changes for the year ending with our
invested weeks. Again, the orders of the weeks on the graphs are as before with the value at risk graphs. We compare the Gaussian and student’s t expected shortfall calculations.

<table>
<thead>
<tr>
<th>Week</th>
<th>$\text{ES}_{90%G}$</th>
<th>$\text{ES}_{90%t}$</th>
<th>$\text{ES}_{95%G}$</th>
<th>$\text{ES}_{95%t}$</th>
<th>$\text{ES}_{99%G}$</th>
<th>$\text{ES}_{99%t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>5.1827%</td>
<td>10.1045%</td>
<td>6.1651%</td>
<td>14.0167%</td>
<td>8.0885%</td>
<td>27.8861%</td>
</tr>
<tr>
<td>Week 2</td>
<td>4.5807%</td>
<td>9.2423%</td>
<td>5.5514%</td>
<td>12.9677%</td>
<td>7.4518%</td>
<td>26.0035%</td>
</tr>
<tr>
<td>Week 3</td>
<td>5.2237%</td>
<td>10.1317%</td>
<td>6.2186%</td>
<td>14.0403%</td>
<td>8.1666%</td>
<td>27.8310%</td>
</tr>
<tr>
<td>Week 4</td>
<td>5.5528%</td>
<td>9.8305%</td>
<td>6.5695%</td>
<td>13.3191%</td>
<td>8.5600%</td>
<td>25.0423%</td>
</tr>
<tr>
<td>Week 5</td>
<td>5.4779%</td>
<td>9.6983%</td>
<td>6.5472%</td>
<td>13.1810%</td>
<td>8.6406%</td>
<td>24.6605%</td>
</tr>
<tr>
<td>Week 6</td>
<td>5.0274%</td>
<td>9.6854%</td>
<td>5.9950%</td>
<td>13.4067%</td>
<td>7.8894%</td>
<td>26.6236%</td>
</tr>
<tr>
<td>Week 7</td>
<td>5.9619%</td>
<td>10.5028%</td>
<td>7.0753%</td>
<td>14.2269%</td>
<td>9.2551%</td>
<td>26.4380%</td>
</tr>
</tbody>
</table>

Table 29: ARMA-GARCH Modeling Expected Shortfall

Figure 34: Weekly Expected Shortfall Time Series for Gaussian and Student’s t Innovations
Once again, the student’s t innovations have a smoother curve compared to the normal. The Gaussian and student’s t innovations appear to follow a previous observation from the assessment of the value at risk and expected shortfall in prior sections. For the Gaussian innovations, as value at risk increases, the difference between the expected shortfall and value at risk decreases. The probability of having a loss smaller than alpha, given that you have already reached alpha, is higher. For student’s t innovations, as the value at risk increases, the difference between the expected shortfall and value at risk increases percentage-wise. From looking at the time series for both models, it seems that for most of the weeks around 15 and after, both risk measures are quite unstable, but the closer time gets to current weeks, value at risk and expected shortfall both smooth out more. The graphs do show that there is a current rise in both risk measure values. Once again the values from the student’s t innovations are considerably higher, especially for the 99% value at risk, than the Gaussian innovations.\textsuperscript{15}

\textbf{2.3.2 Calculating the Daily Value at Risk and Expected Shortfall Using Gaussian Innovations and Student’s t Innovations for the Stock and Option Portfolio}

We have previously in this chapter only considered the weekly loss of the stock portion of our portfolio. We will now model the daily loss of both the combined stock and option portfolio. Since time series of data is needed to perform the necessary risk calculations, the options part of the portfolio brought up an issue. Historical options data cannot be freely obtained, but can be purchased, usually at a costly price. Instead we resorted to collecting options data starting on March 14, 2012. Starting on April 2, 2012, the ARMA-GARCH model was run daily, whereas the days progressed, the data from the previous data was added to the data set. In other words, to estimate the value at risk and expected shortfall for April 2, 2012, the portfolio returns from March 14, 2012 to March 30, 2012 were used. After each day past, the estimation of the value at risk was done based on adding an extra day’s data into the time series. Previously we only modeled the weekly value of the portfolio according to only the stock data. Using the same procedure, we modeled the daily portfolio values with the inclusion of the time series data for options. One of the biggest differences between the previous portfolio modeling process and this particular section is that the weights of the stocks and options were changed by week to match up with the actual weights each stock and option had during its respective week. With the weekly model, the entire series corresponded to the weight of that particular week. To get a clear picture of this explanation, refer to Excel file titled “ARMAGARCHdata.xlsx” to see how this was done. The Matlab code from the previous section was used to calculate the standard deviation, mean, value at risk, and expected shortfall for the daily portfolio losses. The subscript G indicates the Gaussian innovations and the subscript t indicates the student’s t innovations. As done in prior sections, if the degrees of freedom were not greater than 3, then it was reassigned to 3. We first observe the change in the standard deviation and mean of both the Gaussian and student’s t innovations.

\textsuperscript{15} Refer to Appendix K: Matlab Code – ARMAGARCH.m for the Matlab code used for the ARMA-GARCH calculations and graphs.
The mean for both the normal and student’s t distribution were at their peaks during the first day of running the calculation. As more portfolio returns were added into the data set, the mean became less volatile between days for the student’s t distribution. The mean for the normal distribution did not have as dramatic changes as student’s t when there was a smaller data sample of portfolio returns. As the data set got larger, the mean for the normal approaches student’s t and followed the same pattern. As for the standard deviation, student’s t had a larger variation, which is no surprise by now. The normal standard deviation had very small changes. The trend here seems to be that with the addition of a data point from the beginning of the week, there is a small increase in volatility and then it lowers as the week goes on which each new data point added.\footnote{Refer to Appendix L: ARMA-GARCH Daily Computation Tables}

The value at risk generally stayed between 4-7% unless there was a spike in volatility, to which it increased very quickly, and then went back down. Both the student’s t and normal distribution followed this trend. Hence there was a spike in volatility on April 2\textsuperscript{nd}, April 9\textsuperscript{th}, and April 17\textsuperscript{th}. The difference between the value at risk using Gaussian innovations and the value at risk using student’s t innovations at the same alpha level gets larger as alpha increases. For both the value at risk and expected shortfall, though they did respond to increased volatility with larger values, the general increasing trend does so at a smaller growth as more portfolio returns were added. To clarify this statement, an increase in the value at risk at the beginning of the month is larger than the increase of the value at risk towards April 19\textsuperscript{th}.\footnote{Refer to Appendix L: ARMA-GARCH Daily Computation Tables}

The student’s t innovations report much higher values for all alpha levels for value at risk and expected shortfall compared to the Gaussian innovations. A conclusion that can be made through the daily runs is that there is a definite relationship between the standard deviation and the risk values.
Figure 36: Gaussian Daily Run for April 2, 2012 and April 20, 2012

Figure 37: Student's t Daily Run for April 2, 2012 and April 20, 2012
The ARMA-GARCH section ends with a series of value at risk and expected shortfall graphs depicting the daily run of portfolio values. All the graphs on the left are from the first day of the run, April 2, 2012, while all the graphs on the right are from the last day of the run, April 20, 2012. In the first run, we see the typical relationship between the normal and student’s t distribution of normal having lower values than student’s t. For the last run the relationship becomes clearer with more data to work with, as it can be seen how student’s t risk values are more consistent and do not increase and decrease from week to week like the normal values at risk do. Next, we observe the fit of the Gaussian and student’s t innovations.

2.3.3 Model Fit Analysis

When creating statistical models of any type, the problem of potentially over fitting or under fitting the model emerges. How well does the model produced really represent the behavior of a data set? Adding more variables does not necessarily mean the model is better because it just might add random noise in the data. In order to consider both the fit to the data and model complexity of the Gaussian innovations and student’s t innovation of our portfolio returns, we compared both the Akaike’s information criterion (AIC) and Bayesian information criterion (BIC) for each week our portfolio is held. Though we used the built in function in Matlab to compute these values, the AIC and BIC are computed in the following way:

\[
AIC = -2 \log \{ L(\hat{\theta}_{ML}) \} + 2p \\
BIC = -2 \log \{ L(\hat{\theta}_{ML}) \} + \log(n) p,
\]

where

\[
\log \{ L(\hat{\theta}_{ML}) \} \text{ is the maximized value of the log-likelihood,} \\
n \text{ is the sample size, and} \\
p \text{ is the number of parameters in the model.}
\]

Equation 51: AIC and BIC Formulas

The \( p \) term in the AIC and BIC formulas are considered the “complexity penalties,” since the more parameters added to a model, the greater the chance of over fitting and the greater the complexity of the model. It can be seen in the equation how the BIC weights the complexity of a model more heavily than the AIC, resulting the in BIC being more likely to select simpler models. In general, the smaller the AIC and BIC are, the better the model since small values maximize the likelihood function and minimized \( p \).[48] The AIC and BIC for the portfolio during the weeks held are the following:

<table>
<thead>
<tr>
<th>Week</th>
<th>( AIC_{Gaussian} )</th>
<th>( AIC_{Student's t} )</th>
<th>( BIC_{Gaussian} )</th>
<th>( BIC_{Student's t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-208.1150</td>
<td>-216.3974</td>
<td>-196.5240</td>
<td>-202.8746</td>
</tr>
<tr>
<td>2</td>
<td>-204.1642</td>
<td>-212.0194</td>
<td>-192.5733</td>
<td>-198.4966</td>
</tr>
<tr>
<td>3</td>
<td>-203.9115</td>
<td>-211.9559</td>
<td>-192.3206</td>
<td>-198.4331</td>
</tr>
<tr>
<td>4</td>
<td>-204.3663</td>
<td>-211.1689</td>
<td>-192.7754</td>
<td>-197.6461</td>
</tr>
<tr>
<td>5</td>
<td>-199.3455</td>
<td>-204.6723</td>
<td>-187.7546</td>
<td>-191.1495</td>
</tr>
<tr>
<td>6</td>
<td>-201.4079</td>
<td>-206.3854</td>
<td>-189.8169</td>
<td>-192.8626</td>
</tr>
<tr>
<td>7</td>
<td>-205.8721</td>
<td>-215.1600</td>
<td>-194.2811</td>
<td>-201.6372</td>
</tr>
</tbody>
</table>

Table 30: AIC/BIC Comparison
According to the AIC and BIC values calculated, the student’s t innovations for both the AIC and BIC is the best model for every week. The Gaussian values are not that far ahead of the student’s t, but if we are just going for a basic reading on which model is best, student’s t is the winner.

2.4 Chi-Squared Goodness of Fit

In each section we have assumed that our portfolio loss or risk factors have either come from a specific distribution, like the normal or student’s t, or of the form of a specific model, like a polynomial tail or ARMA(1,1)-GARCH(1,1). This assumption enabled us to estimate parameters and calculate the value at risk and expected shortfall for our portfolio, but how well do these models actually fit our data? In order to get an idea of well our data is fitted, we perform a Chi-Squared Goodness of Fit test. The main idea of this test is to test a hypothesis of the following nature:

\[ H_0 \text{(null)}: \text{The sample was taken from a distribution, } F \]
\[ H_1 \text{(alternative)}: \text{The sample was not taken from } F \]

where F signifies the distribution we are testing the fit of our data against.[12] When doing this type of hypothesis testing, two types of errors can occur – type I error and type II error. Type I error is when the null hypothesis is true, but it is rejected. Type II error is when the null hypothesis is not true and we fail to reject it.[48] When testing our data we compute what is known as the Chi-square test statistic, which is the following:

\[ \chi_s^2 = \sum_{i=1}^{k} \frac{(m_i - Np_i)^2}{Np_i} \]

where

- \( m_i \) are the observed frequencies,
- \( N \) is the sample size, and
- \( p_i \) are the probabilities of various classes in a distribution.

Equation 52: Chi-Squared Test Statistic

The test statistic is a measure of the deviation of a sample from expectation. Ideally the desired test statistic is a small one. After having this statistic, we then calculate the p-value, which is the value at which the probability that the distribution assumes a value of \( \chi^2 \) greater than \( \chi_s^2 \). [16] The p-value itself is the lowest level of significance (\( \alpha \)) at which the null hypothesis could have been rejected. Formally, the p-value is the following:

\[ P(\chi^2 \geq \chi_s^2) = \frac{\Gamma \left( \frac{\nu - 1}{2} \right)}{\Gamma \left( \frac{\nu - 1}{2} \right)} \]

where

- \( \nu \) is the degrees of freedom.[16]

Equation 53: P-value Formula

If the null hypothesis is true, then \( \chi^2 \) has a \( \chi^2(n - 1 - p) \) distribution, where \( p \) is the number of parameters estimated from the data.[12]

The Chi-square goodness of fit test was done on whether or not using the normal distribution and student’s t-distribution for modeling the historical returns of our
underlying assets was reasonable. The test was done for the stock portfolio on each underlying asset’s returns for the first week held and last week held using the built-in Matlab function chi2gof. Additionally the test was done on the stock portfolio log returns since the portfolio returns, not individual underlying assets, were modeled with polynomial tails. This function returns the test statistic, p value, and whether to reject or do not reject the null hypothesis. Refer to Appendix B for additional statistical information for each of the individual underlying assets and portfolio. The following table displays whether the null hypothesis was rejected or fail to be rejected for the underlying asset historical returns during the first week we held our portfolio.  

<table>
<thead>
<tr>
<th>Underlying Asset (First Week)</th>
<th>p-value (Normal)</th>
<th>Reject/Do Not Reject Null (Normal)</th>
<th>p-value (Student’s t)</th>
<th>Reject/Do Not Reject Null (Student’s t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>0.3170</td>
<td>Do Not Reject</td>
<td>0.2885</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>DELL</td>
<td>0.7100</td>
<td>Do Not Reject</td>
<td>0.6987</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>DIS</td>
<td>0.9075</td>
<td>Do Not Reject</td>
<td>0.7635</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>XOM</td>
<td>0.6900</td>
<td>Do Not Reject</td>
<td>0.2931</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>GLD</td>
<td>0.2384</td>
<td>Do Not Reject</td>
<td>0.2338</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.0067</td>
<td>Reject</td>
<td>0.0018</td>
<td>Reject</td>
</tr>
<tr>
<td>IBM</td>
<td>0.1239</td>
<td>Do Not Reject</td>
<td>0.0448</td>
<td>Reject</td>
</tr>
<tr>
<td>JPM</td>
<td>0.3435</td>
<td>Do Not Reject</td>
<td>0.1962</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.5143</td>
<td>Do Not Reject</td>
<td>0.6465</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>OIL</td>
<td>0.1710</td>
<td>Do Not Reject</td>
<td>0.1166</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>V</td>
<td>0.0093</td>
<td>Reject</td>
<td>0.0077</td>
<td>Reject</td>
</tr>
</tbody>
</table>

Table 31: Normal and Student’s t Hypothesis Test for First Week Held for Underlying Assets

The p-value is considered as evidence against the null hypothesis if it is small, whereas a large p-value shows that the data is consistent with the null hypothesis.[48] For the sake of our testing, we compare the p-value against $\alpha = 0.05$ to determine whether or not to reject the null hypothesis. As indicated in Table XX, 9 out of 11 of the underlying assets failed to reject the null hypothesis that the historical returns come from the normal distribution. Observing the p-value, we can see that the highest probabilities of the underlying assets being consistent with the normal distribution belong to Dell, Disney, and Exxon, while the smallest probabilities belong to IBM, Oil, and Gold. The lowest probability, IBM with probability 0.1239, is still a distance of 0.0739 away from the rejection region. The biggest difference between the p-values for the normal versus the student’s t is that the student’s t hypothesis testing results have smaller p-values for all assets except Microsoft. For testing with a null hypothesis that the historical returns come from a student’s t distribution, 8 out of 11 of the underlying assets failed to reject the null hypothesis. The p-value for Exxon decreased the most, more than 40%. It’s interesting to note that the asset with the smallest probability that failed to reject the null for the normal was reduced enough to reject the null when tested for student’s t. Overall, between the two distributions that we tested for fitting, if the normal reported a rejection of the null, student’s t did as well (though this does not hold vice versa). The next table displays whether the null hypothesis was rejected or fail to be rejected for the underlying asset historical returns during the last week we held our portfolio.

---

[48] Refer to Appendix M: Matlab Code – Goodness of Fit for the Matlab code used to compute the Chi-Squared Goodness of Fit test with the function. The file is titled “Chi2Testgof.m”.
As before, looking at the p-values for the normal versus student’s t, the most noticeable difference is that the normal values are larger. However, in the first week, the only increase in p-value from normal to student’s t was with Microsoft, whereas in the last week Apple was the only increased value while Microsoft decreased.

Comparing the first week held to the last week held, there were some substantial changes in the p-value for the underlying assets. Looking at the null hypothesis of the returns following a normal distribution, we see that the probability decreased for just under a half of the assets, the largest being Dell. There was a huge probability increase for Microsoft, Gold, and Apple. Disney remains the same as one of the highest probabilities and Visa, Google, and IBM remain among the lowest probabilities compared to the first week. Fundamentally, those assets that decreased in p-value are those whose returns have a smaller chance of coming from a normal distribution and those that increased have a larger chance. As stated, the most significant increases were Microsoft and Gold, so as the weeks went by, the probability of the returns coming from a normal distribution was very likely. Overall, the rejected assets were the same as the first week.

Looking at the null hypothesis of the returns following a student’s t distribution, we see that a majority of the p-value decreased, the largest being Dell once again. The largest increases in probability were Gold and Apple. Those assets that decreased and increased in probability were about the same for student’s t as they were for the normal hypothesis. Fundamentally, those assets that decreased in p-value are those whose returns have a smaller chance of coming from a student’s t distribution and those that increased have a larger chance. Overall, the rejected assets were the same as the first week, except Oil was rejected as following a student’s t this time. Once again, those assets that rejected the null for the normal also rejected the null for student’s t (though this does not hold vice versa). We now look at the stock portfolio as a whole, where the value of the portfolio was modeled as before when calculating the linearized mean and variance for the stocks. The next table displays the results from testing the log returns of the portfolio value.

<table>
<thead>
<tr>
<th>Underlying Asset (Last Week)</th>
<th>p-value (Normal)</th>
<th>Reject/Do Not Reject Null (Normal)</th>
<th>p-value (Student’s t)</th>
<th>Reject/Do Not Reject Null (Student’s t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>0.6208</td>
<td>Do Not Reject</td>
<td>0.6741</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>DELL</td>
<td>0.1321</td>
<td>Do Not Reject</td>
<td>0.1165</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>DIS</td>
<td>0.6376</td>
<td>Do Not Reject</td>
<td>0.5749</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>XOM</td>
<td>0.5588</td>
<td>Do Not Reject</td>
<td>0.1767</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>GLD</td>
<td>0.6985</td>
<td>Do Not Reject</td>
<td>0.5077</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.0082</td>
<td>Reject</td>
<td>0.0018</td>
<td>Reject</td>
</tr>
<tr>
<td>IBM</td>
<td>0.0699</td>
<td>Do Not Reject</td>
<td>0.0186</td>
<td>Reject</td>
</tr>
<tr>
<td>JPM</td>
<td>0.2827</td>
<td>Do Not Reject</td>
<td>0.1625</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.9066</td>
<td>Do Not Reject</td>
<td>0.8241</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>OIL</td>
<td>0.0515</td>
<td>Do Not Reject</td>
<td>0.0336</td>
<td>Reject</td>
</tr>
<tr>
<td>V</td>
<td>0.0216</td>
<td>Reject</td>
<td>0.0167</td>
<td>Reject</td>
</tr>
</tbody>
</table>

Table 32: Normal and Student’s t Hypothesis Test for Last Week Held for Underlying Assets
Table 33: Normal and Student’s t Hypothesis Test for Stock Portfolio

<table>
<thead>
<tr>
<th>Portfolio Log Returns</th>
<th>First Week</th>
<th>Last Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>p-value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0882</td>
<td>0.1769</td>
</tr>
<tr>
<td></td>
<td>Reject/Do Not Reject Null</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>Student’s t</td>
<td>p-value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0337</td>
<td>0.0658</td>
</tr>
<tr>
<td></td>
<td>Reject/Do Not Reject Null</td>
<td>Reject</td>
</tr>
</tbody>
</table>

The hypothesis testing done on the portfolio returns was computed because there is a relationship between polynomial tails and the student’s t distribution with respect to the tail index and a relationship between student’s t to the normal with respect to how high the degrees of freedom is. Recall that a student’s t distribution has a polynomial tail with the tail index equal to the degrees of freedom, and as the degrees of freedom increases, a student’s t distribution approaches a normal distribution. While the null hypothesis that the historical returns of the stock portfolio were sampled from a normal distribution failed to be rejected the first week, the p-value was quite small at 0.0882 and just barely was able to fail to reject the null. The probability of the returns coming from student’s t the first week was even smaller, causing the null to be rejected. Looking at the last week held, both p-values increased, causing a failure to reject both null hypotheses. However, the returns have a better chance of being sampled from a normal distribution than a student’s t distribution, and the failure to object the null for student’s t just barely missed the rejection region.

In addition to using the Matlab function for checking the goodness of fit of the distributions, we created a script file called “Chi2gofCodeTest.m” that computed the p-value, test statistic, and whether to reject or do not reject the null hypothesis that the sample came from a normal distribution, but done so by manually computing the necessary values instead of using a ready made function.\(^\text{19}\) The Matlab function defaults to using 10 bins to organize the frequency of the data values. Since the data set we are testing is only a sample of 52, defaulting the bin size to 10 leaves gives a greater chance that the frequency in some of the bins to be very low. Generally it is preferred to have a bin frequency of at least 5. The results of this are displayed in the table below, where the p-values shown are in the order of using 7 bins and 10 bins in the manually coded file, and then 10 bins with the Matlab function.

<table>
<thead>
<tr>
<th>Underlying Asset</th>
<th>p-value (First Week)</th>
<th>Reject/Do Not Reject</th>
<th>p-value (Last Week)</th>
<th>Reject/Do Not Reject</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>0.5175 / 0.2304 / 0.3170</td>
<td>N/N/N</td>
<td>0.5815 / 0.4683 / 0.6208</td>
<td>N/N/N</td>
</tr>
<tr>
<td>DELL</td>
<td>0.5250 / 0.6614 / 0.7100</td>
<td>N/N/N</td>
<td>0.2374 / 0.1691 / 0.1321</td>
<td>N/N/N</td>
</tr>
<tr>
<td>DIS</td>
<td>0.6453 / 0.8633 / 0.9075</td>
<td>N/N/N</td>
<td>0.5717 / 0.6640 / 0.6376</td>
<td>N/N/N</td>
</tr>
<tr>
<td>XOM</td>
<td>0.0314 / 0.1392 / 0.6900</td>
<td>Y/N/N</td>
<td>0.0180 / 0.0817 / 0.5588</td>
<td>Y/N/N</td>
</tr>
<tr>
<td>GLD</td>
<td>0.3312 / 0.0460 / 0.2384</td>
<td>N/Y/N</td>
<td>0.3201 / 0.2391 / 0.6985</td>
<td>N/N/N</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.2374 / 0.0017 / 0.0067</td>
<td>N/Y/Y</td>
<td>0.1038 / 0.0009 / 0.0082</td>
<td>N/Y/Y</td>
</tr>
<tr>
<td>IBM</td>
<td>0.0685 / 0.1074 / 0.1239</td>
<td>N/N/N</td>
<td>0.0314 / 0.0904 / 0.0699</td>
<td>Y/N/N</td>
</tr>
<tr>
<td>JPM</td>
<td>0.7672 / 0.3731 / 0.3435</td>
<td>N/N/N</td>
<td>0.4658 / 0.4161 / 0.2827</td>
<td>N/N/N</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.4124 / 0.5521 / 0.5143</td>
<td>N/N/N</td>
<td>0.6949 / 0.6149 / 0.9066</td>
<td>N/N/N</td>
</tr>
<tr>
<td>OIL</td>
<td>0.0059 / 0 / 0.1710</td>
<td>Y/Y/N</td>
<td>0.0071 / 0 / 0.0515</td>
<td>Y/Y/N</td>
</tr>
<tr>
<td>V</td>
<td>0 / 0 / 0.0093</td>
<td>Y/Y/Y</td>
<td>0 / 0 / 0.0216</td>
<td>Y/Y/Y</td>
</tr>
</tbody>
</table>

Table 34: Goodness of Fit Number of Bins Comparison

\(^{19}\) Refer to Appendix M: Matlab Code – Goodness of Fit for the Matlab code “Chi2gofCodeTest.m”
We discovered that while there is some consistency between the computations using 10 bins, there were a few assets that did have a significant difference in the p-values. There was an even larger difference between using 7 bins and 10 bins. In the end we relied on using the Matlab function to compute the goodness of fit for our data since it decreased the chance of a mistake being made compared to the manually coded file. In reference to the frequency of the 10 bins, most of the bins created with using the function had at least a count of 5 values. There were a few counts of 3 or 4, but not an alarmingly high amount. Thus we made our decision to focus our analysis based on the Matlab chi2gof function when testing the normal and student’s t distribution.

To summarize this section, most of the underlying assets have a lesser probability of being sampled from a normal distribution or student’s t distribution going from the first week the portfolio was held to the last week. Keep in mind though that even though there were assets where the probability decreased, some of them, like Apple and Disney for example, still had among the highest probabilities to follow said distributions. The same can be said about those assets that increased in probability, where the assets could still be among those that have the lowest chance of following said distributions. Overall the conclusion we come to is it would be deceiving to assume all stocks returns come from the same distribution and to model them that way, especially if the weight of the assets in the portfolio are taken into consideration. Of those assets that rejected the null hypothesis of being sampled from either normal or student’s t distribution, Oil was quite heavily weighted in a short position and Google was quite heavily weighted in long position in the last week the portfolio was held. IBM and Visa, however, had low weights in long positions. The change from the first week to the last week held for Oil and Google were small as well, so the increase for Google and the decrease for Oil did not have a favorable, significant change.

3. Portfolio Performance and Model Conclusions

The performance of our portfolio was better than we initially anticipated after closing our positions. Initially we invested $990,659.54. Our technique of using optimization for the stocks and fairly equality weighted options and risk free assets at the time of the initial investment appeared to have worked up until week 5. Prior to week 5, whenever the stock value increased, the bond value decreased, and vice versa. Our option portfolio did take a big hit during week 5, more so than our stock portfolio, but it
increasing in value by more than $10,000 in week 6. When we closed our positions on Friday, April 20, 2012, we recorded the closing price according to Interactive Brokers Trader Workstation and calculated the value of our portfolio. As mentioned earlier we took a big hit in the options portfolio when closing and only a small overall decrease in the stock portfolio.

<table>
<thead>
<tr>
<th>Week</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Treasuries</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$351,055.45</td>
<td>$137,473.09</td>
<td>$502,131.00</td>
<td>$990,659.54</td>
</tr>
<tr>
<td>2</td>
<td>$344,148.87</td>
<td>$181,508.00</td>
<td>$502,131.00</td>
<td>$1,027,787.87</td>
</tr>
<tr>
<td>3</td>
<td>$349,589.02</td>
<td>$157,190.00</td>
<td>$502,131.00</td>
<td>$1,008,910.02</td>
</tr>
<tr>
<td>4</td>
<td>$356,436.74</td>
<td>$138,777.00</td>
<td>$502,131.00</td>
<td>$997,344.74</td>
</tr>
<tr>
<td>5</td>
<td>$338,541.25</td>
<td>$99,894.00</td>
<td>$502,131.00</td>
<td>$940,566.25</td>
</tr>
<tr>
<td>6</td>
<td>$342,456.60</td>
<td>$112,678.00</td>
<td>$502,131.00</td>
<td>$957,265.60</td>
</tr>
<tr>
<td>7</td>
<td>$342,135.50</td>
<td>$22,779.30</td>
<td>$501,487.00</td>
<td>$866,401.80</td>
</tr>
</tbody>
</table>

Table 36: Actual Weekly Portfolio Valuation

We specifically look at the options instead of the stocks since that was the cause of our concern after closing. The largest overall loss came from IBM and Nokia, though IBM was more significant since it was more heavily weighted than Nokia.

<table>
<thead>
<tr>
<th>Underlying Asset</th>
<th>Strike Price</th>
<th>Starting Investment</th>
<th>Ending Investment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>560</td>
<td>13,244.10</td>
<td>20,020.00</td>
<td>-6,775.90</td>
</tr>
<tr>
<td>AAPL</td>
<td>560</td>
<td>17,564.10</td>
<td>14,700.00</td>
<td>2,864.10</td>
</tr>
<tr>
<td>BAC</td>
<td>8</td>
<td>-129,504.16</td>
<td>-159,323.00</td>
<td>29,818.84</td>
</tr>
<tr>
<td>BAC</td>
<td>9</td>
<td>71508.48</td>
<td>83,384.00</td>
<td>-11,875.52</td>
</tr>
<tr>
<td>BAC</td>
<td>10</td>
<td>37261.48</td>
<td>40,500.80</td>
<td>-3,239.32</td>
</tr>
<tr>
<td>EMC</td>
<td>28</td>
<td>38463.14</td>
<td>24,704.00</td>
<td>13,759.14</td>
</tr>
<tr>
<td>EMC</td>
<td>33</td>
<td>-10,353.68</td>
<td>-4,864.00</td>
<td>-5,489.68</td>
</tr>
<tr>
<td>GLD</td>
<td>167</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GLD</td>
<td>166</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>IBM</td>
<td>190</td>
<td>72,985.23</td>
<td>25,872.00</td>
<td>47,113.23</td>
</tr>
<tr>
<td>IBM</td>
<td>180</td>
<td>-43,273.98</td>
<td>-49,500.00</td>
<td>6,226.02</td>
</tr>
<tr>
<td>NFLX</td>
<td>105</td>
<td>12,728.20</td>
<td>8,840.00</td>
<td>3,888.20</td>
</tr>
<tr>
<td>NFLX</td>
<td>105</td>
<td>20,956.39</td>
<td>16,080.00</td>
<td>4,876.39</td>
</tr>
<tr>
<td>NOK</td>
<td>6</td>
<td>62,926.64</td>
<td>14,035.30</td>
<td>48,891.34</td>
</tr>
<tr>
<td>NOK</td>
<td>7</td>
<td>-27,032.85</td>
<td>-11,669.80</td>
<td>-15,363.05</td>
</tr>
</tbody>
</table>

Table 37: Option Starting and Ending Positions

Over the entire portfolio of stocks, options, and risk-free assets, we experienced a portfolio loss of 12.54%, which does not include the $8,090 cash we did not invest. One may pose the question then: why we started this assessment saying that that we actually did better than anticipated? We are unaware of the reasons why, but according to the “Closing Positions Summary” generated from the company, our option strategy on IBM is the only profit/loss reported item that does not match with what we have calculated in excel document tracking the portfolio value. The values for our put options are reversed. The prices of the options are different between the two. According to historical collection of option data, the values we inputted in Excel should be correct, however there is a chance that we could have been mistaken. Given our activity statement for our portfolio, we actually have a true portfolio value of $914,289 and only loss 7.7% of our portfolio. The timing of when this issue was discovered was after the analysis on the
value at risk and expected shortfall was already done, so all analysis is not in reference to this final portfolio value, but in reference to the $866,401 value.

In closing even though the week 7’s value at risk and expected shortfall would have surely changed due to the issue discovered, we still have concluded that student’s t distribution is a better way to model the losses of our portfolio, whether unconditionally or conditionally, compared to using the normal distribution. Ultimately we feel that the stationarity assumption is too strong and therefore would like to take into account the previous information that the conditional loss distribution enables us to do. Given the opportunity to obtain historical options data of a longer nature, we think that the ARMA-GARCH model for both the stock and option portfolio together would give us a good insight of the portfolio loss.
Bibliography


Appendix A: Matlab Code – Optimization Code

portfolioOptimization.m

% Script to plot efficient frontier for the 11 risky assets of our portfolio

% Import weekly log return data for Stock positions only for the 3 month
% historical period

AAPLr=xlsread('OptimizationData.xlsx','Expected Returns','B2');
DELLr=xlsread('OptimizationData.xlsx','Expected Returns','B3');
DISr=xlsread('OptimizationData.xlsx','Expected Returns','B4');
XOMr=xlsread('OptimizationData.xlsx','Expected Returns','B5');
GLDr=xlsread('OptimizationData.xlsx','Expected Returns','B6');
GOOGr=xlsread('OptimizationData.xlsx','Expected Returns','B7');
IBMr=xlsread('OptimizationData.xlsx','Expected Returns','B8');
JPMr=xlsread('OptimizationData.xlsx','Expected Returns','B9');
MSFTr=xlsread('OptimizationData.xlsx','Expected Returns','B10');
OILr=xlsread('OptimizationData.xlsx','Expected Returns','B11');
Vr=xlsread('OptimizationData.xlsx','Expected Returns','B12');

expReturns = [AAPLr,DELLr,DISr,XOMr,GLDr,GOOGr,IBMr,JPMr,MSFTr,OILr,Vr];
CovMatrix = xlsread('OptimizationData.xlsx','Covariance Matrix','B2:L13');

% Calculate the coefficients and optimal weights for the efficient
% frontier

bone = ones(length(expReturns),1);
iCovMatrix=inv(CovMatrix);
A = bone'*iCovMatrix*expReturns;
B = expReturns'*iCovMatrix*expReturns;
C = bone'*iCovMatrix*bone;
D = B*C - A^2;
bg = (B*iCovMatrix*bone - A*iCovMatrix*expReturns)/D; bh = (C*iCovMatrix*expReturns - A*iCovMatrix*bone)/D;
gh = bg*CovMatrix*bh;
gh = bg*CovMatrix*bh;

muf = 0.09; % from a 3 month T-bill
mumin = -gh/hh;
sdmin = sqrt((gg*(1-gg^2)/(gg*hh)));

expPortfolioReturn = linspace(min(expReturns), max(expReturns),100); sigmaP = zeros(1,100);

for i=1:100
    [optimalWeights] = optimalPortfolio(expReturns,CovMatrix,expPortfolioReturn(i));
    sigmaP(i) = sqrt(optimalWeights'*CovMatrix*optimalWeights);
end

ind = (expPortfolioReturn > mumin); % Efficient Frontier
ind2 = (expPortfolioReturn < mumin); % Indicates Locus below efficient frontier

% Plot - efficient frontier is solid curve and dashed locus is inefficient part

figure(1)
p1 = plot(sigmaP(ind),expPortfolioReturn(ind),'-',sigmaP(ind2),expPortfolioReturn(ind2),'-',sdmin,mumin,');
%Change Graph settings for better appearance
set(p1(1:2),'linewidth',4); set(p1(1:2),'color','blue'); set(p1(3),'markersize',40); set(p1(3),'color','red');
fs = 16;
xlabel('Volatility of Returns','fontsize',fsize); ylabel('Expected Return','fontsize',fsize);
grid;
% Calculation for weights of tangency portfolio
bomegabar = CovMatrix*(expReturns - mu*f*bone);

% Weights for tangency portfolio
tangencyWeights = bomegabar/(bone'*bomegabar)
sumOfTangencyWeights = sum(tangencyWeights);
sigmaT = sqrt(tangencyWeights'*CovMatrix*tangencyWeights);
muT = expReturns'*tangencyWeights;

% optimalPortfolio.m

function[optimalWeights]=optimalPortfolio(expReturns,CovMatrix,expPortfolioReturn)
% returns a n-vector optimalWeights which represent the weights for the
% portfolio with the minimal variance given the expected return of
% expPortfolioReturn.
% INPUTS:
% expReturns: an n-vector of expected returns for n risky assets
% CovMatrix: an nxn covariance matrix for these asset returns
% expPortfolioReturn: the expected return for the entire portfolio

bone = ones(length(expReturns),1);
iCovMatrix = inv(CovMatrix);
A= bone'*iCovMatrix*expReturns;
B= expReturns'*iCovMatrix*expReturns;
C = bone'*iCovMatrix*bone;
D = B*C - A^2;

bg = (B*iCovMatrix*bone - A*iCovMatrix*expReturns)/D;
bh = (C*iCovMatrix*expReturns - A*iCovMatrix*bone)/D;

optimalWeights = bg + (expPortfolioReturn*bh);
Appendix B: Matlab Code - ReturnsPortfolioGraphs.m

%Script to plot graph of portfolio value time series and Log returns against time t

% Retrieve the log returns and value of the portfolio for each time t
Vt = xlsread('LossStockData FILE NUMBER.xlsx','Portfolio'; 'M2:M53');
LogReturns = xlsread('LossStockData FILE NUMBER.xlsx','Weekly Log Returns Chart'; 'B2:L53');

% Assign the stocks to their log returns
LR_AAPL = LogReturns(:,1);
LR_DELL = LogReturns(:,2);
LR_DIS = LogReturns(:,3);
LR_XOM = LogReturns(:,4);
LR/GLD = LogReturns(:,5);
LR_GOOG= LogReturns(:,6);
LR_IBM = LogReturns(:,7);
LR_JPM = LogReturns(:,8);
LR_MSFT = LogReturns(:,9);
LR_OIL = LogReturns(:,10);
LR_V = LogReturns(:,11);

% Plot the weekly portfolio value process and the log returns
T = 51;
dt = 1;
t=[T:-dt:0];

% Plot individual graphs for value of portfolio and log returns for better view
figure(1) % plots the Log Returns
hold on
plot(t,LR_AAPL,'y-');
plot(t,LR_DELL,'m-');
plot(t,LR_DIS,'c-');
plot(t,LR/XOM,'r-');
plot(t,LR/GLD,'g-');
plot(t,LR_GOOG,'b-');
plot(t,LR_IBM,'k-');
plot(t,LR_JPM,'k--');
plot(t,LR_MSFT,'y--');
plot(t,LR_OIL,'m--');
plot(t,LR_V,'c--');
hleg1 = legend('LR_AAPL','LR_DELL','LR_DIS','LR/XOM','LR/GLD','LR_GOOG','LR_IBM','LR_JPM','LR_MSFT','LR_OIL','LR_V');
set(hleg1,'Location','NorthWestOutside');
set(hleg1,'Interpreter','none');
fszize = 16;
xlabel('Time in Weeks','fontsize',fszize);
ylabel('Log Returns','fontsize',fszize);
hold off

figure(2) % plots Vt
hold on
plot(t,Vt,'m-');
hleg1 = legend('Vt');
set(hleg1,'Location','NorthWestOutside');
set(hleg1,'Interpreter','none');
fszize = 16;
xlabel('Time in Weeks','fontsize',fszize);
ylabel('Value of Vt','fontsize',fszize);
hold off
Appendix C: Log Return Graphs for Weeks 2-6
% script file that estimates and plots both the probability density function and cumulative distribution function of the % linearized loss distribution

%get the weekly portfolio value process
Vt = xlsread('LossStockData FILE NUMBER.xlsx','Portfolio',M2:M53);

%get the Expected returns for all the 11 stocks for
ExpectedReturns = xlsread('LossStockData FILE NUMBER.xlsx','Expected Returns',B2:B12);

%get weekly weights for all 11 stocks
Weights = xlsread('LossStockData FILE NUMBER.xlsx','Weights',B2:B12);

%Get the covariance matrix
CovarianceMatrix = xlsread('LossStockData FILE NUMBER.xlsx','Covariance Matrix',B2:L12);

% Calculates the Mean Linearized Loss and Variance Linearized Loss at every time-step
for ii=1:52
% Mean of Linearized loss Distribution
tempMean_Linearized_Loss(ii) = -Vt(ii)*(Weights'*ExpectedReturns);

%Variance vector of loss distribution for 52 weeks
tempVariance_Linearized_Loss(ii) = (Vt(ii).^2)*Weights'*CovarianceMatrix*Weights;
end
Mean_Linearized_Loss=tempMean_Linearized_Loss';
Variance_Linearized_Loss=tempVariance_Linearized_Loss';

%%
% Estimates and plots the probability density function and the cumulative % distribution function of the linearized loss distribution for the most % recent week (Actual Value)

lossSigma=sqrt(Variance_Linearized_Loss(1)); %calculates the standard deviation of the most recent week
Leftplot=-3.5*lossSigma;
Rightplot=3.5*lossSigma;
XInterval=[Leftplot:100:Rightplot]; % creates x values that are 3.5 standard deviations away from the mean

pdfLossValues=normpdf(XInterval,Mean_Linearized_Loss(1),lossSigma);
cdfLossValues=normcdf(XInterval,Mean_Linearized_Loss(1),lossSigma);

figure (1)
hold on;
[haxes,hline1,hline2]=plotyy(XInterval,pdfLossValues,XInterval,cdfLossValues);
 Alec = legend('PDF','CDF');
set(Alec,'Location','NorthWest');
set(Alec,'Interpreter','none');
fsiz = 16;
axes(haxes(1));
ylabel('Probability Density Function','fontsize',fsiz)
axes(haxes(2));
ylabel('Cumulative Distribution Function','fontsize',fsiz)
hold off;
hold on

% Estimates and plots the probability density function of the linearized loss distribution for the most % recent week (Percentage Value)
Leftplot=(-3.5*lossSigma);
Rightplot=(3.5*lossSigma);
XIntervalPercent=((Leftplot:100:Rightplot])*(100/Vt(1)); % creates x values that are 3.5 standard deviations away from the mean

pdfLossValues=normpdf(XInterval,Mean_Linearized_Loss(1),lossSigma);
cdfLossValues=normcdf(XInterval,Mean_Linearized_Loss(1),lossSigma);

figure (2)
hold on;
%plot(XIntervalPercent,pdfLossValues);
[haxes,hline1,hline2]=plotyy(XIntervalPercent,pdfLossValues,XIntervalPercent,cdfLossValues);
hlleg1 = legend('PDF','CDF');
set(hlleg1,'Location','NorthWest');</ref>
set(hlleg1,'Interpreter','none');
fsize = 16;
axes(haxes(1));
ylabel('Probability Density Function','fontsize',fsize)
axes(haxes(2));
ylabel('Cumulative Distribution Function','fontsize',fsize)
hold off;
hold on

% Converts the most recent weeks mean and standard deviation for the linearized loss
% to percentages for comparison with t distribution
Most_Recent_Mean_Linearized_Loss=Mean_Linearized_Loss(1)

Mean_Linearized_Loss_Percent=(Mean_Linearized_Loss(1))*(100/Vt(1))
Variance_Linearized_Loss_Percent=(100/Vt(1))^2*(Variance_Linearized_Loss(1));

lossSigma=sqrt(Variance_Linearized_Loss(1))
lossSigma_Percent=sqrt(Variance_Linearized_Loss_Percent)
Appendix E: Matlab Code – portfolioLoss.m

% script that calculates the linearized loss of a portfolio of stocks and % options for both the Normal and Student's T distribution

% Import the vector of risk factor changes
RiskFactors=xlsread('PortfolioRiskFactors.xlsx','Portfolio Loss Vector','COLUMN 2:COLUMN 23');

% Import the parameters needed for finding the Greeks for on 1 option
OptionParameters=xlsread('Historical Volatility Data and Option Data.xlsx','Option Parameters','B NUMBER:G NUMBER');

% Generate vectors that have the coefficients for the sensitivity to time, % the underlying asset, and volatility

\[ \text{alldelta}=\text{OptionParameters}(,1); \]
\[ \text{allcpr}=\text{OptionParameters}(,2); \]
\[ \text{allS}=\text{OptionParameters}(,3); \]
\[ \text{allK}=\text{OptionParameters}(,4); \]
\[ \text{allr}=\text{OptionParameters}(,5); \]
\[ \text{alltau}=\text{OptionParameters}(,6); \]

% Generate vectors that has the coefficients for sensitivity to time, % the underlying asset, and volatility
\[ \text{AAPLCallTheta},\text{AAPLCallDelta},\text{AAPLCallVega}]=\text{getcallGreeks}(\text{allcpr}(1),\text{allS}(1),\text{allK}(1),\text{allr}(1),\text{alltau}(1)); \]
\[ \text{AAPLPutTheta},\text{AAPLPutDelta},\text{AAPLPutVega}]=\text{getputGreeks}(\text{allcpr}(2),\text{allS}(2),\text{allK}(2),\text{allr}(2),\text{alltau}(2)); \]
\[ \text{BAC8CallTheta},\text{BAC8CallDelta},\text{BAC8CallVega}]=\text{getcallGreeks}(\text{allcpr}(3),\text{allS}(3),\text{allK}(3),\text{allr}(3),\text{alltau}(3)); \]
\[ \text{BAC9CallTheta},\text{BAC9CallDelta},\text{BAC9CallVega}]=\text{getcallGreeks}(\text{allcpr}(4),\text{allS}(4),\text{allK}(4),\text{allr}(4),\text{alltau}(4)); \]
\[ \text{BAC10CallTheta},\text{BAC10CallDelta},\text{BAC10CallVega}]=\text{getcallGreeks}(\text{allcpr}(5),\text{allS}(5),\text{allK}(5),\text{allr}(5),\text{alltau}(5)); \]
\[ \text{EMC28CallTheta},\text{EMC28CallDelta},\text{EMC28CallVega}]=\text{getcallGreeks}(\text{allcpr}(6),\text{allS}(6),\text{allK}(6),\text{allr}(6),\text{alltau}(6)); \]
\[ \text{EMC33CallTheta},\text{EMC33CallDelta},\text{EMC33CallVega}]=\text{getcallGreeks}(\text{allcpr}(7),\text{allS}(7),\text{allK}(7),\text{allr}(7),\text{alltau}(7)); \]
\[ \text{GLD167PutTheta},\text{GLD167PutDelta},\text{GLD167PutVega}]=\text{getputGreeks}(\text{allcpr}(8),\text{allS}(8),\text{allK}(8),\text{allr}(8),\text{alltau}(8)); \]
\[ \text{GLD166PutTheta},\text{GLD166PutDelta},\text{GLD166PutVega}]=\text{getputGreeks}(\text{allcpr}(9),\text{allS}(9),\text{allK}(9),\text{allr}(9),\text{alltau}(9)); \]
\[ \text{IBM190PutTheta},\text{IBM190PutDelta},\text{IBM190PutVega}]=\text{getputGreeks}(\text{allcpr}(10),\text{allS}(10),\text{allK}(10),\text{allr}(10),\text{alltau}(10)); \]
\[ \text{IBM180PutTheta},\text{IBM180PutDelta},\text{IBM180PutVega}]=\text{getputGreeks}(\text{allcpr}(11),\text{allS}(11),\text{allK}(11),\text{allr}(11),\text{alltau}(11)); \]
\[ \text{NFLXCallTheta},\text{NFLXCallDelta},\text{NFLXCallVega}]=\text{getcallGreeks}(\text{allcpr}(12),\text{allS}(12),\text{allK}(12),\text{allr}(12),\text{alltau}(12)); \]
\[ \text{NFLXPutTheta},\text{NFLXPutDelta},\text{NFLXPutVega}]=\text{getputGreeks}(\text{allcpr}(13),\text{allS}(13),\text{allK}(13),\text{allr}(13),\text{alltau}(13)); \]
\[ \text{NOK6CallTheta},\text{NOK6CallDelta},\text{NOK6CallVega}]=\text{getcallGreeks}(\text{allcpr}(14),\text{allS}(14),\text{allK}(14),\text{allr}(14),\text{alltau}(14)); \]
\[ \text{NOK7CallTheta},\text{NOK7CallDelta},\text{NOK7CallVega}]=\text{getcallGreeks}(\text{allcpr}(15),\text{allS}(15),\text{allK}(15),\text{allr}(15),\text{alltau}(15)); \]

\[ \text{ThetaVector}=[\text{AAPLCallTheta};\text{AAPLPutTheta};\text{BAC8CallTheta};\text{BAC9CallTheta};\text{BAC10CallTheta};\text{EMC28CallTheta};\text{EMC33CallTheta};\text{GLD167PutTheta};\text{GLD166PutTheta};\text{IBM190PutTheta};\text{IBM180PutTheta};\text{NFLXCallTheta};\text{NFLXPutTheta};\text{NOK6CallTheta};\text{NOK7CallTheta}]; \]
\[ \text{DeltaVector}=[\text{AAPLCallDelta};\text{AAPLPutDelta};\text{BAC8CallDelta};\text{BAC9CallDelta};\text{BAC10CallDelta};\text{EMC28CallDelta};\text{EMC33CallDelta};\text{GLD167PutDelta};\text{GLD166PutDelta};\text{IBM190PutDelta};\text{IBM180PutDelta};\text{NFLXCallDelta};\text{NFLXPutDelta};\text{NOK6CallDelta};\text{NOK7CallDelta}]; \]
\[ \text{VegaVector}=[\text{AAPLCallVega};\text{AAPLPutVega};\text{BAC8CallVega};\text{BAC9CallVega};\text{BAC10CallVega};\text{EMC28CallVega};\text{EMC33CallVega};\text{GLD167PutVega};\text{GLD166PutVega};\text{IBM190PutVega};\text{IBM180PutVega};\text{NFLXCallVega};\text{NFLXPutVega};\text{NOK6CallVega};\text{NOK7CallVega}]; \]

% Import the stock weights \( V(t)w(t) \) and the number of option shares
stockterm=xlsread('Historical Volatility Data and Option Data.xlsx','StockOptionFactor','COLUMN 2:COLUMN H12');

Optionshares=xlsread('PortfolioRiskFactors.xlsx','Portfolio Loss Vector','N2:N16');

% Generate the coefficient for the sensitivity to the underlying term \( X(t+1,1) \)
AAPLspcterm=[-stockterm(1)-allS(1)*(Optionsshares(1)*DeltaVector(1))+(Optionsshares(2)*DeltaVector(2))];
BACspcterm=[-allS(3)*(Optionsshares(3)*DeltaVector(3))+(Optionsshares(4)*DeltaVector(4))+(Optionsshares(5)*DeltaVector(5))];
DELLspcterm=[-stockterm(2)];
DISspcterm=[-stockterm(3)];
EMCspcterm=[-allS(6)*(Optionsshares(6)*DeltaVector(6))+(Optionsshares(7)*DeltaVector(7))];
XOMspcterm=[-stockterm(4)];
GLDspcterm=[-allS(8)*(Optionsshares(8)*DeltaVector(8))+(Optionsshares(9)*DeltaVector(9))];
GOOGspcterm=[-stockterm(5)];
IBMspcterm=[-stockterm(6)-allS(10)*(Optionsshares(10)*DeltaVector(10))+(Optionsshares(11)*DeltaVector(11))];
JPMspcterm=[-stockterm(7)];
MSFTspcterm=[-stockterm(8)];
NFLXspcterm=[-allS(12)*(Optionsshares(12)*DeltaVector(12))+(Optionsshares(13)*DeltaVector(13))];
NOKspcterm=[-allS(14)*(Optionsshares(14)*DeltaVector(14))+(Optionsshares(15)*DeltaVector(15))];
OILspcterm=[-stockterm(10)];
Vspcterm=[-stockterm(11)];

% Generate the coefficient for the sensitivity to the volatility [X(t+1,3)]
AAPLVterm=[-(Optionsshares(1)*VegaVector(1))+(Optionsshares(2)*VegaVector(2))];
BACVterm=[-(Optionsshares(3)*VegaVector(3))+(Optionsshares(4)*VegaVector(4))+(Optionsshares(5)*VegaVector(5))];
EMCVterm=[-(Optionsshares(6)*VegaVector(6))+(Optionsshares(7)*VegaVector(7))];
GLDVterm=[-(Optionsshares(8)*VegaVector(8))+(Optionsshares(9)*VegaVector(9))];
IBMVterm=[-(Optionsshares(10)*VegaVector(10))+(Optionsshares(11)*VegaVector(11))];
NFLXVterm=[-(Optionsshares(12)*VegaVector(12))+(Optionsshares(13)*VegaVector(13))];
NOKVterm=[-(Optionsshares(14)*VegaVector(14))+(Optionsshares(15)*VegaVector(15))];

% create z vector for dot product with risk factors
zvector=[AAPLVterm;BACVterm;EMCVterm;GLDVterm;IBMVterm;NFLXVterm;NOKVterm;AAPLspcterm;BACspcterm;DELLspcterm;DISspcterm;EMCspcterm;XOMspcterm;GLDspcterm;GOOGspcterm;IBMspcterm;JPMspcterm;MSFTspcterm;NFLXspcterm;NOKspcterm;OILspcterm;Vspcterm];

% Generate a constant vector with respect to the time sensitivity [C(t)^BS]
delta=1/250;
TimeSensitivity=[-(delta*(Optionsshares(1)*ThetaVector(1))+(Optionsshares(2)*ThetaVector(2)))+(delta*(Optionsshares(3)*ThetaVector(3))+(Optionsshares(4)*ThetaVector(4))+(Optionsshares(5)*ThetaVector(5)))+(delta*(Optionsshares(6)*ThetaVector(6))+(Optionsshares(7)*ThetaVector(7)))+(delta*(Optionsshares(8)*ThetaVector(8))+(Optionsshares(9)*ThetaVector(9)))+(delta*(Optionsshares(10)*ThetaVector(10))+(Optionsshares(11)*ThetaVector(11)))+(delta*(Optionsshares(12)*ThetaVector(12))+(Optionsshares(13)*ThetaVector(13)))+(delta*(Optionsshares(14)*ThetaVector(14))+(Optionsshares(15)*ThetaVector(15))];

% Compute the linearized loss
PortfolioLinLoss=(zvector'*RiskFactors)+sum(TimeSensitivity)
Vt=xlsread('Historical Volatility Data and Option Data.xlsx','StockOptionFactor','B NUMBER'); %retrieves the stock and option portfolio %value
PortfolioLinLoss_Percent=(PortfolioLinLoss*100)/(1/Vt)

%% Normal Distribution and Student's T distribution Linearized Mean and Variance
expReturnRiskFactors=xlsread('PortfolioRiskFactors.xlsx','Expected Returns','COLUMN 2: COLUMN 23');
CovarianceMatrix=xlsread('PortfolioRiskFactors.xlsx','Covariance Matrix','B NUMBER:W NUMBER');
% Compute the linearized mean
PortfolioLinMean_Normal=(zvector'*expReturnRiskFactors)+sum(TimeSensitivity)
PortfolioLinMean_Normal_Percent=(PortfolioLinMean_Normal*100)*(1/Vt)

% Compute the linearized variance
PortfolioLinVar_Normal=zvector'*CovarianceMatrix*zvector;
PortfolioLinVar_Normal_Percent=((100/Vt)^2)*PortfolioLinVar_Normal;
PortfolioSigma_Normal=sqrt(PortfolioLinVar_Normal)
PortfolioSigma_Normal_Percent=sqrt(PortfolioLinVar_Normal_Percent)

%% Student T's Distribution
% Calculate the degrees of freedom
estPara=mle(RiskFactors,'distribution','tlocationscale'); % maximum likelihood estimation for parameters with most recent weeks risk factors

tempv=estPara(3) % extracts the degrees of freedom from the mle function vector

% Check to make sure MLE degrees of freedom is above 3, if not, assign it
% to 3
if tempv > 3
  v=tempv
else
  v=3
end

% Compute the linearized mean of multivariate t
PortfolioLinMean_tdist=(zvector'*expReturnRiskFactors)+sum(TimeSensitivity)
PortfolioLinMean_tdist_Percent=(PortfolioLinMean_tdist*100)*(1/Vt)

% Compute the linearized variance of multivariate t
PortfolioLinVar_tdist=(v/(v-2))*(zvector'*CovarianceMatrix*zvector);
PortfolioLinVar_tdist_Percent=((100/Vt)^2)*PortfolioLinVar_tdist;
PortfolioSigma_tdist=sqrt(PortfolioLinVar_tdist)
PortfolioSigma_tdist_Percent=sqrt(PortfolioLinVar_tdist_Percent)
Appendix F: Matlab Code – VaRES_Normal.m

% script file that calculates the value at risk and expected shortfall for
% the Normal Distribution

% Enter the Linearized mean and variance, as well as the value for the
% portfolio for the week you are looking to analyze

Mean_Linearized_Loss= ENTER;
Variance_Linearized_Loss=ENTER;
lossSigma=sqrt(Variance_Linearized_Loss);
Vt=ENTER;

% Compute value at risk at alpha
alpha = [0.95:0.001:0.995];
VaR_alpha = Mean_Linearized_Loss + lossSigma*norminv(alpha)

% Value at risk at 95%
VaR_alpha_95 = VaR_alpha(:,1)
VaR_alpha_95_Percent=(VaR_alpha_95)*(100/Vt) % converts Value at Risk to percentage Value at Risk

% Value at risk at 99%
VaR_alpha_99 = VaR_alpha(:,41)
VaR_alpha_99_Percent=(VaR_alpha_99)*(100/Vt)

%% Plots
% creates x values that are 3.5 standard deviations away from the mean
Leftplot=-3.5*lossSigma;
Rightplot=3.5*lossSigma;
XInterval=[Leftplot:100:Rightplot];

% Estimates and plots the cumulative distribution function of the linearized
% loss distribution (with Actual Value)
cdfLossValues=normcdf(XInterval,Mean_Linearized_Loss,lossSigma);
figure(1)
hold on;
plot(XInterval,cdfLossValues,'g');
plot([VaR_alpha_95,VaR_alpha_95],[0,1],b');
plot([VaR_alpha_99,VaR_alpha_99],[0,1],r');
plot([Mean_Linearized_Loss,Mean_Linearized_Loss],[0,1],m');
fsize=16;
ylabel('Cumulative Distribution Function',fontsize,fsize);
hold off;
hold on
%%
% Estimates and plots the cumulative distribution function of the linearized
% loss distribution (with Percentage Value)
LeftPerplot=((4*lossSigma)/Vt)*100;
RightPerplot=(4*lossSigma)/Vt)*100;
XIntervalPercent=[LeftPerplot:0.1:RightPerplot]; %converts loss values to loss percentages

Mean_Linearized_Loss_Percent=(Mean_Linearized_Loss)*(100/Vt);

Variance_Linearized_Loss_Percent=(100/Vt)*2*(Variance_Linearized_Loss); % adjustment to the variance for multiplying by a constant
lossSigmaPercent=sqrt(Variance_Linearized_Loss_Percent);
cdfLossValuesPercent=normcdf(XIntervalPercent,Mean_Linearized_Loss_Percent,lossSigmaPercent);
ActualLossesPercent=xlsread('Historical Volatility Data and Option Data.xlsx','StockOptionFactor','C19:C24');
figure(2)
hold on;
plot(XIntervalPercent,cdfLossValuesPercent,'g');
plot([VaR_alpha_95_Percent,VaR_alpha_95_Percent],[0,1],'b');
plot([VaR_alpha_99_Percent,VaR_alpha_99_Percent],[0,1],'r');
plot([Mean_Linearized_Loss_Percent,Mean_Linearized_Loss_Percent],[0,1],'m');
plot(ActualLossesPercent,0,'x');
fsize=16;
ylabel('Cumulative Distribution Function','fontsize',fsize);
hold off;
hold on

% Estimates and plots the probability density function of the linearized loss distribution (with Actual Values)

pdfLossValues=normpdf(XInterval,Mean_Linearized_Loss,lossSigma);
figure(3)
hold on
plot(XInterval,pdfLossValues,'g');
plot([VaR_alpha_95,VaR_alpha_95],[0,0.000025],'b');
plot([VaR_alpha_99, VaR_alpha_99],[0,0.000025],'r');
plot([Mean_Linearized_Loss,Mean_Linearized_Loss],[0,0.000025],'m');
fsize=16;
ylabel('Probability Density Function','fontsize',fsize);
hold off;
hold on

% Estimates and plots the probability density function of the linearized loss distribution (with Percentage Value)

figure(4)
LeftPerplot=(-4*lossSigma);
RightPerplot=(4*lossSigma);
XIntervalPercent=([Leftplot:100:Rightplot]*(100/Vt));

pdfLossValues=normpdf(XInterval,Mean_Linearized_Loss,lossSigma);
hold on;
plot(XIntervalPercent,pdfLossValues,'g');
plot([VaR_alpha_95_Percent,VaR_alpha_95_Percent],[0,0.000025],'b');
plot([VaR_alpha_99_Percent,VaR_alpha_99_Percent],[0,0.000025],'r');
plot([Mean_Linearized_Loss_Percent,Mean_Linearized_Loss_Percent],[0,0.000025],'m');
plot(ActualLossesPercent,0,'x');
fsize=16;
ylabel('Probability Density Function','fontsize',fsize);
hold off;
hold on

%% Expected Shortfall
Compute expected short fall of \( L \) for a Normal Distribution
expectedShortfall_LNormal = Mean_Linearized_Loss + ((lossSigma*normpdf(norminv(alpha))).*((1-alpha).^(-1))))
ES_alpha_95 = expectedShortfall_LNormal(:,1)
ES_alpha_95_Percent=(ES_alpha_95)*(100/Vt) % converts Expected Shortfall to percentage Value at Risk

% Value at risk at 99%
ES_alpha_99 = expectedShortfall_LNormal(:,41)
ES_alpha_99_Percent=(ES_alpha_99)*(100/Vt)

%!Plot expected shortfall and VaR of alpha for the week analyzed (Actual)
figure(5)
hold on
plot(alpha,VaR_alpha,'b');
plot(alpha,expectedShortfall_LNormal,'g');
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Value at Risk and Expected Shortfall','fontsize',fsize);
hold off
hold on

%!Plot expected shortfall and VaR of alpha for the week analyzed (Percentage Value)
figure(6)
hold on
VaR_alpha_Percent=(VaR_alpha)*(100/Vt); % converts the weekly Value at Risk to percentage Value at Risk
expectedShortfall_LNormal_Percent=(expectedShortfall_LNormal)*(100/Vt);
alpha_Percent=alpha*100;
plot(alpha_Percent,VaR_alpha_Percent,'b');
plot(alpha_Percent,expectedShortfall_LNormal_Percent,'g');
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Value at Risk and Expected Shortfall','fontsize',fsize);
hold off
hold on
Appendix G: Matlab Code – VaR_ES_T_Dist

% script file that calculates the value at risk and expected shortfall for
% the Student's t distribution

% Enter the linearized mean and variance, as well as the value for the
% portfolio for the week you are looking to analyze and the degrees of
% freedom

PortfolioLinMean_tdist=ENTER; % Portfolio Lin Mean tDist
PortfolioLinVar_tdist=ENTER; % Portfolio Lin Var tDist
lossSigma_tdist=sqrt(PortfolioLinVar_tdist); % Loss Sigma tDist
Vt=ENTER; % T (Portfolio Value)
v=ENTER; % Degrees of Freedom

% Compute value at risk at alpha
alpha = [0.95:0.001:0.995]; % Alpha Values
VaR_alpha_t = PortfolioLinMean_tdist + lossSigma_tdist*tinv(alpha, v); % VaR at tDist

% Value at risk at 95%
VaR_alpha_95 = VaR_alpha_t(:,1)
VaR_alpha_95_Percent=(VaR_alpha_95*100)*(1/Vt)

% Value at risk at 99%
VaR_alpha_99 = VaR_alpha_t(:,41)
VaR_alpha_99_Percent=(VaR_alpha_99*100)*(1/Vt)

% Compute expected short fall of L~
expectedShortfallLtilda = ((tpdf(tinv(alpha,v),v))/(1-alpha))*((v + (tinv(alpha,v).^2))/(v-1));

%Compute expected short fall of L
expectedShortfallL_tdist = PortfolioLinMean_tdist + (lossSigma_tdist*expectedShortfallLtilda);

% Expected Shortfall at 95%
expectedShortfallL_tdist_alpha_95 = expectedShortfallL_tdist(:,1)
expectedShortfallL_tdist_alpha_95_Percent=(expectedShortfallL_tdist_alpha_95*100)*(1/Vt)

% Expected Shortfall at 99%
expectedShortfallL_tdist_alpha_99 = expectedShortfallL_tdist(:,41)
expectedShortfallL_tdist_alpha_99_Percent=(expectedShortfallL_tdist_alpha_99*100)*(1/Vt)

%% Plots

% Estimates and plots the cdf and pdf of the Standard Student's t
XIntervalStandard=[-8:0.01:8]; % Interval Standard
cdfLossValues_tdist=tcdf(XIntervalStandard,v); % CDF Loss Values
pdfLossValues_tdist=tpdf(XIntervalStandard,v); % PDF Loss Values

figure(1) % Standard t distribution graph
hold on;
[haxes,hline1,hline2]=plotyy(XIntervalStandard,pdfLossValues_tdist,XIntervalStandard,cdfLossValues_tdist);
fsiz=16;
axes(haxes(1));
ylabel('Probability Density Function','fontsize',fsiz);
axes(haxes(2));
ylabel('Cumulative Distribution Function','fontsize',fsiz);
hold off;

%%
% Location Scale pdf and cdf plots
VaRInterval=-VaR_alpha_t(1):10000:VaR_alpha_t(1);
pdfLossValuesLocationScale=pdf('tlocationscale',VaRInterval,PortfolioLinMean_tdist,lossSigma_tdist,v);
cdfLossValuesLocationScale = cdf('locationscale', VaRInterval, PortfolioLinMean_tdist, lossSigma_tdist, v);

figure(2) %Location scaled t distribution graph
hold on;
[haxes, hline1, hline2] = plotyy(VaRInterval, pdfLossValuesLocationScale, VaRInterval, cdfLossValuesLocationScale);
fsize = 16;
axes(haxes(1));
ylabel('Probability Density Function', 'fontsize', fsize);
axes(haxes(2));
ylabel('Cumulative Distribution Function', 'fontsize', fsize);
hold off;
hold on

%% % Plot the cdf with the expected loss, 95%, and 99% VaR (Percentage Value)
VaRIntervalPer = ((-VaR_alpha_t(1):10000:VaR_alpha_t(1))*100)*(1/Vt); % Percentage Interval
figure(3)
hold on;
PortfolioLinMean_tdist_Percent = (PortfolioLinMean_tdist*100)*(1/Vt);
plot(VaRIntervalPer, cdfLossValuesLocationScale, 'g');
plot([VaR_alpha_95_Percent, VaR_alpha_95_Percent], [0, 1], 'b');
plot([VaR_alpha_99_Percent, VaR_alpha_99_Percent], [0, 1], 'r');
plot([PortfolioLinMean_tdist_Percent, PortfolioLinMean_tdist_Percent], [0, 1], 'm');
ylabel('Cumulative Distribution Function', 'fontsize', fsize);
hold off;
hold on

%% % Plot the pdf with the expected loss, 95%, and 99% VaR (Percentage Value)
figure(4)
hold on;
plot(VaRIntervalPer, pdfLossValuesLocationScale, 'g');
plot([VaR_alpha_95_Percent, VaR_alpha_95_Percent], [0, 0.000014], 'b');
plot([VaR_alpha_99_Percent, VaR_alpha_99_Percent], [0, 0.000014], 'r');
plot([PortfolioLinMean_tdist_Percent, PortfolioLinMean_tdist_Percent], [0, 0.000014], 'm');
ylabel('Probability Density Function', 'fontsize', fsize);
hold off;
hold on

%% % Plots VAR and expected shortfall (Percentage)
figure(5)
hold on
VaR_alpha_t_Percent = (VaR_alpha_t*100)*(1/Vt);
expectedShortfallL_tdist_Percent = (expectedShortfallL_tdist*100)*(1/Vt);
plot(alpha*100, VaR_alpha_t_Percent, 'b');
plot(alpha*100, expectedShortfallL_tdist_Percent, 'g');
fsize = 16;
xlabel('alpha', 'fontsize', fsize);
ylabel('Value at Risk and Expected Shortfall', 'fontsize', fsize);
hold off;
hold on
% script that calculates the linearized loss of a portfolio of stocks and options for both the Normal and Student's T distribution

% Import the vector of risk factor changes
RiskFactors=xlsread('PortfolioRiskFactors.xlsx','Portfolio Loss Vector','I2:I25');

% Import the parameters needed for finding the Greeks for on 1 option
OptionParameters=xlsread('Historical Volatility Data and Option Data.xlsx','Option Parameters','B104:G118');

alldelta=OptionParameters(:,1);
allcpPrices=OptionParameters(:,2);
alls=OptionParameters(:,3);
allk=OptionParameters(:,4);
allr=OptionParameters(:,5);
alltau=OptionParameters(:,6);

% Generate vectors that has the coefficients for the sensitivity to time, the underlying asset, and volatility

ThetaVector=[AAPLCallTheta,AAPLCallDelta,AAPLCallVega;AAPLPutTheta,AAPLPutDelta,AAPLPutVega;BAC8CallTheta,BAC8CallDelta,BAC8CallVega;BAC9CallTheta,BAC9CallDelta,BAC9CallVega;EMC28CallTheta,EMC28CallDelta,EMC28CallVega;EMC33CallTheta,EMC33CallDelta,EMC33CallVega;GLD167PutTheta,GLD167PutDelta,GLD167PutVega;GLD166PutTheta,GLD166PutDelta,GLD166PutVega;IBM190PutTheta,IBM190PutDelta,IBM190PutVega;IBM180PutTheta,IBM180PutDelta,IBM180PutVega;NFLXCallTheta,NFLXCallDelta,NFLXCallVega;NFLXPutTheta,NFLXPutDelta,NFLXPutVega;NOK6CallTheta,NOK6CallDelta,NOK6CallVega;NOK7CallTheta,NOK7CallDelta,NOK7CallVega];
DeltaVector=[AAPLCallDelta,AAPLPutDelta;BAC8CallDelta,BAC9CallDelta;EMC28CallDelta,EMC33CallDelta;GLD167PutDelta,GLD166PutDelta;IBM190PutDelta,IBM180PutDelta;NFLXCallDelta,NFLXPutDelta;NOK6CallDelta,NOK7CallDelta];
VegaVector=[AAPLCallVega,AAPLPutVega;BAC8CallVega,BAC9CallVega;EMC28CallVega,EMC33CallVega;GLD167PutVega,GLD166PutVega;IBM190PutVega,IBM180PutVega;NFLXCallVega,NFLXPutVega;NOK6CallVega,NOK7CallVega];

% Import the stock weights [V(t+1)] and the number of option shares
stockterm=xlsread('Historical Volatility Data and Option Data.xlsx','StockOptionFactor','I2:I14');
Optionsshares=xlsread('PortfolioRiskFactors.xlsx','Portfolio Loss Vector','N2:N16');

% Generate the coefficient for the sensitivity to the underlying term [X(t+1,1)]
$$AAPLspcterm=-\text{stockterm}(1)\text{-allS}(1)*((\text{Optionshares}(1)\text{)*DeltaVector}(1))+(\text{Optionshares}(2)\text{)*DeltaVector}(2))$$
$$BACspcterm=-\text{allS}(3)*(\text{Optionshares}(3)\text{)*DeltaVector}(3)+((\text{Optionshares}(4)\text{)*DeltaVector}(4))+(\text{Optionshares}(5)\text{)*DeltaVector}(5))$$
$$DELLspcterm=-\text{stockterm}(2)$$
$$DISspcterm=-\text{stockterm}(3)$$
$$EMCspcterm=-\text{allS}(6)*((\text{Optionshares}(6)\text{)*DeltaVector}(6))+(\text{Optionshares}(7)\text{)*DeltaVector}(7))$$
$$XOMspcterm=-\text{stockterm}(4)$$
$$GLDspcterm=-\text{stockterm}(5)\text{-allS}(8)*((\text{Optionshares}(8)\text{)*DeltaVector}(8))+(\text{Optionshares}(9)\text{)*DeltaVector}(9))$$
$$GOOGspcterm=-\text{stockterm}(6)$$
$$IBMpcterm=-\text{stockterm}(7)\text{-allS}(10)*((\text{Optionshares}(10)\text{)*DeltaVector}(10))+(\text{Optionshares}(11)\text{)*DeltaVector}(11))$$
$$JPMspcterm=-\text{stockterm}(8)$$
$$MSFTspcterm=-\text{stockterm}(9)$$
$$NFLXspcterm=-\text{allS}(12)*((\text{Optionshares}(12)\text{)*DeltaVector}(12))+(\text{Optionshares}(13)\text{)*DeltaVector}(13))$$
$$NOKspcterm=-\text{allS}(14)*((\text{Optionshares}(14)\text{)*DeltaVector}(14))+(\text{Optionshares}(15)\text{)*DeltaVector}(15))$$

% Generate the coefficient for the sensitivity to the volatility [X(t+1,3)]

$$AAPLVterm=-((\text{Optionshares}(1)\text{)*VegaVector}(1))+(\text{Optionshares}(2)\text{)*VegaVector}(2))$$
$$BACVterm=-((\text{Optionshares}(3)\text{)*VegaVector}(3))+(\text{Optionshares}(4)\text{)*VegaVector}(4))+(\text{Optionshares}(5)\text{)*VegaVector}(5))$$
$$EMCVterm=-((\text{Optionshares}(6)\text{)*VegaVector}(6))+(\text{Optionshares}(7)\text{)*VegaVector}(7))$$
$$GLDVterm=-((\text{Optionshares}(8)\text{)*VegaVector}(8))+(\text{Optionshares}(9)\text{)*VegaVector}(9))$$
$$IBMVterm=-((\text{Optionshares}(10)\text{)*VegaVector}(10))+(\text{Optionshares}(11)\text{)*VegaVector}(11))$$
$$NFLXVterm=-((\text{Optionshares}(12)\text{)*VegaVector}(12))+(\text{Optionshares}(13)\text{)*VegaVector}(13))$$
$$NOKVterm=-((\text{Optionshares}(14)\text{)*VegaVector}(14))+(\text{Optionshares}(15)\text{)*VegaVector}(15))$$

% create z vector for dot product with risk factors

$$zvector=[AAPLVterm;BACVterm;EMCVterm;GLDVterm;IBMVterm;NFLXVterm;NOKVterm;AAPLspcterm;BACspcterm;DELLspcterm;DISspcterm;EMCspcterm;XOMspcterm;GLDspcterm;GOOGspcterm;IBMspcterm;JPMspcterm;MSFTspcterm;NFLXspcterm;NOKspcterm;OILspcterm;Vspcterm;VXXspcterm;VXZspcterm]$$

% Generate a constant vector with respect to the time sensitivity [C(t)^BS]

$$\delta=1/250,$$

$$\text{TimeSensitivity}=-((\text{delta}*(\text{Optionshares}(1)\text{)*ThetaVector}(1))+(\text{Optionshares}(2)\text{)*ThetaVector}(2)))+((\text{delta}*(\text{Optionshares}(3)\text{)*ThetaVector}(3))+(\text{Optionshares}(4)\text{)*ThetaVector}(4))+(\text{Optionshares}(5)\text{)*ThetaVector}(5)))+((\text{delta}*(\text{Optionshares}(6)\text{)*ThetaVector}(6))+(\text{Optionshares}(7)\text{)*ThetaVector}(7)))+((\text{delta}*(\text{Optionshares}(8)\text{)*ThetaVector}(8))+(\text{Optionshares}(9)\text{)*ThetaVector}(9)))+((\text{delta}*(\text{Optionshares}(10)\text{)*ThetaVector}(10))+(\text{Optionshares}(11)\text{)*ThetaVector}(11)))+((\text{delta}*(\text{Optionshares}(12)\text{)*ThetaVector}(12))+(\text{Optionshares}(13)\text{)*ThetaVector}(13)))+((\text{delta}*(\text{Optionshares}(14)\text{)*ThetaVector}(14))+(\text{Optionshares}(15)\text{)*ThetaVector}(15)))$$

% Compute the linearized loss

$$\text{PortfolioLinLoss}=(zvector*\text{RiskFactors})+\sum(\text{TimeSensitivity})$$

$$Vt=xlsread('Historical Volatility Data and Option Data.xlsx','\text{StockOptionFactor}',\text{'B25'});$$

%retrieves the stock and option portfolio

%value

$$\text{PortfolioLinLoss\_Percent}=(\text{PortfolioLinLoss}\times100)/(1/Vt)$$

%% Normal Distribution and Student's T distribution Linearized Mean and Variance
expReturnRiskFactors = xlsread('PortfolioRiskFactors.xlsx', 'Expected Returns', 'H2:H25');
CovarianceMatrix = xlsread('PortfolioRiskFactors.xlsx', 'Covariance Matrix', 'B170:Y193');
RiskFactorWeights = xlsread('Market Portfolio Project 1.xlsx', 'Risk Factor Weights', 'I2:I25');

%% Normal Distribution

% Compute the linearized mean
PortfoliosRiskMean_Normal = (zvector' * expReturnRiskFactors) + sum(TimeSensitivity);
PortfoliosRiskMean_Normal_Percent = (PortfoliosRiskMean_Normal * 100) / (1 / Vt);

% Compute the linearized variance
PortfoliosRiskVar_Normal = zvector' * CovarianceMatrix * zvector;
PortfoliosRiskVar_Normal_Percent = ((100 / Vt)^2) * PortfoliosRiskVar_Normal;
PortfoliosRiskSigma_Normal = sqrt(PortfoliosRiskVar_Normal);
PortfoliosRiskSigma_Normal_Percent = sqrt(PortfoliosRiskVar_Normal_Percent);

%% Student T's Distribution

% Calculate the degrees of freedom
estPara = mle(RiskFactors, 'distribution', 'tlocationscale'); % maximum likelihood estimation for parameters with most recent weeks risk factors

tempv = estPara(3); % extracts the degrees of freedom from the mle function vector

% Check to make sure MLE degrees of freedom is above 3, if not, assign it to 3
if tempv > 3
    v = tempv;
else
    v = 3;
end

% Compute the linearized mean of multivariate t
PortfoliosRiskMean_tdist = (zvector' * expReturnRiskFactors) + sum(TimeSensitivity);
PortfoliosRiskMean_tdist_Percent = (PortfoliosRiskMean_tdist * 100) / (1 / Vt);

% Compute the linearized variance of multivariate t
PortfoliosRiskVar_tdist = (v / (v - 2)) * (zvector' * CovarianceMatrix * zvector);
PortfoliosRiskVar_tdist_Percent = ((100 / Vt)^2) * PortfoliosRiskVar_tdist;
PortfoliosRiskSigma_tdist = sqrt(PortfoliosRiskVar_tdist);
PortfoliosRiskSigma_tdist_Percent = sqrt(PortfoliosRiskVar_tdist_Percent);
Appendix I: Matlab Code - polynomialTail.m

% script file that computes the value at risk and expected shortfall using % Polynomial Tails

portfolioReturns = xlsread('file', 'Ordered Portfolio Returns', 'C2:C53');

% Select K as 10% of sample size which is 51 in our case
k = [1:5];
n = 51;

% Compute log(-Rk) for the sample k
logRk = log(-portfolioReturns(k))';

% Compute mean of Log Rk
meanLogRk = mean(logRk);

% Compute X for regression
X = log(k/n);

% Compute mean of X
meanX = mean(X);

% Estimate coefficient Beta1
beta1 = (sum(logRk*(X - meanX)))/sum((X - meanX).^2);

% Estimate Beta0
beta0 = meanLogRk - (beta1*meanX);

% Tail Index Regression Estimator
aHat = (-1)/beta1

% Compute constant A
A = exp((beta0*aHat) + log(aHat))

% Compute value at risk at alpha
alpha = [0.9:0.001:1];
alpha0 = 0.9;

% Portfolio Value
Vt=xlsread('file','Weights','J13');

% Import Ordered Portfolio Log Returns for Var(alpha0) computation
portfolioLogReturns=xlsread('file','Ordered Portfolio Returns','E2:E52');

% Compute VarAlpha0 using non parametric estimation
VarAlpha0 = (-Vt)*quantile(portfolioLogReturns,(1-alpha0))

% Compute VarAlpha using semi-parametric estimation
VarAlpha = VarAlpha0*((1-alpha0)/(1-alpha)).^(1/aHat))

% VaR 90,95,99 (Actual and Percentage Value
VarAlpha_90=VarAlpha(1)
VarAlpha_95=VarAlpha(51)
VarAlpha_99=VarAlpha(91)

VarAlpha_90_Percent=VarAlpha(1)*(100/Vt)
VarAlpha_95_Percent=VarAlpha(51)*(100/Vt)
VarAlpha_99_Percent = VarAlpha(91) * (100/Vt)

% Compute Expected Shortfall
expectedShortfallAlpha = (aHat. / (aHat - 1)) * VarAlpha

% ES 90, 95, 99 (Actual and Percentage Value)
expectedShortfallAlpha_90 = expectedShortfallAlpha(1)
expectedShortfallAlpha_95 = expectedShortfallAlpha(51)
expectedShortfallAlpha_99 = expectedShortfallAlpha(91)

expectedShortfallAlpha_90_Percent = expectedShortfallAlpha(1) * (100/Vt)
expectedShortfallAlpha_95_Percent = expectedShortfallAlpha(51) * (100/Vt)
expectedShortfallAlpha_99_Percent = expectedShortfallAlpha(91) * (100/Vt)

% plot value at risk
hold on
figure(1)
plot(alpha, VarAlpha);
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Value at Risk','fontsize',fsize);
hold off

% plot expected shortfall
hold on
figure(2)
plot(alpha, expectedShortfallAlpha);
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Expected Shortfall','fontsize',fsize);
hold off

% Plots value at risk and expected shortfall (Actual)
hold on
figure(3)
plot(alpha, VarAlpha, 'g', alpha, expectedShortfallAlpha, 'g--');
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Value at Risk and Expected Shortfall','fontsize',fsize);
hold off

% Plots value at risk and expected shortfall (Percentage)
VarAlpha_Percent = VarAlpha * (100/Vt);
expectedShortfallAlpha_Percent = expectedShortfallAlpha * (100/Vt);

hold on
figure(4)
plot(alpha * 100, VarAlpha_Percent, 'g', alpha * 100, expectedShortfallAlpha_Percent, 'g--');
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Value at Risk and Expected Shortfall','fontsize',fsize);
hold off
Appendix J: Matlab Code - Polydensity.m

% Script file that graphs the density of the polynomial tail and actual losses as percentage

% Enter in values for aHat, A, and Vt
aHat=;
A=;
Vt=;

% Enter in the k values for the tail of your loss distribution
y=[];

% Calculate the density for y
polydensity=A*(abs(y)).^(-1*(aHat+1))

% Enter all actual losses
allLosses=[];

% plot the density
figure(1)% percentage
hold on
plot(y*(100/Vt),polydensity,'g')
plot(allLosses*(100/Vt),0,'bx')
fs = 16;
xlabel('Percentage Loss (%)','fontsize',fs);
ylabel('f(y)', 'fontsize', fs);
hold off
Appendix K: Matlab Code – ARMAGARCH.m

%get the weekly portfolio log returns
LogReturnsVt=xlsread('FILE', 'Portfolio', 'N2:N52');

spec = garchset('Distribution', 'T', 'P', 1, 'Q', 1, 'R', 1, 'M', 1)
spec2 = garchset('P', 1, 'Q', 1, 'R', 1, 'M', 1)
[specT, errorsT, LLFT, residualsT, sigmasT] = garchfit(spec, LogReturnsVt);
[specN, errorsN, LLFN, residualsN, sigmasN] = garchfit(spec2, LogReturnsVt);

%Compute mu for T dist
muT = LogReturnsVt-residualsT;

%Compute mu for normal dist
muN = LogReturnsVt-residualsN;

estParam=mle(LogReturnsVt,'distribution','tlocationscale')

% Temporary Degrees of freedom for t-dist
tempV = estParam(3);
if tempV > 3
%Degrees of freedom for t-dist
v = tempV
else
v = 3
end

%plot conditional Standard Deviations for T-Dist
figure(1)
subplot(2,1,1)
plot(sigmasT)
xlim([0,length(LogReturnsVt)])
title('Conditional Standard Deviations for T Distribution')

%plot standardized Residuals for T-Dist
subplot(2,1,2)
plot(residualsT./sigmasT)
xlim([0,length(LogReturnsVt)])
title('Standardized Residuals for T Distribution')

%plot mu for T-Dist
figure(2)
plot(muT)
xlim([0,length(LogReturnsVt)])
title('muT for T Distribution')

%plot conditional Standard Deviations for Normal
figure(3)
subplot(2,1,1)
plot(sigmasN)
xlim([0,length(LogReturnsVt)])
title('Conditional Standard Deviations for Normal Distribution')

%plot standardized Residuals for Normal
subplot(2,1,2)
plot(residualsN./sigmasN)
xlim([0,length(LogReturnsVt)])
title('Standardized Residuals for Normal Distribution')

%plot mu for Normal
figure(4)
plot(muN)
xlim([0,length(LogReturnsVt)])
title('muN for Normal Distribution')

%Compare model fits using AIC and BIC
[aic,bic] = aicbic([LLFT,LLFN],[7,6],length(LogReturnsVt))

% Define range of alpha to compute value at risk
alpha = [0.90:0.001:1];
for ii=1:length(alpha);

%Compute Value at risk for t-Dist
VaR_alpha_t(:,ii) = muT + sigmasT.*tinv(alpha(ii), v);

%Compute expected short fall of L~ for t-dist
expectedShortfall_tilda(:,ii) = ((tpdf(tinv(alpha(ii),v),v))/((1-alpha(ii))*((v +(tinv(alpha(ii),v).^2))/(v-1)));

%Compute expected short fall for t-dist
expectedShortfall_tdist(:,ii) = muT + (sigmasT*expectedShortfall_tilda(ii));

%Compute Value at risk for normal-Dist
VaR_alpha_N(:,ii) =  muT + sigmasT*norminv(alpha(ii));

%Compute expected short fall for normal-dist
expectedShortfall_Ndist(:,ii) = muT + (sigmasT*normpdf(norminv(alpha(ii)))).*((1-alpha(ii)).^(-1));
end

% Get the values for 90%,95%,99% Value at Risk and Expected Shortfall
% (Percentage Value) for the most recent week of the time series
VaR_alpha_t_90_Percent=VaR_alpha_t(1,1)*100
VaR_alpha_t_95_Percent=VaR_alpha_t(1,51)*100
VaR_alpha_t_99_Percent=VaR_alpha_t(1,91)*100
expectedShortfall_tdist_90_Percent=expectedShortfall_tdist(1,1)*100
expectedShortfall_tdist_95_Percent=expectedShortfall_tdist(1,51)*100
expectedShortfall_tdist_99_Percent=expectedShortfall_tdist(1,91)*100
VaR_alpha_N_90_Percent=VaR_alpha_N(1,1)*100
VaR_alpha_N_95_Percent=VaR_alpha_N(1,51)*100
VaR_alpha_N_99_Percent=VaR_alpha_N(1,91)*100
expectedShortfall_Ndist_90_Percent=expectedShortfall_Ndist(1,1)*100
expectedShortfall_Ndist_95_Percent=expectedShortfall_Ndist(1,51)*100
expectedShortfall_Ndist_99_Percent=expectedShortfall_Ndist(1,91)*100

%%
%plot value at risk against weeks of t-dist
hold on
figure(5)
surf(alpha*100,[1:1:length(LogReturnsVt)],VaR_alpha_t*100);
fsize = 16;
xlabel('alpha','fontsize',fsize);
ylabel('Weeks','fontsize',fsize);
zlabel('Value at Risk','fontsize',fsize);
ttitle('Value at Risk for t Distribution');
hold off
% plot expectedShortfall against weeks of t-dist
hold on
figure(6)
surf(alpha*100,[1:1:length(LogReturnsVt)],expectedShortfall_tdist*100);
fsSize = 16;
xlabel('alpha','fontsize',fsSize);
ylabel('Weeks','fontsize',fsSize);
zlabel('Expected Shortfall','fontsize',fsSize);
title('Expected Shortfall for t Distribution');
hold off

% plot value at risk against weeks of N dist
hold on
figure(7)
surf(alpha*100,[1:1:length(LogReturnsVt)],VaR_alpha_N*100);
fsSize = 16;
xlabel('alpha','fontsize',fsSize);
ylabel('Weeks','fontsize',fsSize);
zlabel('Value at Risk','fontsize',fsSize);
title('Value at Risk for Gaussian Distribution');
hold off

% plot expectedShortfall against weeks of N dist
hold on
figure(8)
surf(alpha*100,[1:1:length(LogReturnsVt)],expectedShortfall_Ndist*100);
fsSize = 16;
xlabel('alpha','fontsize',fsSize);
ylabel('Weeks','fontsize',fsSize);
zlabel('Expected Shortfall','fontsize',fsSize);
title('Expected Shortfall for Gaussian Distribution');
hold off

% plot modeled loss with actual losses
% Get the actual losses and portfolio value
allLosses=xlsread('FILE','Ordered Portfolio Returns','B2:B52');
Vt=xlsread('FILE','Weights','J13');

ZT=residualsT./sigmasT;
LossesT=muT+sigmasT.*ZT;

ZN=residualsN./sigmasN;
LossesN=muN+sigmasN.*ZN;

hold on
figure(9)
plot([1:1:length(LogReturnsVt)],LossesT*100,'b',[1:1:length(LogReturnsVt)],LossesN*100,'m',[1:1:length(LogReturnsVt)],(allLosses*(100/Vt)),'g');
hold off
### Appendix L: ARMA-GARCH Daily Computation Tables

<table>
<thead>
<tr>
<th>Day</th>
<th>$\sigma_{t,G}$</th>
<th>$\sigma_{t,t}$</th>
<th>$\mu_{t,G}$</th>
<th>$\mu_{t,t}$</th>
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<td>0.4284</td>
<td>0.0069</td>
<td>0.0059</td>
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<tr>
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<th>$VaR_{99%G}$</th>
<th>$VaR_{99%t}$</th>
<th>$VaR_{99%G}$</th>
<th>$VaR_{99%t}$</th>
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<th>$ES_{99%t}$</th>
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<td>69.4652%</td>
<td>149.3508%</td>
</tr>
<tr>
<td>April 10</td>
<td>5.6209%</td>
<td>9.8667%</td>
<td>6.8718%</td>
<td>13.4883%</td>
<td>9.3209%</td>
<td>24.9033%</td>
</tr>
<tr>
<td>April 11</td>
<td>10.4094%</td>
<td>16.2314%</td>
<td>12.2600%</td>
<td>21.2953%</td>
<td>15.8832%</td>
<td>36.9100%</td>
</tr>
<tr>
<td>April 12</td>
<td>5.4329%</td>
<td>9.7648%</td>
<td>6.6163%</td>
<td>13.3956%</td>
<td>8.9332%</td>
<td>25.0971%</td>
</tr>
<tr>
<td>April 13</td>
<td>7.3451%</td>
<td>11.5257%</td>
<td>8.6661%</td>
<td>15.1555%</td>
<td>11.2495%</td>
<td>26.3692%</td>
</tr>
<tr>
<td>April 16</td>
<td>5.0187%</td>
<td>8.6261%</td>
<td>6.1672%</td>
<td>11.7651%</td>
<td>8.4158%</td>
<td>21.4398%</td>
</tr>
<tr>
<td>April 17</td>
<td>33.1582%</td>
<td>54.0611%</td>
<td>39.0711%</td>
<td>71.7192%</td>
<td>50.6479%</td>
<td>128.0441%</td>
</tr>
<tr>
<td>April 18</td>
<td>5.0679%</td>
<td>9.0327%</td>
<td>6.1680%</td>
<td>12.3674%</td>
<td>8.3220%</td>
<td>23.0647%</td>
</tr>
<tr>
<td>April 19</td>
<td>7.5459%</td>
<td>12.9923%</td>
<td>9.0530%</td>
<td>17.5702%</td>
<td>12.0036%</td>
<td>32.2679%</td>
</tr>
<tr>
<td>April 20</td>
<td>8.7228%</td>
<td>14.7027%</td>
<td>10.3149%</td>
<td>19.6873%</td>
<td>13.4321%</td>
<td>35.8747%</td>
</tr>
</tbody>
</table>
Appendix M: Matlab Code – Goodness of Fit

Chi2Testgof.m

% script file that calculates the Chi-square test statistic and the p-value
% for the log returns of the underlying assets, as well as whether to
% reject or do not reject the null

% Import the log returns for the stock portfolio assets
LogReturns =xlsread('LossStockData.xlsx','Weekly Log Returns Chart','B2:L53');

% Specific the degrees of freedom for tcdf
v=3;

% Compute the goodness of fit for Normal and Student's t
for kk=1:length(LogReturns(1,:))
    [hN(kk),pN(kk),statsN(kk)]=chi2gof(LogReturns(:,kk)); %Normal
    [hT(kk),pT(kk),statsT(kk)] = chi2gof(((LogReturns(:,kk)-mean(LogReturns(:,kk)))/std(LogReturns(:,kk))),'cdf', {@tcdf, v}); %Student's t
end

display(hN) %h=0 -> do not reject null, h=1 -> reject null
display(pN)

AAPLstatsN=statsN(1)
DELLstatsN=statsN(2)
DISstatsN=statsN(3)
XOMstatsN=statsN(4)
GLDstatsN=statsN(5)
GOOGstatsN=statsN(6)
IBMstatsN=statsN(7)
JPMstatsN=statsN(8)
MSFTstatsN=statsN(9)
OILstatsN=statsN(10)
VstatsN=statsN(11)

display(hT)
display(pT)

AAPLstatsT=statsT(1)
DELLstatsT=statsT(2)
DISstatsT=statsT(3)
XOMstatsT=statsT(4)
GLDstatsT=statsT(5)
GOOGstatsT=statsT(6)
IBMstatsT=statsT(7)
JPMstatsT=statsT(8)
MSFTstatsT=statsT(9)
OILstatsT=statsT(10)
VstatsT=statsT(11)
Chi2gofCodeTest.m

%get the historical Stock Return
LogReturns =xlsread('FILE.xlsx', 'Weekly Log Returns', 'range');

% Normal
for kk=1:length(LogReturns(1,:));
% Generate 7 bins for the log returns and get the frequencies of the bins
bindist(kk)=(max(LogReturns(:,kk))-min(LogReturns(:,kk)))/10;
edges(kk,:)=[min(LogReturns(:,kk)):bindist(kk):max(LogReturns(:,kk))]; % rows are the edges for each asset

[n(:,kk)]=hist(LogReturns(:,kk),edges(kk,:)); % bar(edges(kk,:),n)
frequencies(:,kk)=n(1:9,kk)+n(10,kk);
% The last two rows are added since the histc adds an extra bin to the end because it takes the frequencies between [yi,yi-1) and the last bin consists of the values on the max edge

% Compute the sample size N
N=length(LogReturns(:,1));

% Compute the probability that a random variable with distribution F takes value in the ith interval
for ii=1:length(n(:,1))-1;
p(ii,kk)=normcdf(edges(kk,ii+1),mean(LogReturns(:,kk)),std(LogReturns(:,kk)))-normcdf(edges(kk,ii),mean(LogReturns(:,kk)),std(LogReturns(:,kk))),p(ii,kk)); % columns correspond to the probabilities for each asset
end

% Compute the Chi^2 Test Statistic
for ii=1:length(p(:,kk))
    Chi2TestStattemp(ii,kk)=((frequencies(ii,kk)-N*p(ii,kk))²)/(N*p(ii,kk));
end
Chi2TestStat(kk)=sum(Chi2TestStattemp(:,kk));
v=10-1-2; %degrees of freedom for a Normal (bins -1-parameters Normal)

% Compute the p-value and decide with to reject or not reject the null hypothesis
pvalue(kk)=1-chi2cdf(Chi2TestStat(kk),v);
end
Chi2TestStat
pvalue

end;
end;
H
Appendix N: Goodness of Fit Tests Statistics

First Week

Normal

AAPLstatsN =
  chi2stat: 3.5289
  df: 3
  edges: [-0.0715 -0.0308 -0.0105 0.0099 0.0303 0.0506 0.1320]
  O: [8 11 11 6 6 10]
  8.5807]

DELLstatsN =
  chi2stat: 2.1404
  df: 4
  edges: [1x8 double]
  O: [6 6 8 11 7 4 10]
  E: [6.1126 5.4858 7.5491 8.6924]
  8.3749 6.7517 9.0336]

DJstatsN =
  chi2stat: 1.0154
  df: 4
  edges: [1x8 double]
  O: [7 7 9 10 8 4 7]
  E: [7.9952 6.8203 8.7805]
  9.2457 7.9629 5.6093]

XOMstatsN =
  chi2stat: 2.2496
  df: 4
  edges: [1x8 double]
  O: [5 4 10 7 9 8]
  E: [5.0616 5.1152 7.4124]
  8.6659 7.3929 7.1982]

GLDstatsN =
  chi2stat: 4.2231
  df: 3
  edges: [1x7 double]
  O: [5 6 8 16 6 11]
  E: [5.7401 6.8410 9.7937]
  10.7163 8.9622 9.9467]

GOOGstatsN =
  chi2stat: 12.2081
  df: 3
  edges: [1x7 double]
  O: [3 5 21 13 5 5]
  E: [5.9441 8.1191 11.5426]
  11.6695 8.3899 6.3349]

IBMstatsN =
  chi2stat: 7.2367
  df: 4
  edges: [1x8 double]
  O: [7 9 8 14 3 5 6]
  E: [9.5583 6.9592 8.4941 8.6659]
  7.3902 5.2679 5.6644]

JPMstatsN =
  chi2stat: 3.3297
  df: 3
  edges: [-0.1220 -0.0474 -0.0225 0.0023 0.0272 0.0521 0.1267]
  O: [6 9 15 8 7 7]
  E: [8.8611 8.4341 10.5088]
  10.0539 7.3855 6.7565]

MSFTstatsN =
  chi2stat: 2.2907
  df: 3
  edges: [1x7 double]
  O: [8 4 12 9 8 11]
  E: [7.0038 6.9279 9.3019]
  9.9133 8.3857 10.4675]

OILstatsN =
  chi2stat: 5.0103
  df: 3
  edges: [1x7 double]
  O: [5 7 9 16 6 9]
  E: [7.7292 7.7490 10.1067]
  10.2084 7.9852 8.2216]

VstatsN =
  chi2stat: 9.3661
  df: 2
  edges: [-0.0577 -0.0338 -0.0098 0.0141 0.0380 0.1816]
  O: [5 8 21 10 8]
  E: [7.3140 9.0646 11.9766]
  11.2141 12.4307]
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**Student's t**

**AAPLstatsT**
- chi2stat: 4.9881
- df: 4
- edges: [-1.8376 -0.9004 -0.4318 0.0368 0.5054 0.9741 2.8485]
- O: [8 11 11 6 6 10]

**DELLstatsT**
- df: 4
- edges: [-2.0381 -0.7619 -0.3366 0.0888 0.5142 0.9396 2.2157]
- O: [12 8 11 7 4 10]

**DISstatsT**
- chi2stat: 1.8437
- df: 4
- edges: [1x7 double]
- O: [7 7 9 10 8 11]

**XOMstatsT**
- chi2stat: 5.5677
- df: 4
- edges: [1x7 double]
- O: [5 6 8 16 6 11]

**GLDstatsT**
- chi2stat: 17.2087
- df: 4
- edges: [1x7 double]
- O: [3 5 21 13 5 5]

**GOOGstatsT**
- chi2stat: 13.8723
- df: 4
- edges: [1x7 double]
- O: [5 8 21 10 3 5]

**IBMstatsT**
- chi2stat: 9.7545
- df: 4
- edges: [1x7 double]
- O: [7 9 8 14 3 11]
Portfolio

statsN =

chi2stat: 6.5378
df: 3
d: edges: [-0.0766 -0.0287 -0.0128 0.0032 0.0192 0.0351 0.0830]
O: [4 11 14 13 3 6]

statsT =

chi2stat: 10.4386
df: 4
d: edges: [-2.7589 -1.0781 -0.5178 0.0425 0.6028 1.1630 2.8439]
O: [4 11 14 13 3 6]

Last Week

Normal

AAPLstatsN =

chi2stat: 1.7733
df: 3
d: edges: [-0.0715 -0.0308 -0.0105 0.0099 0.0303 0.0506 0.1320]
O: [9 8 12 7 6 10]

GLDstatsN =

chi2stat: 1.4299
df: 3
d: edges: [-0.0967 -0.0335 -0.0177 -0.0019 0.0140 0.0298 0.0614]
O: [5 8 9 13 7 10]

DELLstatsN =

chi2stat: 7.0727
df: 4
d: edges: [-0.0867 -0.0495 -0.0310 -0.0124 0.0062 0.0247 0.0433 0.0990]
O: [5 6 9 14 5 10]

GOOGstatsN =

chi2stat: 11.7664
df: 3
d: edges: [-0.1384 -0.0553 -0.0276 1.0044e-004 0.0278 0.0555 0.1386]
O: [2 6 21 13 5 10]

DISstatsN =

chi2stat: 2.5395
df: 4
d: edges: [-0.0983 -0.0422 -0.0236 -0.0049 0.0138 0.0325 0.0511 0.0885]
O: [7 6 9 12 8 3 7]
E: [7.6973 6.7932 8.8472 9.3749 8.0827 5.6699 5.5348]

IBMstatsN =

chi2stat: 8.6716
df: 4
d: edges: [-0.0655 -0.0245 -0.0109 0.0028 0.0164 0.0301 0.0437 0.0711]
O: [6 12 8 12 3 5 6]
E: [9.6685 7.1226 8.6656 8.7630 7.3654 5.1456 5.2693]

JPMstatsN =

chi2stat: 3.8105
df: 3
d: edges: [-0.1220 -0.0474 -0.0225 0.0023 0.0272 0.0521 0.1267]
O: [6 10 14 7 8]
MSFTstatsN =
  chi2stat: 0.5554
  df: 3
  edges: [-0.0790 -0.0314 -0.0155 3.8100e-004 0.0162 0.0321 0.0797]
  O: [7 6 11 9 8 11]

OILstatsN =
  chi2stat: 7.7495
  df: 3
  edges: [-0.1564 -0.0452 -0.0230 -7.4920e-004 0.0162 0.0321 0.0797]
  O: [7 6 11 9 8 11]

Student's t

AAPLstatsT =
  chi2stat: 2.3367
  df: 4
  edges: [-1.8698 -0.9296 -0.4595 0.0106 0.4807 0.9508 2.8312]
  O: [9 8 12 7 6 10]

DELLstatsT =
  chi2stat: 7.3933
  df: 4
  edges: [-2.0596 -0.7524 -0.3167 0.1191 0.5548 0.9905 2.2978]
  O: [11 9 14 5 3 10]
  E: [13.1682 6.9106 8.1900 7.6709 5.7922 10.2682]

DISstatsT =
  chi2stat: 2.8987
  df: 4
  edges: [-2.4193 -1.0449 -0.5868 -0.1287 0.3294 0.7876 2.1620]
  O: [7 6 9 12 8 10]
  E: [9.6937 5.8690 7.9868 8.6000 7.1511 12.6994]

XOMstatsT =
  chi2stat: 6.3166
  df: 4
  edges: [-2.1557 -0.8335 -0.3927 0.0480 0.4887 0.9295 2.2517]
  O: [9 10 8 8 10 7]

VstatsN =
  chi2stat: 7.6664
  df: 2
  edges: [-0.0777 -0.0338 -0.0098 0.0141 0.0380 0.1816]
  O: [5 9 20 10 8]

GLDstatsT =
  chi2stat: 3.3077
  df: 4
  edges: [-3.2367 -1.1550 -0.6345 -0.1141 0.4064 0.9268 1.9677]
  O: [5 8 9 13 7 10]

GOOGstatsT =
  chi2stat: 17.1721
  df: 4
  edges: [-3.1020 -1.2717 -0.6616 -0.0515 0.5586 1.1687 2.9991]
  O: [2 6 21 13 5 3]
  E: [7.6209 6.8223 10.5733 10.9830 7.5002 8.5003]

IBMstatsT =
  chi2stat: 11.8383
  df: 4
  edges: [-2.1932 -0.8930 -0.4596 -0.0262 0.4072 0.8406 2.1408]
  O: [6 12 8 12 3 11]

JPMstatsT =
  chi2stat: 6.5362
  df: 4
  edges: [-2.4442 -0.9443 -0.4443 0.0556 0.5556 1.0555 2.5554]
  O: [6 10 14 7 7 8]
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\[ \text{MSFTstats}_T = \]
\[ \chi^2\text{stat}: 1.5144 \]
\[ \text{df}: 4 \]
\[ \text{edges}: [-2.5586 -1.1100 -0.6271 -0.1443 0.3386 0.8215 2.2701] \]
\[ O: [7 6 11 9 8 11] \]

\[ \text{OILstats}_T = \]
\[ \chi^2\text{stat}: 10.4456 \]
\[ \text{df}: 4 \]
\[ \text{edges}: [-3.6157 -1.0040 -0.4816 0.0407 0.5631 1.0854 1.6078] \]
\[ O: [5 7 11 17 4 8] \]

\[ \text{Vstats}_T = \]
\[ \chi^2\text{stat}: 12.0890 \]
\[ \text{df}: 4 \]
\[ \text{edges}: [-1.6517 -1.0561 -0.4605 0.1351 0.7307 1.3263 4.3044] \]
\[ O: [5 9 20 10 3 5] \]

**Portfolio**

\[ \text{stats}_N = \]
\[ \chi^2\text{stat}: 4.9307 \]
\[ \text{df}: 3 \]
\[ \text{edges}: [-0.0775 -0.0291 -0.0129 0.0032 0.0194 0.0355 0.0840] \]
\[ O: [5 11 15 11 4 5] \]
\[ E: [7.9512 8.5610 10.9594 10.3996 7.3149 5.8138] \]