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Pricing Options with Monte Carlo and Binomial Tree Methods

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Pricing Options with Monte Carlo and Binomial Tree Methods

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Approved:

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Abstract

This report describes our work in pricing options using computational methods. First, I collected the historical asset prices for assets in four economic sectors to estimate model parameters, such as asset returns and covariances. Then I used these parameters to model asset prices using multiple geometric Brownian motion and simulate new asset prices. Using the generated prices, I used Monte Carlo methods and control variates to price call options. Next I used the binomial tree model to price put options, which I was introduced to in the course Math 571: Financial Mathematics I. Using the estimated put and call option prices together with some stocks, I formed a portfolio in an Interactive Brokers paper account[1]. This project was done a part of the masters capstone course Math 573: Computational Methods of Financial Mathematics.

[1] Interactive Brokers, www.interactivebrokers.com , the online discount brokerage firm in th e United States, founded by Thomas Peterffy, 1977

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1. Introduction

In this project, I used Monte Carlo and binomial tree methods to price the European call options and the American put options, respectively. Using historical data for stocks in several economic sectors, I estimated the expected return and volatility values for use as parameters to generate the stock price paths using geometric Brownian motion in Monte Carlo option pricing. Based on the simulated data, I used a variance reduction technique called control variates to increase efficiency by reducing variance of simulation estimates to price the European call options. To price the American put options, I used the binomial tree model. In an Interactive Brokers[2] paper trading account I formed some option positions based on options that have lower computed prices compared with their market prices, along with some stocks. Finally, I tracked the performance of my portfolio.

2. Selecting Assets

There are four sectors composing my portfolio: delivery service industry, shipping industry, retail trading industry and energy resources industry. (The following is the list of the companies in my portfolio)

Delivery Service	United Parcel Service (UPS) FedEx Corporation (FDX) Expeditors International of Was (EXPD) C.H.Robinson Worldwide (CHRW)
Shipping	Overseas Ship holding Group (OSG) Dry Ships (DRYS) Diana Shipping (DSX) Eagle Bulk Shipping (EGLE) Paragon Shipping (PRGN) Excel Maritime Carriers (EXM)
Retail Trading	Wal-Mart Stores (WMT) Target Corporation (TGT) Lowe’s Companies (LOW) Sears Holdings Corporation (SHLD) Best Buy Corporation (BBY)
Energy Resources	United State Oil (USO) ATP Oil & Gas Corporation(ATPG) Frontier Oil Corporation (FTO) Zion Oil &Gas (ZN) Marathon Oil Corporation (MRO)

[2] Interactive Brokers, www.interactivebrokers.com , the online discount brokerage firm in th e United States, founded by Thomas Peterffy, 1977

Delivery Service Industry

The delivery service industry is a stable industry. I selected United Parcel Service (UPS), FedEx Corporation (FDX), Expeditors International of Was (EXPD) and C.H.Robinson Worldwide (CHRW) as companies. These companies all have recognized and admired brands in the delivery service industry in US, and they have developed global business worldwide recent years. This group represented the steady and conservative part of my portfolio.

Shipping Industry

The shipping industry is a special field since it can be affected by many factors such as the weather, the import and export trade, and the economy. I chose this category because the price of crude oil was increasing which meant the costs of the shipping companies were also increasing. I focused on the following companies:

Overseas Ship Holding Group (OSG) is a market leader in energy transportation services. This company transports energy resources such as oil, gas and petroleum products worldwide.

"Dry Ships (DRYS) is a global shipping transportation corporation and specializes in the transportation of dry bulk cargoes such as coal, iron ore for energy and steel production, alumina and other construction materials."[3]

Diana Shipping (DSX) holds similar business to Dry Ships, except that Diana owns 11% of the issued and outstanding shares of Diana Containership which is a company that specializes in the transportation of container cargoes.

"Eagle Bulk Shipping (EGLE) is the largest U.S. based owner of handy max dry bulk vessels which are engaged in the transportation of minor bulk cargoes. Its fleet is mainly composed of larger and supra max class vessels." [4]On Mar 25th, 2011, Eagle Bulk Shipping reached an agreement with Korea Lines Corporation which might enhance the development of the company and attract more investors.

Retail Trading Industry

In retail trading category, I selected Wal-Mart Stores (WMT), the Target Corporation (TGT), Lowe's Companies (LOW), the Sears Holdings Corporation (SHLD) and the Best Buy Corporation (BBY). Wal-Mart Stores focus on selling low price items. Lowe's Companies operates business related to home construction, such as appliances, windows, and doors. Best Buy sells electronic products such as computers, cameras, and mobile phones. These

[3] http://www.dryships.com/

[4] http://www.eagleships.com/phoenix.zhtml?c=189576&p=irol-cglanding

four companies put weight on selling different items which are needed in everyday life, and their customers represent a large portion of the U.S. population. This part, along with the delivery service sector, represented the low risk and low profit investment category in my portfolio.

Energy Resources Industry

The last sector was the energy resources industry related to oil and gas. Since the beginning of 2011, the price of crude oil has continued to increase. ATP Oil & Gas Corporation (ATPG) is engaged in the development and production of oil and gas in the offshore Gulf of Mexico and the North Sea. Frontier Oil Corporation (FTO) is mainly located in Wyoming and Kansas. And Marathon Oil Corporation (MRO) is an integrated international energy company which is based in Houston, Texas and has operations in the US and many other countries such as Canada and The United Kingdom. I hope this field will bring large profits to my portfolio.

3. Pricing the European Call Options

3.1 Collecting Data

I collected the assets' prices from Yahoo Finance[5] and chose the data between Jan 4th, 2011 and Apr 1th, 2011, for a total of sixty-two days. When I observed the historical data, I found this period contained the date of declaration. To include adjustment for dividends, I chose the adjusted close data for my basic assets' prices.

3.2 Estimating Parameters

I turned the basic stock price data into relative return format. I defined S0 as the initial stock price and S1 as the stock price of the next day. The return equaled the difference between S1 and S0 divided by S0. Using the return vector, I computed the expected return vector and covariance.

r= (S1-S0)/S0

The excepted return vector= ∑(ri)/N (N is the number of dates)

Covariance = cov (ri,rj)=E[$(r_i-Er_i)(r_j-Er_j)$]

The results were following:

Er= [-0.00457741080645161;-0.00117337820967742;-0.00298885412903226
0.00172873291935484;-0.00422628943548387;-0.000234178580645161]

[5] The stock price data used in this project was obtained through Yahoo Finance, http://biz.yahoo.com/r/ (May 2, 2011)

Cov=

0.000562	2.33E-04	1.44E-04	-0.00022	0.000355	0.000249
2.33E-04	0.00030971	0.000108	-0.0001	0.000179	0.000155
1.44E-04	0.00010758	0.000198	-4.73E-05	0.000193	9.17E-05
-0.00022	-0.00010333	-4.73E-05	0.000451	-9.91E-05	-7.76E-05
0.000355	0.00017942	0.000193	-9.91E-05	0.000566	0.000212
0.000249	0.00015477	9.17E-05	-7.76E-05	0.000212	0.000232

I defined T=1 as three months. Using the sixty-two trading dates during three months, I converted the excepted value and correlation matrix in terms of days.

Adjusted Er= Er*62

[-0.283799470000000;-0.0727494490000000;-0.185308956000000
0.107181441000000;-0.262029945000000;-0.0145190720000000]

Adjusted Cov = Cov*(62) ^.5

0.0044	0.0018	0.0011	-0.0017	0.0028	0.0020
0.0018	0.0024	0.0008	-0.0008	0.0014	0.0012
0.0011	0.0008	0.0016	-0.0004	0.0015	0.0007
-0.0017	-0.0008	-0.0004	0.0036	-0.0008	-0.0006
0.0028	0.0014	0.0015	-0.0008	0.0045	0.0017
0.0020	0.0012	0.0007	-0.0006	0.0017	0.0018

3.3 Simulating Stock Price Path

Based on the parameters estimated above, I modeled the six different asset prices together using multidimensional geometric Brownian motion to simulate the stock price paths. Using **MA573PROJECT.m** function, I generated the new stock prices that satisfied the GBM. The following is the distribution format[6] :

$$S_i(t_{k+1}) = S_i(t_k)e^{\left(\mu_i - 0.5*\sigma_i^2\right)(t_{k+1}-t_k)+\sqrt{(t_{k+1}-t_k)*(\sum_{j=1}^{d} A_{ij}Z_{k+1,j})^2}}$$

$$Z_k = [Z_{k1}, Z_{k2,\ \ldots\ ,} Z_{kd}] \sim N(0, I)$$

A was computed from the Cholesky factorization of the adjusted covariance matrix.

k=0, 1, 2, 3…, N-1

I =0, 1, 2, 3…, d

3.4 Control Variates

I used control variates to increase the efficiency by reducing the variance of the

[6] Paul Glasserman , 2003, Monte Carlo Methods in Financial Engineering, Springer, New York, 596P

simulation estimates of the call option prices. I chose r, the risk-free rate, to be 0.03 which is the yield rate of the 3- month bond.

Define K as the strike price and x as the stock prices generated by geometric Brownian motion. I found the discounted payoff, Y= (y_1, y_2,..., y_n) using exp(−r*T)*max(x−k, 0). Define b= cov (x, y)/var (x) and y (b) =Y−b*(X−EX), where X =x*exp (−r*T) and EX=S0.[7] Then, I estimated the call option prices as Σy (b)/N.

Calculating EXM call option price:

%use the data generated from GBM

```
r=0.03; k=4; S=S';
S_EXM=S(:,1);
Y_EXM=exp(-r*T)*max(S_EXM-k*ones(63,1),0);
B_EXM=cov(S(:,1),Y_EXM);
c_EXM=var(S(:,1));
X_EXM=S_EXM*exp(-r*T);
b_EXM=B_EXM(1,2)/c_EXM;
y_EXM=mean(Y_EXM-b_EXM*(X_EXM-S0(1,1)*ones(63,1)));
```

EXM call option=1.8329

Calculating EXM call option price:

```
r=0.03; k=5; S=S';
S_DRYS=S (:, 2 ) ;
Y_DRYS=exp (-r*T)*max (S_DRYS-k*ones (63,1),0);
B_DRYS=cov (S (:,2) ,Y_DRYS);
c_ DRYS=var (S(:,2));
X_DRYS=S_DRYS*exp (-r*T);
B _ DRYS=B_DRYS (1, 2) /c_ DRYS;
y_ DRYS=mean (Y_ DRYS –b _DRYS *(X_ DRYS -S0 (2,1)*ones(63,1)));
```

OSG call option=0.3335

4. Pricing the American Put Options

When using binomial tree model to price the American put options (V), I had to solve for up jump factor u, downward jump factor d, and the probability p of an upward movement. This pricing process was quite different from the European options, since a buyer of an American option has the right to exercise the contracts at any time before maturity.

Using what were introduced in the book-Implementing Models in Quantitative Finance: Methods and Cases[8], I set

u=exp (sigma*(delta) ^0.5+(r-0.5*sigma^2)*delta)

d=exp (-sigma*(delta) ^0.5+ (r-0.5*sigma^2)*delta)

p=1/2+(r-0.5*sigma^2)/2*sigma*(delta) ^0.5

[7] Paul Glasserman , 2003, Monte Carlo Methods in Financial Engineering, Springer, New York, 596P

[8] Gianluca Fusai and Andrea Roncoroni, 2008, Implementing Models in Quantitative Finance: Methods and Cases, Springer, New York City, 631 P.

$V_{j,} = \max[\max(K-S_{j},0),\exp(-r*delta)*(p_* V_j+(1-p)V_{j+1})]$

Then using **binomialtree.m**[9], I computed the put option prices.

Function [putoptionprice]=binomialtree(S,K,T,r,sigma,N)

PRGN: binomialtree(3.44,2,1,0.03,0.046,30);

OSG: binomialtree(35.86,31,1,0.03,0.045,30);

DRYS: binomialtree(4.99,3,1,0.03,0.056,30);

5. Performance

My portfolio included two call options, three put options, and fifteen stocks.

The following two call options were selected:

Option	Pricing	Real	K	T(=1three months)	Bid Price	Ask Price
EXM	1.8329	0.05	4	0.3	0.25	0.30
DRYS	0.3335	0.13	5	0.3	0.13	0.14

The following three put options were selected:

Put	Pricing	Real	K	T(=1three months)	Bid Price	Ask Price
PRGN	1.6983	0.85	2	0.33	0.75	1.05
OSG	2.7654	0.25	31	0.33	0.25	0.30
EGLE	27	0.95	3	0.33	0.45	0.65

Since the estimated option prices were higher than the real prices, I chose to buy EXM, DRY, PRGN, OSG and EGLE. Then I ordered 100 shares for each stock and 1 contract for each option. I found the performances of retail trading service assets were stable. For example, I bought TGT at 50.26. The bid and ask prices were 50.25 and 50.27 respectively. When I observed my IB account, I found the change of its quantity to be small, just between 0.88% and 1%.

When choosing the delivery sector, I bought UPS, FDX, EXPD and CHRW at 73.98, 93.74, 53.73 and 79.05. And these assets brought losses to my portfolio, except on Apr 20th when the whole stock market increased in value.

Overall the energy sector brought more profits than the other sectors. I bought MRO at 51.23, USO at 42.88, FTO at 28.15, and Z 5.19 on Apr 19. These respectively grew to 52.66, 44.48, 28.25 and 5.46 on Apr 26. The reason for this might be that the price of crude oil has continued to increase since the beginning of 2011. When using the Monte Carlo and the

[9] Desmond J. Higham , 2002, Nine Ways to Implement the Binomial Method for Option Valuation in MATLAB, Society for Industry and Applied Mathematics, Vol.44, p661-667

binomial tree methods, I chose five options with prices slightly higher than the real prices. DRYS and EXM had increased of 13% and 15% respectively, and brought profits. However, OSG had decreased by 10.45%.

6. Conclusion

Challenges

In this project, one of the challenges I had met was that I had trouble deciding how far back to use historical data. In my opinion, the larger amount of data, the greater the accuracy of the estimated data; however, most of my options will mature in May, 2011. Hence large numbers did not mean a high degree of accuracy. I should have considered collecting the data close to the maturity date, which might estimate the parameters more effectively. Then I selected the most recent 62 days as my sample to estimate the parameters more accurately.

The performances of my portfolio

Overall the energy sector brought more profits than the other sectors. The reason might be that the price of crude oil has continued to increase since the beginning of 2011. When using the Monte Carlo method, my pricing gives prices that were higher than market prices; however, this did not surely mean that I would acquire profits. The option prices might have been affected by other factors which cannot be easily measured in models.

What needs to be done

In this project I chose the weights of my portfolio by comparing the stock prices and the performances over the last two months. Hence the weights of each asset might be not reasonable. If I were to do this project again, I would use portfolio optimization to choose each weight for these assets.

7. References

1. Gianluca Fusai and Andrea Roncoroni, 2008, Implementing Models in Quantitative Finance: Methods and Cases, Springer, New York City, 631 P.
2. Paul Glasserman , 2003, Monte Carlo Methods in Financial Engineering, Springer, New York, 596P
3. Desmond J. Higham , 2002, Nine Ways to Implement the Binomial Method for Option Valuation in MATLAB, Society for Industry and Applied Mathematics, Vol.44, p661-667
4. Dry Ship Company , http://www.dryships.com/ (May 2, 2011)
5. Eagle Bulk Shipping Company, http://www.eagleships.com/phoenix.zhtml?c=189576&p=irol-cglanding (May 2, 2011)
6. Interactive Brokers, www.interactivebrokers.com , the online discount brokerage firm in the United States, founded by Thomas Peterffy, 1977
7. The stock price data used in this project was obtained through Yahoo Finance, http://biz.yahoo.com/r/ (May 2, 2011)

8. Appendix

MATLAB
codes:

1. This file is used to generate the new stock prices which satisfied GBM.

```
function [S]= MA573PROJECT(N,T,S0)
% MU.xlsx is the excepted value matrix of the shipping sector
% shipcorrelationmatrix.xlsx is the correlation matrix of the shipping matrix
%Use the estimated parameters to generate multiple GBM
% MA573PROJECT(62,1,[5.72;5.19;3.44;35.86;4.99;12.12])
[mu,txt]=xlsread('MU.xlsx','A1:A6');
[Sigma,txt]=xlsread('shipcorrelationmatrix.xlsx');
[muRows,muCols] = size(mu);
if muCols > 1
  mu = mu';
end
[d, muCols] = size(mu); d
[S0Rows,S0Cols] = size(mu);
if S0Cols > 1
        S0 = S0';
end
if (muCols ~= 1) % do nothing if mu is not a proper vector
        error('Mu shoud be a vector.')
end
if (S0Cols ~= 1) % do nothing if mu is not a proper vector
        error('S0 shoud be a vector.')
end
% Check Sigma
[SigmaRows, SigmaCols] = size(Sigma);
% Check that Sigma is square
if (SigmaRows ~= d) || (SigmaCols ~= d)
error('Sigma should be square.')
end
% Check that Sigma is symmetric
if Sigma ~= Sigma
error('Sigma must be symmetric.')
end
% Check whether Sigma is positive definite
if min(eig(Sigma))<= 0
error('Sigma must be positive definite.')
end
% compute the Cholesky factorization of Sigma:
A = zeros(d,d);
v = zeros(d,1);
```

```
for jj = 1:d
        for ii = jj:d
                v(ii) = Sigma(ii,jj);
                for kk = 1:(jj-1)
                        v(ii) = v(ii) - A(jj,kk)*A(ii,kk);
                end
                A(ii,jj) = v(ii)/sqrt(v(jj));
        End end
deltaT = T/N;
% Generate matrix of standard normals
Z = randn(d,N+1);
% Initialize the d dimensional GBM matrix
S = zeros(d,N+1);
S(:,1) = S0;
% Recursively generate d dimensional geometric Brownian motion
for ii = 1:d
for kk = 1:N
S(ii,kk+1) = S(ii,kk)*exp((mu(ii)-0.5*Sigma(ii,ii))*deltaT + sqrt(deltaT)*(A(ii,:)*Z(:,kk+1)) );
end      end
```

2. Use variance reduction technique

```
%use the data S generated from function [S]= MA573PROJECT(N,T,S0)
[mu,txt]=xlsread('MU.xlsx','A1:A6');
[Sigma,txt]=xlsread('shipcorrelationmatrix.xlsx');
N=62;
T=1;
S0=[5.72;5.19;3.44;35.86;4.99;12.12]
[muRows,muCols] = size(mu);
if muCols > 1 mu = mu';
end
[d, muCols] = size(mu);
d
[S0Rows,S0Cols] = size(mu);
if S0Cols > 1
        S0 = S0';end
if (muCols ~= 1) % do nothing if mu is not a proper vector
        error('Mu shoud be a vector.')
end
if (S0Cols ~= 1) % do nothing if mu is not a proper vector
        error('S0 shoud be a vector.')     end
% Check Sigma
[SigmaRows, SigmaCols] = size(Sigma);
% Check that Sigma is square
if (SigmaRows ~= d) || (SigmaCols ~= d)
```

```
    error('Sigma should be square.')
end
% Check that Sigma is symmetric
if Sigma ~= Sigma
error('Sigma must be symmetric.')
end
% Check whether Sigma is positive definite
if min(eig(Sigma))<= 0
    error('Sigma must be positive definite.')
end
% compute the Cholesky factorization of Sigma:
A = zeros(d,d);
v = zeros(d,1);
for jj = 1:d
        for ii = jj:d
                v(ii) = Sigma(ii,jj);
                for kk = 1:(jj-1)
                        v(ii) = v(ii) - A(jj,kk)*A(ii,kk);
                end
                A(ii,jj) = v(ii)/sqrt(v(jj));
End end
deltaT = T/N;
% Generate matrix of standard normals
Z = randn(d,N+1);
% Initialize the d dimensional GBM matrix
S = zeros(d,N+1);
S(:,1) = S0;
% Recursively generate d dimensional geometric Brownian motion
for ii = 1:d
        for kk = 1:N
                S(ii,kk+1) = S(ii,kk)*exp((mu(ii)-0.5*Sigma(ii,ii))*deltaT +
sqrt(deltaT)*(A(ii,:)*Z(:,kk+1)) );
        End end
            r=0.03;
            k=4;
            S=S';
            S_EXM=S(:,1);
            Y_EXM=exp(-r*T)*max(S_EXM-k*ones(63,1),0);
            B_EXM=cov(S(:,1),Y_EXM);
            c_EXM=var(S(:,1));
             X_EXM=S_EXM*exp(-r*T);
            b_EXM=B_EXM(1,2)/c_EXM;
            y_EXM=mean(Y_EXM-b_EXM*(X_EXM-S0(1,1)*ones(63,1)));
  % Calculate DRYS call option price:
```

```
%use the data S generated from function[S] = MA573PROJECT(N,T,S0)
        r=0.03;
        k=5;
        S=S';
        S_DRYS=S(:,2);
        Y_DRYS=exp(-r*T)*max(S_DRYS-k*ones(63,1),0);
        B_DRYS=cov(S(:,2),Y_DRYS);
        c_DRYS=var(S(:,2));
         X_DRYS=S_DRYS*exp(-r*T);
        b_DRYS=B_DRYS(1,2)/c_DRYS;
        y_DRYS=mean(Y_ DRYS -b_DRYS *(X_ DRYS -S0(2,1)*ones(63,1)));
```

3. Binomial tree model[10]

```
function [putoptionprice]=binomialtree(S,K,T,r,sigma,N)
delta = T/N;
V=exp(-r*delta)+exp((r+sigma^2)*delta);
U=sqrt((.5*V)^2-1)+V/2;
p=(exp(r*delta)-(1/U))/(U-(1/U));
l=[N:-1:0]';
j=[0:N]';
path_down=(1/U).^l;
path_up=U.^j;
for ii = N:-1:1
Z=max(K-S*path_down.*path_up,0);
St=S*path_down(N-ii+2:N+1).*path_up(1:ii);
P=max(max(K-St,0),p*exp(-r*delta)*Z(2:ii+1)+(1-p)*exp(-r*delta)*Z(1:ii));
End
putoptionprice=P;
```

[10] Desmond J. Higham , 2002, Nine Ways to Implement the Binomial Method for Option Valuation in MATLAB, Society for Industry and Applied Mathematics, Vol.44, p661-667