January 2009

Modeling RF Excitation of a CO2 Laser

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Modeling RF Excitation of a CO_2 Laser

A Major Qualifying Project
submitted to the Faculty
of the
WORCESTER POLYTECHNIC INSTITUTE
in partial fulfillment of the requirements for the
Degree of Bachelor of Science
by

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Date: 1/14/2009

Approved:

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Abstract

In this project I used a Matlab model to predict the ion sheath thickness of a CO_2 gas laser excited with an RF discharge. This sheath region is unique to the RF discharge, and inhibits laser performance. I predicted the sheath thickness to within 4% of experimentally measured values, which is a negligible error when making laser gain measurements.

Acknowledgements

I would like to thank Professor Garcia for taking on a project outside of his normal research and assisting with the writing and submission of this MQP. I would also like to thank Hsian Chou for all of the guidance and time he put in helping me understand the aspects of CO_2 lasers and pointing me in the right direction when conducting the extensive research which went into this project. Lastly, I would like to thank Alex Zozulya for helping me with the programming aspect and sharing with me his extensive knowledge of the Matlab software packages.

Table of Contents

Table of Equations

Table of Figures

1. Introduction

1.1 CO_2 Lasers

CO_2 lasers have a myriad of uses in the industrial world, ranging from cutting steal to communications. These lasers are also capable of producing very high powers, and can be used as either a pulsed source or a continuous wave (CW) laser. As a laser it is considered very efficient; approximately 27%. Demand for lasers to be more powerful and more efficient has ultimately driven extensive studies and the gathering of huge amounts of information about CO_2 lasers [8]. Around the end of the 20th century much advancement in the cost and longevity of the CO_2 laser has sparked further interest in finding more uses for the device. As these lasers grow more powerful, and more practical, the stage is set for many new and versatile applications [6]. The reason for such an efficient and useful laser material lies in the unique vibrational structure of CO_2, which is covered in the following sections.

1.2 CO_2 Laser process

A CO_2 laser has more than 200 vibrational-rotational (VR) transitions through which radiation can occur in the 8-18 micrometer range. Figure 1.1 is a visual aid to understand the laser process of a CO_2:N_2:He laser. It shows the relevant energy levels of both CO_2 and N_2 (He has a first excited state at a much higher energy). CO_2 has three characteristic vibrations of the classical linear model shown at the top of the chart. The 100 and 001 states each have one quanta in the ν_1 (symmetric stretch) and ν_3 (asymmetric stretch) modes respectively. The 020 vibrational state has two quanta in the ν_2 (bending mode). Lasing only takes place in transitions between the odd rotational levels of 001 and the even rotational states of 100 or 020. In order to get a CO_2 laser to work, the upper state must be filled while the 100 and 020 states remain nearly empty, this situation is called a population inversion and is crucial for the system.

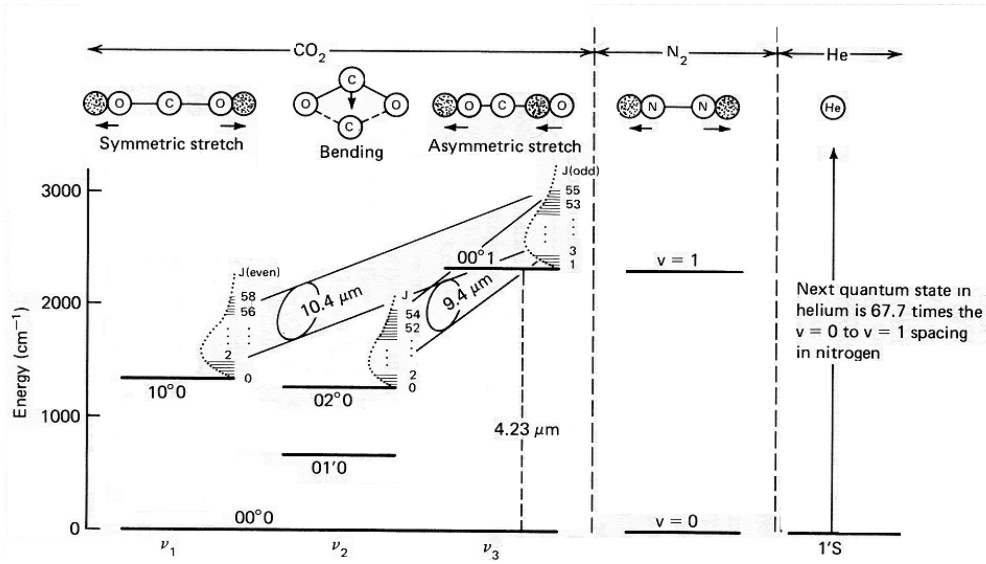

Figure 1.1 Laser process in a CO_2:N_2:He gas laser

It is very easy to compare the first excited state of N_2 with the upper laser level of CO_2, and it can also be seen that radiation emission only occurs between the odd rotational levels of the 001 state and the even rotational levels of the 100 and 020 states [7].

The gas mixture in the CO_2 laser is critical for the lasing process. Exciting CO_2 by electric discharge directly would simply fill all of the states, and not allow for the population inversion necessary for lasing to occur. It should be noted that the first vibrational level of N_2 is very close to the upper laser level of CO_2, however because N_2 is a homonuclear molecule radiative transitions between vibrational levels are not allowed. This means that if the N_2 molecules are in this vibrationally excited state they can transfer energy to the upper laser level of CO_2 through molecular collisions. The excited state of He is much too high in energy to fit on the diagram. This is used in "cleaning out" the 100 and 020 states. The energy from these excited states can be transferred to the He molecules also by collisions, which prevents these states from being saturated, which would in turn hinder the lasing process [7].

The rotational levels in each vibrational state are caused by thermodynamic distributions, and are dependent on the temperature of the gas. This allows for small variations in the wavelength that is produced by the laser and a varying amount of gain for each rotational level. To achieve the desired population inversion and gain, the laser material or gain medium must be pumped by some outside energy source. The next section will discuss some of the methods of exciting the gain medium in order to achieve proper population inversion.

1.3 Pumping a CO_2 Laser

The process of pumping a CO_2 laser begins with electrical power delivered to the electrons by an electric field. This energy is then transferred to the neutral gas (CO_2/N2/He mix), in one of three ways. Gas heating can be caused by inelastic collisions with the more massive neutral atoms. Vibration excitement can transfer some of this energy either to the CO_2 or the N_2. Exciting the N_2 is ideal because when it collides with CO_2 it will only fill the upper laser state. The third way electrical energy is transferred to the gas is by using it to maintain an ionized state, and is typically an insignificant amount. Approximately 60% of the electrical power can be used to pump the upper level, and since the quantum efficiency of the laser is about 45% the overall efficiency of the laser can be expected to approach 27% [7].

In order to get the best results from the laser the value of E/N should be optimized to produce the best electron excitation. Where E is the electric field and N is the neutral gas density. Figure 1.2 shows how crucial the E/N value can be in determining the efficiency of the laser excitation process. The figure shows two different mixes of CO_2,N_2, and He, and how narrow the ideal E/N region can be.

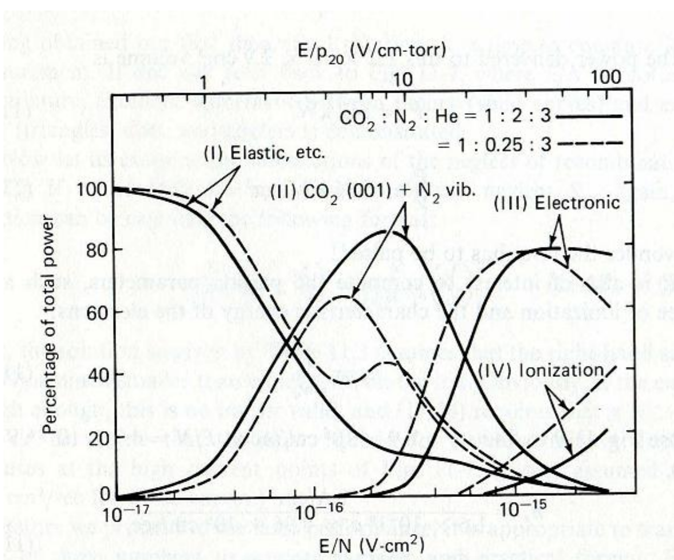

Figure 1.2 Energy distributions for different E/N values

It is clear to see that the E/N value should be optimized in order to achieve efficient laser performance [7].

There are four differential equations which are traditionally used to model the gas discharge. First are the continuity equations for electrons (e) and positive ions (+)

$$\frac{\partial n_e}{\partial t} + \frac{\partial j_e}{\partial x} = \mu_e n_e E \alpha - k_r n_e n_+ \qquad (1)$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial j_+}{\partial x} = \mu_+ n_+ E \alpha - k_r n_e n_+ \qquad (2)$$

The electron and ion densities are given by n, the electric field E, the particle flux, j, and mobility, μ. Equation (1) and (2), explain how electrons and ions are produced through ionization, and lost through recombination. The constant α is the ionization coefficient and k_r is the recombination coefficient. These constants will be explained further in the Methodology section. The charged particles have a drift velocity defined by the product of the mobility and the electric field. Another important equation is the Poisson equation for the system which can be written in the form

$$\frac{\partial E}{\partial x} = 4\pi(n_+ - n_e) \qquad (3)$$

where e is the charge of an electron. Equation (3), by definition, is used to calculate the Voltage across the laser gas. The fourth equation which is needed to understand this system is the heat balance equation. With c_p being the heat capacity, N the neutral gas density, and λ the thermal conductivity of the gas this equation is given by

$$N c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = e(\mu_e n_e + \mu_+ n_+)E^2 \qquad (4)$$

.

Modeling this system of equations would result in a complete understanding of the gas discharge for the CO_2 laser system. However, a complete model of these equations would be very difficult due to the fact that three different types of PDE's are present. The continuity equations are hyperbolic, the Poisson equation is elliptic, and the heat balance equation is parabolic. Because most numerical approaches require the definition of a mesh size mapping these equations together is nearly impossible. In an attempt to simplify this problem these equations can be reduced to a set of linear ODE's which allows for easier application of numerical methods [1]. Before this simplification can be made, the physics of the excitation method must be discussed.

1.4 Existing Model

Prior to this project there existed a model written in FORTRAN which predicts laser performance of a DC discharge situation. A Boltzmann code calculated electron energy distribution, transport coefficients (such as temperature and mobility), and vibrational excitation rates in gases with applied electric field. The transport coefficients were then used as parameters to predict gain and laser performance using the sponsor's Kinetic code [5]. The outputs from the Boltzmann code are used as input parameters for the model that was created by this project to predict the ion sheath thickness that was measured in experiments published in the paper by Vitruk, Baker, and Hall [1].

Due to the complexity of the equations mentioned in previous sections it is necessary to make some approximations in order to simplify the coding process and to create a well behaved system of functions to solve. The four original PDE's that discuss the general problem, can be normalized into a system of 5 ODE's which are more easily handled by the Matlab solvers. The mathematics of this simplification is very tedious and would distract from the purpose of this paper and so they have been omitted. Instead an explanation of the most significant will be presented. This is the equation which defines the location of the boundary between the plasma region and the ion sheath. The ion sheath is an ion dense layer of the gas formed near the electrodes and is characteristic of the RF excitation method. Since this ion sheath is the major topic of this paper section 1.6.1 is dedicated to defining it. Also the following sections will discuss some approximations of key parameters including the input current as a function of input power, and the ionization coefficients of the gas mixture. In order to model laser performance the electron temperature needs to be known, along with the electron density, and the E/N value [4]. Understanding how the electrons are actually excited, and modeling that excitation in Matlab is the main goal of this project.

1.5 DC Discharge

There are several ways that a CO_2 laser can be pumped [3]. Two methods which have been explored are a DC discharge and an AC discharge. Both of the methods have very distinct characteristics which will be discussed in the next couple of sections.

In either of the two discharge cases the basic set up will consist of two electrodes separated by a distance d and containing a laser gas mix (CO_2:N_2:He) under pressure P (or neutral gas density N). Another concept which is common to both discharges is that the Voltage V developed across the laser gas is nearly independent of the discharge current, which means that it cannot be increased just by changing the input power. This, coupled with

$$Pd = CONSTANT$$

which is a similarity relationship for this system, leads to the notion that for any change in P or d, the value E/P is maintained, because E = V/d [4]. A similarity relationship is a way to show how two specific parameters (in this case electrode separation, d and pressure, P) are constrained by each other.

The DC discharge was first used to pump waveguide CO_2 lasers in the early 1970's, and produced a considerable increase in performance. Output power and gain were both increased and the laser was able to operate at a much higher pressure than typical CO_2 lasers at the time. These results were due to a smaller d, and a higher molecular density. In a DC discharge the electrons are produced at one electrode and lost in the other which requires a constant generation of electrons. In order to maintain constant E/N, which is required for optimal laser performance, very high voltages are needed to keep up a DC discharge [4]. These high voltages require very large power supplies and lasers which are less commercially valuable.

1.6 RF Discharge

Lower Voltages are needed to maintain a discharge using RF excitation. Operating at a frequency f the electrons oscillate with a spatial amplitude A given by

$$A = \frac{\mu_e E}{2\pi f} \qquad (5)$$

where μ_e is the electron mobility as before. This means that electrons will not be sent from one electrode to the other provided A < d, and are ideally not lost in this manner. A new similarity relationship can therefore be determined for RF Discharge. Because f can be tuned and E/P must remain the same for any P and d in order to optimize performance, it can now be said that

$$Pd = C_1$$
$$P / f = C_2$$
$$fd = C_3.$$

Where C_1, C_2 and C_3 are empirical constants. For optimal laser performance, these are the scaling factors that should be used in determining the electrode gap, gas pressure and frequency of oscillation [4]. Lower voltages which allow for smaller and more efficient power supplies make the RF discharge a very desirable method of laser gas excitation. There are however certain factors which make understanding the system much more complex. The next section will discuss the ion sheath which is formed near the electrodes.

1.6.1 The Ion Sheath

Up until now the geometry of the gas has not been discussed. The excited laser gas is contained between two electrodes which produce the electric discharge. As mentioned above the electrons will oscillate with given amplitude, A. Any electrons which are within this distance of the electrodes have a greater chance of being absorbed into the electrode surface, which creates and ion dense region which is known as the ion sheath, and is characteristic of the RF discharge method of excitation [4]. Figure 1.3 shows a visible emission intensity reading.

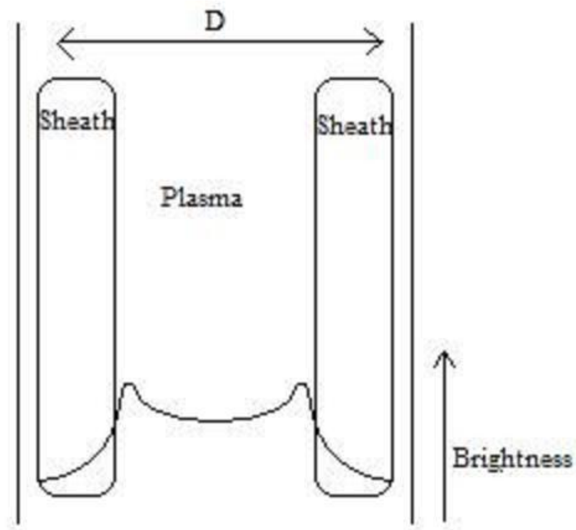

Figure 1.3 Ion sheath and visible emission
Between the electrodes there is a distribution of visible emission from high energy electrons and the dark regions are defined as the ion sheath which has a smaller amount of these electrons [4].

Visible emission is a result of the presence of high energy electrons. From this it can be inferred that the dark ion sheath region will not have much laser excitation, and therefore take away from the gain region [2]. As an attempt to help understand this laser system, the new model should calculate the sheath thickness for different input parameters, in order to show how size the gain medium is affected.

2. Methodology

In order to achieve the goal of implementing a new Matlab code to help understand the RF discharge, some simplifications of Eqs. (1) – (4) must be made. Some other approximations must be made for some of the constants and parameters of the system. These methods are explained in the following sections.

2.1 Finding the Ion sheath

A key parameter that can be calculated and compared to experiment using this code is the ion sheath thickness, and therefore an understanding of how the equations are manipulated to define its location is necessary. Equations (1) – (2) can be expressed in terms of ion and electron densities and the electric field by writing the flux term as

$$j = \mu n E - D\frac{\partial n}{\partial x} \qquad (6)$$

where D is the diffusivity coefficient and it is typical to consider the first term on the right hand side as the drift term and the second as the diffusion term. When Eq. (5) is inserted into the continuity equation, the differential equation takes the form of the wave equation with a mix of both time and coordinate derivatives.

The solutions of this type of differential equation oscillate as a function of distance from the electrode, and using the idea that the sheath is defined by an area that is ion dense, its location should be some kind of sine wave. The best way to define the sheath location is to define a phase, ψ, where the boundary crosses the x coordinate. Now using Eq. (6) for the amplitude and the fact that the current is alternating the electric field is

$$E = E_0 \sin(2\pi f + \psi) \qquad (7)$$

In the plasma region the phase should be π and it should be between 0 and π in the sheath. The differential equation used to define this quantity is

$$\frac{\partial \psi}{\partial x} = \frac{A n_+}{n_0 \sin \psi}\frac{1 - Sign\,(\psi - \pi)}{2} \qquad (8)$$

which is a step function that has a singularity when sin(ψ) is 0. In Eq. (8), A is the amplitude defined earlier and n_0 is the ion density in the center of the plasma which are both used here to normalize the equation creating a dimensionless first order ordinary differential equation [1]. This is one ODE from the system of five that are used as a simplification of the PDE's which define the system. It should be mentioned that at the electrode (x = 0) the boundary conditions define ψ as 0, which causes the equation to blow up.

In order to keep the equation from blowing up at 0, a variable substitution is used such that

$$\varphi = -\cos\psi$$

$$\frac{\partial\varphi}{\partial x} = \frac{An_+}{n_0}\frac{1-Sign\,(\cos^{-1}\psi-\pi)}{2}$$

The boundary value problem solver that was used requires that an initial guess for the form of the equation is made in order to predict a mesh size. Since ψ is a step function which varies in the sheath region and changes to π in the plasma region a form of an arctan(x) was guessed. Finding the sheath region from the solution given for ψ was just a matter of finding the region that $\psi < \pi$.

2.2 Ionization Coefficient

The ionization coefficient for the gas mix is the first Townsend Coefficient α such that

$$\frac{\alpha}{N} = A\frac{|E|}{N}e^{-B\frac{N}{E}}$$

where A and B are empirical constants [1]. Using a curve fitting method to find A and B, the value of the ionization coefficient can be determined. Figure 2.1 shows results from using a data grabber on an α/N vs. E/N curve for a known gas mixture (CO_2:N_2:He = 1:7:30 [7]). This was also done for two other known mixtures and the most appropriate set of constants for this case was used in the model to determine value of this coefficient.

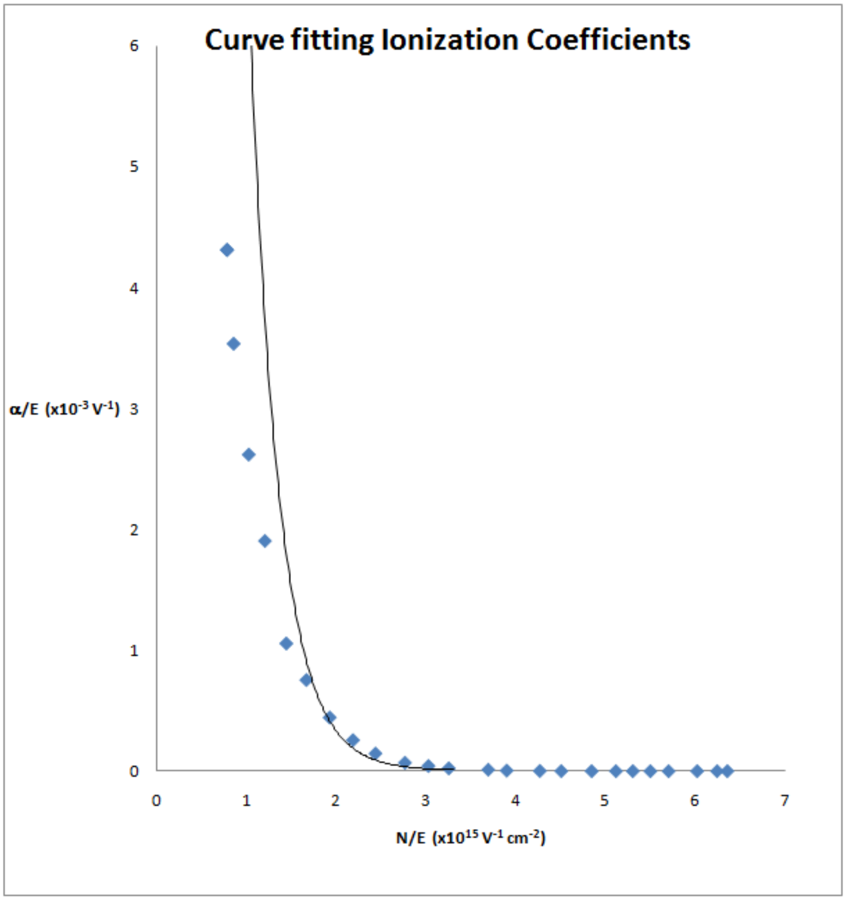

Figure 2.1 Curve fitting used to find Townsend Coefficient

Constants A and B can approximated from the trend line which in turn allows for the calculation of a, the ionization coefficient. The gas mixture CO_2:N_2:He = 1:7:30 is the case considered here[7].

The rest of the constants and parameters come from the design of the laser system or from outputs from the Boltzmann code, and are input for the calculations in the Matlab model.

2.3 Software tools

The key product of this project was a Matlab program used to solve the above model consisting of Eqs. (1)-(4), but there were a few other software tools that were used to obtain and process results. The system of equations was solved using the boundary value problem solver called *bvp4c.m* that is standard as part of the Matlab software package. In order to retrieve data from hard copies and PDF's of scientific paper the data grabber in OriginLab was used. The charts were produced using OriginLab. The results of these efforts are presented and discussed in the final two sections of the paper.

3. Results

Once all of the approximations are made the model gives values for the sheath thickness which differ from experimentally measured values by 4% on average. The experiment that is modeled is the sheath measurement for different input RF powers, and frequencies described in the Baker, Hall, and Vitruk paper written in 1994. Figure 3.1 shows the experimental data measured in the paper, and the solid lines are the values predicted by the model. Two very evident relationships that can be seen are that the sheath thickness tends to decrease as both the frequency, and the input power are raised. As the input power is increased however the thickness appears to approx a minimum value for each frequency.

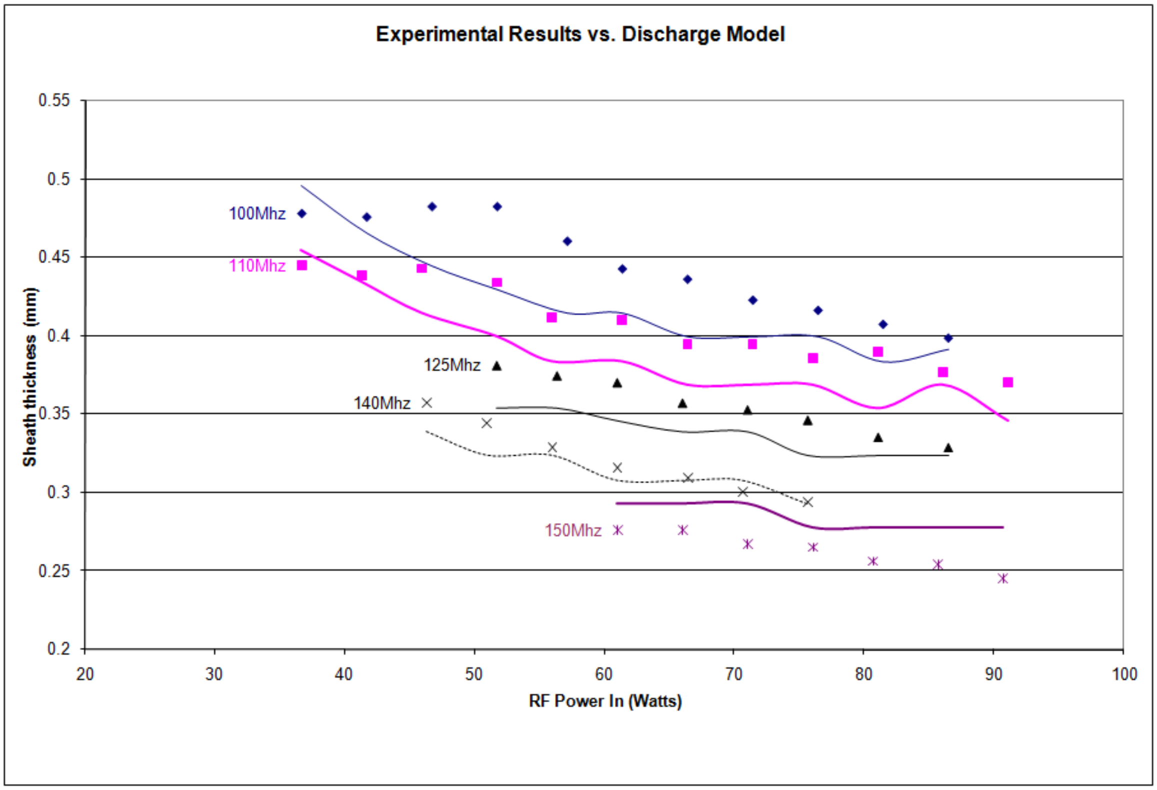

Figure 3.1 Model predictions for Sheath thickness vs. RF input power compared to experiment

The model predictions for various frequencies are shown by the lines and the data points are taken from the Baker Hall and Vitruk 1994 experiment. The trends are accurately modeled and the values vary from experiment by about 4%.

When the frequency dependence is looked at on its own another experimentally supported trend can be noticed. Figure 3.2 shows the sheath thickness as a function of frequency for different input powers calculated by the model, and it is very evident that the sheath thickness is proportional to some constant C and the square of the frequency f, or

$$d_s \propto Cf^2.$$

Where d_s is the sheath thickness and C is less than 1. A best fit second degree polynomial gives an R^2 value of 0.997 for the three cases, showing a strong correlation. This trend is consistent with the 1984 paper by He and Hall.

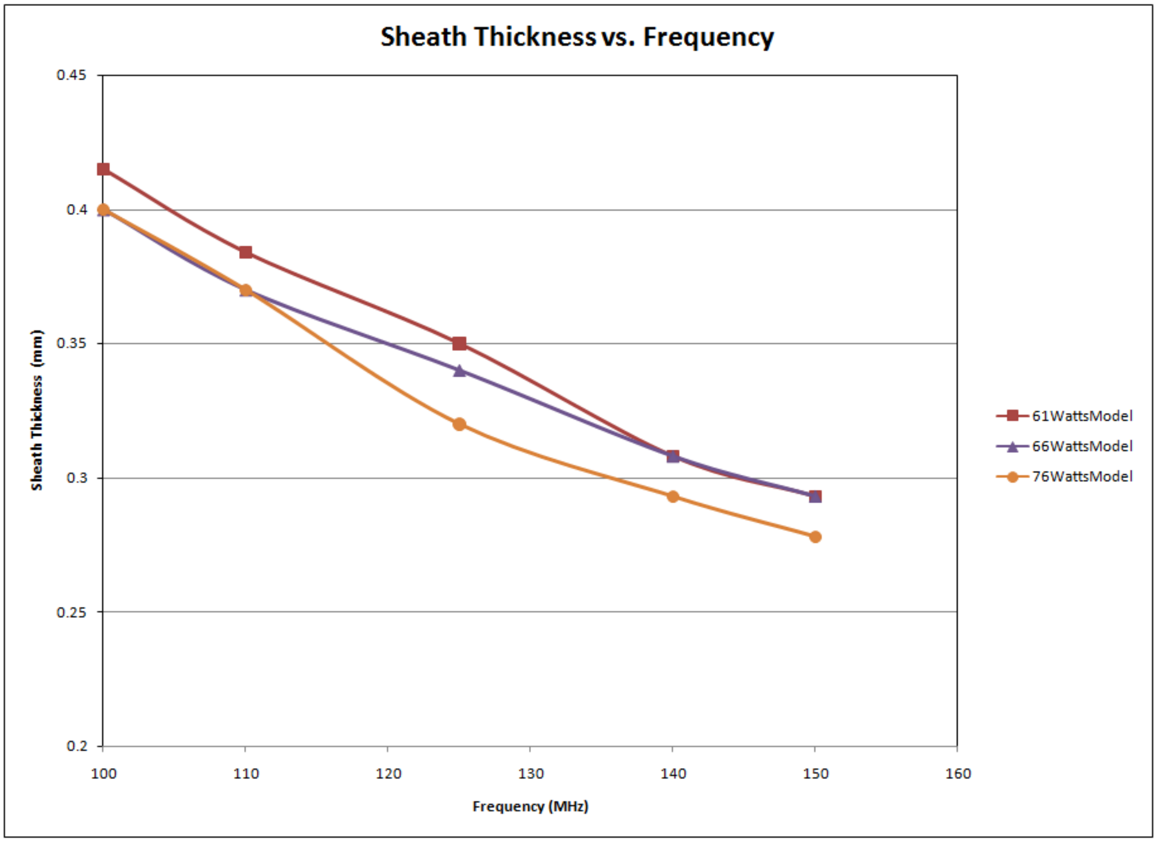

Figure 3.2 Model predictions for Sheath thickness vs. frequency

This figure shows an accurate prediction by the model for the sheath thicknesses when the frequency is varied for constant input power. The input powers used are those which were used experimentally in [1].

4. Conclusions

It has been demonstrated in Fig. 3.1 that the model accurately predict the size of the ion sheath as measured experimentally differing by only 4%. The error of 4% is very good for this experiment. Some of the key input parameters, such as the drive current and the ionization coefficient (Fig. 2.1), were approximated to no better than 10% accuracy, due to the curve fitting methods and available data used. If these values are varied within that 10% then the sheath thickness varies respectively inside the 4% error range. The distance between the electrodes is 2.8 mm for this experiment. So, an error of 4% in a measurement of 0.5 mm is only about 0.7% of the electrode separation, which is nearly negligible for gain measurements in a laser [5].

Knowing the sheath thickness allows for a better understanding of the losses in the laser cavity of an RF excited CO_2 laser. There is understandable error in the calculations due to some approximations made in order to simplify the governing equations of the system such as the curve fitting used for the ionization coefficient, but it has been shown that the trends are very accurate, and coincide with experimentally demonstrated results. There are a few other physical observables such as the electron spatial distribution and temperature profile that can be obtained using the same differential equations which allow for future study on this subject matter, and open the door for some expansion to this model.

As mentioned earlier knowing the temperature profile would be a vital piece of information in creating an all encompassing model of the RF discharge excitation process. In future versions of the model due to the relationship of the ion sheath to the ion density, it would be very easy to see the model producing calculations for a spatial distribution of the ions across the discharge. Since ions have less kinetic energy than electrons by simply being more massive, it is easy to see how the temperature distribution could be obtained from the ion distribution. There are some clear obstacles that were encountered that must first be overcome in order to expand the model in such a manner.

The governing equations naturally do not mesh well together due to the nature of the properties of the PDE's. This means that the system is in general very stiff, and slight alterations in the input parameters can lead to extreme discontinuities in the solution, which cannot be handled numerically by a computer program. There could perhaps be another method to solving the equations, which might include a finite difference analysis of the system. Despite the approximations and more formal mathematical analysis, this predicts accurately the sheath thickness, and displays a great potential to be the basis for a future tool to completely analyze the RF discharge excitation process.

5. References

[1] H. J. Baker, D. R. Hall, and Peter P. Vitruk. Similarity and Scaling in Diffusion-Cooled RF-Excited Carbon Dioxide Lasers. Journal of Quantum Electronics. VOL. 30, NO. 7. July 1994.

[2] D. He, D. R. Hall. Frequency Dependence in RF Discharge Excited Waveguide CO_2 Lasers. Journal of Quantum Electronics. VOL. QE-20, NO. 5. May 1984.

[3] Henri Brunet, Emmanuel Clavier, Michel Gastaud. Amplifying behaviour of an A.C. excited high power CO2 laser. SPIE Vol. 1810 Gas Flow and Chemical Lasers. p. 117. 1992.

[4] Andrew Alexander Cameron. High Pressure Slab Waveguide CO2 Lasers. The British Library. West Yorkshire, United Kingdom. September 1997.

[5] Hsian Chou, Ph.D. Interview about CO_2 Laser systems. 08-11-2008.

[6] Hidekazu Hatanaka, Katsumi Midorikawa, Minoru Obara, and Hideo Tashiro. Multikilowatt TEA CO2 laser driven by an all-solid-state exciter. SPIE Vol. 1810 Gas Flow and Chemical Lasers. p 125. 1992.

[7] Joseph T. Verdeyen. Laser Electronics. Copyright 1981 Prentice-Hall Inc. Englewood Cliffs NJ.

[8] W.J. Witteman. The CO2 Laser. Copyright 1987. Springe-Verlag, Berlin Heidelberg.

Appendix A. Bibliography

This section gives a brief overview of the references used in this paper.

H. J. Baker, D. R. Hall, and Peter P. Vitruk. Similarity and Scaling in Diffusion-Cooled RF-Excited Carbon Dioxide Lasers. Journal of Quantum Electronics. VOL. 30, NO. 7. July 1994.

The paper examines RF excitation of a CO2 laser gas mixture over a range of frequencies. Voltage-current-power characteristics of the RF discharges are experimentally proven to follow the model based on an analytical approach and an equivalent circuit model.

D. He, D. R. Hall. Frequency Dependence in RF Discharge Excited Waveguide CO_2 Lasers. Journal of Quantum Electronics. VOL. QE-20, NO. 5. May 1984.

In this paper experimental data is taken to explore the frequency dependence of the Ion sheath thickness. Calculations are also made to compare the sheath thickness to the impedance of the gas. These numbers are very important when designing efficient electrical circuits.

Henri Brunet, Emmanuel Clavier, Michel Gastaud. Amplifying behaviour of an A.C. excited high power CO2 laser. SPIE Vol. 1810 Gas Flow and Chemical Lasers. p. 117. 1992.

Different methods of Energy loading into the plasma are discussed and A.C. discharge excitation is explored and tested. The results show that the A.C. excitation will help to limit the current in the case of decreasing impedance of the load.

Andrew Alexander Cameron. High Pressure Slab Waveguide CO2 Lasers. The British Library. West Yorkshire, United Kingdom. September 1997.

The second chapter of this thesis gives an overview and explanation of RF excited CO2 Lasers. This includes discussion of the discharge similarity laws.

Hsian Chou, Ph.D. Interview about CO_2 Laser systems. 08-11-2008.

Chou does the CO2 laser modeling for the sponsor using the FORTRAN language.

"A Boltzmann code is used to calculate electron energy distribution, transport coefficients, and vibrational excitation rates in gases with applied electric field. The transport coefficients such as electron temperature, drift velocity, mobility, and vibrational excitation rates are then used as the input to TSC's CO_2 kinetic code to predict gain and laser performance." - Chou

Hidekazu Hatanaka, Katsumi Midorikawa, Minoru Obara, and Hideo Tashiro. Multikilowatt TEA CO2 laser driven by an all-solid-state exciter. SPIE Vol. 1810 Gas Flow and Chemical Lasers. p 125. 1992.

High Rep Rate (HHR) multi-kilowatt CO2 laser was demonstrated. The results show long life and practicality of a HHR CO2 laser which will open up many industrial applications.

Joseph T. Verdeyen. Laser Electronics. Copyright 1981 Prentice-Hall Inc. Englewood Cliffs NJ.

This book talks about the Laser excitation process in a CO2 laser. Charts and tables are used to aid the explain electron transitions and population inversions. Also an example is explained and a detailed solution is given to an energy loading problem. In another section the Gas Discharge phenomenon is discussed.

W.J. Witteman. The CO2 Laser. Copyright 1987. Springe-Verlag, Berlin Heidelberg.

This is *the* book on CO2 lasers. Witteman has compiled just about everything ever known about the lasers to date in this text.