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An In-Depth Look at the Information Ratio

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An In-Depth Look at the Information Ratio

by

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A Thesis

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Abstract

The information ratio is a very controversial topic in the business world. Some portfolio managers put a lot of weight behind this risk-analysis measurement while others believe that this financial statistic can be easily manipulated and thus shouldn't be trusted. In this paper, an attempt will be made to show both sides of this issue by defining the information ratio, applying this definition to real world situations, explaining some of the negative impacts on the information ratio, comparing this ratio to other statistical measures, and showing some ways to improve a portfolio manager's information ratio.

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1 Introduction

There are many ways that an investor can measure the performance of a portfolio. The simplest, and the most obvious measure of performance is return on investment. Did the portfolio make money? Did the portfolio make as much money as possible? This simple view was never able to provide a satisfactory explanation for the simple fact that diversification is a good investment strategy. The simplest view was not able to make a direct comparison between the risk and return for different portfolios. New approaches have developed that compare the performance of one portfolio against appropriate benchmarks. Many of these new approaches are part of what has been called *Quantitative Active Management*.

Modern Portfolio Theory brought economics, quantitative methods, and a scientific perspective to the study of investments. Harry Markowitz is credited as having founded Modern Portfolio Theory with his seminal paper [11] (see also [12]). In his very brief paper, Markowitz gave the first clear, mathematically precise, definition for *risk* and gave, for the first time, a theoretical justification for the value of diversification as an investment strategy. Markowitz argued that a rational investor should consider both return and variance in return when comparing investment options. The return, or expected return, is the simplest measure of performance. The variance in return is a measure of the risk inherent in the investment.

Most of Modern Portfolio Theory is built on economic theory of efficient markets. If the theory is truly correct, then an investor should not be able to do better than investing in the market and no portfolio manager should not expect to be paid for managing a portfolio. *Active Portfolio Management* takes the point of view that an intelligent investor can use information to outperform the market. What really matters to the active manager is how his or her portfolio performs relative to a specified benchmark. What matters is *active return*—does the portfolio perform as well as or better than the benchmark? What matters is *active risk*—does the portfolio carry more or less risk than the benchmark? Perhaps the best overview of this “post-modern” view

of portfolio analysis is given by Grinold and Kahn [6].

The information ratio. It's amazing how those three little words can cause so much controversy in the business world. To this day, many portfolio managers still dispute what the information ratio actually is and how it is calculated. Some investors put a lot of weight on what the information ratio (IR) tells them and may even use this ratio to determine whether to hire or fire their portfolio managers. Others feel that this ratio can easily be manipulated and therefore this statistic should not be trusted. For this reason, it is important to get a good grasp on what this ratio is and what it measures.

The information ratio is built on Markowitz's mean-variance analysis, which states that mean and variance are satisfactory measures for characterizing an active investment portfolio. The information ratio is a single number that summarizes the mean-variance properties of a portfolio.

In this paper, we will attempt to more clearly define what the information ratio is, apply this definition to real world situations, explain some of the negative impacts on the information ratio, compare this ratio to other statistical measures, and show some ways to improve a portfolio manager's information ratio.

2 Background

Each stock has its own return and variance in return. In comparing two stocks with the same return, the one with the smaller variance is preferable (to a rational investor). In comparing two stocks with the same variance (risk), the one with the larger return is preferable. When the comparison is not so simple, as for example when one stock has both higher return and variance than another stock, the choice depends on the investor's level of risk tolerance. The investor must judge whether the additional return is worth the additional risk.

Markowitz showed how portfolios of stocks could be compared in exactly the same way. He showed how to find portfolios which are “good” in the sense that

- no other portfolio has higher return for the same risk,
- no other portfolio has lower risk for the same return.

He called these portfolios *efficient* and developed good computer algorithms for finding all efficient portfolios from a given set of stocks. This set defined a curve in the (risk,return)-plane and Markowitz named this curve the *efficient frontier*. Expected returns are estimated from observing stock returns (or prices) over a period of time. The same data can be used to estimate the covariance matrix for the stock returns.

This vector of returns and matrix of covariances are the inputs needed to find efficient portfolios. Our project focuses on the problem of obtaining good estimates for the covariance matrix of stock returns. There are many ways to estimate the covariance matrix. We will study the sample covariance matrix, single-index covariance matrix, and a convex combination of these two using a shrinkage method. We will also develop ways to test these estimation methods for the covariance matrix and determine which method is the best.

2.1 Modern Portfolio Theory

Harry Markowitz was an economist who suggested, as we mentioned before, that investors should use expected return and variance in return to identify well-diversified portfolios. His major contribution to Modern Portfolio Theory was the system he had for defining the performance of an investment portfolio. Markowitz evaluated a portfolio's performance by observing its return value and variance. The return value for a stock is the percentage increase of the stock price over a certain time frame. The following formula represents the return for stock i from a time interval $(t - 1, t]$.

$$R_i(t) = \frac{P_i(t) - P_i(t - 1)}{P_i(t - 1)}$$

where $P_i(t)$ is the price for stock i at time t . Note that this is a random variable and we denote the expected value by

$$\mu_i(t) = E [R_i(t)].$$

A portfolio can be made up of several stocks. The expected return for a portfolio is then

$$\mu_P = E \left[\sum_{i=1}^N x_i R_i \right] = \sum_{i=1}^N x_i \mu_i = \mathbf{x}^T \boldsymbol{\mu}$$

where x_i represents the percentage of the investment that is invested in stock i .

The variance of a stock price is a measure of how much the price changes. For an individual stock, the variance in return is

$$\text{Var}(R_i) = E [(R_i - \mu_i)^2],$$

and the covariance in return for two stocks is

$$\sigma_{ij} = E [(R_i - \mu_i)(R_j - \mu_j)].$$

The variance in return for a portfolio is given by:

$$\text{Var}(R_P) = \sigma_P^2 = \sum_{i,j} x_i \sigma_{ij} x_j = \mathbf{x}^T \boldsymbol{\Sigma} \mathbf{x}$$

Markowitz defined a portfolio to be *efficient* if the portfolio had the highest possible return for the given risk or variability.

The problem of constructing a portfolio is reduced to the following optimization problem:

$$\text{Minimize } \frac{1}{2} \mathbf{x}^T \Sigma \mathbf{x} - \frac{1}{\lambda} \mathbf{x}^T \boldsymbol{\mu} \quad (1)$$

$$\begin{aligned} \sum_i x_i &= 1 \\ x_i &\geq 0 \quad \text{for all } i \end{aligned}$$

In this problem the objective to be minimized contains a term that measures risk as well as a term that measures return. Risk should be small while return should be large (return is subtracted). The parameter $\lambda > 0$ multiplying the second term is a measure of *risk tolerance*. If λ is small (close to zero), then the second term dominates and the investor puts more weight on return than on risk; he or she is risk tolerant. If, on the other hand, λ is large, then the second term is given less weight in the objective and the investor is risk averse.

Much of the work that has followed Markowitz has addressed practical issues facing the portfolio manager. For example, the theory does not include the impact of transaction costs. It does not include adjustments for the error inherent in statistical estimations for return and covariance. The theory developed in the next section focuses on perhaps the most important of the practical challenges to the Markowitz mean-variance analysis.

3 Definitions of the Information Ratio

Loosely defined, the information ratio (IR) is a measure of portfolio management's performance against risk and return relative to a benchmark. The benchmark is a reference portfolio for active managers and it should be the goal of management to outperform the benchmark. If an active manager took no risk in a portfolio, his/her performance would duplicate the results of the benchmark. By taking risks, an active portfolio manager has the potential to exceed the performance of the benchmark. However, taking risks can also backfire, resulting in a loss for the manager's clients.

It is important that the benchmark that an active manager chooses represents the stocks that the manager has in his/her portfolio. For example, an active manager has decided to invest in international equities. If this manager chose the Russell 2000 for its benchmark, he/she would be comparing the performance of their portfolio in the international market to the smallest two thousand United States stocks that are actively traded. Since these two portfolios are unrelated, the Dow Jones Industrial would be a poor choice of a benchmark. A better choice for this asset class would be the Morgan Stanley Capital International – Europe, Australasia, and Far East (MSCI EAFE). Some other examples of asset classes and appropriate benchmarks are listed in Table 1.

Asset Class	Benchmark
U.S. Fixed-Income	Lehman Aggregate
International Equity	MSCI EAFE
U.S. Large-Cap	S & P 500
U.S. Small-Cap	Russell 2000
Emerging Markets Equity	IFCO EME

Table 1: Benchmarks for Different Asset Classes

There are several different methods for calculating the information ratio. This paper describes three of the more popular definitions. The first definition looks back

at historical data and measures the success (or failure) of the active portfolio manager. The second definition looks into the future and calculates the information ratio based on forecasted information. The third definition is a theoretical estimation of the information ratio.

3.1 The First Definition

One way to calculate the IR is by dividing excess returns by the risk (or standard deviation) of the excess returns. Mathematically, the information ratio is

$$\text{IR} = \frac{\text{ER}}{\sigma}$$

and is typically an annualized ratio. The excess returns (ER) are found by subtracting the return on the benchmark during time i from the return on the stock at time i . To find the annualized return, use the following formula:

$$\left(\prod_{i=1}^N (1 + r_i) \right) - 1.$$

σ is the annualized standard deviation of excess returns, which is also known as the tracking error.

To demonstrate how this definition works, a three-year analysis Meridian Stock (MERDX) was performed. This stock was tracked on a monthly basis from January 2001 till December 2003. The monthly returns were found and are listed in Table 5. The annual returns for 2001, 2002, and 2003 were found using the above formula. A similar process was done for the benchmark (S & P Mid-Cap). The excess returns for each year were found by subtracting the annualized benchmark returns from the annualized stock returns and the results are shown in Table 5. The three year return is found using the formula $(\prod_{i=1}^N (1 + r_i))^{1/\text{years}} - 1$. For the example above, the three year return for MERDX is

$$((1 + .4792) \cdot (1 - .1775) \cdot (1 + .0374))^{1/3} - 1 = .0807 .$$

Using the same formula, the three year return for the S & P Mid-Cap index is 3.69%. Thus, the excess return is 4.38% (8.07% - 3.69%). The tracking error was found by taking the standard deviation of the excess returns for 2001, 2002, and 2003 and in this example, $\sigma = 8.10\%$. The information ratio is $\frac{.0438}{.0810} = .5408$.

3.2 The Second Definition

In the previous definition, we were looking back at historical data (*ex post*). The information ratio can also be used to look forward (*ex ante*). Looking forward, the information ratio is the expected level of annual residual return per amount of annual residual risk. When looking forward, the information ratio is also known as the “alpha-omega ratio” since the new equation for the information ratio becomes $IR = \frac{\alpha}{\omega}$ where these two variables are found using regression techniques. Alpha is a forecast of residual return. A residual return is defined as return independent of the benchmark and is calculated by taking the excess return and subtracting beta times the benchmark excess return, that is

$$(R_{Pi} - R_f) = \alpha_P + \beta \cdot (R_{Bi} - R_f).$$

If θ is a residual return, then $\alpha_i = E(\theta_i)$. β is a measure of the sensitivity of a stock to the benchmark. ω is the expected residual return on stock i .

When looking at a portfolio of stocks,

$$\alpha_P = \sum_{i=1}^T x_i \cdot \alpha_i$$

where x_i s are the weight for asset i and α_i s are the expected residual returns for asset i (i.e. $\alpha_i = E(\theta_i)$). ω_P is the standard deviation of θ_P . The information ratio for this portfolio looks like: $IR_P = \alpha_P/\omega_P$. A manager’s personal information ratio is the maximum IR obtained from all portfolios (i.e. $IR = \max IR_P/P$) (Grinold [6] 114).

The above approach is generally used to avoid rewarding managers from taking on more risk than the benchmark portfolio. If the estimated β is greater than one, then the estimated alpha would decrease in comparison to the alpha computed with $\beta = 1$.

The estimated ω would be greater than the sigma computed if $\beta = 1$ since a $\beta > 1$ indicates a higher level of risk. These two changes result in a lower information ratio for the estimated portfolio. However, if the estimated β is less than one, the overall result would be an increase in the information ratio due to an increase in the estimated α and a decrease in ω . Therefore, if the financial manager takes on less risk than the benchmark, he/she is rewarded with a higher information ratio. Since one of the key ideas is that the benchmark closely resembles the systematic risk of the estimated portfolio, it makes sense to fix $\beta = 1$. The above information shows how easily the information ratio can be manipulated to achieve the desired results. It implies that this version of the information ratio is most useful and accurate when the benchmark has been carefully chosen to match the style of the financial manager.

3.3 The Third Definition

A third way of estimating the information ratio is more of a theoretical approach. In this method, the information ratio is broken down into components: the information coefficient and the *breadth* (Grinold [6], page 148). The information coefficient, denoted IC, is a measure of the manager's skill. It is defined as the correlation between the actual returns and the manager's forecasted returns. The breadth is the number of times per year that the managers use their skill and is defined as the number of independent forecasts of exceptional returns. The information ratio is defined as

$$\text{IR} = \text{IC} \cdot \sqrt{\text{breadth}}$$

. This equation is known as the fundamental law of active management. It has been found that the IR calculated *ex ante* is a rough upper bound for the IR calculated *ex post* in definition one of this paper (Goodwin [4], page 3).

3.4 A “Good” Information Ratio

With the three definitions given above, it may be confusing for managers to decide which definition would be best for them to use. Basically, it depends on what type of data the manager is looking at. If a manager wants to see how well he/she has performed over time, then the first definition (ex post) is the appropriate one to use. If, on the other hand, a manager wants to make predictions on how he/ she will do in the future, then the ex ante (definition 2) is the most appropriate. The third definition is used as a rough estimation of the information ratio.

Now that the IR has been defined, the meaning of a “good” information ratio needs to be determined. It is important to note that the information ratio can be, and frequently is, a negative value. Since the numerator of the ratio is the excess returns and excess returns are negative when the stock does worse than the benchmark, negative information ratios are a common occurrence. In the world that we live in, bigger is usually better and this philosophy carries over to the information ratio as well. According to Grinold and Kahn [6], the information ratio is analogous to a normal bell-shaped curve with an $IR = 0$ as the mean of the distribution. An information ratio greater than zero shows that a manager has performed in the top 50% of the population while a manager with an information ratio less than zero is performing in the bottom half of the active portfolio managers. Table 2 breaks this concept down even farther.

Percentile	Information Ratio
90	1.0
75	0.5
50	0.0
25	-0.5
10	-1.0

Table 2: Ranking of Information Ratios

This table implies that a manager who is performing in the top quartile has a “good” information ratio of 0.5. An $IR = 1.0$ is an exceptional number and it should be the goal of management to reach this level.

4 Applying the Definition

Now that the information ratio has been defined, let's see how it works in applies to the real life situations of opportunities available to managers and the manager's ability to add value to the portfolio.

4.1 Opportunities Available

As previously mentioned, the information ratio defines the opportunities available to the manager and thus a residual frontier is created which describes these opportunities. Figure 1 shows the residual frontier for three different values of the information ratio. Suppose we have two managers. Manager A has an IR = 0.5 and Manager B has an IR of 0.75. With the higher IR ratio, Manager B has more opportunities available than Manager A, for instance portfolio P1 on Figure 1. What this means is that Manager A does not have the information required to achieve P1. This doesn't mean that Manager A cannot hold P1 but rather that with the information Manager A has, P1 does not exist.

Figure 1 also shows that once the information ratio has been determined, the manager can move up and down that residual frontier and still have the same IR. This means that if a manager wants to increase the expected residual return (alpha) and keep the same value for the information ratio, then a corresponding increase in residual risk also has to occur.

4.2 Value Added

The square of the information ratio shows the manager's ability to add value. One of the goals of active managers is to maximize the value added from residual return where the potential value added is defined as

$$VA = \alpha - \lambda \cdot \omega^2$$

w here λ is the risk aversion coefficient (Grinold [6], page 119). In this equation, alpha is found using the ex ante definition of the information ratio, which is a forecast of residual returns found using regression techniques. By rearranging this “alpha-omega” definition in terms of alpha and plugging it into the equation of value added given above, we get

$$VA = \omega \cdot IR - \lambda \cdot \omega^2$$

Now, the problem has been put completely in terms of omega. Since one of the goals of active management is to maximize value added, the derivative of the above equation needs to be taken with respect to ω , resulting in

$$\frac{\partial VA}{\partial \omega} = IR - 2 \cdot \lambda \cdot \omega .$$

Setting this equation equal to zero gives the maximum value of omega as

$$\omega = \frac{IR}{2 \cdot \lambda} .$$

Let’s see an example of this. Let’s say a manager has an information ratio is 0.5 and a risk aversion coefficient(λ) of 0.15. Plugging in these values for IR and lambda into the above formula for maximum omega results in $\omega = \frac{0.5}{2 \cdot 0.15} = 1.67$. Using this value of omega in the equation for value added results in $VA = 1.67 \cdot 0.5 - 0.15 \cdot 1.67^2 = .417$.

Now, let’s look at this graphically. If we graph omega and value added for IR = 0.5 and $\lambda = 0.15$, the results are a concave curve with a maximum at approximately $\omega = 1.6$ and value added = 0.4. Figure 3 demonstrates this example.

If we substitute this maximum value of omega back into the initial equation for value added, we get $VA = \frac{IR^2}{4 \cdot \lambda}$. This equation is very important in active management because it states that in order to maximize your value added you need to choose a strategy that will give you the highest information ratio possible, regardless of the level of risk aversion. Table 3 gives the value added for three levels of the information ratio and three choices for risk aversion.

Info Ratio	Risk Aversion		
	Aggressive(0.05)	Moderate(0.15)	Conservative(0.25)
Exceptional(1.0)	5.00	1.67	1.00
Very Good(0.75)	2.81	0.94	0.56
Good (0.5)	1.25	0.42	0.25

Table 3 - Value Added

In this table, if a manager has an information ratio of 0.5 and a moderate risk aversion of 0.15, then this manager could deliver at most 42 basis points a year over the benchmark. However, a manager with an information ratio of 1.0 and has a more aggressive risk aversion of 0.05 can return up to 281 basis points a year. As demonstrated above, the information ratio has much more importance than the risk aversion. The table also shows that for the same level of risk aversion, the value added increases as the information ratio increases. Figure 2 shows the constant value added lines for $VA = 2.5\%$, 1.4% , and 0.625% .

One of the goals of active portfolio management is to choose portfolios that maximize the value added. If we look back at the residual frontier for $IR = 0.75$ from Figure 1 and transpose this line onto Figure 2 with our constant value added lines, we will get Figure 4. Obviously, we would like to have the maximum value added, in this case the top curve with value added equal to 2.5% , but as the graph demonstrates, that is not an option since the residual frontier does not touch this line. However, the 1.4% curve is tangent to the residual frontier at a point Q. Therefore, this level of value added will maximize our portfolio.

5 Negative Impacts on the Information Ratio

There are several things that could have a negative impact on the information ratio. Two items that we will take a deeper look at are negative excess returns and transaction costs.

5.1 Negative Excess Returns

Since excess returns can frequently be negative, the information ratio is frequently negative. However, the negative excess returns can sometimes manipulate what the IR represents. The following example will illustrate this. Stock A has an 8 year annualized excess return of -2.74% and tracking error of 4.26%. With these two values, the information ratio for Stock A is -0.64. Stock B has an 8 year annualized excess return of -6.87% and tracking error of 11.58%, resulting in an information ratio of -.59. Since Stock B has the higher information ratio, one would think that this is the better of the two stocks. However, Stock A has higher excess return and lower standard deviation, which by real world standards means that Stock A is the more promising stock. As we can see, there is a serious flaw with the information ratio when the excess returns are negative.

A modification is needed to the information ratio formula in order to take into consideration negative excess returns. One possibility that has been suggested is to change the formula to the following:

$$\text{Modified IR} = \frac{ER}{\sigma^{ER/|ER|}}$$

(See [9].) When excess returns are positive, using either formula will produce the same information ratio. When excess returns are negative, the resulting information ratios are quite different when using the two different formulas. While the modification is better than the original in rewarding higher returns and lower risk, it also severely distorts the value of the information ratio by making those IRs with negative excess returns better (more positive) than when calculated in the first formula. This modifi-

cation can be used to rank stocks but shouldn't be used to put a numerical value on a portfolio manager's performance.

To illustrate this point, a five year study of twenty-five mutual funds was conducted. All of the mutual funds were randomly chosen from the Mid-Cap Growth Asset Class and the benchmark used was the S & P MidCap 400. Two of the mutual funds had to be thrown out due to insufficient data. A brief summary of all the mutual funds used is given in the appendix. During the five year time period, the annual returns and excess returns were calculated. Then, the five year return, five year excess return, and tracking error were computed and the resulting information ratios were found. The results are found in Table 5 on page 32.

Using the modification formula described above, the new Information Ratios were found for the same group of stocks. The stocks were ranked according to their information ratios with 1 being the highest and 23 the lowest. The results are listed in Table 6 on page 33. As seen in this table, the new information ratios are significantly different from the original ratios. There needs to be a way to compromise between the two methods so that the information ratios are more realistic as with the original ratios but have the ranking scheme of the modified information ratios.

Another possibility is to use beta in the calculation of the Information Ratio when the excess returns are negative. Beta, β , is a measure of how sensitive a stock is to the benchmark. For instance if a stock has $\beta = .9$, then for every one percent return that the benchmark has, the stock will have 0.9 percent return. So if the market moves up 20%, this stock will move up 18% ($.2 \cdot .9$). With this in mind, it seems that beta would be a useful variable in determining the information ratio since both beta and the information ratio are comparing a stock to a benchmark. One possible modification is

$$IR = \begin{cases} \frac{ER}{\sigma} & ER > 0, \\ \frac{ER}{\sigma^{-\beta}} - .15 & ER < 0. \end{cases}$$

By using beta instead of $ER/abs(ER)$ in the exponent of the denominator, there is a better measure of how the stock and the benchmark are related. Since the goal of

the information is to show how well or poorly a manager is performing compared to a benchmark then it is a reasonable assumption to use a variable that measures that relationship. Beta is a perfect choice for that. The use of the scalar -.15 is to make the newly calculated information ratios more realistic. In the first modification, almost all of the new information ratios got better in that they became closer to zero. The main objective of the modification is to reward managers with higher excess returns and lower risk and penalize those with lower excess returns and higher levels of risk. Thus, the modification should result in information ratios that more negative for those managers who have poor returns are have high levels of risk. By imposing a constant penalty of -.15 to all ratios with negative excess returns, there are more realistic information ratios but the ranking of the information ratios would not change since the ranking is decided by the first part of the formula ($\frac{ER}{\sigma-\beta}$).

This new modification was applied to the same group of mutual funds given in the previous two examples. Five year betas were found for each stock using information from the Yahoo!Finance website. Table 6 shows the results of applying all three formulas to the data. As seen in the table, the third formula gives almost the exact same rankings as the second formula but has information ratios that more closely resemble those in the first formula.

5.2 Transaction Costs

Up to this point, we have looked at the information ratio net of any fees and expenses that occur when conducting a transaction in the real world. Transactions costs include brokerage fees, on-line trading commissions, administrative costs, and other various expenses. Brokerage costs vary from broker to broker. The same is true for on-line trading firms (such as E*trade and Ameritrade). With these varying costs, it is difficult to find an industry wide standard for these expenses. For this reason, these types of fees will not be included in the analysis that follows.

Administrative costs, investment advisory fees, and other expenses are easier to

determine. Every year companies must report their expense ratio to the SEC. The expense ratio is the percentage of a fund’s assets that go to the expense of running the fund and includes management fees, investment advisory fees, 12b-1 distributions (for mutual funds), fees for directors, etc. Grinold and Kahn [6, ?] presented their empirical observation on this topic. In this study, three hundred U.S. active equity mutual funds were analyzed from January 1991 till December 1993. The before fees and after fees information ratios are given in Table 8. This table shows that the after fees information ratios are similar to the information ratios given in Table 2 for what are “good” information ratios. These results show how much fees and expenses can negatively impact the information ratio.

	Before	After
Percentile	Fees	Fees
90	1.33	1.08
75	0.78	0.58
50	0.32	0.12
25	-0.08	-0.33
10	-0.47	-0.72

Table 8

6 Comparison to Other Ratios

The information ratio can be compared to several other statistical performance measures such as the t -statistic and the Sharpe ratio.

6.1 IR vs. t -statistic

Looking ex post, the information ratio can be compared to the t -statistic since there is a connection between the statistical significance of excess returns and statistical significance of an information ratio. As a refresher from statistics, here is a quick review of this statistical value. The t -statistic has a t distribution with $T - 1$ degrees of freedom where T is the number of time periods. This statistic is based on a hypothesis test and the result of this test can be determined by standard t -tables. When testing the statistical significance of the information ratio, one would choose a hypothesis test where the null hypothesis would be that the excess returns over the benchmark portfolio would be zero and the alternative hypothesis would be that the excess returns are positive. The formula for computing the t statistic is as follows:

$$\begin{aligned} t - statistic &= \frac{ER}{\sigma_{ER}/\sqrt{T}} \\ &= \frac{IR}{1/\sqrt{T}} \\ &= \sqrt{T} \cdot IR \end{aligned}$$

Using this formula, a manager with an information ratio of 0.4 based on returns from 24 time periods has a t -statistic of 1.96. The t -statistic for 23 degrees of freedom with a 95% confidence level is 1.71. Since the computed value of 1.96 is higher than the 95% critical value of 1.71, the result is significant. From this it can be concluded that with 95% confidence, this manager's excess returns will be positive.

The number of time periods plays an important role in this comparison. For example, if we use the same information ratio of 0.4 but change the number of periods

to nine, the corresponding t -statistic is 1.2 but the 95% confidence value is 1.86 for eight degrees of freedom. Thus, the result is not significant in this example. This demonstrates that statistical testing can show how confident a manager can be with the calculated information ratio but the length of time that the IR was calculated over plays a very important role in the significance of the t -test.

6.2 IR vs. Sharpe Ratio

Another ratio that is closely related to the information ratio is the Sharpe ratio. This statistic was introduced by William Sharpe in 1966 and is defined as the excess return on a portfolio over a risk-free asset, such as a T-bill, divided by the risk of the portfolio:

$$SR = \frac{ER_{rf}}{\sigma}$$

The exact connection between the Sharpe ratio and the IR is quite controversial. Sharpe states that the information ratio is a “generalized Sharpe ratio” [15]. Others state that the Sharpe ratio is actually the squared information ratio. While there is a lot of confusion over the actual connection between these two ratios, you can see that there is a relationship between these two ratios.

7 IR Improvement Techniques

There are several things to keep in mind when using the information ratio. First, consistency is key. In order to have the most accurate information ratio possible, the same type of data must be used for both the funds and the benchmark. If daily returns are used for the funds, then daily returns need to be used for the benchmark. If different types of returns are used (i.e. weekly returns for funds and monthly returns for the benchmark) then the information ratio could be negatively affected.

It is also important that the funds being analyzed are mostly from the same asset class. By choosing stocks from different asset classes it becomes a greater challenge to find a benchmark that accurately reflects both asset classes, thus resulting in a lower information ratio. That leads us to the next improvement technique, choosing a benchmark that corresponds to the asset class the funds are in. If a poor choice of benchmark is picked, the data can easily be manipulated to produce a higher information ratio than in reality.

Another way to improve the information ratio is just to practice, practice, practice. Take a look back at the third definition of the information ratio given in this paper. Here, the IR is defined as skill (IC) multiplied by the square root of breadth. It can be difficult to increase your skill value so in order to increase the information ratio using this definition, breadth has to be increased. Since breadth is the number of times that a manager uses his/her skill, it is easy for breadth to be increased. Also, by using skill over and over to increase breadth, a manager is inevitably going to increase their skill level so it is a win-win situation.

Another strategy is given by Kahn and takes into consideration before and after fees information ratios (Kahn [10]). Kahn's paper focuses on bond managers but his ideas can be applied to active portfolio managers as well. In his approach, estimates are given for fees/expenses and excess returns. For instance, let's say a manager thinks he/she will have fees/expenses of 40 basis points and excess returns of 75 basis points, net of fees and expenses during the next time period. With these estimates, this manager

would have to produce 115 ($40 + 75$) basis points of excess returns, gross of fees and expenses. If this manager is a top-quartile manager, then his/her information ratio is around 0.5. In order for this manager to maintain that information ratio, he/she will have to take on 230 ($115/0.5$) basis points of active risk. Now let's look at this from a slightly different angle. Let's say this same manager wants to increase the information ratio from 0.5 to 0.6. To achieve this improved information ratio, the manager has three possible options: (a) increase excess returns (gross of fees and expenses) to 138 and keep the same risk level of 230, (b) increase the level of risk to 192 and keep the same target of excess returns at 115, or (c) slightly increase both of these two variables (one possibility is excess returns equal to 125 and risk equal to 208). Deciding which of the strategies to implement depends on a variety of things including the manager's individual strengths and weaknesses and the market conditions at that time.

8 Conclusion

As its name suggests, the information ratio can be a very “informative” tool. It also can be a deceiving tool. There are both advantages and disadvantages for using this statistic. On one hand, the information ratio can provide some insight into how well an active portfolio manager is performing with respect to a chosen benchmark. On the other hand, transaction costs and negative excess returns can significantly impact the information ratio in unflattering ways.

In this paper, three different definitions of the information ratio are given. While each one is theoretically correct, it is not clear whether one definition is better than any of the other definitions. The Brandes Institute [1] recommends defining an industry-wide set of standards to help clear up the confusion. As of this time, no such regulations have been implemented but it is something that should further investigated.

There are several other aspects of the information ratio that could be topics of future research projects. One is finding a better way to handle negative excess returns. As shown in this paper, the negative excess returns can manipulate the information ratio. Two modifications were given regarding this problem but further research might give and improved modification that would work better than the two given here. Another area of further research is finding a clearer relationship between the information ratio and the Sharpe ratio. A brief look at the connection between these two ratios was given in this paper but more research is needed to find a more concise relationship the information and Sharpe ratios.

Overall, the information ratio can provide a good measurement of how a portfolio manager is performing but should not be the only statistic used due to some ambiguities in calculating this statistic.

A Appendix: Stock Summaries

(All information provided by Yahoo!Finance)

MERDX - Meridian Growth Fund seeks long-term growth of capital. The fund invests primarily in equity securities. Investments may include common stocks, convertible securities, and warrants. When evaluating companies, the advisor considers such factors as growth rates relative to P/E ratios, financial strength, quality of management, and relative value compared with other investments. The fund may invest up to 25% of assets in companies with fewer than three years of operating history. It may also invest in securities of foreign issuers.

AASCX - Thrivent Mid Cap Stock Fund seeks long-term capital growth. The fund normally invests at least 65% of assets in mid-sized company stocks. The advisors of the fund define mid-sized companies as those with market capitalization ranging from \$100 million to \$7.5 billion, within this category the advisor generally will focus on companies with market capitalizations ranging from \$500 million to \$3.5 billion. It may invest the balance in additional mid-cap stocks, large-cap stocks, and convertibles.

CVGRX - Calamos Growth Fund seeks long-term capital growth. The fund normally invests in common stocks, though there are no limitations on the amount of assets that may be allocated to various types of securities. The investment-selection process emphasizes earnings-growth potential coupled with financial strength and stability. The fund may invest no more than 5% of assets in the securities of unseasoned issuers. It may also invest up to 25% of assets in foreign securities and may engage in various futures and options strategies.

FISGX - First American Mid Cap Growth Opportunities Fund seeks capital appreciation. The fund normally invests at least 80% of assets in common stocks of mid-capitalization companies. These are defined as companies that constitute the S &

P MidCap 400 Index and with market capitalizations between \$278 million and \$10.9 billion. The fund may invest up to 25% of assets in foreign securities.

HMCAX - Heritage Mid Cap Stock Fund seeks long-term capital appreciation. The fund normally invests at least 80% of assets in equity securities of medium capitalization companies. It may invest in common and preferred stocks, warrants or rights exercisable into common or preferred stock, and securities convertible into equities. The fund may invest up to 100% of assets in high-quality and short-term debt instruments. It may engage in short-term transactions under various market conditions to a greater extent than certain other mutual funds with similar investment objectives.

NVEA X - Wells Fargo Growth Equity Fund seeks long-term capital growth while moderating annual return volatility. The fund normally invests at least 65% of assets in equities. It divides assets among three equity investment styles represented by other Norwest funds: 35% in Large Company Growth, which invests in issuers with market caps of at least \$500 million and with unrecognized value; 35% in Small Company Growth, which invests in issuers with market caps of \$750 million or less; and 30% in International.

FGRW X - Hartford Growth Opportunities Fund seeks short and long-term capital appreciation. The fund primarily invests in equity securities covering a broad range of industries, companies and market capitalizations. It may invest up to 20% of assets in foreign issuers and non-dollar securities.

AAGF X - AIM Aggressive Growth Fund seeks long-term growth of capital. The fund invests in common stocks of companies whose earnings the fund's portfolio managers expect to grow more than 15% per year. It typically invests in securities of small and medium-sized growth companies. The fund may also invest up to 25% of assets in

foreign securities and may hold a portion of assets in cash or cash equivalents.

INV PX - AXP Equity Select Fund seeks growth of capital. The fund invests at least 80% of assets in equity securities. It primarily invests in growth stocks of medium-sized companies. It may also invest in small- and large-sized companies. Management chooses equity investments by identifying companies with effective management, financial strength, growth potential and a competitive market position.

ADEGX - Advance Capital I Equity Growth Fund seeks long-term capital growth. The fund normally invests at least 65% of assets in equities, not including stock index futures and options. It invests primarily in small growth companies with market capitalizations of \$1.5 billion or less. Management selects companies that are in the developmental stage of its life cycle and that have demonstrated or are expected to achieve long-term earnings growth which reaches new highs per share during each major business cycle. The fund may invest the balance in preferred stocks, convertible debt securities, stock index futures and options.

OCAAX - BB&T Mid Cap Growth Fund seeks long-term growth of capital. The fund normally invests at least 65% of assets in equities issued by companies with established growth records. These companies typically have capitalizations in excess of \$1 billion and revenues in excess of \$500 million. To select investments, the advisor also considers development of new products, business restructuring, new management, and the potential for increased institutional ownership.

NESBX - CDC Nvest Star Advisers Fund seeks long-term growth of capital. The Fund primarily invests in equity securities. It may also invest in securities offered in initial public offerings (IPOs), real estate investment trusts (REITs), convertible preferred stock and convertible debt securities, fixed-income securities, including U.S.

government bonds and lower-quality bonds. The Fund may also invest in options enter into futures, swap contracts and currency hedging transactions and hold securities of foreign issuers.

DFDIX - Delaware Growth Opportunities Fund seeks long-term capital growth. The fund primarily invests in equities that are selected based on the financial strength of the company, the expertise of its management, the growth potential of the company, and the growth potential of the industry itself. The fund may invest up to 25% of assets in foreign securities.

EMGFX - Eaton Vance Growth Fund seeks capital growth; income is a secondary consideration. The fund primarily invests in common stocks of U.S. growth companies. Although it invests primarily in domestic companies, the fund may invest up to 25% of assets in foreign companies. It may at times engage in derivative transactions (such as futures contracts and options) to protect against price declines, to enhance return or as a substitute for the purchase or sale of securities.

VCGBX - JP Morgan Capital Growth Fund seeks long-term capital growth. The fund normally invests at least 80% of assets in equity securities of companies with market capitalizations equal to those within the universe of Russell Midcap Growth Index at the time of purchase. It may invest in common stocks, preferred stocks, convertible securities, depositary receipts and warrants to buy common stocks. The fund may also use derivatives for hedging. It is nondiversified.

LBMGX - Thrivent Mid Cap Growth Fund seeks long-term growth of capital. The fund normally invests at least 80% of assets in common stocks issued by companies with market capitalizations between \$1 billion and \$5 billion. Management seeks to identify companies that have a track record of earnings growth or the potential for

continued above-average growth. The fund may also purchase bonds, preferred stocks, convertible bonds and preferreds, warrants, and ADRs. In addition, it may also invest in common stocks of companies falling outside of the mid-capitalization range.

OTCCX - MFS Mid-Cap Growth Fund seeks long-term growth of capital. The fund normally invests at least 80% of assets in equity securities. It may invest in common stocks and related securities, such as preferred stock, convertible securities and depositary receipts, of companies with medium market capitalizations with above average growth potential. The fund may invest in securities traded in the over-the-counter markets, foreign securities, emerging markets and foreign currency exchange contracts. It may also engage in active and frequent trading.

NAGBX - Nicholas-Applegate Growth Equity Fund seeks capital appreciation. The fund normally invests at least 90% of assets in equity securities, primarily in common stocks and convertible securities of companies with market capitalizations corresponding to the middle 90% of the Russell Midcap Growth Index. It may sell securities short.

OENAX - Oppenheimer Enterprise Fund seeks capital appreciation. The fund normally invests in common stocks of growth companies. It may invest without limit in companies in any market capitalization range. The fund may invest without limit in foreign securities and in any country. It may also purchase investment-grade debt securities. The fund may use leverage, invest in special situations, and engage in derivatives strategies designed to enhance total return.

POEGX - Putnam OTC Emerging Growth Fund seeks capital appreciation. The fund normally invests at least 80% of assets in common stocks traded in the over-the-counter market and in common stocks of emerging growth companies listed on

securities exchanges. It may also invest in foreign securities.

SGWAX - SunAmerica Growth Opportunities Portfolio seeks capital appreciation. The fund invests at least 65% of assets in equities issued by small-capitalization companies with operating histories of least five years that the advisor considers to have the potential for substantial earnings growth or value. The fund may invest in unlisted securities that have an established over-the-counter market.

PMEGX - T. Rowe Price Institutional Mid-Cap Equity Growth Fund seeks long-term capital appreciation. The fund normally invests at least 80% of assets in mid-capitalization companies. These companies typically have market capitalizations between \$1 billion and \$12 billion. The advisor expects the earnings of these companies to grow at an above-average rate. The fund mainly invests in U.S. common stocks. It can, however, invest in foreign securities, convertibles, and warrants when considered consistent with the fund's objective. The fund may also buy and sell options and futures.

VAGAX - Van Kampen Aggressive Growth Fund seeks capital growth. The fund primarily invests in equities issued by small- and medium-capitalization companies. Using a bottom-up approach, the advisor seeks issuers that are likely to produce high future earnings through new product developments or industry and market changes. It may invest without limit in issuers involved in special situations, such as new management, mergers, or liquidations. The fund may invest up to 25% of assets in foreign securities.

B Appendix: Tables

		Annualized	Benchmark	Excess
Date	Returns	Returns	Ann. Returns	Returns
12/01/03	0.009474	47.92%	34.02%	13.90%
11/03/03	0.041375			
10/01/03	0.104394			
09/02/03	-0.03165			
08/01/03	0.067587			
07/01/03	0.039668			
06/02/03	0.007613			
05/01/03	0.094127			
04/01/03	0.096848			
03/03/03	0.027217			
02/03/03	-0.00653			
01/02/03	-0.03941			
12/02/02	-0.08596	-17.75%	-15.45%	-2.31%
11/01/02	0.091112			
10/01/02	0.035615			
09/03/02	-0.07011			
08/01/02	-0.01525			
07/01/02	-0.10939			
06/03/02	-0.04709			
05/01/02	-0.01348			
04/01/02	0.022109			
03/01/02	0.061154			
02/01/02	-0.05109			
01/02/02	0.009208			
12/03/01	0.066379	3.74%	-1.63%	5.38%
11/01/01	0.092704			
10/01/01	0.036477			
09/04/01	-0.20312			
08/01/01	-0.01467			
07/02/01	0.003153			
06/01/01	0.040087			
05/01/01	0.028871			
04/02/01	0.078447			
03/01/01	-0.03512			
02/01/01	-0.04827			
01/02/01	0.029041			

3-Year Return	3-Year Benchmark	3-Year Excess	Tracking Error	3-Year IR
8.07%	3.69%	4.38%	8.10%	0.5408

Table 4

Stock	5 Year Return	5 Year Excess Ret	Tracking Error	5 Year Info Ratio
MERDX	12.91%	4.92%	7.10%	0.6933
AASCX	5.60%	-2.38%	7.85%	-0.3036
CVGRX	19.99%	12.01%	28.42%	0.4226
FISGX	7.05%	-0.93%	7.12%	-0.1305
HMCAX	12.80%	4.81%	12.20%	0.3945
NVEAX	2.42%	-5.57%	11.30%	-0.4926
FGRWX	4.31%	-3.67%	25.60%	-0.1434
AAGFX	10.70%	2.72%	54.48%	0.0499
INVPX	3.24%	-4.74%	16.06%	-0.2952
ADEGX	4.01%	-3.97%	13.53%	-0.2934
OCAAX	-1.52%	-9.50%	25.51%	-0.3726
NESBX	2.06%	-5.92%	23.50%	-0.2521
DFDIX	5.93%	-2.06%	30.06%	-0.0684
EMGFX	-4.35%	-12.33%	10.42%	-1.1831
VCGBX	3.14%	-4.85%	5.41%	-0.8953
LBMGX	5.41%	-2.57%	20.69%	-0.1244
OTCCX	1.42%	-6.57%	37.21%	-0.1764
NAGBX	-4.75%	-12.74%	50.51%	-0.2522
OENAX	-8.15%	-16.14%	57.83%	-0.2791
POEGX	-10.47%	-18.46%	69.49%	-0.2656
SGWAX	-0.20%	-8.18%	43.90%	-0.1864
PMEGX	7.91%	-0.07%	8.47%	-0.0085
VAGAX	1.92%	-6.06%	63.57%	-0.0954
S&P MidCap	7.98%			

Table 5

Stock	IR	Rank	New IR	New Rank
MERDX	0.6933	1	0.6933	1
AASCX	-0.3036	19	-0.0019	7
CVGRX	0.4226	2	0.4226	2
FISGX	-0.1305	9	-0.0007	6
HMCAX	0.3945	3	0.3945	3
NVEAX	-0.4926	21	-0.0063	12
FGRWX	-0.1434	10	-0.0094	14
AAGFX	0.0499	4	0.0499	4
INVPX	-0.2952	18	-0.0076	13
ADEGX	-0.2934	17	-0.0054	10
OCAAX	-0.3726	20	-0.0242	17
NESBX	-0.2521	13	-0.0139	16
DFDIX	-0.0684	6	-0.0062	11
EMGFX	-1.1831	23	-0.0129	15
VCGBX	-0.8953	22	-0.0026	8
LBMGX	-0.1244	8	-0.0053	9
OTCCX	-0.1764	11	-0.0244	18
NAGBX	-0.2522	14	-0.0643	21
OENAX	-0.2791	16	-0.0933	22
POEGX	-0.2656	15	-0.1282	23
SGWAX	-0.1864	12	-0.0359	19
PMEGX	-0.0085	5	-0.0001	5
VAGAX	-0.0954	7	-0.0385	20

Table 6

Stock	IR	Rank	New IR	Rank	New IR #2	Rank
MERDX	0.6933	1	0.6933	1	0.6933	1
AASCX	-0.3036	19	-0.0019	7	-0.1525	7
CVGRX	0.4226	2	0.4226	2	0.4226	2
FISGX	-0.1305	9	-0.0007	6	-0.1508	6
HMCAX	0.3945	3	0.3945	3	0.3945	3
NVEAX	-0.4926	21	-0.0063	12	-0.1566	12
FGRWX	-0.1434	10	-0.0094	14	-0.1577	14
AAGFX	0.0499	4	0.0499	4	0.0499	4
INVPX	-0.2952	18	-0.0076	13	-0.1576	13
ADEGX	-0.2934	17	-0.0054	10	-0.1535	9
OCAAX	-0.3726	20	-0.0242	17	-0.1739	18
NESBX	-0.2521	13	-0.0139	16	-0.1648	16
DFDIX	-0.0684	6	-0.0062	11	-0.1549	11
EMGFX	-1.1831	23	-0.0129	15	-0.1594	15
VCGBX	-0.8953	22	-0.0026	8	-0.1533	8
LBMGX	-0.1244	8	-0.0053	9	-0.1548	10
OTCCX	-0.1764	11	-0.0244	18	-0.1649	17
NAGBX	-0.2522	14	-0.0643	21	-0.2024	21
OENAX	-0.2791	16	-0.0933	22	-0.2332	22
POEGX	-0.2656	15	-0.1282	23	-0.2455	23
SGWAX	-0.1864	12	-0.0359	19	-0.1769	19
PMEGX	-0.0085	5	-0.0001	5	-0.1500	5
VAGAX	-0.0954	7	-0.0385	20	-0.1882	20

Table 7

C Appendix: Figures

Figure 1

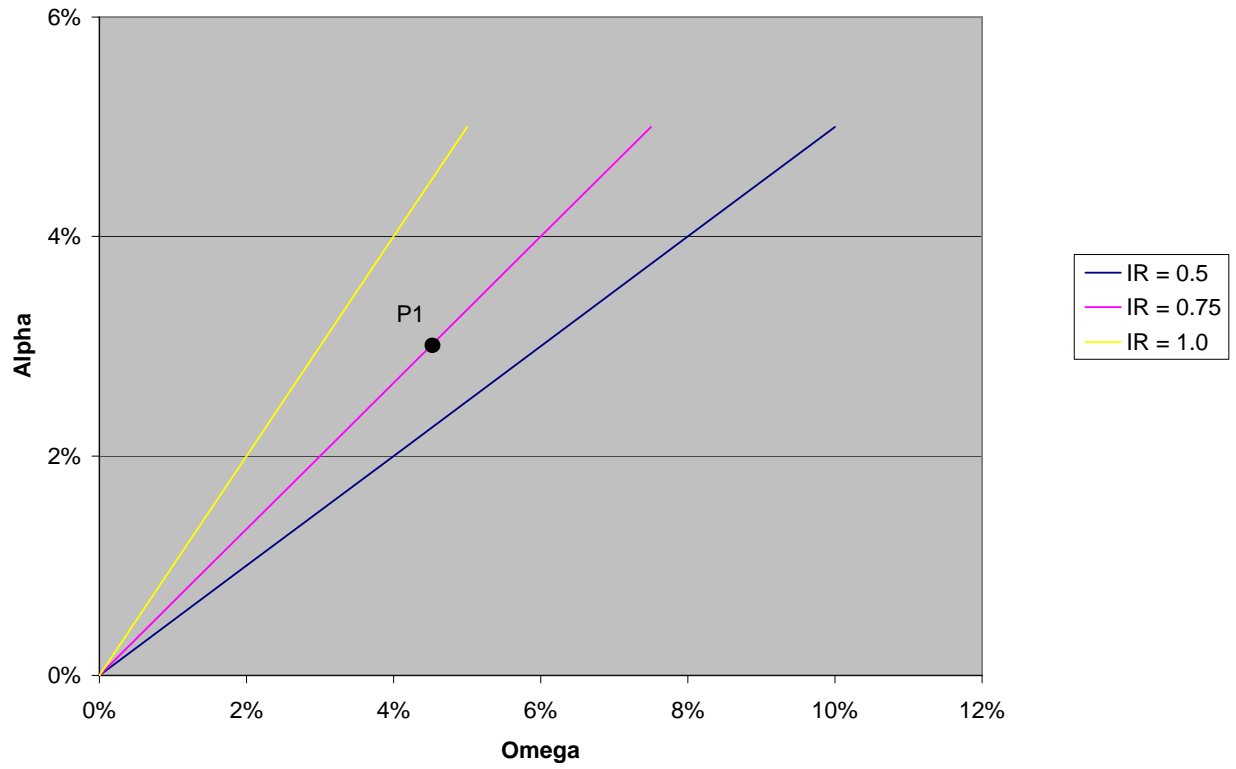


Figure 2

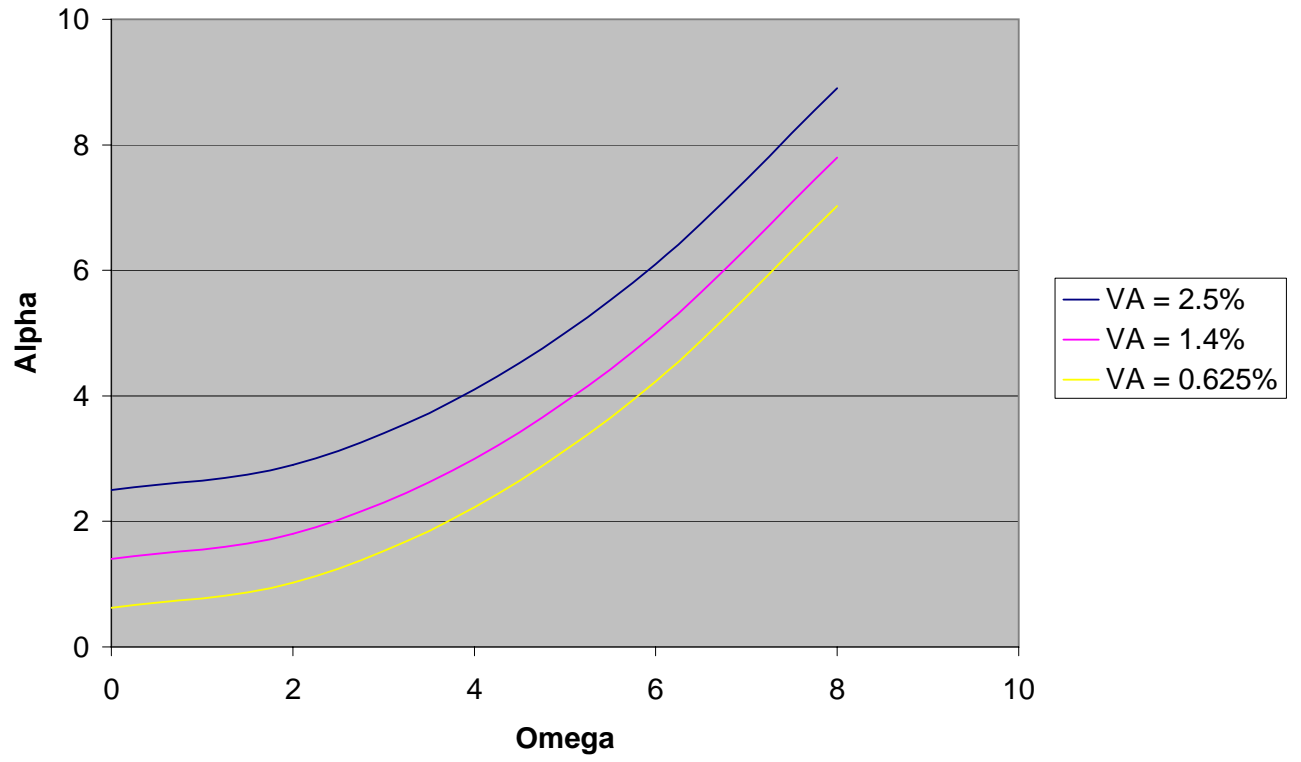


Figure 3

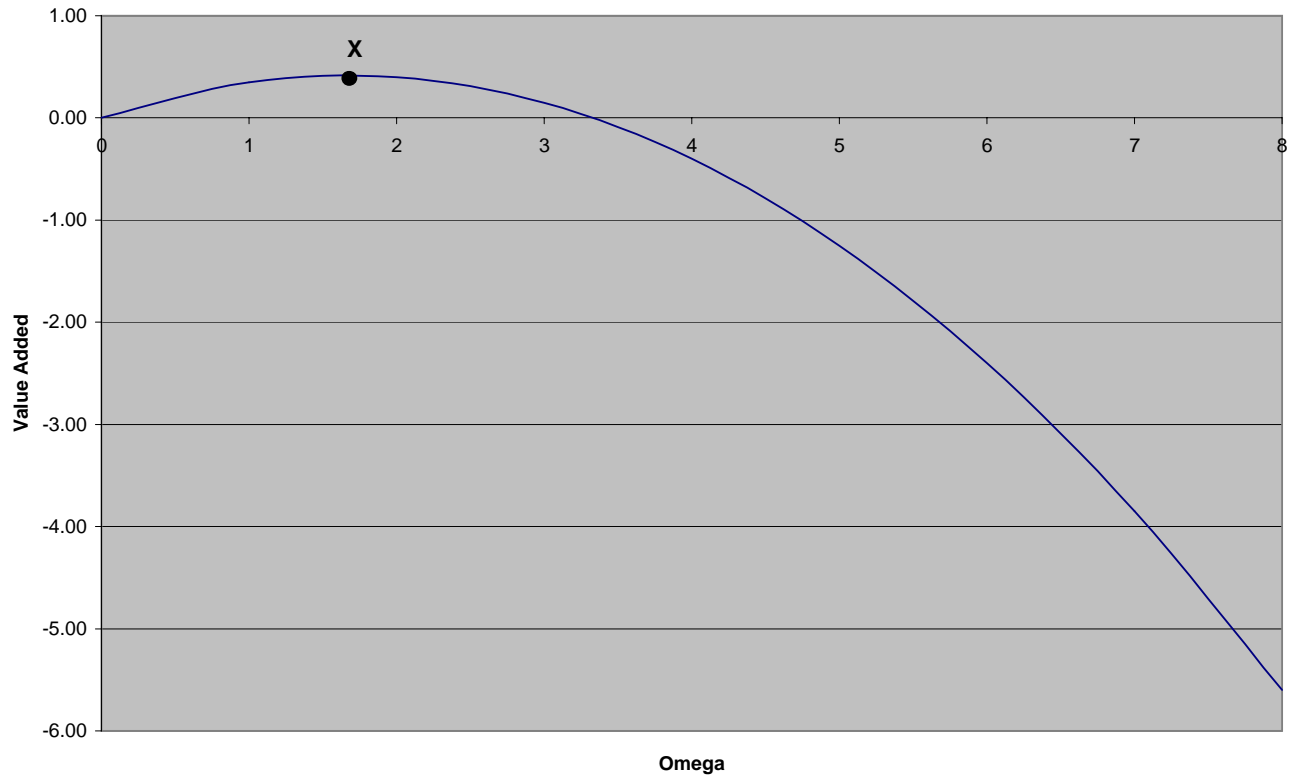


Figure 4

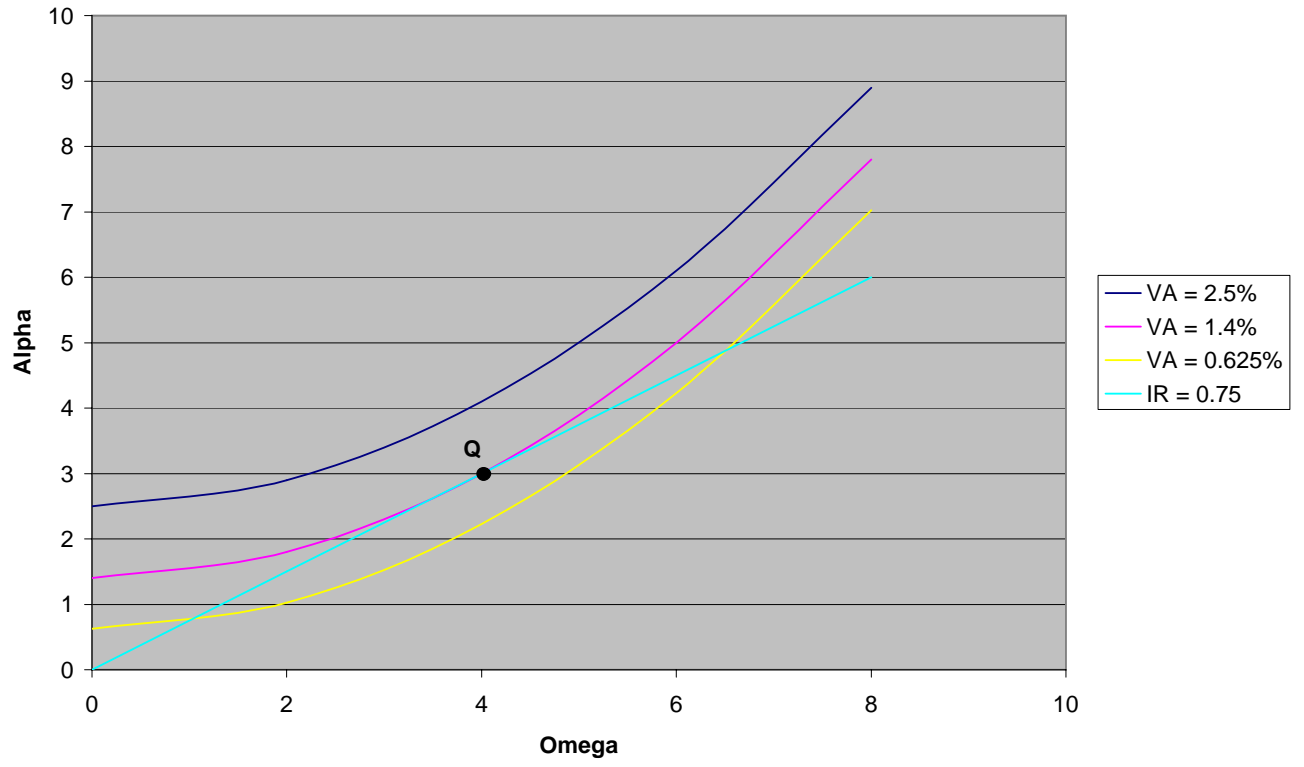


Figure 5 - Rankings

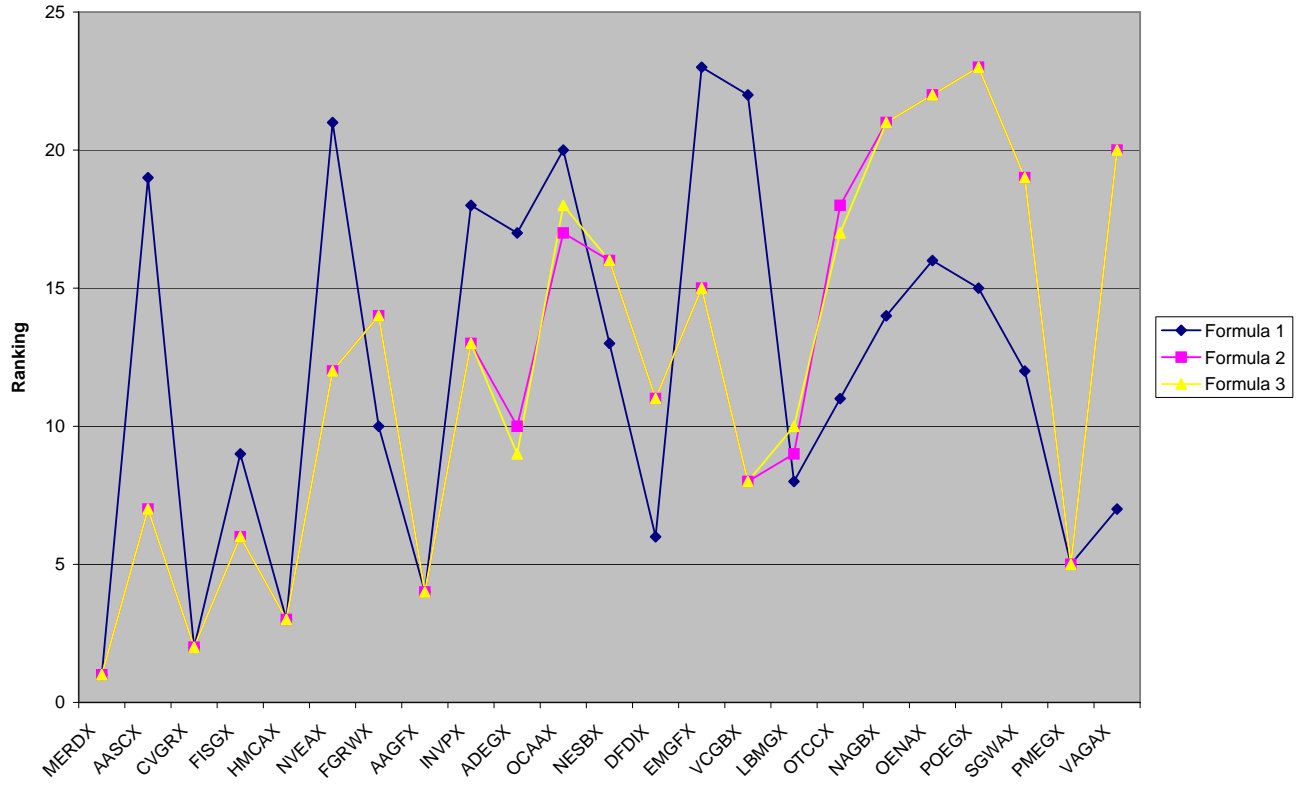
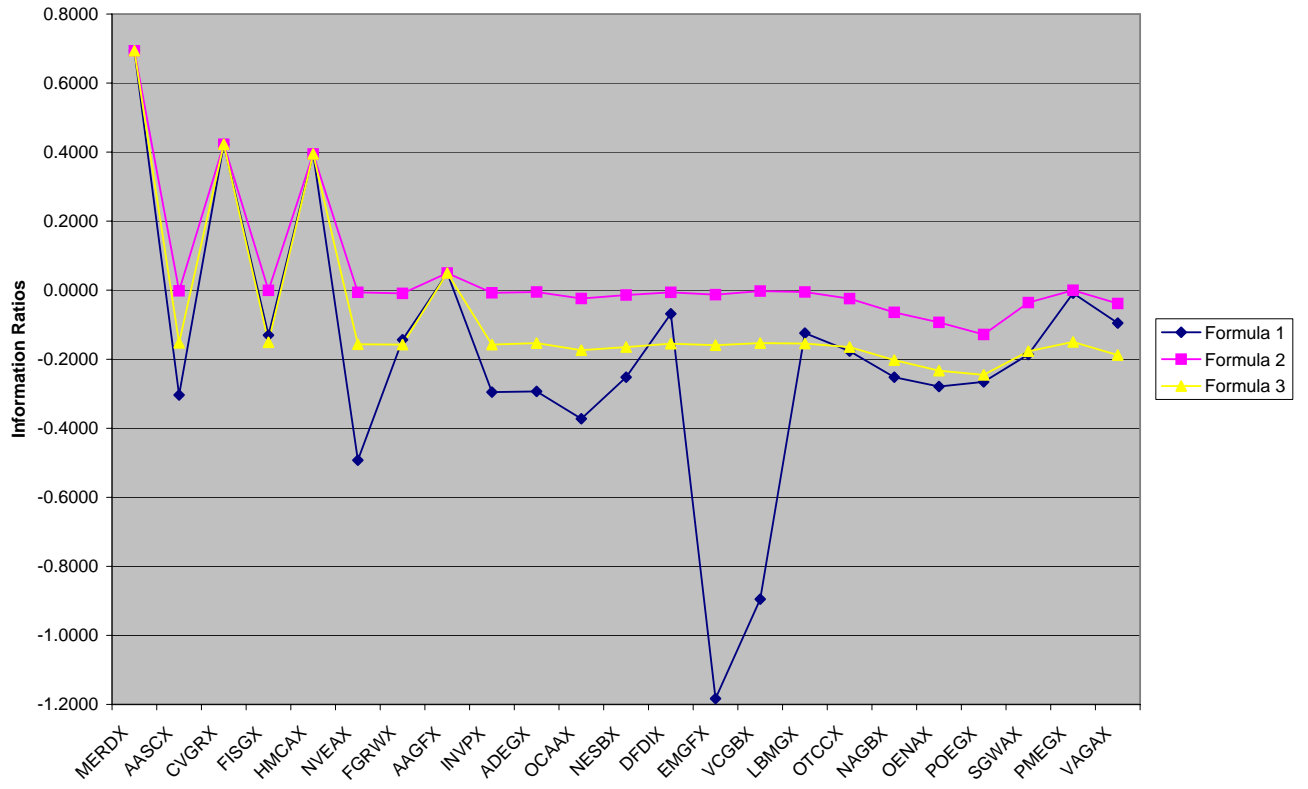


Figure 6 - Information Ratios



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