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Exploring Measurement Estimation Through Learners Actions, Language, and Gestures

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Exploring Measurement Estimation Through Learners’ Actions, Language and Gestures

by

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A Thesis

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Abstract

This thesis intends to advance educational research by providing exploratory insights about the roles of, and relationships between, the actions, language, and gestures of college and elementary-aged students surrounding measurement estimation. To the best of my knowledge, prior research has examined the role of speech and gestures as they relate to areas of mathematics such as algebra and geometry, however, this work has not been extended to the area of measurement. Similarly, language and gesture have been explored but the three-way interplay between actions during problem-solving, and the language and gestures observed during explanations after problem solving has not been investigated in mathematics. To actualize the findings from this research in practice, this thesis uses the findings from two studies on behavior during measurement tasks to propose text and image support for an elementary-aged measurement game, EstimateIT!, to support students as they practice how to measure objects and develop conceptual skills through embodied game play. Specifically, this thesis intends to provide 1) a synthesis of the work on gestures in mathematics as well as the research methods used to study gestures, 2) a coding guide to analyze the gestures of mathematics learners, as well as their actions and language, 3) an application of the coding guide to explore the behavior of college and elementary students during measurement estimation tasks, and 4) proposals for action-guiding support for EstimateIT! to help elementary students develop and reinforce an understanding of measurement during gameplay based on the more mature strategies demonstrated by college students as they complete similar tasks.

Keywords: Embodied cognition, gestures, measurement estimation, learning technologies
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# Table of Contents

Abstract 2  
Acknowledgements 3  
Table of Contents 4  

**Part 1: Theoretical Framework and Data Analysis Tool** 5  
Chapter One: The Roles and Study of Gesture in Mathematics Learning 6  
Chapter Two: The Development of a Novel Coding Guide for Behavior of Learners 23  
Chapter Three: The Embodied Cognition Task Analysis Coding Guide 31  

**Part 2: Exploring Student Behavior in Measurement Estimation Tasks** 52  
Chapter Four: Exploring Measurement Strategies of College Students 54  
Chapter Five: Analyzing the Behavior of College Students During Measurement Estimation Tasks 62  
Chapter Six: Analyzing the Behavior of Elementary Students During Measurement Estimation Tasks 71  
Chapter Seven: Identifying Age-Based Differences in Problem-Solving Behavior in Estimation Tasks 85  

References 101  

Appendices 108  
Appendix A. College Estimation Tasks 108  
Appendix B. Helpful Hints Worksheet 110  
Appendix C. Questionnaire for College Study 111  
Appendix D. Questionnaire for Elementary Study 112  
Appendix E. Agreement Scale for Elementary Study 112  
Appendix F. Reference Images for Elementary Study 114  
Appendix G. Elementary Tasks 115
Part 1: Theoretical Framework and Data Analysis Tool

Early mathematical skills are predictors of academic success (VanDerHeyden & Burns, 2009) and differences in math achievement can lead to substantial differences later in life such as academic degree attainment and employment (Lee, 2013; Parsons & Bynner, 2005). The importance of early math achievement, in parallel with persisting achievement gaps in K-12 classrooms, calls for new approaches to identify and support struggling students early on in their mathematics education. One approach is to explore students’ behavior while problem-solving to learn about students’ mathematical abilities and shortcomings in a given topic then develop support to foster productive actions and strategies which reinforce conceptual knowledge.

The work presented in this thesis is grounded in the theory of embodied cognition. While critics point out that the theory of embodied cognition does not account for all cognitive processes (Goldinger, Papesh, Barnhart, Hansen, & Hout, 2016), there is substantial evidence that our physical experiences in the world influence our cognitive processes, including our mathematical competence (Foglia & Wilson, 2013). Gestures, specifically, have been shown to improve the ability to process new math concepts (Goldin-Meadow, Cook, & Mitchell, 2009), indicate students’ readiness to learn, and reveal cognitive processes that are not necessarily reflected in language (Congdon, Novack & Goldin-Meadow, 2018). However, less is known about the relationship between the actions, language and gestures of math learners as well as the shifting role of gestures in math over time (Novack & Goldin-Meadow, 2015).

This thesis applies the theory of embodied cognition to mathematics learning by focusing on the study of students’ physical behavior (actions, language, and gestures) to inform support for a hands-on measurement game for elementary students. The first section of this thesis presents the theoretical framework for this body of work and a proposed coding guide and classification system to measure student behavior in hands-on measurement tasks. This section has two main objectives: 1) synthesize the study of gestures in the past 50 years with a call to focus on developing a common classification system and 2) introduce a tool for video data analysis which could be expanded and used across similar projects to study students’ action, language and gestures.
Chapter One: The Roles and Study of Gesture in Mathematics Learning

Whether intentional or unintentional, it is common to “talk with your hands.” As a product of excitement, to search for a word just beyond the tip of your tongue, or to paint a better visual of the point you want to articulate, our bodies tend to aid communication by extending expression beyond verbal dialogue through purposeful or spontaneous hand movement, hereafter referred to as *gestures*. Gestures, considered here as movements of hands and body parts produced while an individual is thinking or speaking (McNeill, 1992), are a common byproduct of speech from birth (Novack & Goldin-Meadow, 2015). In situations from casual conversations to classroom lessons, across languages and cultures (Kita, 2009), gestures are prevalent and even exhibited by blind speakers (Goldin-Meadow & Alibali, 2013; Iverson & Goldin-Meadow, 1998). Indeed, gesturing is a ubiquitous behavior and research on gestures has shown that these movements have value beyond augmenting speech in multiple domains such as speech production (Krauss, Chen, & Gottesman, 2000) language acquisition (Goldin-Meadow & Alibali, 2013; Iverson, Capirci, Volterra, & Goldin-Meadow, 2008; Macedonia & von Kriegstein, 2012) and processing (McNeill, 1992; Wilson & Foglia, 2011), science (Roth, 2001), and, the focus of this chapter, mathematics learning.

While gesture has been studied in the context of mathematics learning over decades of research (Alibali & Nathan, 2012; Church & Goldin-Meadow, 1986; Gunderson, Spaepen, Gibson, Goldin-Meadow, & Levine, 2015; Perry, Church, & Goldin-Meadow, 1988), the methodologies used to quantify and typify gestures have continued to shift across projects without achieving a common, consistent system for classifying gestures. This inconsistency in methodology and language across similar lines of research hinders the ability to directly compare findings on gestures in mathematics learning and create generalizable tools for classifying gestures in future projects. To that end, this chapter aims to advance research on gestures in mathematics learning by 1) defining gesture through foundational research in this area, 2) providing an overview of the role of gestures in mathematics learning and 3) reviewing prior classification systems and methodologies used to quantify gestures in research.

1.1 Defining Gesture

Although gestures are ubiquitous across lifespans and cultures, the field of gesture research has only flourished within the past four to five decades, giving rise to various perspectives on overlapping nonverbal behaviors. Ekman & Friesen’s (1969) comprehensive work on the classifications of nonverbal behavior gave way to later frameworks for the study of gestures. For instance, after Kendon (1980) asserted that speech and gesture should be considered together because of the co-occurrence between verbal and nonverbal facets of conversation, McNeill (1992) proceeded to demonstrate the various connections between the language used in conversation and the gestures that accompany the language itself, such as through the timing and meaning of hand movements. This work supported the notion to consider gestures as an important nonverbal
component of conversation. Since the publication of these foundational pieces of literature on nonverbal behavior and gestures, specifically, a plethora of similar definitions have arisen to describe what constitutes a gesture (Chu & Kita, 2011; Ekman & Friesen, 1969; Goldin-Meadow, 2003; Kendon, 1980; McNeill, 1992).

Considered here, gestures are meaningful movements of hand and body parts produced while an individual is thinking or speaking. To elaborate, gestures are considered hand and body parts that 1) have meaning, 2) are produced while an individual is thinking or speaking, 3) can be produced intentionally or unintentionally, and perhaps as a novel amendment, 4) can be produced even while holding an external object. Each qualification for this definition of gesture is discussed in detail below and represented as the four fingers of gesture (Figure 1-1).

![Figure 1-1. The four tenants, or fingers, of gesture as defined in this chapter.](image)

**Figure 1-1.** The four tenants, or fingers, of gesture as defined in this chapter.

**Gestures are hand or body movements which provide emphasis or meaning alongside an individual’s speech.** This criterion for gestures does discount some of the hand movements that are typically observed by speakers on a day-to-day basis in conversation or lecture, such as behavioral tics and waving in greeting. Specifically, gestures, at least in the context of research, are distinct from nonverbal behavior that is merely habitual or meaningful independent of speech (Ekman & Friesen, 1969; Goldin-Meadow, 2003). Habitual movements, coined *adaptors*, include actions such as face-scratching or hair twisting which, while common behavior, do not add any meaning to speech. Conversely, universal signals that convey meaning independent of speech, coined *emblems*, include actions such as the gestures used between divers to communicate while underwater or the ubiquitous act of scribbling in the air to signal “check, please!” to a passing server (Ekman & Friesen, 1969). Within Ekman and Friesen’s (1969) framework of nonverbal behavior, gestures are situated along a spectrum of nonverbal behavior between these adaptors and
Gestures are movements produced while an individual is thinking or speaking. By using the term ‘in the context of speech’ in the previous paragraph, I allow room for gestures that may not directly accompany speech but occur while an individual is thinking, problem-solving, or formulating a sentence. This extension varies slightly from Ekman & Friesen’s (1969) original definition of illustrators as “movements which are directly tied to speech, serving to illustrate what is being said verbally” or Goldin-Meadow’s (2003) definition of gesture as “hand movements that co-occur with speech.” While being inclusive of co-speech gestures, I extend the definition to include co-thought gestures which “are hand movements produced in silent, non-communicative, problem-solving situations” such as during spatial transformation tasks (Chu & Kita, 2011). Because the Gesture as Simulated Action (GSA) framework for gesture considers these movements to be simulated actions and perceptual states generated as a speaker is thinking (Hostetter & Alibali, 2018), it follows that gestures can be produced while thinking, independent of speech, if the mental simulation of a spatial representation or motor activity exceeds a cognitive threshold for an individual. This follows the line of reasoning from the GSA framework that gestures are produced when the motor activation from a particular line of speech exceeds a threshold, causing the individual to physically simulate an action or representation in addition to the mental simulation (Hostetter & Alibali, 2008). That said, gestures produced when an individual is thinking are less (or not) interpretable without contextual speech. Please note that while including gestures that accompany thought and problem-solving in the overall definition of gestures, this chapter focuses mostly on observing the physical actions (directed movement with objects during problem-solving, without speech) of learners during problem-solving processes and the gestures (with or without objects) observed after problem-solving during, and surrounding, a verbal explanation of the problem-solving process.

In further regards to timing, by the same standards as Roth (2011), a unique gesture is considered as a hand or body movement which starts and stops in a place of rest with a clear peak of activity in between the beginning and end of the gesture.

Gestures can be produced both intentionally and spontaneously without conscious awareness. While gestures are considered purposeful in augmenting speech, they are not necessarily intentionally produced by the speaker (Congdon, Novack, & Goldin-Meadow, 2018; McNeill, 1992). To this end, while “speakers may not be completely aware of having produced hand movements, they are very aware of having spoken. Their gestures are in the service of communication and, in this sense, are deliberate” (Goldin-Meadow, 2003). While gestures deliberately aid cognitive processes and communication, these gestures could be communicative, to consciously send messages, or informative, with meanings that are not necessarily intentional though able to be decoded by an observer (Ekman & Friesen, 1969). In short, gestures add meaning to speech or thought without meeting a threshold of intentionality in their production.

Gestures can be hand or body movements as well as simulated actions that involve the use of objects. While previous work has not extended the definition of gestures to include movements involving external objects (to the best of my knowledge), I posit that movements involving tool
use can still be classified as gestures by applying a framework for physicality to gestures. Traditionally regarded gestures (hand movements) and movements involving objects or tools can be classified through Melcer and Isbister’s (2016) framework for embodied learning systems, specifically under the dimension of physicality. Melcer and Isbister (2016) posit that there are five ways in which individuals can physically interact within a learning environment: embodied, enacted, manipulated, surrogate, and augmented. I focus here on applying embodied, enacted, manipulated, and surrogate interactions to gestures; for a full review of the framework, please see the paper by Melcer and Isbister (2016).

I postulate that traditional gestures, unaccompanied by anything other than hand or body movements, can be classified under the category “Direct Embodied: Body-centered” or “Enacted: Body-in-action” as defined by Melcer and Isbister (2016). Embodied interactions constitute actions and movements that enable the body to “physically represent learning concepts” (Melcer & Isbister, 2016; Johnson-Glenberg Birchfield, Tolentino, & Koziupa, 2014). Applying this construct to gestures in mathematics, embodied movements could include those that create geometric shape outlines or depict concepts such as slopes or angles through hand/body representations. Enacted interactions include “acting/enacting out knowledge through physical action of statements or sequences” (Melcer & Isbister, 2016). I believe this definition could encompass gestures such as those that represent subsequent movement or measurement along a plane.

Parallel to embodied and enacted physicality, manipulated and surrogate forms of embodiment have similar definitions but involve the use of tangible objects. I believe that gestures which employ the use of a tool or object accompanying a movement would be classified as “Manipulated: Object-centered” or “Surrogate: Object-in-action” (Melcer & Isbister, 2016; Melcer, 2016). Manipulated interactions are similar to embodied but instead of the individual’s body representing a learning concept, an object becomes the representative. I believe this could qualify gestures which use an object as the tangible entity to represent a concept, such as holding an object and tilting it from side to side to demonstrate the idea of “wobbly” during speech. Similarly, I posit that under manipulated gestures, objects can be used as extensions of the body during gesturing, such as pointing with a tool in hand or identifying “this” object while holding it up for the listener to see. Lastly, surrogate interactions are similar to the enacted form of physicality but with a tangible object acting out knowledge rather than the individual. To continue the parallel comparison of surrogate to enacted physicality, I believe this definition of surrogate physicality could encompass gestures such as those that represent subsequent movement or measurement along a plane that is represented by the movement of an object. Hand movements that represent an action or movement, even aided by an object as a tool or extension of the body, could still be largely considered simulated if it is in the context of an explanation or reenacting a previous action, or series of actions, taken. By this rationale and under this framework for modes of physicality (Melcer & Isbister, 2016), gestures can be inclusive of hand and body movements that employ the use of tangible objects.
Figure 1-2. Distinctions between gesture and action.

**Action and Gesture**

Consistent with prior work in this area, gestures, as defined here, are a subclass of action (Göksun, Goldin-Meadow, Newcombe, & Shipley, 2013). As Goldin-Meadow and Alibali (2013) defined gestures as “individual actions in space”, the current work considers gestures to be distinct movements. As shown above in Figure 1-2, there is substantial overlap between the characteristics of action and gestures. For instance, both actions and gestures may occur in the context of thought and/or speech (Chu & Kita, 2011). However, actions are commonly distinguished from gestures by being classified as movements that effect change on objects or the environment (Goldin-Meadow & Feldman, 1975; Son, Ramos, DeWolf, Loftus, & Stigler, 2018). Under the Gestures as a Simulated Action framework, gestures, on the other hand (or should I say, by the other hand), are a product of a threshold in the motor system being exceeded during the process of motor and perceptual simulation which induces the need to physically simulate that action (Hostetter & Alibali, 2008). For instance, Son and colleagues (2018) classify drawing with an ink pen as an action whereas pointing with a capped pen would be considered a gesture. The current work focuses on gestures in the context of speech and considers gestures more broadly, encompassing demonstrative actions which augment communication by illustrative movement and action, even if that action effects change on the immediate environment. Specifically, I posit that gestures may effect change but are distinct from actions because they are actions produced to 1) augment speech and communication, and 2) display contextually meaningful information to the audience.

**1.2 The Roles of Gesture in Mathematics Learning**

Since the work of Ekman and Friesen (1969), Kendon (1980) and McNeill (1992) among others, a large body of research has been done to explore the role, importance, and meaning, of gesture across domains, particularly in mathematics learning (Alibali, Flevares, & Goldin-Meadow, 1997; Chu & Kita, 2011; Cook, Friedman, Duggan, Cui, & Popescu, 2016; Goldin-
Meadow, Cook, & Mitchell, 2009; Singer & Goldin-Meadow, 2005; Wakefield, Novack, Congdon, Franconeri, & Goldin-Meadow, 2018). A commonality across projects in this line of research is the implication that gestures do play one or more roles in mathematics learning and as such, can be used for assessments as well as student support. That said, mathematics is a subject that has traditionally been taught through textbooks and studied through pen-and-paper assignments. In such a nonphysical subject, how can gesture, a physical expression, be situated as a factor in mathematics learning?

The theory of embodied cognition posits that physical experiences in the world, as well as the actions and perceptions of an individual, shape cognition and thinking skills, extending to the development of mathematical thinking (Foglia & Wilson, 2013; Shapiro, 2010). Within this theoretical framework, gestures of an agent also contribute to mathematics learning, from gestures produced by the learners themselves (Chu & Kita, 2011; Goldin-Meadow et al., 2009) to gestures performed by instructors and witnessed by learners (Cook et al., 2016; Wakefield et al., 2018). From this framework, I apply the theory of embodied cognition to mathematics learning by modeling mathematics learning as a multimodal, potentially cyclical process, stemming from the reciprocal relationship between perception and action. Under the broader premise that “perception is for action…. the ability to perceive evolved from a need to interact with the world” (Hostetter & Alibali, 2008 citing Gibson, 1979; Sperry, 1952) I focus on how this relationship applies to mathematics learning. Specifically, I build off Alibali and Nathan’s (2012) stance that mathematical cognition is “based in perception and action, and it is grounded in the physical environment,” to posit that within a learning context, an individual’s environment shapes her perceptions that inform her cognitive processes to then act on her environment again to develop mathematical skills and thinking (Figure 1-3). For instance, the lecture on algebraic expressions that a learner observes from a teacher improves the learner’s ability to recognize patterns in the following assignment. This influence from the environment on the learner’s perceptions then influences the learner’s problem-solving process for each algebraic expression in the assignment. As the learner asks questions for clarification and literally points to areas of confusion within a given expression, prompting the teacher to provide feedback. Once again, this feedback from the learner’s environment informs the learner’s perceptions of the content and will shift the way in which the learner will act on the environment in the future. Acknowledging that the mathematics learning process might not resemble a purely cyclical process, I posit that in order to fully learn and develop mathematical thinking skills, learners engage in this cyclical relationship between the mind, body and environment to form connections and develop mathematical skills. As such, researchers and educators can think about these connections to better assess students’ mathematical abilities as well as create and implement appropriate instructional activities that capitalize on interaction with the learners’ environment.

Below, the roles of gestures in mathematics learning are highlighted from both the role of gestures observed by learners as well as gestures produced by learners in the teaching-learning process. This review is by no means exhaustive, rather, a brief overview of notable contributions
to this field that should motivate efforts to advance methodologies and unify the language used for gesture research.

![Theoretical model of embodied cognition as it pertains to the role of gestures in learning mathematics.](image_url)

**Figure 1-3. Theoretical model of embodied cognition as it pertains to the role of gestures in learning mathematics.**

**Gestures Observed by Learners**

As gesturing is a natural behavior across cultures and contexts, it is prevalent in instructional settings (Alibali et al., 2014) as teachers utilize pointing and other movements to direct attention, explain concepts, and connect ideas for students. Particularly in mathematics, teachers tend to utilize pointing gestures during lessons (Alibali & Nathan, 2012). Pointing gestures have been found to improve students’ performance on a posttest after a kindergarten lesson on symmetry was taught with or without pointing gestures (Valenzeno, Alibali, & Klatzky, 2003). Such gestures have been theorized as beneficial to learning by connecting speech and visuals, thereby reducing learners’ cognitive load (Alibali & Nathan, 2012).

As teachers integrate gesture with speech during lessons and explanations, the gestures that learners observe from instructors have been shown to foster learning, suggesting that just seeing others gesture is beneficial for learning in mathematics. For instance, Goldin-Meadow et al. (2009) found that teaching problem solving strategy with incorporated gestures can lead to improved learning over teaching strategy through speech only. Students who were taught a strategy with a partially-correct gesture still outperformed students who were taught the correct strategy with solely speech, no gestures, suggesting that teaching with gestures supports learning above and beyond teaching through speech only. To build off similar prior research (Singer & Goldin-Meadow, 2005) and investigate how instructors’ gestures facilitate learning, Wakefield et al. (2018) explored the role of gestures as visual attention directors. They found that gestures from
instructors do direct learners’ attention towards the problem rather than the speaker, and this focus on “following along with speech” predicted learning outcomes, suggesting that the synchronization between attention towards speech and gestures facilitates learning.

**Gestures Performed by Learners**

Just as learners benefit from observing others gesture in the context of mathematics, producing gestures is also beneficial to mathematics learning. As potentially unintentional movements (McNeill, 1992), gestures are observable behaviors that can provide insights about learners’ underlying cognitive processes (Congdon et al., 2018). Specifically, gestures have been shown to improve the ability to process new mathematics concepts (Goldin-Meadow et al. 2009), reduce learners’ cognitive load (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001; Ping & Goldin-Meadow, 2010), and reveal cognitive processes that are not necessarily reflected in language (Congdon et al., 2018), such as indicating students’ implicit knowledge and readiness to learn (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, 2003; Goldin-Meadow & Singer, 2003). This work supports the finding that educators and researchers can assess students’ knowledge through more than speech and written skills by observing students’ hands (Alibali et al., 1997) and demonstrates the overall importance of gestures in mathematics learning.

Gestures aid expression of mathematical understanding. Considering that gestures may be produced intentionally or unintentionally suggests that feelings and knowledge that are not conveyed through speech can be gleaned through the observation of gestures (Congdon et al., 2018; Williams-Pierce et al., 2017). Kendon’s (1980) foundational work was among the first to suggest that the consideration of gestures alongside language is telling of learners’ knowledge. For instance, Church and Goldin-Meadow (1986) found that students who produced conceptually correct gestures with incongruent, incorrect speech benefited more from instruction than students who may have produced an incorrect explanation in tandem with congruent (and incorrect) gestures, suggesting that students with discordant gestures were more “ready to learn” a new concept than their peers. Similarly, Alibali and Goldin-Meadow (1993) discovered that as students acquired an understanding of a new concept, the students first demonstrated incorrect, and then discordant gestures with incorrect explanations. Finally, students produced gestures that were congruent with a correct explanation of the concept, suggesting that gesture-speech mismatch is a transitional stage in learning. More recently, Gunderson and colleagues (2015) found that children who hadn’t learned all the number words could more accurately label sets of items with gestures than speech alone. Overall, these lines of research on gestures produced by mathematics learners suggests that gestures promote the learning process and expression of students’ understanding of mathematics concepts.

In addition to contributing to cognitive processes, aiding speech, and suggesting implicit knowledge, gestures also play a causal role in mathematics learning (Cook, Mitchell, & Goldin-Meadow, 2008). Learners retain more knowledge if they gesture spontaneously during an activity (Alibali & Goldin-Meadow, 1993; Cook & Goldin-Meadow, 2006) which suggests that gesturing contributes to learning. To study the causal role of gestures in mathematics learning, Cook and
colleagues (2008) taught children how to complete equations by making each side of the equation equal, controlling for gesture production during instruction. During instruction, children mimicked the instructor’s speech, gestures, or both. They found that students who were required to perform gestures linked to the content while learning retained more knowledge than peers who learned without being required to gesture (Cook et al., 2008), suggesting that gesture production does play a causal role in mathematics learning and emphasis should be placed on observing and encouraging productive student behavior while learning.

These examples from this body of literature highlight the importance of gestures in mathematics learning although more questions about the role of gestures in mathematics learning still remain. For instance, while gestures have been studied across different ages, does the importance of gesturing change with age? (Novack & Goldin-Meadow, 2015). Thinking about the implications of this question for mathematics learning, does the role of gesture for learners shift with age, expertise, or exposure to mathematics practice? And while student behavior plays a strong role in learning (Cook et al., 2008), most work has focused on studying behavior through students gestures rather than incorporating other action such as physical behavior during problem-solving. For instance, whereas a large body of work has focused on gesture-speech mismatch as it pertains to learning, to the best of my knowledge, not much is known about whether readiness to learn could also be inferred by the congruence between students’ physical problem-solving strategies and post-solving gestures that accompany speech when asked to give an answer justification. Perhaps students who are able to accurately explain their problem-solving strategy with congruent gestures have a better grasp of mathematics content than students who display a totally different problem-solving strategy through a verbal explanation and gestures.

Additionally, to the best of my knowledge, while an extensive body of literature demonstrates the importance of gestures in mathematics domains, not much of this work has been done on gestures surrounding measurement. In order to fully understand the role and importance of gesture in mathematics as a whole, it is important to study the role of gestures in multiple mathematics topics, and I would assume that gestures produced by learners during algebraic problem-solving would differ from those produced during measurement tasks. Lastly, most research on gestures in mathematics has focused on gestures in traditional settings between learners and instructors. However, with new learning technologies that bring mobility and movement into mathematics classroom, the contexts to examine gesture are expanding, posing questions such as how do gestures vary between student-teacher interactions and student-device interactions?

**Gesture Use in Technologies for Mathematics Learning**

In addition to observing learners’ gestures in traditional, mathematics learning environments such as the classroom, new mathematics activities and technologies also open the door to studying learners’ gestures in new contexts. Over the past several years, multiple dynamic technologies for mathematics learning have surfaced, inviting students to explore and practice mathematics concepts in action- and movement-based platforms (i.e. Harrison et al., 2018). Online
technologies such as the Mathematics Imagery Trainer (Howison, Trninic, Reinholz, & Abrahamson, 2011), Graspable Math (Weitnauer, Landy & Ottmar, 2016), Dragon Box (Siew, Geoffrey & Lee, 2016), and Geogebra (Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008) all integrate mathematics concepts with action and movement by the learner to make mathematics more tangible through experience. While varying in terms of user experience and physical interaction, these technologies exemplify the expanding landscape to learn mathematics.

As more dynamic technologies and physically interactive mathematics activities are used in practice, the importance of distinguishing such technologies from one another becomes more essential to dissecting why such learning technologies are effective for mathematics learning. To this end, the framework proposed by Melcer & Isbister (2016) can classify different embodied learning technologies for mathematics by distinguishing how users engage with the technology and their environment, such as whether users engage in mathematical practice within a computer interface or whether the technology augments users’ experience with objects in their environment to practice mathematics concepts. Additionally, as opportunities for learners to utilize gestures in mathematics expands through dynamic learning technologies, structures will be needed to characterize the use of gestures in these settings and determine whether the use of gestures is helpful to learning in these contexts. To that end, a common language to classify gestures across different learning environments would help solidify a unifying system to compare learning technologies through users’ actions and gestures in the future.

Figure 1-4. Top: children playing EstimateIT! at Worcester Polytechnic Institute. Bottom: Example prompt on player device.

The Wearable Learning Cloud Platform and EstimateIT!

The Wearable Learning Cloud Platform (WLCP) (Micciolo, 2018; Micciolo, Arroyo, Harrison, Hulse & Ottmar, 2018), developed at Worcester Polytechnic Institute, extends the
boundaries of learning technologies for students by being a visual programming tool that allows users to both create and play original, active games for mathematics learning with mobile devices, creating opportunities to explore the use of different types of embodiment in games created by educators and students. Two of the most played embodied games on the WLCP to date include the Tangrams Race (Agbaji, 2019) and the targeted game for this line of research, EstimateIT! (Rountree, 2015).

Tangrams Race is a physically active game where individuals race to complete a tangible puzzle the fastest. The Tangrams Race was modified to have students work in teams of three and compete against other teams to build the puzzle. Each player uses a smartphone, attached to an armband, which delivers instructions about which geometric shapes to retrieve in a team-based relay race. A recent study explored the effects of embodiment on learning by examining differences in student learning after playing the Tangrams race 1) using small, traditional Tangrams pieces of an inch or two in length, 2) using large Tangrams pieces of roughly a foot in length, or 3) by completing a Tangrams puzzle online, without the team-based relay race and physical manipulation of puzzle pieces. There was a pre- to post-test improvement in mathematical understanding for participants who played using the small or large pieces, while the digital condition group did not improve, suggesting that physical activity and interaction with tangible pieces are important for mathematical learning (Agbaji, 2019). The Tangrams Race demonstrates how the WLCP provides opportunities to create and refine games for mathematics learning that productively use movement and interaction with the players’ environment.

Similarly, EstimateIT! was designed by a graduate student at Worcester Polytechnic Institute to reinforce students’ measurement estimation skills by estimating the length, width, height and diameter of 3D objects such as prisms, cubes, and spheres in a team-based geometry scavenger hunt (Rountree, 2015). EstimateIT! is played on mobile devices that deliver instructions and hints to players (Figure 1-4). The game is designed to provide players with hands-on experience with tasks that bridge qualities of physical measurement and measurement estimation. Recently, EstimateIT! has been used in a line of research on computational thinking to introduce users to the WLCP through game play prior to designing novel games (Harrison et al., 2018). Previous research on the effectiveness of EstimateIT! in both the U.S. and the Philippines has shown that playing the game is engaging for elementary-aged students and leads to learning gains, especially when paired with a relevant lecture (Arroyo, Micciollo, Casano, Ottmar, Hulse, & Rodrigo, 2017). This research suggests that EstimateIT! is effective for engagement and learning although further development to the game through an embodied lens could increase benefits of game play such as learning gains with measurement estimation skills.

For instance, in order to strengthen players’ conceptual understanding of measurement and procedural measurement skills, future iterations of EstimateIT! could explore the use of video and images hints that demonstrate effective actions and gestures for measurement estimation for players. As evidence suggests that watching and performing instructed gestures influences learning (i.e. Cook et al., 2008; Goldin-Meadow et al., 2009; Wakefield et al., 2018), EstimateIT! could incorporate action- and gesture-guiding hints that support learning through demonstration and
practice. In order to do so, evidence is first needed to suggest what kind of measurement strategies and techniques should be taught to and encouraged for elementary-aged students to strengthen their understanding of measurement.

1.3 The Study of Gesture in Mathematics Learning

As research on gestures in mathematics learning accumulates, a systematic review of the methodologies to study gesture is needed to work towards developing a common language and approach to typify and quantify gestures which the goal of being able to unify projects and findings in the field. A large body of work has been done in educational research to classify and identify gestures to better understand the role of gestures in learning (Congdon et al., 2018; Goldin-Meadow & Alibali, 2013). Numerous approaches have been developed by researchers that vary in methodology to identify and classify gestures in specific empirical studies (Walkington et al., 2014) as well as authentic learning settings (Alibali & Nathan, 2012) and broader contexts of gesture (Ekman & Friesen, 1969; McNeill, 1992; Krauss et al., 2000; Suppes, Tzeng, & Galguera, 2015).

Ironically, while gesture-speech mismatch (Church & Goldin-Meadow, 1986), or “discordance” (Perry et al., 1988) is a major topic of focus within research on gestures, there seems to be a prevailing gesture-classification mismatch between projects across the field. While the various categories and definitions do broadly match up across researchers and projects, this inconsistency in a classification system leaves the field without a universal language for gesture classification. As such, this leaves remaining holes for categorization definitions (especially across mathematics domains such as algebra to geometry) and creates issues for comparing findings across studies for a holistic understanding of the role that gestures play in mathematics learning. Like previous work on learner’s progression from a transitional state of discordance between gestures and speech during the learning process to matching, correct gesture and speech (Alibali & Goldin-Meadow, 1993), hopefully the field of research on gestures can utilize this compilation of previous systems to reconcile a common language and classification system to use in accordance with one another.

Gesture Typologies

A growing body of research on gestures demonstrates a multitude of classification systems for gestures. Across domains, gestures are most commonly, and broadly, classified as deictic, iconic, metaphoric, and beat gestures. While the terms for each category vary from system to system across researchers and projects, the definitions for each term are largely consistent across projects. That said, the following terms may be considered dimensional classifications rather than categorical because of the possibility for one gesture to embody characteristics of multiple categories (McNeill, 2005).

Deictic gestures (Ekman & Friesen, 1969), or pointing (Alibali & Nathan, 2012), are gestures which identify an entity through the use of a body part, typically a finger or hand. Deictic
gestures are those which indicate objects, people and locations (McNeill, 1992), including pointing or reaching for something (Iverson et al., 2008). Such gestures are considered grounding as they connect speech to objects in the immediate environment (Alibali & Nathan, 2012).

*Iconic* gestures (McNeill, 1992) depict semantic content directly through motion trajectory of the hand(s), such as tracing a shape in the air. These gestures extend to representing an object’s location in space, and/or movement (Goksun et al., 2013), such as moving hands apart to represent “spreading” coins on a table. Ekman and Friesen (1969) originally posed iconic gestures as those comprised of *spatial*, *kinetographic*, and *pictographic* movements. Specifically, spatial gestures were defined as considered movements that depict spatial relations, kinetographic gestures as gestures which indicate action by the individual, and pictographic gestures as those which “draw” referent objects or shapes in the air (Ekman & Friesen, 1969).

*Metaphoric* gestures (McNeill, 1992) are gestures that occur when an individual creates a physical representation of an abstract idea or concept (Andric & Small, 2012). Alibali and Nathan (2012) consider iconic and metaphoric gestures to both be perception and action gestures which are representational of the referent object. Iconic gestures are considered literal representations and metaphoric gestures are considered a subclass of representational gestures that use the body to represent semantic content or conceptual metaphors. For instance, a metaphoric gesture would be representing weighing the pros and cons of an idea by creating a scale with both palms face up and moving inverse to one another.

Lastly, *beat* (McNeill, 1992), or *baton* gestures (Ekman & Friesen, 1969), are rhythmic movements that “time out, accent, or emphasize a particular word or phrase” (Ekman & Friesen, 1969) but do not directly identify or represent any object referred to in speech.

![Figure 1-5](image.png)

*Figure 1-5. A static gesture representing the sphere (left) and a dynamic gesture depicting pushing objects closer together through hands moving towards each other (right).*

**Gesture Typologies: Static and Dynamic Gestures**

Individuals have been found to vary in the quantity and types of gestures they use by cognitive skills; namely, spatial ability and verbal skills, such as phonemic and semantic fluency. For instance, Hostetter and Alibali (2007) found that individuals with low verbal skills but high
Spatial ability were more likely to produce gestures than individuals with the opposite skills (low spatial; high verbal) when describing a cartoon clip. To better understand how cognitive skills relate to gesture production, researchers have looked at not just the types of gestures produced by individuals (as described previously) but also the static versus dynamic nature of individuals’ gestures. For instance, Göksun and colleagues (2013) found that high-spatial ability individuals produced more dynamic gestures than low-spatial ability individuals on a mental rotation task, suggesting that individual differences in cognitive skills do relate to the types of gestures used by individuals. To study the dynamic nature of individuals’ gestures, multiple classification systems have analyzed gestures by typifying the referent object in speech, as well as the gesture as either static or dynamic (Göksun et al., 2013; Walkington et al., 2014).

Learners may reference an object in speech accompanied by static or dynamic gestures (Figure 1-5). Static gestures are considered hand movements that identify an object or a property of that object, such as the shape or location of the object in space. For instance, pointing to a dowel while saying, “I used that dowel…” or creating the outline of a sphere with both hands while saying, “The sphere…” would be examples of static gestures. Dynamic gestures, on the other hand, are those in which include a movement that accompanies dynamic speech for the referent object. For instance, pinching fingers as if holding an object then sliding hands along a plane while saying, “I slid the measuring tool along the prism to measure it” would be a dynamic gesture as the focus of the gesture is on the simulated action.

<table>
<thead>
<tr>
<th>Deictic gestures</th>
<th>Deictic gestures</th>
<th>Deictic gestures</th>
<th>Pointing gestures</th>
<th>Deictic gestures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial movement gestures</td>
<td>Kineticographic gestures</td>
<td>Iconic gestures</td>
<td>Lexical gestures</td>
<td>Iconic gestures</td>
</tr>
<tr>
<td>Pictographic gestures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ideographic gestures</td>
<td>Metaphoric gestures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baton gestures</td>
<td>Beat gestures</td>
<td>Motor gestures</td>
<td>Beats gestures</td>
<td></td>
</tr>
</tbody>
</table>

_Figure 1-5. A select example of shifting gesture classification systems over almost fifty years._
Gesture Typologies: An Overview of Classification Systems

Figure 1-6, above, modifies an older visualization of gesture classifications (Goldin-Meadow, 2003) to provide a limited example of overlapping, yet different, gesture classification systems that have been used over the past five decades, namely varying in the precision or breadth of gesture categories. In Figure 1-6, gesture categories along a horizontal plane are similar, if not synonymous, with one another, such as deictic and pointing gestures or spatial movement and iconic-deictic gestures.

While some approaches to coding and studying gestures are specific to a given project or mathematics concepts (i.e. Walkington et al., 2014), many researchers use or modify the classification systems posed first by Ekman and Friesen (1969) and later McNeill (1992). Ekman and Friesen’s (1969) classification system for gestures remains the most fine-grained approach with six distinct categories for gesture. McNeill’s (1992) typology of gestures (pointing or deictic, iconic, and metaphoric) one of the cornerstone pieces of work on gestures, has since been modified by other researchers, such as by combining iconic and metaphoric gestures under the umbrella of lexical (Krauss et al., 2000) or representational gestures that represent a concept through literal or figurative actions (Nathan & Alibali, 2012). Despite broadly thinking of gestures in the same vein, individual projects and researchers have modified existing gesture classification systems to fit the needs of their current work.

Figure 1-7. Benefits of fine and coarse-grained classification systems.

1.4 Discussion

Research on gestures, particularly regarding mathematics learning, benefits from applying uniquely crafted classification systems and coding protocol for different projects, although the benefits vary by the granularity of the classification system. By modifying existing classification systems for the project at hand, research teams are likely to be more economic and precise with their data. For instance, Alibali and Nathan (2012) modified previous classification systems to
exclude the category of beat gestures, presumably because beat gestures emphasize speech without providing any additional contextual information or having been linked to implicit knowledge in other work. By doing so, this classification system eliminates time spent on identifying and analyzing a gesture type that not relevant to their work. Similarly, by grouping iconic and metaphoric gestures into one category of representational gestures for a total of just three gesture types, Alibali and Nathan (2012) mitigate risks of coding error and likely benefit from quicker judgments about gesture type when coding participants’ gestures. McNeill (1992) and Krauss and colleagues (2000) also exemplify the same affordances to their own classification systems by grouping gestures into the umbrella terms of iconic gestures and lexical gestures. On the other hand, Suppes et al. (2015) implemented a more granular system with five gesture types, similar to those proposed by Ekman and Friesen (1969). These classification systems benefit from the precision of identifying more gesture types, providing the opportunity to compare frequencies and relations between more gesture types, such as iconic-deictic (Suppes et al., 2015) and pictographic gestures (Ekman & Friesen, 1969).

While each classification method has its merits and was appropriate for the context in which it has been presented, there is yet to be a standard classification system for gesture types across research studies or an unifying approach to systematically code gestures through video or real-time analysis. The lack of a common classification system creates a demand to modify or construct unique coding guides and classification systems specific to individual gesture studies even though doing so creates obstacles to compare findings across studies for better generalizations about findings on gestures (Göksun et al., 2013). The theory, context, and focal points of research studies on gestures in mathematics learning vary which makes it difficult to pinpoint common definitions and methodologies for gesture typification. However, such studies would benefit from a common language and classification system for the time saved on coding procedure as well as the ability to more easily compare findings across studies. With time, perhaps the refinement of a common classification system would also lend itself to drawing clearer distinctions between categories to diminish the dimensional nature of some overlapping gesture types. Despite affordances provided by adapting existing classification systems for specific research projects, the field of research on gestures in mathematics learning would benefit from foundational attempts to create a common classification system that could be expanded and refined over time for different contexts and purposes.

1.5 Contributions and Conclusion

This chapter aimed to advance research on gestures in mathematics learning by making two contributions to this area of research. First, I provided a novel definition of gesture based on existing literature. The proposed definition expands upon previous descriptions to include gestures that involve the use of tangible objects. Second, this chapter provided a brief overview of the role of gestures in mathematics learning and a review of overlapping methodologies that have been used to classify gestures over five decades of research. These reviews were in no way inclusive of
all major contributions to research on gestures in mathematics learning but intended to 1) provide
eexamples of the work that has been done in this area and 2) highlight the call to combine previous
classification methodologies into a common system that would facilitate research on learners’
gestures across projects in this area.
Chapter Two: The Development of a Novel Coding Guide for Behavior of Learners

Given the large body of evidence suggesting that gestures play a strong role in mathematics learning, as well as the continued shift in classifying gestures, I currently aim to develop a theoretically-driven classification system to typify and quantify mathematics learners’ gestures in rich detail. The previous section outlined fifty years of research on gestures and calls for a common way to study gesture moving forward, considering the substantial work on gestures to date, as well as the lack of research connecting students’ actions with their language and gestures to gain insight about student knowledge. The proposed coding guide aims to build off established classification systems to strike a balance between specificity towards this project with room for generalizing this classification system to other projects on physical behavior, verbal behavior, and gestures.

The Embodied Cognitive Task Analysis (ECTA) Coding Guide, presented in Chapter Three, expands the data measured by previous efforts by incorporating items to identify and classify 1) learners’ physical strategies and actions taken during problem-solving, and 2) whether learners’ gestures and language used during answer justifications are congruent with observed actions during problem-solving. Whereas learning through actions alone (manipulating objects) rather than gesturing could result in shallow learning that does not generalize to other contexts (McNeill & Uttal, 2009; Novack & Goldin-Meadow, 2015), this coding guide allows researchers to examine the potential relationship between actions during problem-solving as well as the gestures that accompany language afterwards as students explain their problem-solving process.

To the best of my knowledge, prior research has examined math learners’ problem-solving language and gestures during and after problem-solving but has yet to explore the three-way interplay between actions during problem-solving, and language and gestures observed during explanations after problem solving. Previous research on the coupling between speech and actions in communication has shown a stronger relationship between speech and gesture than between speech and action in the context of language production and language comprehension (Church, Kelly & Holcombe, 2014; Kelly, Healy, Özyurek, & Holler, 2015). However, this line of inquiry has not been extended to mathematics to explore the relationship between action, speech, and gesture as they relate to learning. This coding guide incorporates tools to measure learners’ actions during problem-solving to compare those behaviors to learners’ language and gestures during answer justifications. This will allow us to explore whether the relations between actions and speech and/or gestures is also indicative of readiness-to-learn or another underlying cognitive processes related to mathematics. As such, this coding guide could advance educational research by providing a tool to gain insights about the roles of, and relations between, action, language, and gestures in math learning that could be applicable across research projects on gestures in learning.

In addition to expanding previous efforts to classify the actions and gestures of learners, the ECTA Coding Guide aims to unify previous efforts in gesture classification to offer a refined coding guide that will be appropriate to use with the studies in the following chapter with the potential for other researchers to expand on these codes and definitions for use with conducting video analysis of learner’s gestures during measurement tasks. More broadly, the current coding...
guide intends to integrate definitions of gesture classification to provide a gesture classification system that could be relevant across projects on gestures in learning across mathematics domains, working towards the much larger goal of constructing a common coding tool for research on gestures in learning.

Table 2-1. A list of the coding items in the ECTA Coding Guide.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to Answer</td>
<td>Overall Verbal Strategy</td>
<td>Overall Gesture Strategy</td>
</tr>
<tr>
<td>Dowel Use</td>
<td>Change Answer</td>
<td>Total Gestures</td>
</tr>
<tr>
<td>External Tool Use</td>
<td>Correct Reasoning</td>
<td>Verbal-Action Match</td>
</tr>
<tr>
<td>Autonomous Tool Use</td>
<td>Precise Language</td>
<td>Gesture-Action Match</td>
</tr>
<tr>
<td>Number of Tools Used</td>
<td>Dynamic Speech</td>
<td>Verbal-Gesture Match</td>
</tr>
<tr>
<td>Description of Tools Used</td>
<td></td>
<td>Gesture-Verbal-Action Match</td>
</tr>
<tr>
<td>Multiple References</td>
<td></td>
<td>Gestures in Speech:</td>
</tr>
<tr>
<td>Placeholder</td>
<td></td>
<td>1. Speech</td>
</tr>
<tr>
<td>Start Point Marker</td>
<td></td>
<td>2. Gesture Description</td>
</tr>
<tr>
<td>End Point Marker</td>
<td></td>
<td>3. Gesture Type</td>
</tr>
<tr>
<td>Perspective</td>
<td></td>
<td>4. Referent Object</td>
</tr>
<tr>
<td>Proximity</td>
<td></td>
<td>5. Speech Classification</td>
</tr>
<tr>
<td>Double-Check</td>
<td></td>
<td>6. Gesture Classification</td>
</tr>
<tr>
<td>Problem Decomposition</td>
<td></td>
<td>7. Object Held</td>
</tr>
<tr>
<td>Overall Action Strategy</td>
<td></td>
<td>8. Dowel Use</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9. Speech Match</td>
</tr>
</tbody>
</table>

2.1 Construction of the Embodied Cognitive Task Analysis Coding Guide

The Embodied Cognitive Task Analysis (ECTA) Coding Guide, outlined in the following section, was developed through the compilation of previous methodologies in related research on gestures and speech, as well as novel efforts to identify the physical strategies and actions demonstrated by participants during problem-solving. The third section of the coding guide was originally drafted by compiling a classification system based on previous methods to study learners’ gestures and then iteratively developed and edited to bridge classification definitions with the behaviors demonstrated in our particular research project. To do so, the coding guide effectively partitions the coding scheme into three sections: actions, language, and gestures (Table 2-1; Figure 2-1).
Part One: Problem-Solving Strategy

The first section, “Part 1: Problem-Solving Strategy” is comprised of 15 coding items that capture the physical behavior of participants for the duration of problem-solving. That is, from the time that the researcher states the problem until the subject delivers his/her final answer. This section is relatively novel in that previous work on gestures in learning has focused on gestures during problem solving (Chu & Kita, 2009) and the congruence between language and gestures after problem-solving (Alibali & Goldin-Meadow, 1993; Church & Goldin-Meadow, 1986) but not the physical behaviors, potentially aided by tools, indicating strategy during problem-solving.

The majority of items in the first section aim to identify the general characteristics of physical behavior (i.e. identifying any tool use) as well as components of successful measuring strategy (i.e. marking a clear start- and end-point for measurement). The last item, “Overall Action Strategy” categorizes the participant’s approach to problem-solving solely through the physical actions observed to infer the type of mathematical reasoning used by the participant. For instance, a participant observed subsequently moving a reference tool along the edge of the object being measured would be coded as demonstrating a part-part-whole strategy that involves using the reference tool as parts to make up the whole of the object. This section was designed to capture the actions surrounding students’ problem-solving in geometry tasks and allow the researcher to compare the behavior observed during problem-solving to the behavior observed after problem-solving while the participant provides an answer justification, captured in the following sections of the coding guide. Please note that language during problem-solving is not included for coding in this version of the coding guide because the study which prompted the development of the ECTA Coding Guide did not prompt participants to think aloud. However, the items for the following section, “Part 2” could be used to capture language during problem-solving as well.

Part Two: Problem-Solving Explanation

The second section, “Part 2: Problem-Solving Explanation” focuses on the participant’s language during answer justification as an indication of his/her mathematical understandings. These six coding items namely attempt to capture the problem-solving strategy stated by the
participant as well as the accuracy and precision of the language used for the answer justification.
For instance, verbal problem-solving strategies include whether the participant *eyeballed* the object of measurement, identified a point of reference to make an *estimation*, or used either the measuring tool or object dimension to describe a ratio of the relationship between the measuring tool and object.

Prior methodologies have included coding learners’ verbal mathematical reasoning/procedure as sound or not, usually to analyze gesture accordance/discordance (Perry et al., 1988; Alibali & Goldin-Meadow, 1993), this coding guide expands on that to capture whether the explanation has mathematically correct reasoning as well as whether the language used to articulate the reasoning is precise or vague. Separating “Correct Reasoning” into two questions (“Correct Reasoning” and “Precise Language”) provides the opportunity to examine congruence between mathematical reasoning skills and language precision. The last item, “Dynamic Speech,” builds on prior coding efforts to analyze whether explanations solely include static references to objects, such as location, or dynamic references to the movement of objects during problem-solving (Göksun et al., 2013). This affords the opportunity to examine how performance on measurement tasks corresponds with gestures and language use.

This section can be coded using participant transcriptions stored separately from video data. Using participant transcriptions will mitigate potential bias from coding speech while viewing associated gestures as captured in the collected videos. The ECTA Coding Guide details the use of NVivo software as a means to separate and organize cases for analysis as well as record transcriptions that could be used independently to code the items in this section of the coding guide.

<table>
<thead>
<tr>
<th>Transcription</th>
<th>Gesture</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Researcher:</em> Estimate the diameter of the sphere. <em>Participant:</em> 8 inches. <em>Researcher:</em> How did you get that? <em>Participant:</em> Just using the lines on the basketball, I tried to line it up as accurately as possible <em>[kinda using your hands as a parallel]</em> and whatever was left was the remainder and I know my thumb is about an inch so I used that.</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 2-2. Example of problem-solving explanation transcription and one recorded gesture by a participant.*

**Part Three: Problem-Solving Explanation Gestures**

The third section, “Part 3: Problem-Solving Explanation Gestures” is also to be used during the answer justification portion of a problem. The first five items aim to capture the overall use of gestures in the explanation as well as whether the gestures match 1) the accompanying speech and 2) the physical actions observed during problem-solving. Prior work has coded gesture-speech match/mismatch (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, 2003; Goldin-Meadow &
Singer, 2003) and the current coding guide extends this work to also capture whether subjects are consistent with their physical actions to solve problems and the gestures used to later convey those actions—capturing the difference between actions while thinking and gestures used while explaining and acting almost as an instructor. Analyzing students’ actions during problem-solving will provide data to analyze whether consistency between actions used during problem solving and gestures exhibited during answer justification are related to performance in measurement tasks. The final item, “Gestures in Speech” captures the majority of information on each, individual gesture observed. This item was primarily shaped by the work of Göksun and colleagues (2013), who created a table to capture congruence between language and gesture in speech. Similarly, this item breaks each gesture into nine sub-items (refer to Table 2-1, above) to code that utilize participant speech transcriptions to link language with each unique gesture (Figure 2-2), describe and identify the type of gesture exhibited, whether gesture and speech match, whether the participant’s referral to the referent object associated with a gesture, along with the gesture itself, is static or dynamic (Göksun et al., 2013), and whether objects or measuring tools were used during each gesture and in what capacity.

Table 2-2 defines each gesture type included in the coding guide as well as the source from which the definition most closely aligns. While multiple researchers have used a common body of terms, such as “iconic,” to classify gestures, the definitions of gesture types have varied slightly under the same name. In an effort to unify similar definitions, the ECTA coding guide attempted to combine definitions for each gesture category. After careful consideration of existing classification categories and definitions that have been established, the categories and definitions included in the ECTA Coding Guide are considered to be largely mutually exclusive and collectively exhaustive definitions for gestures likely to be observed during mathematics, specifically geometry and measurement, tasks. Note that beat gestures were not included for analysis within this coding guide, consistent with prior classification systems (Alibali & Nathan, 2012) since the rhythmic movements are not connected to any mathematical concepts.

<table>
<thead>
<tr>
<th>Gesture Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iconic: Spatial</td>
<td>Gestures that depict spatial relations (Ekman &amp; Friesen, 1969)</td>
</tr>
<tr>
<td>Iconic: Kinetographic</td>
<td>Gestures that indicate bodily actions (Ekman &amp; Friesen, 1969), usually corresponding with dynamic speech such as “move”, “flip”, or “balance”</td>
</tr>
<tr>
<td>Iconic: Pictographic</td>
<td>Gestures used to draw a referent object in the air (Ekman &amp; Friesen, 1969)</td>
</tr>
</tbody>
</table>
Deictic Gestures that indicate objects, people and locations (McNeill, 1992), including pointing, showing an object, or reaching for something (Iverson et al., 2008) and extending to picking up an object as well as identifying a trait of an object

Metaphoric Gestures that occur when an individual creates a physical representation of an abstract idea or concept (Andric & Small, 2012), including dimensions such as length, width, or height

Unsure/Out of View Gestures that are present in the video but largely out of view

Lastly of note, the first item of the third section, “Overall Gesture Strategy” questions whether a given participant used gestures during the explanation or if he/she largely used a measurement tool to reenact his/her problem-solving strategy. This item affords the ability to identify movements that accompany speech under the sixth item, “Gestures in Speech”, even if those movements utilize a tool rather than solely being hand gestures. To the best of my knowledge, while this approach has yet to be taken under research on gestures in learning, this approach affords participants more flexibility with action and movement during the problem-solving explanation portion of a task if he/she is able to use or ignore the available resources for problem-solving.

2.2 Affordances and Challenges of the ECTA Coding Guide

This coding guide was developed with the purpose of unifying previous efforts in this line of research to provide a richer compilation of data on the actions, language, and gestures exhibited by students during problem-solving, specifically during measurement estimation tasks. In addition to attempting to incorporate items of interest across established coding systems for gestures, the ECTA coding guide presents a few unique affordances. Namely, the coding guide provides items to 1) compare between actions during problem-solving to behaviors observed after problem-solving, 2) delve deeper into gesture-speech congruence by also examining speech precision, and 3) incorporating the identification of gestures that involve the use of tangible objects. These affordances are all situated in a coding guide designed for analyzing behavior during measurement estimation tasks because, to my knowledge, learner behavior has not been studied in the context of measurement estimation tasks.

To capture as much physical behavior as possible, the ECTA Coding Guide specifies to code the physical movements that accompany speech either solely by gesture or when participants demonstrate an action with an object. This sustains the organic quality of participants’ behavior during problem-solving explanations by not specifying whether participants should or should not use any objects to recreate their problem-solving process. As stated in section 1.1 on defining gesture, I believe that gestures produced with and without objects can be categorized under Melcer
and Isbister’s (2016) framework. This coding guides affords the opportunity to distinguish between such gestures and allows for greater flexibility in study procedures by allowing participants the freedom to use surrounding objects authentically.

While this coding guide has been appropriate for the studies presented in the following chapter, challenges have become apparent. Because the ECTA Coding Guide attempts to provide rich detail about individuals’ behavior with an extensive set of items, achieving interrater reliability for coders can be difficult to achieve. To that end, although this system will provide more extensive detail about types of gestures demonstrated by individuals than classification systems that group these categories into larger, umbrella categories, the five classifications of gestures can be difficult to discriminate in observation. As McNeill (2005) noted about his own classification system, the definitions for gesture types proposed in this coding guide are more dimensional than mutually exclusive categories. The proposed gesture types, while thorough in categories, leave room for a gesture to include some overlapping characteristics between categories. Lastly, in its current state, the coding guide is in no way universally applicable to all mathematics research on gestures without modification. Rather, this is meant to be a foundation for future projects to expand upon as needed within the given items and category definitions.

Although this coding guide is a refinement of previous gesture coding methods, and an appropriate tool for the research project in the following chapter, this coding guide could be expanded to measure learners’ actions and gestures in even more detail and in turn, provide deeper insights about learning. For instance, to better understand students’ problem-solving strategies, this coding guide could be expanded to define each component of a problem-solving strategy with the option to mark which, if any or all, components of a strategy that a student exhibits during problem-solving and later, through language and gesture, during his/her problem-solving explanation. Expanding the coding guide to encompass strategy components would allow researchers to observe 1) which problem-solving strategies students are employing and describing, 2) quantify how well students are able to execute a given strategy, and 3) identify strategy components to target for student support through scaffolding in related games and activities. The ECTA coding guide is meant to be modified and expanded for such purposes to help unify research on gestures through a common language derived from decades of research before this guide.

2.3 Contributions and Conclusion

In an attempt to reconcile differing approaches and language used across research on gestures, this chapter introduced the Embodied Cognitive Task Analysis Coding Guide. The coding guide expands upon previously established classification systems and uniquely captures actions observed by learners during problem-solving as well as the language and gestures exhibited by learners during answer justifications in a mathematics context. While this is a first attempt at proposing a unifying classification system and is still structured around measurement tasks in particular, the coding guide can serve as a foundation for a common classification system of gestures that could be expanded and applied across gestures in other domains for coding
purposes. Overall, this chapter aimed to advance research on gestures, particularly in mathematics learning, by proposing a tool to quantify and classify gestures that could be applied broadly to any video analyses of gesture.
Chapter Three: The Embodied Cognition Task Analysis Coding Guide

This chapter is the official copy of the Embodied Cognition Task Analysis Coding Guide. The coding guide was developed by me, Richard Valente, Luisa Perez, Hannah Smith, and the Embodied Cognition Research Group at Worcester Polytechnic Institute. Only the table of contents, references, and appendices have been removed from the original document as the information is still available within this thesis.

This coding scheme is designed to provide rich, quantitative data about the actions, language, and gestures observed by students as they 1) complete estimation tasks with geometric objects and then 2) explain the strategy they used to complete each task. This builds off previous work on gesture analysis by Ekman and Friesen (1969), McNeil (1992), Alibali & Nathan (2012), and Goldin-Meadow, Cook and Mitchell (2009), among others, with the intent to capture gesture type, action/speech/gesture congruence, and the mathematical accuracy of speech/gesture through video footage.

This coding scheme should enable researchers to transform data into quantifiable data to answer multiple research questions, including but not limited to:

1) During the problem-solving phase:
   a) What kind of problem-solving strategies do students exhibit through their actions to approximate 2D dimensions of physical, geometric 3D objects?
   b) Do these strategies vary by performance?

2) During the explanation phase:
   a) What kinds of verbal problem-solving explanations do students exhibit?
   b) What kinds of gestures do students exhibit?
   c) Do gestures during problem-solving explanations map onto the verbal explanations?
   d) How do explanations, gestures, and congruence vary by performance?

3) Across the problem-solving and explanation phase:
   a) How congruent are student actions, language, and gestures across tasks?
      i) Do students provide verbal explanations that match their actions during problem-solving?
      ii) Do students exhibit gestures during the explanation that largely match their actions during problem-solving?
      iii) Are problem-solving strategies, language and/or gestures correlated with performance on estimation tasks?
   b) Does action, language, and gesture congruence vary by performance?
About the College Study

Understanding a) the role of embodiment in mathematics learning, and b) the opportunities to capitalize on embodied learning through action-based games and activities with emerging technologies pushes us to explore the deeper features of embodied cognition in mathematics. Our goal is to better realize classroom instruction and activities for mathematics students by discovering how fine-grained gestures and actions influence learning during problem-solving activities. Our first study is a pilot study to collect information on the fine motor actions utilized by college students, presumably K-12 mathematics “experts,” before analyzing physical behavior during problem solving among elementary students to identify age-based differences in conceptual and procedural knowledge of geometry concepts.

In this study, we recruited 44 undergraduate students from Worcester Polytechnic Institute using the online platform, SONA, to participate in a series of tasks challenging students to estimate the measurements of geometric objects. Students were offered an unmarked 6-inch or 12-inch dowel to use as a tool for estimating the measurement of geometric objects such as prisms, spheres and cylinders, of various sizes. Students were then asked to complete 18 tasks such as 1) choosing an object (of three) that has sides of a certain length and 2) estimating the length of a given object. Video data was obtained for 27 of the 44 participants.

Materials

The main materials for this study included a brief questionnaire and Helpful Hints worksheet, 12 objects of varying geometric shapes and sizes, two wooden dowels, and an answer sheet to mark participant responses. Additionally, a video camera was used to record the actions of participants during each task as well as the language and gestures observed afterwards, while participants explained their problem-solving strategy for each task. The questionnaire contains 12 items related to background information and interest in mathematics. The Helpful Hints worksheet was used as a distractor task between estimation tasks, asking participants to create a helpful hint for a student who might be struggling with completing the previous task. The 12 geometric objects include a variety of spheres, rectangular prisms, cylinders, and cubes that range from having 2-inch side lengths to 24-inch side lengths. The wooden dowels are 6 inches and 12 inches, respectively. And the answer sheet was a printed form for the researcher to mark verbal participant responses during the study.

Measures

Participant responses from the questionnaire, verbal responses to each task, and videotaped actions, speech, and gestures will be used to answer our aforementioned research questions. The questionnaire contains background information as well as attitude constructs based on a 7-point Likert scale adapted from previously accepted measures (Eccles, Wigfield, Harold, & Blumenfeld, 1993; Arroyo, Shanabrook, Burleson, & Woolf, 2012) to identify student feelings regarding mathematics and self-concept. The verbal responses to each task were recorded by the researcher during the study, including discrete responses from multiple choice questions as well
as estimations that can be measured on a continuous scale. And we will use this coding scheme to transform video data into variables for analysis.

**Design and Procedure**

The research design of this study is experimental. Participants were randomly assigned to one of two dowel conditions, the 6-inch condition or the 12-inch condition. Additionally, participants were randomly assigned to one of two predetermined task orders.

Participants were run individually in 30-minute time slots. Upon entering the lab, participants reviewed and signed the informed consent form then filled out the brief questionnaire. After the questionnaire was completed, we began videotaping. The participant was offered, though not required, to use an unmarked dowel that is 6 inches or 12 inches for the 16 estimation tasks.

There were two types of tasks, multiple choice and estimating in inches. For example, participants will be asked to choose which of three objects meets a certain size requirement and to estimate the length of an object. Some tasks were two-fold, asking participants to select the correct object from three options and then giving an estimate regarding the chosen shape. Note, participants did not receive any feedback. In the case of two-fold tasks, if participants initially selected the wrong object in the first part of the task, they were directed to use the intended object for the second part of the task without accuracy feedback. Between tasks, the participant was asked to face the other direction and write a “helpful hint” for another student who might struggle with the task just completed. While the participant wrote a hint, the next set of objects was arranged for the following estimation task. After completing every task and finishing the “helpful hints” worksheet, we recorded the participant’s height and length of dominant hand before debriefing the participant.
Instructions for Coders

Training Coders

To begin the process of training coders for this guide, prospective coders should follow the instructions on the following page to download, access, and learn to use NVivo in the context of this dataset and coding goals. Please seek help and direction from team members as needed. After feeling confident in your ability to use NVivo with fluency for the purpose of this project, please return to this section.

Participant videos will be coded task by task, largely as indicated by the NVivo software. A “task” here refers to one task performed by one participant. (i.e. Participant 5, Task A). A task is defined as the video segment from the beginning of a problem-solving segment (when the researcher states the problem) to the end (when the researcher dismisses the participant to fill out a Helpful Hint at the back of the room).

The initial coders for this project should choose five tasks to work through as a group. Together, coders should review instructions for coding video segments, view a task video, then work through each coding item. For each coding item, coders should discuss which response is appropriate, why other responses are not appropriate, and come to a consensus. While doing so, complete a Google Form to record responses. Only one form should be filled out per task.

Note: If a consensus cannot be reached for a particular item, or if there is debate over the wording of a coding item, note these points of controversy in a Google Doc under the CAREER Studies folder to discuss in the following lab meeting and continue with other items for that task. Then, wait to code other tasks until the coding guide has received further refinement. Repeat as needed while working through the five selected tasks.

After collaboratively coding responses for five tasks, the coders should decide whether 1) the coding guide needs further editing, 2) the coders need more collaborative practice, or 3) the coders agree that they are largely in agreement on most coding items and feel ready to continue independently.

1) If the coding guide needs further refinement, discuss changes in the next lab meeting then select three more tasks to work through as a group and repeat the procedure above.

2) If the coders are not in agreement on items most of the time, select three more tasks to work through as a group and repeat the procedure above.

3) If the coders are largely in agreement on items and feel confident moving forward, proceed to the following instructions.

Next, prospective coders should choose five tasks (different from the tasks used in the previous training section but the same tasks collectively) for each coder to work through independently.
Coders should individually watch the videos for the selected tasks and mark responses to each item on the paper version of the answer sheet. Then, prospective coders should meet to:

1. Calculate and record, in a dated document in the CAREER Studies Google Drive folder, percent agreement for each task as well as the average percent agreement across all five tasks.
   a. **Percent agreement** is calculated by comparing (previously, independently recorded) responses to each item. Each item marked with the same response by all coders receives 1 point. Tally the total number of items agreed upon and divide that number by the total numbers of items coded. If there is a discrepancy in total due to one coder identifying more gestures than the other, divide the items agreed upon by the highest number of items coded. Items related to timing can be marked as agreed upon within 3 seconds of difference.
   b. To calculate the **average percent agreement**, sum each percent agreement for the five tasks then divide that sum by five.

2. Discuss each coding item for each of the five tasks, filling out one Google Form per task as responses are agreed upon.

3. For two master coders, if the average percent agreement is below 80% for the five tasks, repeat this section of training with five new tasks. If the average percent agreement is 80% or higher, proceed to split up videos/tasks to be coded and continue to follow instructions for “How to Code a Video”, below. For three or four master coders, 75% of coders should be in agreement at least 80% of the time before moving forward.

Once two or more coders have reached this point and are independently coding videos, **additional prospective coders** should meet with coders for training as they join the project. Coders should review the coding guide, NVivo, and accessing data with prospective coders, code two videos together, then select five tasks (already coded in Google Forms) for the prospective coder to work through independently, uploading responses on paper as well as the Google Form. In the following meeting, the coder should review and document percent agreement of the prospective coder’s responses to the master coding responses in Google Forms then discuss each item as needed. If the average percent agreement is below 80% for the five tasks, repeat this section of training with five new tasks. If the average percent agreement is 80% or higher, the coder should proceed to assign videos/tasks for the new coder to work through independently by following instructions for “How to Code a Video”, below.

**Materials Needed for Coding**

To code videos and record data as accurately as possible, coders will need:

1. The coding guide: An online or paper version of this coding guide to reference
2. **Answer sheet(s):** A printed version of the answer sheet found in the Appendix
3. Pen
4. Internet, NVivo, and Google Form access
How to Code a Video

1. To beginning coding, access the data, including videos and transcription.

2. Videos will be coded task by task. The beginning of a task is when the researcher states the problem and the end is when the researcher dismisses the participant to fill out a Helpful Hint at the back of the room.

3. Begin to code responses on the printed version of the answer sheet. First, play the problem-solving video segment and code responses to the items in Part 1, below. Pause and replay the video as needed to respond to items as accurately as possible. Do not watch the video for Part 2, only use the transcription recorded through NVivo to code items in Part 2 before watching the problem-solving explanation portion of the video. Then, play the rest of the video, the problem-solving explanation portion, to code responses for Part 3, following the instructions stated at the beginning of each section.

4. Double check responses to ensure that all items are filled in as needed.

5. If in training, follow the protocol described in the earlier section. If you are independently coding this task or participant, use responses from the paper version of the answer sheet to enter responses into the Google Form.

6. Store the paper version of the answer sheet in an agreed upon location.
NVivo Software

Download NVivo

Download NVivo 12 here: http://www.qsrinternational.com/nvivo/support-overview/downloads
**Must be Windows version**
Activate NVivo with the license key

User Resources

Visit www.qsrinternational.com/nvivo-training to view free, available courses to get started with NVivo.

Visit www.qsrinternational.com/learning to access other free resources for help with NVivo software.

Linked Materials

Source Data: The NVivo file has many ways of grouping and viewing data. The source data for all of these views can be found in the Data tab on the right. Clicking it will lead it to expand and then clicking on Files will allow users to view the CSV Data, transcription text, as well as the videos themselves that have the transcription data synced with the video timeline for each Task.

Data Organization: Nodes and Cases

Although it may be useful to view the source data, one of the most powerful uses for NVivo is being able to organize the data into Nodes and Cases.
**Nodes:** A node allows for data to be grouped together for ease of access, in the screenshot below the Task nodes are used to compile the transcription data for each participant into a file that contains all of the participant responses for a task, *(Task A for example)*, allowing for word searches or graph generation to be done on the node. Nodes can also be nested similarly to a folder in a file system for further organization.

**Cases:** Cases differ from nodes in the way that they are typically used to represent literal entities, in our case the participants. Each case allows for all of the data relating to a Participant to be aggregated and connected to the case. In the screenshot below, the participant transcriptions as well as their written helpful hints can be viewed in one area. The participant CSV data can also be seen by right clicking the participant and clicking “Open Classification Sheet” the CSV data is connected to the participant case as attributes.
Video Access

Navigate to the Data/Files/Videos folder within NVivo. Then double click on the video you would like to transcribe.

Issues Loading: (If the video gets stuck like this triple click on the loading icon and it will start)

In this view you will be able to view the video as well as the transcriptions for each task.

Scroll Transcriptions: Click the synchronized play mode depicted above in order for the transcription text to scroll with the video.

Play Specific Task: If you want to play a specific task click on the task on the gray sidebar to select it in blue then click “Play Selected Rows”. If it doesn’t immediately start at that point press the play button and it should work.
Part 1: Problem-Solving Strategy
(Actions observed during problem-solving)

This portion of the coding scheme pertains to the segments of participant video, for each estimation task, between 1) the time that the researcher states the estimation task or question and 2) the participant provides a final, verbal response to the prompt. This portion of the coding scheme is to be used with the observable actions that participants exhibit during problem-solving.

Participants were allowed to change their answer during each task and some participants did so after being prompted to justify their response. We recorded the final answer provided. To account for this, code actions observed during problem-solving up until participants provide a justification for their final answer.

Below are the detailed questions and explanations of coding choices for each item. Please use this guide as you watch the videos and code each item in the Google form.

Coder: __________
Participant #: ______
Task (letter): ______
Task category:
- Comparison task (2-3 objects)
- Diameter (of cylinder or sphere)
- LWH (length, width, height)

Video Segment (time): _____ to ______

1. Time to Answer: How many seconds is the problem-solving video segment? Or if the video segment on NVivo does not match up perfectly, how many seconds are there from the end of the researcher stating the task to when the participant delivers an answer prior to giving their explanation? If the participant asks for clarification, start recording the time to answer after this clarification has been made, unless the participant has already started the problem-solving process.

2. Dowel: Does the participant use the dowel at any point during the problem-solving process?
   -1. Unsure/Out of view 0. No 1. Yes

3. External Tool: Does the participant use another external tool at any point during the problem-solving process? (i.e. pen, paper, another object)
   -1. Unsure/Out of view 0. No 1. Yes
4. **Autonomous Tool:** Does the participant use any body part as a reference tool for measurement? (i.e. finger, thumb, hand, arm, torso) Note: This does not include using body parts as markers.
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

5. **Tool Specification:** If you answered yes to any of the previous three questions, please specify the number of tools and give a one-word description of each tool (i.e. dowel, thumb, etc.). If you answered no or unsure, please move on to the next item.
   - a. Number of external tools used: ______________________
   - b. One-word description of each tool: ______________________

6. **Multiple References:** Does the participant use two or more tools for multiple references?
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

7. **Placeholder:** Does the participant use a placeholder (i.e. finger, thumb, dowel, etc.) to mark a specific point during subsequent measuring? For example, if the participant lays the dowel along the length of a cylinder with excess length along the cylinder, does she use her finger to mark the position of the end of the dowel on the cylinder before moving the dowel to continue measuring?
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

8. **Start Point Marker:** Does the participant mark the designated start point of measurement? This is recognizable by the edge of the object lining up with the end of the reference tool (such as the end of the dowel and the bottom of the object resting on the table to measure upwards for height). The start point marker may be observed prior to the end point marker.
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

9. **End Point Marker:** Does the participant mark the designated end point of measurement? A start point marker is usually observed to identify an end point. This is recognizable by the participant clearly marking a point opposite the start point marker. (i.e. if the participant marks a spot on a dowel that corresponds with the opposite edge of the object that he/she is measuring)
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

10. **Perspective:** From what angle does the participant appear to examine the object? Specifically, how does the participant’s eye-level compare to the object(s) in question?
    - 1. Unsure/Out of view
    - 0. **Eye-level:** participant appears to be at an even level with the object
    - 1. **High view:** above the object at an angle
    - 2. **Bird’s eye:** directly above object, leaning over the table

11. **Proximity:** How close to the object(s) does the participant appear to be while measuring/examining?
    - 1. Unsure/Out of view
    - 0. **Near:** Within roughly 1 foot of the object
1. **Moderate**: Roughly 1-2 feet away from the object

2. **Far**: Roughly more than 2 feet away from the object

12. **Double-Check**: Does the participant display at least two different strategies, or the same strategy more than once, before providing their answer? (Do they appear to double-check their work?)

   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

13. **Problem Decomposition**: Does the participant appear to break the problem into 2 or more separate tasks? This is recognizable by observing the participant measure two or three dimensions/segments separately before delivering a composite answer or using process of elimination to complete the tasks involving the selection of one object out of three.

   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

14. **Overall Action Strategy**: Please mark the primary physical strategy observed during problem-solving.

   - 1. Unsure/Out of View
   i. Actions used during problem-solving were obscured from the camera by the objects or participant, preventing the coder from making a confident observation; or, the strategy does not match one of the categories below.

   0. **Eyeballing**
   ii. This strategy involves zero use of tools. The participant may touch the object(s) in question but without distinctly measuring any dimensions or making any observed comparisons to other objects.

1. **Part-Part-Whole (Whole=Dowel or Tool)**
   iii. This strategy should be used primarily to estimate dimensions shorter than the length of the dowel or reference tool(s) used. This strategy is recognizable by the participant subsequently measuring the object along the length of the dowel or reference tool.

2. **Part-Part-Whole (Whole=Object(s) to be measured)**
   iv. This strategy should be used primarily to estimate dimensions longer than the dowel length or reference tool(s) used. This strategy is recognizable by the participant subsequently measuring the length of the dowel or reference tool along the specified dimension of the object.

3. **Part-Part-Whole: 2+ References**
   v. This strategy deploys the use of two or more references (dowel, hand, other object, etc.) and is seen through two or more displays of estimating the object(s).

4. **Memorized or External Object Reference**
   vi. This strategy involves the participant using a memorized reference point such as a thumb (~1”) or dowel (6” or 12”). This is recognizable by the participant holding one reference tool in close proximity to the object (without subsequent measuring) before drawing a conclusion.
Part 2: Problem-Solving Explanation
(Language observed during explanation of problem-solving strategy)

This portion of the coding scheme pertains to the segments of participant video, for each estimation task, between 1) the time that the researcher asks the participant to explain why/how he/she arrived at the answer given and 2) the researcher directs the participant to create the next Helpful Hint at the back of the lab. This portion of the coding scheme is to be used with the NVivo transcriptions that record the audible *speech* that participants provide during explanations.

**Note:** Do NOT watch the video prior to, or during, coding items in this section. This section is to be completed only using the NVivo transcription of dialogue to avoid any bias from seeing participant gestures that accompany speech.

1. **Overall Verbal Strategy:**
   - 1. Little to no explanation
     1. *Proportion of Object*: Fraction/ratio of object to reference tool(s)
        i. “This looks like it’s about half of the dowel so it must be 6 inches”
        ii. “I can slide this cube along the dowel four times so it’s about 3 inches”
     2. *Proportion of Tool*: Fraction/ratio of reference tool(s) to object
        iii. “The dowel is longer than half of the cylinder but not by much so maybe the height is 7 inches”
        iv. “The candle (measurement tool) fits into the height three times so it must be 24 inches”
     3. *Estimation*: Identifying a point of reference and using that to make a guess
        v. “It’s a little wider than my thumb and my thumb is about an inch so I think it’s 2 inches”
        vi. “It’s just a little bit shorter than the dowel so it is 6 inches”
     4. *Guessing*: An explanation without a strategy
        vii. “It looks like x inches”
        viii. “I don’t know, I just think it is x inches”
     5. *Other*: An explanation that cannot be considered one of the above categories

2. **Describe Other:** If you answered “other” to the previous question, please describe the verbal strategy in your own words with concise language. If not, please move on to the next item.

3. **Change Answer:** Does the participant change his/her final answer during the course of explaining his/her strategy?
   0. No
   1. Yes, but it is less accurate than the original answer
   2. Yes, and it is more accurate than the original answer
4. **Correct Reasoning:** Does the participant provide an explanation that is mathematically correct? The explanation can be correct even if the estimation is not. For instance, an explanation of “the height was half of the 12-inch dowel so it is 6 inches” or “Since the height of the short cylinder was 8 inches and the height of the tall cylinder was 24 inches, I added those together so the combined height is about 30 inches” would both be mathematically correct reasoning. Conversely, “it looks like the length from the center of the sphere to the outside is about 4 inches so the diameter is about 4 inches” would be incorrect mathematical reasoning.

   0. No  
   1. Yes

5. **Precise Language:** Is the language used by the participant to articulate his/her reasoning precise? Precise language would be seen in a largely complete explanation with a clear thought process that arrives at the answer, such as “I stood the dowel on the table against the cube then marked where the top of the cube hit the dowel. It looks like about half of the dowel so it’s 6 inches”. Conversely, an explanation with imprecise language would be unclear without the video, such as, “I put the dowel up against and saw where it hit, and then flipped it over to see if it matched up, but it was a little bit longer than six inches. I also measured the medium one but it seems much shorter than the big cube.”

   0. No  
   1. Yes

6. **Dynamic Speech:** Does the participant’s explanation of his/her answer refer to solely the objects (including reference tools) involved and their locations (static) or also reference the movement of those objects, such as moving a dowel along an object to measure it (dynamic)?

   0. Static  
   1. Dynamic
Part 3: Problem-Solving Explanation Gestures
(Gestures observed during explanation of problem-solving strategy)

This portion of the coding scheme pertains to the segments of participant video, for each estimation task, between 1) the time that the researcher asks the participant to explain why/how he/she arrived at the answer given and 2) the researcher directs the participant to create the next Helpful Hint at the back of the lab. This portion of the coding scheme is to be used with the observable gestures that participants exhibit during their verbal explanations.

Please read the following table for details on gesture classification and address any confusion prior to coding this section.

1. **Overall Gesture Strategy**: Throughout the participant’s verbal explanation of his/her strategy, what role do gestures play?
   - 1. *Unsure*: Any gestures exhibited by the participant are obscured from the camera.
   - 0. *None*: The verbal explanation is unaccompanied by any gestures
   - 1. *Reenactment*: The participant primarily uses the dowel or other objects to reenact their problem-solving strategy while explaining the process
   - 2. *Simulated reenactment*: The participant primarily uses gestures (without holding anything) to convey his/her problem-solving strategy

2. **Total Gestures**: Please count the total number of unique gestures observed during the problem-solving explanation.

3. **Verbal-Action Match**: Does the participant’s verbal explanation largely match the physical actions observed during problem-solving?
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

4. **Strategy and Gesture Match**: Do the participant’s gestures observed during problem-solving explanation largely match the physical actions observed during problem-solving?
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

5. **Verbal and Gesture Match**: Do the participant’s gestures observed during problem-solving explanation largely match the participant’s verbal explanation overall?
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

6. **Overall Match**: Does the participant’s verbal explanation AND gestures observed during problem-solving explanation largely match the physical actions observed during problem-solving?
   - 1. Unsure/Out of view
   - 0. No
   - 1. Yes

7. **Gestures in Speech**: Please use the information from the following table on classifying gestures to transcribe the participant’s speech accompanied by gestures and to code
observable gestures to the best of your ability. Repeat this for each gesture observed in problem-solving explanations. If you run out of space on the answer sheet, please continue to legibly mark each item on the back of the paper.

a. **Speech:** Please transcribe the speech that directly accompanies a gesture.

b. **Gesture Description:** Please choose the most fitting description of the observed gesture from the table below or describe the gesture if description is not included above. If gesture is “Unsure/Out of view,” please mark that and proceed to the next gesture, leaving the following items blank.

c. **Gesture Type:** Please mark the gesture type which most closely matches the observed gesture and corresponds to the gesture description within the table.

d. **Referent Object:** Please mark whether the referent object is an object, tool, or something else, “Other,” such as balance, length, the participant, unclear, etc.

   0. Object   
   1. Tool    
   2. Other

e. **Speech Classification:** Please mark whether the referent object in speech is static or dynamic.

   0. Static     
   1. Dynamic

f. **Gesture Classification:** Please mark whether the gesture itself is static or dynamic.

   0. Static     
   1. Dynamic

g. **Object Held:** Please mark whether or not the referent object was held during the gesture.

   0. No Object  
   1. Object    
   2. Dowel     
   3. Other Tool

h. **Dowel Use:** If the dowel was used during the gesture, please mark whether the dowel was used and if so, how.

   0. No Dowel  
   1. Identified Object  
   2. Measuring Tool    
   3. Extension of Body

i. **Speech Match:** Please mark whether or not the gesture observed matches the audible speech associated with the gesture. In cases where gestures are obstructed from view, or it is unclear whether the gesture and speech match, possibly from lack of speech associated with the gesture, mark “Unsure/Out of view.”
Table 1. Classifying Gestures in Speech.

<table>
<thead>
<tr>
<th>Gesture Type</th>
<th>Definition</th>
<th>Gesture Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iconic: Spatial</td>
<td>Gestures that depict spatial relations (Ekman &amp; Friesen, 1969)</td>
<td>Marking a location on an object&lt;br&gt;“This looks like the halfway point”&lt;br&gt;Referencing a location on an object&lt;br&gt;“I lined this up to the edge”&lt;br&gt;Downward slicing with hand to mark a division&lt;br&gt;“This would be the halfway point”&lt;br&gt;Placing a tool against an object while referencing a location&lt;br&gt;Representing a spatial relation between two distances&lt;br&gt;“This is shorter/longer/closer than that”</td>
</tr>
<tr>
<td>Iconic: Kinetographic</td>
<td>Gestures that indicate bodily actions (Ekman &amp; Friesen, 1969), usually corresponding with dynamic speech such as “move”, “flip”, or “balance”</td>
<td>Placing a tool against an object&lt;br&gt;Using the dowel to measure an object x times&lt;br&gt;Downward slicing with hand to mark a division&lt;br&gt;“I split the dowel in half”&lt;br&gt;Hands move apart or together&lt;br&gt;Consecutive marking location with (finger or dowel)&lt;br&gt;“I slid the dowel along this four times”&lt;br&gt;Consecutive marking location with hand&lt;br&gt;An action accompanied by dynamic speech</td>
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<tr>
<td>Iconic: Pictographic</td>
<td>Gestures used to draw a referent object in the air (Ekman &amp; Friesen, 1969)</td>
<td>Tracing a shape outline&lt;br&gt;Creating a shape outline</td>
</tr>
<tr>
<td>Deictic</td>
<td>Gestures that indicate objects, people and locations (McNeill, 1992), including pointing, showing an object, or reaching for something (Iverson, Capirci &amp; Goldin-Meadow, 2008) and extending to picking up an object and identifying a trait of an object</td>
<td>Pointing with dowel&lt;br&gt;Pointing with finger&lt;br&gt;Pointing with hand&lt;br&gt;Picking up/holding object for identification</td>
</tr>
<tr>
<td>Metaphoric</td>
<td>Gestures that occur when an individual creates a physical representation of an abstract idea or concept (Andric &amp; Small, 2012), including dimensions such as length, width, or height</td>
<td></td>
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<td>------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td></td>
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<tr>
<td></td>
<td>Creating a segment for length/width/height/diameter and identifying that dimension</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Creating a segment or arbitrary length with fingers and thumb or hand and placing it along an object to describe a measuring process</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Creating unit of measurement</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pointing to consecutive places on an object and identifying a unit of measurement</td>
<td></td>
</tr>
<tr>
<td></td>
<td>“This looks about an inch and I measured it 4 times so it is 4 inches”</td>
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</tr>
<tr>
<td></td>
<td>Motioning to convey a mathematical concept such as height</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Motions towards objects involved in a mental mathematical procedure</td>
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</tr>
<tr>
<td></td>
<td>“Then I added them up”</td>
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<tr>
<td>Unsure / Out of view</td>
<td>Gesture is not clearly visible</td>
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</tr>
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**Part 3: Problem-Solving Explanation (Gestures)**

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**Gesture 1: Speech**

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**Gesture 2: Speech**

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Total responses which match other coder(s) or master answer sheet: Matched responses divided by total items:
Part 2: Exploring Student Behavior in Measurement Estimation Tasks

Exploring the physical behaviors (actions, language, and gestures) of math learners can provide insight about conceptual and procedural understandings, as well as procedural and linguistic shortcomings. For instance, prior research has shown that learners who demonstrate gestures that do not match their speech are more ready to learn that learners who demonstrate gestures that are congruent with their speech (i.e. see Congdon, Novack, & Goldin-Meadow, 2018 for a review), suggesting that in some cases, learners are able to understand concepts without the ability to verbally articulate a point or concept. This line of research has demonstrated that holistic assessments of math learners, utilizing gestures, can provide detailed insights about learners’ understandings and shortcomings that are not solely dependent on the learners’ verbal or written communication abilities. Continuing to analyze the language and gestures used by various levels of math learners will contribute to the current understanding of how language and physical behaviors are related to math learning.

That said, prior research in this area has focused on observing the language and gestures of math learners’ during problem-solving or afterwards, during justifications of answers given. Little to no work has considered the relationship between learners’ language, gestures, and actions observed during problem-solving for activities such as measurement tasks. In the following chapters, I aim to apply the previously introduced coding guide and expand the body of research on gestures in mathematics learning by analyzing the three-way interplay between learners’ action, language, and gestures surrounding problem-solving, specifically focusing on measurement estimation. Additionally, I aim to identify how the roles of physical behavior in measurement tasks shift over age and math experience. Ultimately, this work intends to answer the question: Can we use the behavior of college students as models to develop behavior-based support for measurement games designed for elementary-aged students?

Measurement Estimation and Behavior

Measurement skills extend beyond mathematics classrooms in everyday tasks. Given its importance, what kind of measurement strategies be taught to, and practiced by, elementary students to reinforce procedural and conceptual understanding of measurement? The following chapters explore the behavior surrounding successful measurement strategies by experienced math learners to see how they apply measurement skills and effectively communicate those processes as well as how elementary-aged children differ in behavior during similar tasks. The findings have implications for developing instructional activities that foster successful measurement strategies among elementary students that will be discussed in Chapter Seven.

The project presented in the following chapters analyzes the physical behaviors of college students and elementary students exhibited during hands-on measurement estimation tasks. Behavior is first analyzed within each population then compared across the college and elementary
population to identify age-based differences in problem-solving behaviors. To better understand the role of action, language, and gesture in hands-on problem-solving for estimation tasks, this project explores the physical behaviors observed during problem-solving as well as afterwards, during the problem-solving explanation provided by each participant. This should provide new insights about the relationships between actions, language, and gestures as they relate to cognitive processes during problem-solving as well as conceptual understanding demonstrated during the problem-solving explanations. From this exploration of age-based differences, research-driven embodied scaffolding (action and gesture-based hints) can be developed to help elementary students develop geometry skills through guided practice with measurement tasks.
Chapter Four: Exploring Measurement Strategies of College Students

This work was originally submitted as a research report to a national conference with co-authors Hannah Smith, Richard Valente, Luisa Perez, Erin Ottmar and Ivon Arroyo.

Measurement and estimation skills are components of everyday life beyond mathematics classrooms. We estimate dimensions without necessarily considering about how we are applying mathematical skills and past experiences to make judgments. For instance, we estimate the length of parallel parking spots, gauging whether or not our cars can fit in a given space. We estimate whether furniture is too wide to be squeezed through a door frame, and how to approximate cups and teaspoons of ingredients while cooking. Across two- and three-dimensional quantities, we apply our understanding of measurement on a daily basis.

Because measurement skills and concepts are applied in everyday life, it is crucial for elementary curriculum to focus on developing an understanding of measurement concepts as well as strategies for application beyond physical measurement (NCTM, 2000). The act of measuring and conceptual understanding of measurement differ (Mullins & LaCroix, 2018). Measuring, or physical measurement, refers to the process of making direct comparisons between objects or using a unitized ruler. Concepts of measurement, on the other hand, refer to the understanding of inverse relationships, unitizing, and ultimately, rational numbers. (Lamon, 2007). For instance, as individuals estimate dimensions, they mentally represent magnitudes on mental number lines based on knowledge and experience with individual unit sizes and reference points (Joram, Gabriele, Bertheau, Gelman & Subrahmanym, 2005). We posit that students should practice measurement estimation with tools other than a marked ruler to shift from measuring objects to applying concepts of measurement for measurement estimation.

To create new approaches for teaching and practicing measurement strategies that reinforce knowledge about measurement concepts, we begin by exploring what successful measurers are doing and how they think about measurement tasks without the aid of a marked ruler. Specifically, the current study explores how college students, with years of experience in mathematics and measurement, successfully measure geometric objects by analyzing their behavior while completing tasks that are a stepping-stone between physical measurement and measurement estimation tasks. In the next stage of this project, these findings will be compared to the behavior of elementary students, to explore how novice learners differ from learners with years more of experience. That information will then be used to create learning interventions that help students better explore and learn measurement concepts through gesture and action.

4.1 Theoretical Framework

Physical measurement involves the use of tools to measure an object whereas measurement estimation is measurement in the absence of tools (Bright, 1976). While measurement estimation skills are applied in everyday life, an individual must have knowledge of physical measurement concepts in order to properly apply them in measurement estimation. For instance, in order to
estimate a distance, the measurer must apply skills to segment a continuous distance into countable units. This strategy of unit iteration is a commonly used practice in measurement estimation (Joram et al., 2005). Further, Siegel, Goldsmith and Madson (1982) found that participants use available benchmarks to estimate objects and if there are no benchmarks, they subdivide objects to fit into their mental or physical benchmarks, essentially dividing a reference point into a smaller, more precise unit for measurement. Increased experience with measurement may increase the number of mental reference points and fine-tune iteration skills. These examples demonstrate measurement concepts necessary to understand in order to progress beyond physical measurement tasks.

While physical measurement is an important foundation for understanding concepts of measurement, the ultimate goal is to apply concepts of measurement in measurement estimation tasks without the use of measurement tools. To help students successfully transition from the process of raw measurement to applying measurement concepts in the process of making estimates, we propose the use of hands-on instructional activities for measurement without marked rulers available for use. By transitioning from measuring objects with rulers to measuring objects with unmarked tools, students may practice physical measurement while integrating concepts of measurement estimation into their problem-solving process.

To create effective instructional activities that focus on this transition from physical measurement to measurement estimation, we first explore which aspects of physical measurement are still important in measurement tasks without a unitized tool such as a ruler. Because physical behavior while problem-solving (Clark, 2014) and gestures specifically, reveal implicit knowledge (Goldin-Meadow, 2003), we posit that observing students’ physical actions while completing measurement tasks will reveal implicit knowledge of measurement concepts. This project analyzes the physical behavior of college students to explore how experienced measurers approach tasks with the option to use an unmarked tool. These measurement tasks bridge the gap between physical measurement and measurement estimation by providing unmarked tools for measurement.

**Current Study**

The current study examines the question: *What kind of measurement strategies and techniques do students exhibit through their actions to approximate dimensions of 3D geometric objects such as cubes, spheres, and rectangular prisms?* A task analysis study was conducted at a northeastern technical university to observe and analyze students’ physical behavior as they completed geometry-based measurement estimation tasks. The aim of this exploratory study was to collect data about the actions, language, and gestures observed in college students surrounding measurement tasks. Observing how students, with years of exposure to math concepts, solve measurement estimation tasks through their physical behavior provides a reference point to observe how behavior may differ among lower level students during the same tasks. Specifically, we consider how our findings could suggest new approaches to teaching and fostering the practice of measurement skills with elementary students through hands-on learning interventions.

**4.2 Methods**
Design and Procedure

Over the 2017-2018 academic year, 45 undergraduate students (12 male, 16 female, 1 nonbinary) from a northeastern technical university were recruited through an online research platform to participate in a series of measurement tasks challenging participants to estimate the dimensions of geometric objects. The 12 geometric objects included a variety of spheres, rectangular prisms, cylinders, and cubes that range in size from 2- to 24-inch side lengths. At the start of the experiment, participants were offered, though not required to use, an unmarked 6-inch or 12-inch dowel, which were randomly assigned, as a tool for estimating the measurement of objects. Students were then asked to complete 18 tasks such as choosing the cube (of three) that has sides of six inches or estimating the diameter of a sphere. Participants were not given any accuracy feedback. After verbally providing an answer for a given task, participants were asked to explain how they arrived at their answer. For the answer justification portion of the task, no restrictions were placed on participants behavior. Specifically, participants were not instructed whether to use or not use objects to aid explanations, allowing for authentic behavior and interactions with the objects and tool provided. A video camera was used to record the actions of participants during each task as well as the language and gestures observed afterwards, while participants explained their problem-solving strategy for each task. These videos were then analyzed (see video coding procedures below).

<table>
<thead>
<tr>
<th>Researcher Instruction: “Estimate the height of the cylinder.”</th>
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<tbody>
<tr>
<td>Coded Measurement Strategy: Part-Part-Whole: Whole = Object (↔)</td>
</tr>
<tr>
<td>Coded Techniques Applied: Dowel Use (D), Start-Point Marker (S)</td>
</tr>
<tr>
<td>Placeholder (✓), End-Point Marker (✗)</td>
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</tbody>
</table>

Participant Answer: “Twenty-three and a half (23 ½) inches”

Figure 4-1. Sample Measurement Task and Coded Behavior.

Video Coding and Data Analysis Procedure

Video data from 29 participants and up to 16 tasks were used, resulting in a total of 442 measurement tasks captured on video. Participant videos were analyzed with the coding guide designed for this project to classify physical and verbal behavior 1) during problem solving and then 2) during problem solving explanations. For the current study, we focus on analyzing the physical behavior of participants while problem-solving (Figure 4-1). Four coders regularly met
for two months to collaboratively code videos and discuss any discrepancies. Once the coders obtained 75% agreement (3 out of 4 coders) across 80% of items on seven cases, the coders individually coded the remaining videos while continuing to meet and discuss any unclear cases.

Coders were assigned to individual measurement tasks. For this study, we only focused on the codes from the Problem-Solving Strategy domain of the coding guide which coded the physical behavior displayed by participants while completing a given measurement task. We then calculated performance on each task (accuracy = |estimate - given length| / given length) and averaged performance on all tasks to create an overall performance score for each student. Participants ranged in overall accuracy from 72.33% to 97.04% with an average performance of 92.52% and median of 94.78%, supporting the use of college students as experienced, high-performers on measurement tasks. Next, the types of physical problem-solving strategies and techniques that students exhibit during measurement tasks were explored with the coded data with descriptive statistics to answer the stated research question.

<table>
<thead>
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<th>Description</th>
<th>Example</th>
<th>% Tasks Completed Using Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eyeballing</td>
<td>Participant uses no tools or strategies to inform estimate</td>
<td></td>
<td>12%</td>
</tr>
<tr>
<td>External Reference</td>
<td>Participant uses one tool one time to inform estimate</td>
<td></td>
<td>49%</td>
</tr>
<tr>
<td>Part-Part-Whole: Whole = Tool</td>
<td>Participant measures number of times the object fits into the tool</td>
<td></td>
<td>9%</td>
</tr>
<tr>
<td>Part-Part-Whole: Whole = Object</td>
<td>Participant measures number of times the tool fits into the object</td>
<td></td>
<td>20%</td>
</tr>
<tr>
<td>Part-Part-Whole: Multiple References</td>
<td>Participant uses two or more tools for measurement</td>
<td></td>
<td>4%</td>
</tr>
</tbody>
</table>

Figure 4-2. Measurement Strategies Displayed by College Students.
4.3 Results

Measurement Strategies

For overall strategies used during problem-solving, the coding guide captured five different, observable approaches. Figure 4-2 (above) provides strategy examples and descriptive statistics of how frequently each strategy was applied across all tasks.

First, *eyeballing (1)* referred to cases in which the participant appeared to use no strategy for measurement. This strategy involved zero use of tools. The participant may have touched the object(s) in question but without distinctly measuring any dimensions or making any observed comparisons to other objects. Next, an *external reference (2)* strategy was a case in which the participant held up a body part or object as a reference tool, without subsequent measurement. This strategy involved the participant using one memorized reference point such as a thumb (~1”) or dowel (6” or 12”) and using that reference tool to estimate the object in question in comparison. This was recognizable by the participant holding one reference tool in close proximity to the object (without subsequent measuring) before providing an answer. This strategy was the most frequently displayed across all tasks.

We also observed strategies that derived an estimate based on the comparison of one object to another in a part-part-whole strategy. The *part-part-whole: whole=tool (3)* strategy was used primarily to estimate dimensions shorter than the length of the dowel or reference tool(s) used. This strategy was recognizable by the participant subsequently measuring the object along the length of the dowel or reference tool. Conversely, the *part-part-whole: whole=object(s) to be measured (4)* strategy was used primarily to estimate dimensions longer than the dowel length or reference tool(s) used. This strategy was recognizable by the participant subsequently measuring the length of the dowel or reference tool along the specified dimension of the object. In cases where the participant employed a part-part-whole strategy with more than one tool, the strategy was labeled as *part-part-whole: 2+ references (5)*. This strategy was displayed the least among participants across all tasks.

Measurement Techniques

In addition to overall strategies, we explored whether participants used specific measurement techniques for each task, such as lining up a tool to an object’s edge or double-checking their measurement. Table 4-1 (below) outlines each technique and descriptive statistics about how they were applied to each measurement task in the study. Notably, out of the nine observed techniques, participants used the provided *dowel* as a reference tool in the majority of tasks completed, indicated a clear *start-point* for measuring a given dimension in the majority of tasks, and indicated a clear *end-point* for measuring a given dimension in roughly half of the tasks. While uncommon, other reference tools (external tool use and autonomous tool use) or a combination of multiple tools (multiple references) were also used on a small percentage of measurement tasks as participants used available sheets of paper, a knuckle, etc. as reliable reference points for measurement.
### Table 4-1. Measurement Techniques Applied Across All Tasks.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Description</th>
<th>Used in Percent of Tasks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dowel Use</td>
<td>The participant uses the dowel at any point during the problem-solving process.</td>
<td>82%</td>
</tr>
<tr>
<td>Start Point Marker</td>
<td>The participant marks the designated start point of measurement on an object or tool.</td>
<td>76%</td>
</tr>
<tr>
<td>End Point Marker</td>
<td>The participant marks the designated end point of measurement on an object or tool.</td>
<td>49%</td>
</tr>
<tr>
<td>Placeholder</td>
<td>The participant uses a placeholder such as a thumb, finger, or dowel, to mark a specific point during subsequent measuring.</td>
<td>32%</td>
</tr>
<tr>
<td>Double-Check</td>
<td>The participant displays at least two measurement processes, either by completing the same process twice or by trying more than one measurement strategy.</td>
<td>20%</td>
</tr>
<tr>
<td>Decomposition</td>
<td>The participant measures two dimensions before delivering a composite estimate such as measuring the lengths of two objects before estimating the combined length.</td>
<td>19%</td>
</tr>
<tr>
<td>External Tool Use</td>
<td>The participant uses an object other than the provided dowel as a measurement tool such as a pen or sheet of paper.</td>
<td>8%</td>
</tr>
<tr>
<td>Multiple References</td>
<td>The participant displays the use of two or more tools for one measurement task.</td>
<td>8%</td>
</tr>
<tr>
<td>Autonomous Tool Use</td>
<td>The participant uses a body part as a tool for direct measurement such as a thumb or hand.</td>
<td>4%</td>
</tr>
</tbody>
</table>

*Note: Percentages add up to over 100% as participants may apply more than one technique on each task

### 4.4 Discussion

Overall, college students displayed measurement estimation strategies that fell into five distinct categories. Students most frequently used the external reference strategy to complete measurement tasks by placing a tool beside an object just once before reaching an estimate. Looking more closely at students’ behavior within each task, the use of nine different measurement
techniques were coded. College students 1) used the provided dowel as a reference tool in the majority of measurement tasks and 2) marked start- and end-points for measurement at least in roughly half of the tasks. These results provide insight about successful measurement strategies and techniques used by college students in tasks that go beyond physical measurement by removing unitized tools. In turn, the behavior and language demonstrated in this study can be used to create instructional activities for hands-on measurement practice among elementary students.

Observing the physical behavior of college students as they measured the length, width, or height of various cubes, spheres, and prisms revealed patterns of successful strategies and techniques for accurate measurement. Across all measurement tasks, college students most often demonstrated a measurement strategy of using one external reference tool, such as the unmarked dowel or a pen, to gauge a dimension of an object, without any subsequent measurement. While this was the most popular method for measurement estimation, holding one object against another to estimate a length is not the most thorough strategy that could be used for measurement. A possible explanation for the use of this strategy among college students could be experience; as individuals gain experience with measurement tasks in and out of the classroom, perhaps more thorough strategies like part-part-whole measurement are unnecessary to make accurate estimates.

The strategy of breaking up a given reference point (such as an unmarked tool) into subsequently smaller pieces to increase the preciseness of measurement requires knowledge of proportional reasoning (Lamon, 2007). College students who have proportional reasoning skills may have split the tool or object into smaller, comparable pieces, by placing the tool and object against one another and gauging where “half” or “a third” of the tool may be. This form of unitizing is a valuable measurement strategy that could be promoted through this type of task bridging measurement and measurement estimation to fine tune proportional reasoning skills with the use of unmarked tools. To this end, we speculate that less-experienced elementary students might benefit from learning and practicing how to apply a part-part-whole strategy with a reference tool as a more precise method of measurement that reinforces an understanding of proportional reasoning skills.

Looking at the specific techniques applied within each measurement task, participants did use the provided dowel as a reference tool for help with measurement on the majority of tasks, and clearly denoted a beginning- and end-point for measurement. This behavior suggests that understanding how to use a tool for reference with measurement, and clearly denoting the dimension of measurement, are key points of successful measurement. As such, we intend to explore how elementary-aged students use the same strategies and measurement techniques and whether the use of such approaches are related to performance on measurement tasks.

Beyond its exploratory nature, the current study is not without limitations. Namely, the video data was quantified through the use of a coding guide developed for this project so there are no validity measures for the tool. While extensive time was spent to ensure coder consensus, there could be inconsistency in how the data was coded and there is always the risk of human error.

Moving forward, this project will explore the behavior of elementary-aged students, as they complete the same measurement tasks, and compare differences in behavior to the behavior
exhibited by college students. We will identify gaps among elementary-aged students in applying successful measurement strategies and techniques, as well as their ability to provide correct and precise answer explanations for their estimates. Then, we will create instructional support for students to complete similar measurement tasks with the assistance of technology. The instructional support will model the successful strategies and techniques offered by college students, such as a part-part-whole strategy and use of a tool, to foster the use of such strategies with a conceptual understanding of how they work, by elementary students. As the project develops, we intend to explore the effectiveness of such instructional support and use the findings from this research to create other instructional activities to promote elementary-aged students understanding of measurement concepts and physical strategies for measurement.

4.5 Conclusions and Contributions

This chapter has presented the foundation of an exploratory project on measurement by exploring the behavior of college students as they complete measurement tasks. College students most frequently employed a strategy for measurement that placed a reference tool against the object of interest to inform estimates. However, participants may have made accurate estimates using this strategy due to years of experience with measurement rather than from this strategy being the most useful for measurement tasks. Participants also used a reference tool and marked the dimension for measurement in the majority of measurement tasks, suggesting the importance of applying these techniques for accurate measurement. These findings will be used to explore differences in behavior between college students and elementary students during measurement tasks. Identified gaps in behavior and verbal explanations among elementary students will inform the construction of image and video hints in a game to practice hands-on measurement skills and help elementary students develop concepts of measurement as they transition from physical measurement tasks to broader measurement estimation.
Chapter Five: Analyzing the Behavior of College Students During Measurement Estimation Tasks

The current chapter builds on the research questions in the previous chapter to continue exploring the physical behavior and language used by college students in the same study. This chapter provides an extended version of the methods used for the study as well as results for additional research questions. Whereas the previous chapter explored the physical actions demonstrated by college students while completing measurement estimation tasks to explore effective measurement estimation strategies and techniques, the current chapter seeks to explore the language and gestures demonstrated by college students as they provide a post-hoc explanation of their measuring process. Specifically, this chapter aims to answer the following questions from participants’ post-hoc explanations.

1) What kinds of verbal problem-solving explanations do college students offer for measurement estimation processes?
2) What kinds of gestures do college students exhibit during post-hoc explanations for measurement estimation tasks?

Additionally, this chapter will explore the relations between action, language, and gesture by trying to explore the congruence between student actions, language, and gestures across measurement estimation tasks. Specifically, this chapter will analyze behavior observed while participants completed the measurement estimation tasks as well the behavior displayed during the post-hoc explanations to answer the following questions: How congruent are participants’ actions, explanations, and gestures across tasks?

5.1 Methods

Over the 2017-2018 school year, an embodied cognition task analysis study was conducted at a northeastern university to observe and analyze the physical behavior of college students. Specifically, this study sought to analyze students’ behavior as they solved problems and justified answers to geometry-based estimation tasks. The aim of this study was to collect data about the actions, language, and gestures observed in college students surrounding problem-solving in mathematics to serve as a baseline for students with years of exposure to, and practice with, geometry and math concepts. Observing how college students, with years of exposure to math concepts, solve geometry problems with actions, specific language, and gestures, provided a reference point to observe how behavior differs among lower level students during the same measurement tasks. This study served as our pilot study for the protocol and materials, as well as the first study to be analyzed with our novel coding guide.

Design and Procedure
Forty-five (45) undergraduate students from Worcester Polytechnic Institute were recruited through the online research platform, SONA, to participate in a series of tasks challenging students to estimate the measurements of geometric objects. Prior to beginning data collection, participants were randomly assigned to one of two predetermined task orders (Appendix A) as well as one of two conditions. The conditions designated the length of the wooden tool provided for students to use for help with estimation tasks: a 6-inch wooden dowel or 12-inch dowel.

Participants were run individually in 30-minute time slots. Upon entering the lab, participants reviewed and signed the informed consent form then filled out the brief questionnaire. After the questionnaire was completed, we began videotaping as permitted. Participants were offered, though not required to use, the unmarked 6-inch or 12-inch dowel to use as a tool for estimating the measurement of geometric objects such as prisms, spheres and cylinders, of various sizes. Students were then asked to complete 18 tasks such as 1) choosing an object (of three) that has sides of a certain length and 2) estimating the length of a given object. Note, participants did not receive any accuracy feedback. After verbally providing an answer for a given task, participants were asked to explain how they arrived at that answer. For the answer justification portion of the task, no restrictions were placed on participant behavior. Specifically, participants were not instructed whether to use or not use objects to aid explanations, allowing for authentic behavior and interactions with the objects and tool provided. Between tasks, the participant was asked to face the other direction and write a “helpful hint” for another student who might struggle with the task just completed. While the participant wrote a hint, the next set of objects was arranged for the following estimation task. After the study, we recorded the participant’s height and length of dominant hand before debriefing the participant.

Materials

The main materials for this study included a brief questionnaire and Helpful Hints worksheet, 12 objects of varying geometric shapes and sizes, two wooden dowels, and an answer sheet to mark participant responses. Additionally, a video camera was used to record the actions of participants during each task as well as the language and gestures observed afterwards, while participants explained their problem-solving strategy for each task. The questionnaire contains 12 items related to background information and interest in mathematics. The Helpful Hints worksheet was used as a distractor task between estimation tasks, asking participants to create a helpful hint for a student who might be struggling with completing the previous task (Appendix B). The 12 geometric objects include a variety of spheres, rectangular prisms, cylinders, and cubes that range from having 2-inch side lengths to 24-inch side lengths. The wooden dowels are 6 inches and 12 inches, respectively. And the answer sheet was a printed form for the researcher to mark verbal participant responses during the study.

Measures

Participant responses from the questionnaire, verbal responses to each task, and videotaped actions, speech, and gestures were used to answer the stated research questions. The questionnaire...
contains background information as well as attitude constructs based on a 7-point Likert scale adapted from previously accepted measures (Eccles et al., 1993; Arroyo et al., 2012) to identify student feelings regarding mathematics and self-concept (Appendix C). The verbal responses to each task were recorded by the researcher during the study, including discrete responses from multiple choice questions as well as estimations that can be measured on a continuous scale. Video data was obtained for 30 of the 44 participants.

Exclusions
Data from one participant was excluded from analyses due to the participant’s age, leaving 44 participants. Due to a server crash, some video data was lost during data collection, leaving 30 participants with video recordings and consent to use the data for analysis. However, one participant misunderstood directions and their data was culled, leaving N=29 participants. While participants completed a total of 17 measurement tasks, data from one task (M) was excluded from analyses due to inconsistency in researcher protocol. Similarly, four cases (individual participants’ tasks) were culled due to coding error. For each research question, data from 29 participants and up to 16 tasks were used, resulting in a total of 442 measurement tasks captured on video.

Video Coding Procedure
Participant videos were analyzed using the coding guide that designed for this project (see Section 1 for more detail). Four coders met on a regular basis to discuss specific cases as well coding items that warranted editing, resulting in an iteratively designed coding guide that extensively captures the rich video data. The four coders regularly met over two months to collaboratively code participant videos and discuss any points of controversy. Once the four coders felt confident in their consensus and obtained 75% agreement (3 out of 4 coders) across 80% of items on seven given cases, the coders individually coded the remaining video data while continuing to meet and discuss any unclear cases.

Approach to Analyses
To analyze the types of actions, language, and gestures demonstrated by participants, I looked at the frequencies and descriptive statistics of the coding items related to each type of behavior (action, language gesture) observed across all tasks completed. Given the exploratory nature of this work, descriptives were primarily used to describe the behavior of participants.

5.2 Results

Behavior During Answer Explanations
As participants provided a rationale for their answer, I explored 1) the types of verbal problem-solving explanations that students exhibit and 2) the kinds of gestures that students exhibit during post-hoc answer explanations.
First, Table 5-1 below provides an overview of the types of verbal explanations observed by participants. The descriptions of each verbal strategy include examples from actual participants. Of the 423 tasks that included video data of the verbal explanations, only one case was marked as “Little to no explanation”. Frequencies reveal that participants most often identified a point of reference to inform their answer, referred to as the “Estimation” strategy. This strategy includes references to knowledge of an inch, the length of the dowel, or another tool such as a piece of paper which is the primary reference to inform the current measurement of an object. Most interestingly, one participant used an “Estimation” strategy by picturing the size of a sub sandwich from Subway. Specifically, he explained that, “For this I imagined a five-dollar footlong... not a five-dollar footlong, a six-inch, but it seemed a little bit bigger than a six-inch so then I added .5” reach his answer of 6.5 inches for the height of a cylinder.

Also of note, the second-highest verbal strategy exhibited among college students involved tendencies to thinking proportionally about the size of two objects relative to each other. In the “Proportion of Object” or “Proportion of Tool” strategies, participants reported comparing the size of one object to another to inform their estimate of a dimension. By combining these strategies under one umbrella of “Proportional Reasoning”, participants reported using a proportion strategy in 37.8% of the tasks.

<table>
<thead>
<tr>
<th>Verbal Strategy</th>
<th>Description</th>
<th>Used in Percent of Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation</td>
<td>The participant identifies a point of reference that informed their answer. “I tried to guess where it would be six. And then move it up and see where it would be like one to six and this one looks like it seemed around a four mark.”</td>
<td>41.6%</td>
</tr>
<tr>
<td>Proportion of Object</td>
<td>The participant compares the size of the object to the size of the reference tool. “The large purple cylinder is two dowel lengths so twenty-four”</td>
<td>19.6%</td>
</tr>
<tr>
<td>Proportion of Tool</td>
<td>The participant compares the size of the measurement tool to the size of the object. “Just using the 12-inch dowel and measuring how many of them fit along each block.”</td>
<td>18.2%</td>
</tr>
<tr>
<td>Other</td>
<td>The participant uses a strategy not specified by the coding guide such as process of elimination or referencing a measurement from a previous task. “…to my memory, this was twenty-seven inches”</td>
<td>15.8%</td>
</tr>
</tbody>
</table>
Guessing
The participant provides an answer without a clear strategy.
“I’m just eyeballing it, I’m just guessing”

Next, I broadly analyzed how participants gesture while describing their measurement estimation process. Looking at the behavior of all participants across all tasks, participants ranged from not using any gestures to producing up to nine unique gestures for a given explanation ($M=2.27$, $SD=1.55$). Out of a total of 448 tasks, these “gesture strategies” were classified as either “Simulated Reenactment” (12.9%) for explanations that were accompanied by gestures that largely did not use any objects or “Reenactment” (77.1%) to classify explanations that were largely accompanied by gestures that involved the use of nearby objects to illustrate points of speech.

Table 5-2. Percentage of Gesture Types Observed by College Students.

<table>
<thead>
<tr>
<th>Gesture Type</th>
<th>Description</th>
<th>Percent of Total Gestures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deictic</td>
<td>Gestures that indicate objects, people and locations (McNeill, 1992), including pointing, showing an object, or reaching for something (Iverson et al., 2008) and extending to picking up an object as well as identifying a trait of an object</td>
<td>40.3%</td>
</tr>
<tr>
<td>Iconic: Kinetographic</td>
<td>Gestures that indicate bodily actions (Ekman &amp; Friesen, 1969), usually corresponding with dynamic speech such as “move”, “flip”, or “balance”</td>
<td>28.7%</td>
</tr>
<tr>
<td>Iconic: Spatial</td>
<td>Gestures that depict spatial relations (Ekman &amp; Friesen, 1969)</td>
<td>21.5%</td>
</tr>
<tr>
<td>Metaphoric</td>
<td>Gestures that occur when an individual creates a physical representation of an abstract idea or concept (Andric &amp; Small, 2012), including dimensions such as length, width, or height</td>
<td>7.4%</td>
</tr>
<tr>
<td>Iconic: Pictographic</td>
<td>Gestures used to draw a referent object in the air (Ekman &amp; Friesen, 1969)</td>
<td>1.4%</td>
</tr>
<tr>
<td>Unsure/Out of view</td>
<td>Gestures that are present in the video but largely out of view</td>
<td>.8%</td>
</tr>
</tbody>
</table>

As defined in Section 1 (Chapter Two and Chapter Three), gestures were categorized as five distinct gesture types. See Table 5-2, above, for the percentage of gesture types produced by participants during answer explanations out of a total of 923 gestures. Notably, deictic gestures were most frequently used to indicate a tool or object during answer explanations whereas pictographic gestures were the least frequently used gesture type, with few instances of participants drawing shapes or figures in the air to convey meaning.
Behavior Congruence

Across the problem-solving and explanation stages of each task, we analyzed the congruence of participants’ action, language, and gestures. Specifically, we explored how congruent participants’ actions, language, and gestures were during measurement estimation tasks and post-hoc explanations. Table 5-3, below, highlights the findings showing congruence between action, speech, and gesture with the strongest relation between speech and gesture. For each item, 5.9-8.6% of cases were labeled as “Unsure/Out of view”.

Table 5-3. Behavior Congruence of College Students During Measurement Estimation Tasks.

<table>
<thead>
<tr>
<th>Congruence</th>
<th>Description</th>
<th>Mismatch Across All Tasks</th>
<th>Congruence Across All Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speech-Gesture</td>
<td>The participant’s gestures observed during problem-solving explanation largely match the participant’s verbal explanation overall.</td>
<td>4.6%</td>
<td>89.5%</td>
</tr>
<tr>
<td>Action-Speech</td>
<td>The participant’s verbal explanation largely matches the physical actions observed during problem-solving.</td>
<td>9.0%</td>
<td>85.1%</td>
</tr>
<tr>
<td>Action-Gesture</td>
<td>The participant’s gestures observed during problem-solving explanation largely match the physical actions observed during problem-solving.</td>
<td>15.9%</td>
<td>75.8%</td>
</tr>
<tr>
<td>Action-Speech-Gesture</td>
<td>The participant’s verbal explanation AND gestures observed during problem-solving explanation largely match the physical actions observed during problem-solving.</td>
<td>15.4%</td>
<td>76%</td>
</tr>
</tbody>
</table>

The relation between speech and gesture was further examined using the “Verbal and Gesture Match” binary item coded from the Embodied Cognitive Task Analysis. The item labels whether the participant’s gestures observed during a problem-solving explanation largely match the participant’s overall verbal explanation for his/her answer. Across 409 tasks with this information recorded, 89.5% of participants’ gestures were congruent with their overall verbal explanations for their measurement estimation process. Only 4.6% of tasks included gestures which were not consistent with the participant’s verbal explanation and 5.9% of tasks were labeled as “Unsure / Out of view”. A crosstabs analysis in SPSS showed that these cases with unsure or mis-matching speech and gestures were spread across over half of the total participants, showing that instances of gesture-speech mismatch occurred on one to three measurement tasks with
multiple participants rather than one or two participants consistently displaying gesture-speech mismatch on the majority of tasks.

5.3 Discussion

Overall, the results show evidence for the common ways that college students conceptualize their measurement estimation processes through their speech and accompanying gestures. Notably, the majority of participants reported that, in a given task, they identified a point of reference for measurement estimation, such as knowledge of an inch, then compared that reference to the magnitude of the dimension at hand. To supplement these rationales, each explanation was accompanied by a couple gestures on average. These gestures most frequently served to identify referent objects in speech and largely involved the use of relevant objects. Lastly, our findings suggest that largely, college students do have congruent action strategies, verbal explanations, and gestures although this relationship is strongest between verbal explanations (speech) and gestures.

Interestingly, the verbal strategies most used by college students match up with the action strategies most displayed by college students analyzed in the previous chapter. The “estimation” verbal strategy maps onto the “External Reference” action strategy, both of which were the most common approaches taken by college students in measurement estimation tasks. This finding is also backed by the congruence of action and speech strategies in the majority of tasks, suggesting that college students largely complete measurement estimation tasks then are able to explain their conceptual reasoning consistently rather than displaying one approach to measurement estimation while completing the task and then describing their rationale differently. For instance, college students are likely to display a measurement process that involves using one point of reference, and then explain their process as using a reference point to inform their answer rather than appearing to eyeball the object then describe a proportional reasoning strategy.

In addition to action-speech congruence, the majority of tasks also displayed speech-gesture congruence. This finding supports previous research on speech-gesture congruence (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, 2003; Goldin-Meadow & Singer, 2003) as speech-gesture congruence among high-performing college students supports evidence that experts, with a better understanding of content, are likely to communicate with matching speech and gesture. This shows that speech-gesture congruence extends to high-performers while explaining problem-solving for a measurement estimation task and sets the stage to examine how speech-gesture congruence varies, if at all, for younger, lower-performing children on measurement estimation tasks.

And like prior research on the congruence between action-speech-gesture (Church et al., 2014; Kelly et al., 2015), the speech-gesture relation was more tightly linked than that of action-speech, action-speech-gesture, or action-gesture although all four combinations did reveal congruence between behaviors in the majority of tasks. This shows that the relations between
action, speech, and gesture may be similar across speech production, comprehension, and mathematics tasks for communication.

Similarly, the majority of gestures were also produced while holding or using an object to help illustrate a point to the researcher. While movements involving objects and effecting some sort of change on the surrounding environment have not been previously considered gestures to my knowledge, perhaps providing learners with the options to use props during task analyses or interviews allows learners to demonstrate more authentic behavior rather than focusing on how to convey knowledge in the absence of props. While gesturing in the absence of objects has been more strongly associated with communication than actions have (Church, et al., 2014), seeing that learners opt to use objects available to them rather than gesturing only with their hands suggests that allowing the use of props to observe learners behavior may be more organic and helpful outside of contexts where the goal is to directly support learning through the task at hand.

The gestures produced by participants during answer explanations were mostly used to help identify objects of interest in deictic gestures by pointing to or holding up the referent object. The use of so many deictic gestures could be explained by the tendency to gesture for help with speech production (Krauss et al., 2000) with words likely not in the participants’ day-to-day repertoire. For instance, the unmarked stick given to participants as a reference tool was referred to as a “dowel,” and the objects for measurement included a hodgepodge assortment of cubes, spheres, prisms, and cylinders. Perhaps participants used identifying gestures to help summon the terms used by the researchers to describe the objects involved with the study. Conversely, the gestures least used in explanations for measurement processes were pictographic gestures. Because the objects were left on the table for participants to use freely and participants were describing the processing of linear measurement rather than something more abstract or a 3D measurement, perhaps this eliminated the need to create shape outlines and draw images in the air.

As mentioned in the previous chapter, this study did have limitations. This study was the first application of the Embodied Cognitive Task Analysis coding guide so there are no validity measures for the tool and there could be inconsistency in how the data was coded and there is always the risk of human error in data collection and coding. Additionally, the video data revealed that the camera was not placed optimally to capture participants’ physical behaviors, resulting in a loss of some data because of an obscured view. And importantly, participants’ strategies and explanations for the measurement estimation tasks might have matched the quality of answer they needed for the moment without fully applying their measurement abilities. For instance, college students might have applied more rigorous strategies and provided more detailed explanations if performance on the tasks was more meaningful for participants than credit for partial course fulfillment. Lastly, the ambiguous wording for the instructions of one of the tasks (M) led to inconsistent set-up among researchers and error in completion among participants, leading data from that task to be dropped from analyses.

This exploration of behavior among college students during measurement estimation tasks serves as the high-performing baseline to later compare against the behavior of elementary students in the same tasks with the hopes of identifying age- and performance-based differences in
approaches to measurement estimation as well as explanations behind measurement processes. Exploring how high-performing college students complete measurement estimation tasks and effectively communicate those processes will help create measurement estimation support for elementary-aged children as they complete similar tasks in a game.

5.4 Conclusions and Contributions

This chapter has provided insight about how high-performing college students describe measurement estimation processes in speech and with gesture. Namely, participants explained how they measured an object by gesturing with the tools and objects available to them. This chapter also provided evidence for the three-way relation between action, speech, and gesture which had previously not been explored in a learning context. The current findings do support prior research in that there is an existing relationship between actions, speech, and gestures although the relationship between speech and gestures may be stronger than either behavior with action while completing measurement estimation tasks. This information about the behavior of high-performing students surrounding measurement tasks can be used to create instruction and scaffolding for younger students to develop procedural and conceptual measurement estimation abilities.
Chapter Six: Analyzing the Behavior of Elementary Students During Measurement Estimation Tasks

This chapter aims to extend the research presented in the previous chapters by exploring the behavior of elementary-aged children as they complete and explain their measuring processes for measurement estimation tasks with the ultimate goal of developing instructional activities and scaffolds to foster and reinforce children’s understanding of measurement.

The physical act of measuring and concepts of measurement are not synonymous. Physical measurement involves the use of tools to make direct comparisons about magnitude whereas measurement estimation is the process of measuring in the absence of tools by applying concepts of measurement (Bright, 1976). Beyond the ability to procedurally measure, a conceptual understanding of measurement is important for children to develop over elementary school as these skills continue to be applied in everyday routine. As Richard Lehrer wrote, it is important to help children beyond developing procedural measurement skills “so that procedures and concepts are mutually boot-strapped” (2003).

To help elementary-aged children successfully develop conceptual foundations of measurement to transition from the process of raw measurement with rules to applying measurement concepts in the absence of tools, we propose the use of hands-on measurement tasks that provide children with unmarked rulers available for use during measurement estimation tasks. By transitioning from measuring objects with rulers to measuring objects with unmarked tools, children may practice physical measurement techniques while integrating concepts of measurement estimation into their problem-solving processes. Our research team has developed the measurement estimation scavenger hunt, EstimateIT!, for elementary-aged students to practice these skills in a collaborative, active game (Rountree, 2015). As the game develops, we aim to construct and refine hints within the game to guide students’ behaviors during measurement estimation tasks in order to reinforce concepts of measurement.

To create effective hints and support within EstimateIT! that focus on this transition from physical measurement to measurement estimation, we currently analyze the behavior of elementary students to explore which procedural and conceptual aspects of measurement are present and absent in measurement tasks without a unitized tool such as a ruler. Because physical behavior while problem-solving (Clark, 2014) and gestures specifically, reveal implicit knowledge (Goldin-Meadow, 2003), we posit that observing students’ physical actions, language, and gestures while problem-solving in measurement tasks then providing an answer justification will reveal some implicit knowledge of measurement concepts that can be targeted in support to help reinforce concepts of measurement. Specifically, we pose the following problems below.

As participants are solving estimation tasks, I ask: What kind of measurement strategies and techniques do students exhibit through their actions to approximate dimensions of 3D geometric objects such as cubes, spheres, and rectangular prisms?

Next, to follow the questions posed for the College Study presented in Chapter Five, I explore the behavior of participants as they justify each answer given after a measurement estimation task. Specifically, I intend to answer:
1) What kinds of verbal problem-solving explanations do participants exhibit?
2) What kinds of gestures do participants exhibit?
3) How congruent are student actions, language, and gestures across tasks?

6.1 Methods

In the fall of 2018, an adapted version of the College Study was completed at a local after-school program with elementary-aged children in Massachusetts. Like the College Study, the current study was designed to capture participants’ behavior during geometry-based measurement estimation tasks with the ultimate goal of comparing behavior across the college and elementary populations to learn about effective measurement estimation strategies demonstrated by college students as well as gaps in procedural techniques demonstrated by elementary students.

Participants

Thirty (30) elementary-aged children in a local after-school program participated in this study. Prior to data collection, the staff at the after-school program collected consent forms for participants and provided demographic information to our research team.

Participants (17 female, 13 male) ranged from 8-11 years old. The children varied in grade from third through sixth grade with the majority of participants in fourth grade. Specifically, the participant pool included 7 participants in third grade, 16 participants in fourth grade, 5 participants in fifth grade, one participant in sixth grade, and one participant with grade unknown.

Design and Procedure

Data collection was completed in two-hour sessions at the after-school program each week in late September and October. Researchers worked one-on-one with participants in a room of the after-school program for 15-20 minutes apiece. For each participant, the researcher verbally delivered the same math-attitude questionnaire administered to participants in the College Study (Eccles et al., 1993; Arroyo et al., 2012; Appendix D). To help participants answer to the best of their abilities, an agreement scale with smiley and frowny faces was used for students to choose the face they most agree with for each statement (Appendix E). The researcher then provided participants with an unmarked, 12-inch dowel to use for help with estimation tasks. Note that participants in the current study were only provided with a 12-inch dowel rather than a randomized assignment to a 12-inch dowel or 6-inch dowel to match the conditions of EstimateIT!. The researcher then reviewed the terms “length,” “height,” and “diameter” while giving the participant reference pictures to use during the measurement estimation tasks (Appendix F). The researcher then asked participants to complete 10 estimation tasks involving a variety of geometric objects (Appendix G). For example, a participant may have been asked to “estimate the width” of a 6x6-inch cube. The tasks were selected from the College Study based on performance among college students. Order A presented the tasks in order of easiest (highest performance) to hardest (lowest performance) while Order B presented the tasks in the opposite order. Scaffolding instruction was
given as needed if the participants appeared confused or asked for help. The researcher recorded
the participant’s answer and prompted the participant to explain how he/she arrived at that answer.
Like the College Study, no restrictions were placed on behavior during the answer explanations,
allowing participants to demonstrate and pick up objects freely. Participants were compensated
with a toy and debriefing letters were distributed to parents.

Materials
The materials for this study included a questionnaire, 11 objects of varying geometric
shapes and sizes, a 12-inch wooden dowel, an answer sheet to mark participant responses, and a
video camera. Video recordings captured the actions of consenting participants during each task
as well as the language and gestures observed afterwards while participants explained their
problem-solving strategies. The questionnaire contained 12 items related to background
information and interest in mathematics, as described above. The objects included a variety of
spheres, rectangular prisms, cylinders, and cubes that range from having 2-inch dimensions to 24-inch dimensions, similar to the objects used in EstimateIT!

Measures
Demographic information about each participant was obtained from the after-school
program, including age, gender, and grade level. Verbal responses to each questionnaire item and
task were recorded by the researcher. These responses include discrete responses to multiple
choice questions as well as estimations that can be measured on a continuous scale. These verbal
responses and videotaped actions, speech, and gestures of each participant were used to answer
the stated research questions. Video data was analyzed using the same coding guide and protocol
that was designed for this project after iterative design and application with the College Study data.

Video Coding Procedure
Video recordings were obtained for 28 of the participants for analysis. Twenty-five (25)
videos included the full duration of the study while three videos contained partial recordings due
to the camera battery dying or researcher error during the study. As participants completed up to
ten tasks during the study, a total of 214 individual tasks were coded. Three coders, who also coded
the College Study videos, met regularly for a month to collectively create master codes and discuss
any items of disagreement. The majority of master codes for analysis were created by two coders
independently coding a task and then collaboratively creating a master code for the given task.
This process ensured that the majority of coders agreed on all cases.

Approach to Analyses
To analyze the types of actions, language, and gestures demonstrated by participants, I
approached each research question the same way as for the College Study, first calculating overall
performance for each participant and then utilizing descriptive statistics of the coding items related
to each type of behavior (action, language gesture) observed across all tasks completed.
6.2 Results

Performance

Performance was calculated with the same formula used for participants in the College Study. Specifically, we calculated overall accuracy on the ten tasks as the average absolute deviation from accuracy on each task, recorded as a percentage. The percent error for free-response measurement tasks were calculated as:

\[
\% \text{ Error} = \left| \frac{\text{Correct Answer} - \text{Given Answer}}{\text{Correct Answer}} \right|
\]

Accuracy for multiple choice tasks (i.e. “Choose which object…”) was considered binary right/wrong. Because a small minority of students did not always provide a serious answer, a floor (0%) and ceiling (100%) were set for accuracy on individual tasks by capping incorrect responses at 100% error rate. By this metric, double the correct answer was the maximum possible value for a given task. The formula used to calculate average participant accuracy is:

\[
\text{Accuracy} = 1 - \frac{\sum \% \text{ Error Free Response Tasks} + \sum \text{Incorrect Multiple Choice Tasks}}{\text{Count (All Tasks Completed)}}
\]

From these calculations, participants ranged in overall accuracy from 8.49% to 91.37% (\(M=64.13\%, \ SD=22.90\%\)) with a median accuracy of 69.45% (Figure 6-1). Using a median split, participants were then binned into low- and high-performing groups to compare differences in behavior between performance levels.

Figure 6-1. Average Participant Performance Across all Measurement Estimation Tasks.
Behavior During Problem-Solving

For overall strategies used during problem-solving, the coding guide captured five different, observable approaches as shown in Chapter Four. Table 6-1, below, provides strategy descriptions and descriptive statistics of how frequently each strategy was applied across all tasks. Out of 214 tasks, 1% was labeled as missing due to an obscured view or researcher error. Most frequently, participants displayed the “External Reference Strategy,” statically comparing the magnitude of the dowel (or another tool) to the dimension of interest, before providing an estimate. Interestingly, multiple tools were used for measurement on only one task and no one attempted to estimate a dimension by seeing how many times the object could stack alongside the tool.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
<th>% Tasks Completed Using Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Reference</td>
<td>Participant uses one tool one time to inform estimate</td>
<td>73.4%</td>
</tr>
<tr>
<td>Eyeballing</td>
<td>Participant uses no tools or strategies to inform estimate</td>
<td>18.7%</td>
</tr>
<tr>
<td>Part-Part-Whole: Whole = Object</td>
<td>Participant measures the number of times the tool fits into the object</td>
<td>6.5%</td>
</tr>
<tr>
<td>Part-Part-Whole: Multiple References</td>
<td>Participant uses two or more tools for measurement</td>
<td>.5%</td>
</tr>
<tr>
<td>Part-Part-Whole: Whole = Tool</td>
<td>Participant measures number of times the object fits into the tool</td>
<td>0%</td>
</tr>
</tbody>
</table>

In addition to overall strategies, we explored whether participants used specific measurement techniques for each task, such as lining up a tool to an object’s edge or double-checking their measurement. Table 6-2, below, outlines each technique and descriptive statistics about how they were used.

Notably, out of the nine observed techniques, participants used the provided dowel as a reference tool for measurement in the majority of cases and identified a clearly designated start point for measurement on an object. Conversely, participants clearly designated the end point of measurement in less than 40% of cases. Less than 25% of the time, participants demonstrated decomposing problems (although few tasks were optimal for decomposition), double-checked their work, or used a placeholder while subsequently measuring an object. And aside from dowel use, a hand was used as a reference tool on a task only twice.

<p>| Table 6-2. Measurement Techniques Applied Across All Tasks in Elementary Study. |</p>
<table>
<thead>
<tr>
<th>Technique</th>
<th>Description</th>
<th>Used in Percent of Tasks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dowel Use</td>
<td>The participant uses the dowel at any point during the problem-solving process.</td>
<td>80.4%</td>
</tr>
<tr>
<td>Start Point Marker</td>
<td>The participant marks the designated start point of measurement on an object or tool.</td>
<td>69.2%</td>
</tr>
<tr>
<td>End Point Marker</td>
<td>The participant marks the designated end point of measurement on an object or tool.</td>
<td>39.3%</td>
</tr>
<tr>
<td>Decomposition</td>
<td>The participant measures two dimensions before delivering a composite estimate such as measuring the lengths of two objects before estimating the combined length.</td>
<td>17.3%</td>
</tr>
<tr>
<td>Double-Check</td>
<td>The participant displays at least two measurement processes, either by completing the same process twice or by trying more than one measurement strategy.</td>
<td>15.9%</td>
</tr>
<tr>
<td>Placeholder</td>
<td>The participant uses a placeholder such as a thumb, finger, or dowel, to mark a specific point during subsequent measuring.</td>
<td>9.3%</td>
</tr>
<tr>
<td>Multiple References</td>
<td>The participant displays the use of two or more tools for one measurement task.</td>
<td>.9%</td>
</tr>
<tr>
<td>Autonomous Tool Use</td>
<td>The participant uses a body part as a tool for direct measurement such as a thumb or hand.</td>
<td>.9%</td>
</tr>
<tr>
<td>External Tool Use</td>
<td>The participant uses an object other than the provided dowel as a measurement tool such as a pen or sheet of paper.</td>
<td>0%</td>
</tr>
</tbody>
</table>

*Note: Percentages add up to over 100% as participants may apply more than one technique on each task

**Behavior During Answer Explanation**

As participants provided a rationale for their answer, I explored 1) the types of verbal problem-solving explanations that students exhibit and 2) the kinds of gestures that students exhibit during post-hoc answer explanations.

First, Table 6-3, below, provides an overview of the types of verbal explanations observed by participants. The descriptions of each verbal strategy include examples from actual participants. Of the 214 tasks, 10.3% of tasks were marked as “Little to no explanation” or missing. Frequencies
reveal that participants most often described a category that did not meet the criteria for a predefined category in the coding guide, considered an “Other” description. Of these 121 instances, 48 tasks were described as the participant “counting” to describe their measurement process while the others included explanations that referenced previous measurements, used process of elimination, etc. After the “Other” category, participants most frequently used an “Estimation” strategy of identifying a point of reference used to make a judgment about an object’s dimension.

Table 6-3. Observed explanations provided by participants after each task.

<table>
<thead>
<tr>
<th>Verbal Strategy</th>
<th>Description</th>
<th>Used in Percent of Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other</td>
<td>The participant uses a strategy not specified by the coding guide such as process of elimination, referencing a measurement from a previous task, or counting. “Because I thought this one was too big to be six and this one was the perfect thing and this one was too small.” “I counted by going up.”</td>
<td>56.5%</td>
</tr>
<tr>
<td>Estimation</td>
<td>The participant identifies a point of reference that informed their answer. “Because I know what an inch is and I counted inches on the sticks all the way to the end of the cube.” “Because the line is like the same height as the stick.”</td>
<td>19.2%</td>
</tr>
<tr>
<td>Guessing</td>
<td>The participant provides an answer without a clear strategy. “Looking at it. Two.”</td>
<td>9.3%</td>
</tr>
<tr>
<td>Proportion of Object</td>
<td>The participant compares the size of the object to the size of the reference tool. “I lined up the edge and I put my fingers near the tips and then I do this but I don't use my fingers that I put near my tip and is it complete, then I would see that this is one feet right? Twelve inches. So it would be two feet because I measured it [the object] two times.”</td>
<td>2.3%</td>
</tr>
<tr>
<td>Proportion of Tool</td>
<td>The participant compares the size of the measurement tool to the size of the object. “If I put this [dowel] right here, and you're saying this is twelve inches, it's about half of it [the cylinder]. ... So one plus one, one plus one inch, or one foot plus one foot...two feet.”</td>
<td>2.3%</td>
</tr>
</tbody>
</table>
Next, I explored how participants gesture while describing their measurement estimation process. Looking at the behavior of all participants across all tasks, participants ranged from not using any gestures to producing up to 19 unique gestures for a given explanation ($M=2.57$, $SD=2.582$). On each of the 214 total tasks, these “gesture strategies” were classified as either “Simulated Reenactment” (12.7%) for explanations that were accompanied by gestures that largely did not use any objects or “Reenactment” (76.1%) to classify explanations that were largely accompanied by gestures that involved the use of nearby objects. On 8.5% of tasks, participants provided verbal explanations that did not involve the use of any gestures. In total, 731 unique gestures were produced, with the breakdown of gesture types demonstrate below (Table 6-4). Most often, students used deictic gestures to identify objects in speech or kinetographic gestures to indicate actions made during the measurement task. Conversely, there were only 17 instances of pictographic gestures used to draw or trace object outlines.

**Table 6-4. Percentage of Gesture Types Observed by Elementary Students.**

<table>
<thead>
<tr>
<th>Gesture Type</th>
<th>Description</th>
<th>Percent of Total Gestures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deictic</td>
<td>Gestures that indicate objects, people and locations (McNeill, 1992), including pointing, showing an object, or reaching for something (Iverson et al., 2008) and extending to picking up an object as well as identifying a trait of an object</td>
<td>31.9%</td>
</tr>
<tr>
<td>Iconic: Kinetographic</td>
<td>Gestures that indicate bodily actions (Ekman &amp; Friesen, 1969), usually corresponding with dynamic speech such as “move”, “flip”, or “balance”</td>
<td>27.9%</td>
</tr>
<tr>
<td>Iconic: Spatial</td>
<td>Gestures that depict spatial relations (Ekman &amp; Friesen, 1969)</td>
<td>24.4%</td>
</tr>
<tr>
<td>Metaphoric</td>
<td>Gestures that occur when an individual creates a physical representation of an abstract idea or concept (Andric &amp; Small, 2012), including dimensions such as length, width, or height</td>
<td>10.1%</td>
</tr>
<tr>
<td>Unsure/Out of view</td>
<td>Gestures that are present in the video but largely out of view</td>
<td>3.2%</td>
</tr>
<tr>
<td>Iconic: Pictographic</td>
<td>Gestures used to draw a referent object in the air (Ekman &amp; Friesen, 1969)</td>
<td>2.4%</td>
</tr>
</tbody>
</table>

**Behavior Congruence**

Across the problem-solving and explanation stages of each task, I explored how congruent elementary-aged children’s actions, language, and gestures are in measurement tasks. Specifically, I explored how congruent participants’ actions, language, and gestures were during measurement estimation tasks and post-hoc explanations. Table 6-5, below, highlights the findings showing
congruence between action, speech, and gesture with the strongest relation between speech and gesture. For each item, 13.6-20.1% of cases were labeled as missing.

Table 6-5. Behavior Congruence of Elementary Students During Measurement Estimation Tasks.

<table>
<thead>
<tr>
<th>Congruence</th>
<th>Description</th>
<th>Mismatch Across All Tasks</th>
<th>Congruence Across All Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speech-Gesture</td>
<td>The participant’s gestures observed during problem-solving explanation largely match the participant’s verbal explanation overall.</td>
<td>11.2%</td>
<td>69.2%</td>
</tr>
<tr>
<td>Action-Speech</td>
<td>The participant’s verbal explanation largely matches the physical actions observed during problem-solving.</td>
<td>23.4%</td>
<td>63.1%</td>
</tr>
<tr>
<td>Action-Gesture</td>
<td>The participant’s gestures observed during problem-solving explanation largely match the physical actions observed during problem-solving.</td>
<td>27.6%</td>
<td>58.9%</td>
</tr>
<tr>
<td>Action-Speech-Gesture</td>
<td>The participant’s verbal explanation AND gestures observed during problem-solving explanation largely match the physical actions observed during problem-solving.</td>
<td>30.8%</td>
<td>49.1%</td>
</tr>
</tbody>
</table>

6.3 Discussion

This discussion provides a brief summation of the findings and interpretations from this study. Chapter Seven will further reflect on interpretations from this study as the findings compare to those of the College Study with implications for future directions and applications for these findings.

Behavior During Measurement Tasks

The elementary-aged participants provided an array of identifiable actions while completing the measurement estimation tasks. For overall measurement strategies, most tasks were completed by participants applying an “External Reference” strategy by placing a tool next to the object once before ultimately giving an estimate. This strategy was followed by roughly a fifth of the total tasks being completed with an eyeballing strategy. Conversely, there were no instances of any “Part-Part-Whole: Whole = Tool” strategies. Looking at specific measurement techniques applied during the measurement estimation process, the majority of tasks involved the elementary-aged children using the dowel as a measurement tool, as well as marking a start-point for
measurement. Conversely, participants never used other objects (such as a pen or piece of paper) as an alternative tool for measurement, and rarely used a body part for a reference tool.

For measurement techniques, the majority of participants did appear to use the dowel as a reference tool by placing it alongside an object while completing a measurement estimation task. Many participants also continued to hold the dowel throughout the study between tasks and seemed to enjoy having something to wield and toy with during the study. Given this, and the researcher prompt that participants were allowed, although not required, to use the dowel for help raises the possibility that while participants held up the dowel in a way that looked productive, the children might not have known how to utilize the dowel for help with measurement. For instance, some of the children placed the dowel against an object but then counted, presumably in inches, along the side of an object without using the dowel. This strategy did not actually benefit from the use of an unmarked dowel at all. However, because the problem-solving portion of the measurement estimation tasks did not require thinking out loud, we cannot distinguish whether or not participants understood how to use the dowel, only whether or not the participants did place the dowel along an object while appearing to measure a dimension.

**Behavior During Answer Explanations**

During post-hoc answer explanations, most participants used a verbal explanation for their measurement process “Other” than the predefined categories created for the coding guide. About a third of those explanations included or consisted of the participant counting out loud, without including units (i.e. “One, two...” rather than “One inch, two inches...”). Counting was followed by “Estimation” explanations in which participants identified a point of reference from which they made an estimate for a given dimension. Conversely, very few tasks were described as being completed using a proportionality strategy. Anecdotally, counting is the least sophisticated type of answer explanation whereas describing proportions is the most complex. From this logic, it follows that elementary students, with varying ranges of performance, would tend to use strategies such as counting, process of elimination, or estimation to explain their measurement processes rather than strategies indicating proportional reasoning.

The majority of elementary-aged children did use objects while gesturing to explain their measurement processes. While the definition of gesture presented in this thesis may be controversial, extending to actions which effect change on the environment, the prevalence of use of objects while gesturing by elementary-aged children suggests that permitting the use of objects during post-hoc answer explanations might lead to more organic behavior and insight into students’ understanding by allowing participants more freedom to express ideas. I believe this rationale is supported by the types of gesture most prevalently demonstrated by participants. Specifically, the majority of gestures were used to identify objects or demonstrate actions taken during the measurement process so perhaps the affordance of touching and holding the objects while gesturing help to relieve cognitive load. For instance, not all participants referred to the dowel or geometric objects by name, opting instead to hold up the object and say “this” in
reference. In the future, it would be interesting to compare the gestures observed surrounding measurement estimation tasks when participants are and are not permitted to use objects.

**Behavior Congruence**

Participants overall showed a congruence between their actions while completing measurement tasks, their explanations for their measurement process afterwards, and the gestures used to augment their explanations. Speech and gesture were congruent the majority of the time while action, speech, and gesture together only matched in about half of the tasks. This is similar to the findings from the College Study and support prior research providing evidence of a stronger connection between speech and gesture than the connection between either of those behaviors with action (Church et al., 2014; Kelly et al., 2015). Like the College Study, these findings suggest that the relations between action, speech, and gesture may be similar across speech production, comprehension, and mathematics tasks for communication (Church et al., 2014; Kelly et al., 2015). That said, the current work only investigated general patterns of behavior congruence without examining the relationship between behavior congruence and performance on measurement tasks. Chapter Seven will examine the relationship between behavior congruence and performance to discern any patterns of speech-gesture, action-speech, action-gesture, and action-speech-gesture between lower performing elementary-aged children and high-performing college students to provide insight about the relationships between action, speech, and gesture as they relate to measurement estimation tasks.

![Figure 6-2](image)  
*Figure 6-2. Participants’ behavior during problem-solving may have been influenced by researcher gestures.*

**Limitations**

This study was not without limitations. The study was conducted in a busy after school program which led to disruptions during data collection. The room was near the lobby which meant that the noise level fluctuated, sometimes resulting in video recordings where the participants’ voices were inaudible for periods of time. Some participants waited to participate in the same room in which the study was being conducted, distracting the current participant. And a couple participants did not complete the study due to parent pick-up.
During data collection, the researchers realized that the reference image given to children to remind them what the diameter of an object is was actually misleading. The image for diameter showed two intersecting arrows along the diameter of a circle (Appendix F) and some participants interpreted the image to mean that the diameter meant measuring the height and width of a sphere then adding those together. Also, during data collection, the researchers realized that the prompt for Task F, asking children to “estimate the length of the longest sides” of a prism (see Appendix G), confused participants. Many children focused on identifying which side of the prism they considered to have the longest length before being redirected to estimate that length in inches. The researchers also realized that the Likert scale used in the questionnaire was labeled in the opposite direction of the agreement scale used by participants to answer each question (see Appendices D and E). This required recoding by the researchers but ultimately did not affect the data in any way.

Another limitation of the study involves the researchers’ behavior. Gestures performed by the researcher may have influenced the participant’s behavior during problem solving. However, the researcher was not intentionally filmed (the camera was directed only on the participant) so it is unclear how often a researcher gestured in front of the participant during data collection. For instance, on the task “Estimate the combined length of the three cubes”, if the researcher ever pushed their hands together to represent “combined”, the participant may have been more inclined to push cubes together for one composite measurement rather than estimating the length of the three cubes individually then adding those values together. Similarly, if the researcher clarified what the diameter of a sphere is by referencing the widest points of the ball (Figure 6-2), the participant may have been more inclined to measure the diameter from side-to-side rather than the top-to-bottom of the sphere, which seems to be easier.

Applying the Embodied Cognitive Task Analysis Coding Guide, designed for the College Study, to the Elementary Study data worked well overall although the categories and codes were not exhaustive of elementary-aged participants’ behavior. Namely, over half of the verbal explanations were coded as “other” rather than one of the four other speech categories. Although the coders took notes about what that “other” speech was like, the lack of more exhaustive categories means a loss of information about how elementary-aged children describe their measurement estimation processes.

Lastly, no protocol was taken to assess the participants’ construct of an inch or a foot, or the participants’ level of seriousness during participation in the study. Participants were instructed to give their answer in inches and despite researcher clarification, some participants provided answers that may have been based on their idea of a foot rather than an inch. Other participants were extremely playful during the study and provided answers such as “One million” and “One hundred” and no protocol was in place to assess how sincere an answer might have been or how well a given answer represented a participant’s knowledge of conventional units of measurement.

Reflections on the Interview Process

This study was the first in our line of research on measurement strategies to conduct one-on-one interviews with children. As such, there were components of the study that were planned
and executed well as well as areas for improvement and consideration moving forward. I consider
the guidelines for conducting clinical interviews with children posed by Ginsburg (1997) to frame
how our interview process went with the elementary-aged children.

As Ginsburg (1997) discusses, interviews with children can provide a deep level of insight
to their knowledge and cognitive processes if their behavior is investigated thoroughly and
approached with the assumption that the children are acting with some degree of rationale even if
that rationale isn’t immediately clear to the interviewer. The role of the interviewer is active,
developing trust between interviewer and interviewee, and effectively responding to the
interviewee’s answers and questions, in order to develop rapport and draw out sincere answers
from the child (Ginsburg, 1997). To do so, Ginsburg (1997) highlights techniques for successful
interviews that are relevant to the presented study. Namely, our protocol effectively posed tasks
with slight variation in content and answer type, keeping children engaged. Students were asked
both open-ended and focused questions, followed by a non-leading, open-ended form of the
question, “How did you get that answer, what did you do?” In general, the researchers aimed to
engage the children and keep them feeling supported and encourage more verbalization. Our goal
was to generally identify characteristics of behavior surrounding measurement estimation tasks
and to do so, our research team prepared for the study and one-on-one interviews but did not
extensively cover relevant theory and normative behavior to look for beforehand. As, Ginsburg
(1997) highlights, “An open mind is not an empty mind.” If another study with one-on-one
interviews with children will be run with this line of research, adhering to this set of guidelines
and delving into the theory and normative behavior surrounding the development of measurement
understanding would be beneficial to the project.

Future Directions

This study served as one of two exploratory, foundational studies to explore the behavior
of high- and low-performing individuals from a college and an elementary population as
participants complete measurement estimation tasks. Moving forward, this project will integrate
findings from both studies (discussed in Chapter Seven) and use this information to create support
for elementary-aged children to develop and reinforce measurement estimation skills. Importantly,
from observing how some, but not all, students are able to exemplify foundational concepts of
measure (Lehrer, 2003), perhaps behaviors representative of those concepts should be targeted in
support for activities to reinforce measurement estimation skills. Such as hints targeted to support
the concept of additivity (Lehrer, 2003) with text such as “The dowel is 12 inches, right? You
could check your answer by seeing if the remaining length adds up to 12 inches.” This line of work
for continuation of the project will be explored in greater detail in Chapter Seven.

6.4 Conclusions and Contributions

This chapter has provided insight about how elementary-aged children complete
measurement estimation tasks, as well as how they describe their processes in speech and with
gesture. Namely, participants explained how they measured an object by simply counting or identifying a point of reference. To complement their explanations, participants primarily gestured with the objects available to them to identify relevant objects and demonstrate actions taken during measurement tasks. These behaviors may be precursors to more mature, efficient strategies for measurement estimation. These findings about the behavior of elementary-aged children surrounding measurement tasks will be compared to those of high-performing college students to identify behavioral differences in strategies and explanations surrounding measurement estimation tasks. Successful behaviors modeled by college students that are not observed by elementary-aged children will be used to create instruction and scaffolding for measurement estimation games to develop procedural and conceptual measurement estimation abilities of elementary students.
Chapter Seven: Identifying Age-Based Differences in Problem-Solving Behavior in Estimation Tasks

This chapter intends to synthesize the findings from the College Study and the Elementary Study, presented in the previous chapters, to explore age-based differences in performance and behavior on measurement estimation tasks. I will identify any shifts in action, language, and gesture depending on math experience (age), by comparing the data collected from the College Study to the data collected from the Elementary Study. This chapter will pose suggestions for behavior-guiding player support in EstimateIT!, a measurement estimation game, based on the findings of this project, as well as explore other future directions for this line of research. This chapter concludes with the major contributions of this thesis as well as overarching conclusions for consideration.

Identifying age-related differences in measurement estimation problem-solving behaviors will indicate potential gaps in conceptual and procedural knowledge of elementary, and/or low-performing, students who are just acquiring conceptual understandings of measurement. Conversely, comparing these behaviors to those of college students might reveal more mature, efficient behaviors and strategies for measurement that instructional support for elementary students could target in related activities. Isolating procedural and conceptual knowledge gaps through the actions demonstrated in problem-solving, language used by participants, and gestures performed by participants will then allow us to target specific points of instructional support through video and image scaffolding in active, technology-augmented math games. Specifically, this project allows us to ask how we can move elementary students towards more mature measurement estimation explanations and strategies as seen by high-performing college students.

I hypothesize that college students, with more years of experience and exposure to math, will demonstrate higher performance on hands-on geometry tasks than elementary students. I also hypothesize that that college students will demonstrate behavior distinct from elementary students, such as being more likely to demonstrate proportional reasoning through their actions, language, and gestures.

Specifically, I intend to explore whether we can infer any gaps in conceptual or procedural knowledge from the behavioral differences observed between college students and elementary-aged children by exploring:

1) How do problem-solving strategies, as observed through actions, shift between college students and elementary students?
2) How do problem-solving explanations shift between college students and elementary students?
3) How do the gestures used during problem-solving explanations shift between college students and elementary students?

From exploring these questions, the discussion considers how we can apply these findings and consider scaffolding techniques for measurement estimation activities that focus on physical
modeling of measurement estimation concepts to foster engagement and understanding in late elementary school geometry students.

7.1 Approach to Analyses

Using the datasets described in previous chapters from the College Study (N=32 participants; N=445 tasks) and the Elementary Study (N=30 participants; N=214 tasks), I will apply descriptive statistics and basic statistical analyses to explore differences in performance between each population and then explore each of the research questions posed above that delve into differences in behavior between college students and elementary-aged children.

7.2 Results

Differences in Performance

Performance was calculated with the same formula for participants in the College Study and the Elementary Study. Specifically, we calculated overall accuracy on the tasks as the average absolute deviation from accuracy on each task, recorded as a percentage. The percent error for each free-response measurement task was calculated as:

\[
\% \text{ Error} = \left| \frac{\text{Correct Answer} - \text{Given Answer}}{\text{Correct Answer}} \right|
\]

Accuracy for multiple choice tasks (i.e. “Choose which object…””) was considered binary right/wrong. Because a small minority of students in the Elementary Study did not always provide a serious answer, a floor (0%) and ceiling (100%) were set for accuracy on individual tasks by capping incorrect responses at 100% error rate by this metric (given answer * 2). Average participant accuracy was calculated as:

\[
\text{Accuracy} = 1 - \frac{\sum \% \text{Error Free Response Tasks} + \sum \text{Incorrect Multiple Choice Tasks}}{\text{Count (All Tasks Completed)}}
\]

An independent samples t-test revealed that college students \((M=92.52\%, SD=6.00\%)\) performed significantly higher than elementary-aged children \((M=64.13\%, SD=22.90\%)\), \(t(61)=6.83, p<0.001\).

Differences in Problem-Solving Actions

The table below (Table 7-1) presents the percentage of tasks in the College Study (N=448) and the Elementary Study (N=214) that were completed using each of these predefined measurement estimation strategies. The table is presented in ascending order from the least sophisticated strategy (eyeballing) to the most sophisticated strategy for measurement estimation (Part-Part-Whole: Multiple References). Each of the Part-Part-Whole measurement strategies indicate what appears to be the use of proportional reasoning by the participant using subsequent
measurement with the unmarked reference tool or object in question. Interestingly, both college and elementary students most often applied an “External Reference” strategy for measurement by placing a reference tool against the object of measurement just once before making an estimate. However, college and elementary students differ in the use of other strategies. Whereas college students were more likely to apply a proportional reasoning strategy, elementary students were more likely to rely on eyeballing and very rarely applied a proportional reasoning approach to measurement estimation.

### Table 7-1. Comparison of Measurement Strategies Displayed.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
<th>College: % Tasks Completed Using Strategy</th>
<th>Elementary: % Tasks Completed Using Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eyeballing</td>
<td>Participant uses no tools or strategies to inform estimate</td>
<td>12.1%</td>
<td>18.7%</td>
</tr>
<tr>
<td>External Reference</td>
<td>Participant uses one tool one time to inform estimate</td>
<td>49.4%</td>
<td>73.4%</td>
</tr>
<tr>
<td>Part-Part-Whole: Whole = Tool</td>
<td>Participant measures number of times the object fits into the tool</td>
<td>9.2%</td>
<td>0%</td>
</tr>
<tr>
<td>Part-Part-Whole: Whole = Object</td>
<td>Participant measures the number of times the tool fits into the object</td>
<td>19.6%</td>
<td>6.5%</td>
</tr>
<tr>
<td>Multiple References</td>
<td>Participant uses two or more tools for measurement</td>
<td>4.3%</td>
<td>.5%</td>
</tr>
</tbody>
</table>

Interestingly, the measurement techniques identified in the Embodied Cognitive Task coding guide were applied in similar patterns across both populations. In roughly 80% of the total tasks, participants did use the provided, unmarked dowel as a measurement tool. And in more than half of the measurement estimation tasks, participants demonstrated marking a start-point for measurement, such as lining up the edge of an object and the edge of the dowel along a table. Table 7-2, below, highlights the specific measurement techniques applied to all of the measurement estimation tasks in both the college population and the elementary population.

### Table 7-2. Comparing measurement techniques applied across all tasks.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Description</th>
<th>College: Used in Percent of Tasks*</th>
<th>Elementary: Used in Percent of Tasks*</th>
</tr>
</thead>
</table>

87
Differences in Answer Explanations

College and elementary participants demonstrated remarkably different verbal strategies to explain their measurement processes for each task (Table 7-3). Notably, college students most frequently vocalized using a reference point such as a finger, the length of the dowel, or another object, to inform their estimates for a given task. Elementary-aged children, on the other hand, used strategies unclassified by the Embodied Cognitive Task Analysis Coding Guide in 56.5% of the total tasks. Of these 121 instances of an “Other” verbal strategy coded within the elementary student tasks, 48 tasks were described as the participant counting to describe their measurement
process without using units while other descriptions included explanations that referenced previous measurements, used process of elimination, etc. After the “Other” category, participants most frequently used the “Estimation” strategy of identifying a point of reference used to make a judgment about an object’s dimension, like the college students. Anecdotally, it would appear that whereas elementary students tended to rely on counting to demonstrate their measurement processes, college students leaned towards using proportional reasoning strategies to explain their rationale for each estimate.

**Table 7-3. Observed explanations provided by participants after each task.**

<table>
<thead>
<tr>
<th>Verbal Strategy</th>
<th>Description</th>
<th>College: Used in Percent of Tasks</th>
<th>Elementary: Used in Percent of Tasks</th>
</tr>
</thead>
</table>
| Estimation        | The participant identifies a point of reference that informed their answer.  
“I tried to guess where it would be six. And then move it up and see where it would be like one to six and this one looks like it seemed around a four mark.” | 41.6%                             | 19.2%                                |
| Proportion of Object | The participant compares the size of the object to the size of the reference tool.  
“The large purple cylinder is two dowel lengths so twenty-four” | 19.6%                             | 2.3%                                 |
| Proportion of Tool | The participant compares the size of the measurement tool to the size of the object.  
“Just using the 12-inch dowel and measuring how many of them fit along each block.”  | 18.2%                             | 2.3%                                 |
| Other             | The participant uses a strategy not specified by the coding guide such as process of elimination or referencing a measurement from a previous task.  
“...to my memory, this was twenty-seven inches”  | 15.8%                             | 56.5%                                |
| Guessing          | The participant provides an answer without a clear strategy.  
“I’m just eyeballing it, I’m just guessing”  | 4.5%                              | 9.3%                                 |

**Differences in Gestures Produced During Answer Explanations**

An independent samples t-test revealed that elementary students ($M=2.57$ gestures, $SD=2.58$) produced more gestures in each answer explanation than did college students ($M=2.27$ gestures, $SD=2.13$).
gestures, $SD=1.55$), $t(619)=-1.86$, $p<0.001$. Table 7-4, below, highlights the frequency of gesture types produced by both populations across all tasks. Notably, college students and elementary students followed the same pattern for types of gesture use. Most often, participants produced deictic gestures to identify an object used in speech as well as kinetographic gestures to depict a motion or action. Conversely, only in a few cases did participants produce pictographic gestures by outlining any shapes or objects in the air.

Table 7-4. Comparing Gesture Types Observed by College and Elementary Students.

<table>
<thead>
<tr>
<th>Gesture Type</th>
<th>Description</th>
<th>College: Percent of Total Gestures</th>
<th>Elementary: Percent of Total Gestures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deictic</td>
<td>Gestures that indicate objects, people and locations (McNeill, 1992), including pointing, showing an object, or reaching for something (Iverson et al., 2008) and extending to picking up an object as well as identifying a trait of an object</td>
<td>40.3%</td>
<td>31.9%</td>
</tr>
<tr>
<td>Iconic: Kinetographic</td>
<td>Gestures that indicate bodily actions (Ekman &amp; Friesen, 1969), usually corresponding with dynamic speech such as “move”, “flip”, or “balance”</td>
<td>28.7%</td>
<td>27.9%</td>
</tr>
<tr>
<td>Iconic: Spatial</td>
<td>Gestures that depict spatial relations (Ekman &amp; Friesen, 1969)</td>
<td>21.5%</td>
<td>24.4%</td>
</tr>
<tr>
<td>Metaphoric</td>
<td>Gestures that occur when an individual creates a physical representation of an abstract idea or concept (Andric &amp; Small, 2012), including dimensions such as length, width, or height</td>
<td>7.4%</td>
<td>10.1%</td>
</tr>
<tr>
<td>Iconic: Pictographic</td>
<td>Gestures used to draw a referent object in the air (Ekman &amp; Friesen, 1969)</td>
<td>1.4%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Unsure/Out of view</td>
<td>Gestures that are present in the video but largely out of view</td>
<td>0.8%</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

7.3 Discussion

Exploring the performance, and behavior, of college and elementary students surrounding measurement estimation tasks revealed differences in both performance and participants’ actions, language and gestures. My hypothesis, that college students would demonstrate higher
performance on measurement estimation tasks than elementary students, was supported by an independent samples t-test. Additionally, my hypothesis that college students would demonstrate different behavior than elementary students through their actions, language, and gestures, seems to be supported by qualitatively comparing behavior across both populations.

**Performance**

By confirming that college students, with years more of experience and expertise than elementary-aged students, perform more accurately on measurement estimation tasks, it follows that the behavior of college students is likely more mature and sophisticated than the behaviors of elementary students. Consequently, the behaviors observed by college students could be used to identify effective strategies and behaviors that should be taught to, and practiced by, elementary students to reinforce concepts of measurement and help those students develop more effective strategies for measurement estimation.

**Behavior During Measurement Tasks**

While creating the coding guide that was used to identify behaviors in these measurement tasks, the measurement strategies were anecdotally considered to have a hierarchical structure in terms of rigor and sophistication. Eyeballing was considered to be the least sophisticated technique for measurement estimation, followed by the external reference strategy, while the part-part-whole (proportionality) techniques were considered the most sophisticated. Particularly, we considered the use of multiple tools to apply proportional reasoning to measurement to be the most intensive strategy for measurement estimation.

College students and elementary-aged children most often applied the same measurement strategy of using an “External Reference” of one tool, just once, to inform judgments about the length, width, height, or diameter of an object. While it was not surprising to find that elementary students tended to use a less sophisticated method of measurement estimation, it was surprising that college students were more likely to use this strategy over proportional reasoning, as that would be a more efficient, and likely more accurate, strategy. However, this finding could be explained by the dimensions of the objects being measured. Most of the objects of measurement were under 12 inches and as such, proportional reasoning might have taken more effort to apply well. Similarly, the college students might have applied measurement estimation strategies that matched the importance of the situation. In other words, perhaps college students are all (or mostly) able to apply proportional reasoning to measurement estimation tasks but for a brief study that awards a credit towards a class, perhaps using the “External Reference” strategy for a quick judgment seemed more worthwhile than taking the time to apply proportional reasoning.

Interestingly, no elementary students ever applied a strategy of “Part-Part-Whole: Whole = Tool” to measure objects smaller than the dowel. Perhaps applying proportional reasoning with the reference tool (12-inch dowel) and larger objects is a precursor to considering the dowel as a whole and using the object of measurement as a unit.
Similarly, students appeared to use the dowel as a measurement tool in the majority of tasks just like their college student counterparts. The elementary students appeared excited by the prospect of having a prop to hold in general and may not have actually used the dowel as a measurement tool. This possibility is supported by the instances of elementary students counting along the edge of an object while holding the dowel against the object but never actually using it for reference. However, by placing the dowel against the objects of measurement, elementary students demonstrated an understanding of being able to use an unmarked tool for reference, even if they were unable to integrate the unmarked tool into their measurement processes. This highlights the gap between physical measurement and applying a conceptual understanding of measurement to complete measurement estimation tasks. And elementary students, like college students, tended to use start-point markers more often than not which suggests that participants understood at least a precursor to the concept of tiling, which is the understanding of successive measurement without gaps between units (Lehrer, 2003).

Some of the measurement estimation tasks lend themselves to some of the strategies and techniques more than others. For instance, some objects, such as the two-foot cylinder, allow the 12-inch dowel to neatly fit twice along the height of the cylinder, making it more appropriate for a proportional reasoning strategy. Conversely, the task of estimating the combined length of three cubes measures out to 14-inches. Because that measurement is so close to the length of the dowel, perhaps the external reference tool strategy is more appropriate and efficient than attempting to apply proportional reasoning. Similarly, for tasks with object dimensions less than the length of the dowel, the use of a placeholder in measurement would be unnecessary, potentially explaining why the technique was not used often by either population.

Additionally, the coding does not account for how well each technique was applied. For instance, “Placeholder” was labeled if the participant clearly marked a spot in between subsequent measurements with a tool. However, no code was applied for whether the participant precisely lined up subsequent measurement segments (demonstrating tiling; see Lehrer, 2003) or left gaps between the first measurement and second measurement. Because the use of measurement techniques follows the same pattern of frequency between college students and elementary-aged children, I am inclined to think that these findings may be more indicative of the techniques appropriate to each measurement task rather than differences in procedural techniques between older and younger students.

Differences in Answer Explanations

Anecdotally observing shifts in behavior between elementary and college students, there appeared to be the most substantial differences in the answer explanations used by participants. Whereas college students typically used an “Estimation” explanation or proportional reasoning, elementary students most often provided explanations not captured by the ECTA coding guide such as counting along an object. The coding guide was iteratively refined using the video data from the College Study which explains why explanations such as counting and process of elimination were not taken into consideration as unique categories.
The prevalence of counting explanations offered by elementary students, in contrast to the proportional reasoning demonstrated by college students, could suggest that counting, without using units, is a precursor to understanding and applying measurement concepts such as proportional reasoning which are ultimately more efficient approaches to measurement estimation. Categories of answer explanations based on those most used by elementary-aged children should be incorporated into the coding guide for future use.

**Differences in Gestures Produced During Answer Explanations**

Elementary students tended to gesture more on each measurement estimation task than the college students. That said, both populations averaged 2-3 gestures per explanation while the standard deviation was higher for elementary students. With further analyses, I would suspect that the number of gestures used by students in answer explanations is directly tied to overall performance or even performance on a given task. Specifically, I suspect that higher-performing students tend to gesture less to explain measurement processes than do lower-performing students. This hypothesis should be investigated in coming iterations of this work.

Observing the types of gestures demonstrated by participants revealed a similar breakdown of gesture types by both college and elementary students, with deictic gestures used most often and pictographic gestures used the least. This suggests the possibility that types of gestures used to indicate a problem-solving process could be related to the task at hand rather than differences in age or performance. For instance, pictographic gestures would be the least relevant to measurement estimation tasks, especially given that participants were still able to use the objects during their answer explanations rather than needing to outline the geometric objects or tools. Instead, participants were recreating actions taken in specific spatial locations along objects. The Gesture as a Simulated Action (GSA) framework (Alibali & Hostetter, 2008) posits that the likelihood of gesturing increases as action simulation, and the use of spatial imagery, increases. Stemming from that framework, perhaps the types of gestures used are also related to the amount of action simulation involved in thought and speech surrounding gestures.

Investigating the differences in gesture use and types of gestures demonstrated by college and elementary students suggests overall that there are shifts towards using fewer gestures with age, possibly as students accrue an understanding of measurement and experience with measurement estimation. However, there do not appear to be any shifts across age groups between the types of gestures used to explain measurement estimation processes, suggesting that the use of gestures may vary by age and performance whereas the types of gestures are more closely related to a given situation.

7.4 Lessons Learned: Limitations of the Project

While this project was intended to be exploratory and qualitative in nature, there are still limitations to the overall project. Many of the limitations of each individual study have been
discussed in the previous chapters although many of these limitations are overlapping for both studies. These limitations are discussed below.

Most importantly, this project was exploratory and observed the behaviors of college students and elementary-aged children during measurement estimation tasks. To the best of my knowledge, this is new territory for research surrounding mathematics learning, measurement, and embodied cognition. Consequently, our team developed terms and constructs for the physical and verbal behavior observed by participants. The behaviors coded may not be categorized well or representative of all important behaviors related to measurement estimation across age groups. For instance, many elementary-aged children explained their measurement process as “counting” although the coding guide was iteratively refined while exploring the behavior of college students, almost none of whom described their strategy as counting. Future studies would benefit from reconsidering behaviors to look for during measurement estimation tasks.

Future conversations about the study of gesture would also benefit from a discussion about the affordances and challenges of using speech to code gestures. The constructs modified and refined in this project largely relied on the language surrounding a given gesture in order to classify that gesture type. For instance, if a participant pointed at the end of the dowel, that gesture could have been labeled *deictic* if the participant was merely identifying the dowel, or it could have been labeled as a *spatial* gesture if the participant was referring to lining up the *end* of the dowel to another object. While the use of speech made labeling gestures easier, this method also prevents our research team from being able to dig into speech and gesture congruence on specific tasks.

A few other limitations of the project include the tedious nature of the project, materials, and procedure. As this project relied on the development, refinement, and application of a coding guide, this project was very time-consuming, tedious, and despite hours of coder training, relying on coded video data inevitably leaves the chance of human (coding) error. In terms of materials, some of the objects used for measurement tasks in both studies were used in more than one task. For instance, college participants were asked to measure the height of a cylinder and later, to measure the combined height of that cylinder with another cylinder (see Appendix A). From using the same objects in multiple tasks, participants may have relied on previous answers rather than applying a more thoughtful measurement strategy. Because of this, as well as differences in participant effort in general, the measurement strategies and explanations observed may not be entirely indicative of participants’ understanding of measurement. Lastly, this project demonstrated the value of good camera use during data collection. Some data was lost due to the camera view being obscured by the participant or objects on the table and some data was lost due to the camera battery draining during data collection. Future studies related to the observation of student behavior during measurement estimation tasks would benefit from time dedicated to finding an optimal spot for camera placement as well as charging protocol for all researchers. Since this project was intended to be exploratory and serve as a foundation for developing support for a measurement estimation game, we consider the limitations mentioned to be lessons learned.

7.5 Application of Findings: Developing Support for a Measurement Game
Vygotsky (1978) once said that "children can imitate a variety of actions that go beyond the limits of their capabilities". With this theme in mind, we can use the conceptual and procedural shortcomings of elementary students that identified in the discussed studies to propose action and gesture-based video and image tutorials and hints for *EstimateIT!*, a geometry scavenger hunt for elementary students played on cell phones. Players complete measurement estimation tasks that map onto the estimation tasks completed in the college and elementary studies. Currently, players may request text hints for help during tasks in the game, but these hints could be developed further to be more effective by targeting specific aspects of measurement concepts and skills, as well as utilizing action and gestures in image and video hints to model behavior for players.

To design hints that target specific actions for the act of measuring, as well as reinforcing a broader understanding of measurement, the hints will be modeled after specific concepts of measurement. Concepts of measurement is a broad term that does not inherently suggest what kinds of conceptual understanding children need to develop in order to understand measurement. However, Lehrer (2003) posed eight conceptual foundations for measurement (referred to as *measure*): unit-attribute relations, iteration, tiling, identical units, standardization, proportionality, additivity, and origin (zero-point). These facets of measurement can be considered specific areas of measurement to focus on for elementary-aged players.

<table>
<thead>
<tr>
<th>Lehrer’s (2003) Concept of Measure</th>
<th>Description</th>
<th>Proposed <em>EstimateIT!</em> Support Text</th>
<th>Proposed <em>EstimateIT!</em> Support Image</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit-Attribute Relations</strong></td>
<td>Measuring should be done with an appropriate unit for the dimension.</td>
<td>Since we are measuring small objects, we are measuring in inches.</td>
<td>An image of a finger and thumb creating an inch-long segment</td>
</tr>
<tr>
<td><strong>Iteration</strong></td>
<td>A length can be measured as multiple units from a start- to end-point.</td>
<td>Remember, you can measure an object by seeing how many times it fits along the stick.</td>
<td>A gif of a six-inch cube being moved along the dowel twice for measurement</td>
</tr>
<tr>
<td><strong>Tiling</strong></td>
<td>Measuring can be done successively but without any gaps between units.</td>
<td>When you measure an object, line up your stick directly against the edge of the object without any space between.</td>
<td>An image of a dowel next to an object with a hand showing no space between the dowel and object</td>
</tr>
<tr>
<td><strong>Identical Units</strong></td>
<td>One unit should be consistent so that measuring could be done by counting that unit.</td>
<td>Your knuckle is about an inch long. An inch is always the same length.</td>
<td>An image of two knuckles stacked next to a ruler</td>
</tr>
<tr>
<td><strong>Standardization</strong></td>
<td>Conventional units are standard and allow communication about measurement.</td>
<td>Remember, your stick is twelve inches which is equal to one foot.</td>
<td>An image of hands facing each other, creating a segment of a foot over a dowel</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Proportionality</strong></td>
<td>Different-sized units can be used to represent the same quantity.</td>
<td>If your stick is 12 inches long, each half of your stick is 6 inches.</td>
<td>An image of a flat hand “slicing” the middle of the dowel</td>
</tr>
<tr>
<td><strong>Additivity</strong></td>
<td>A total distance is equivalent to the sum of two segments along that plane</td>
<td>You are looking for a prism 11 inches long. That means it should be one inch shorter than your 12-inch stick.</td>
<td>An image of a finger and thumb creating an inch-long segment along the edge of a dowel</td>
</tr>
<tr>
<td><strong>Origin (Zero-Point)</strong></td>
<td>Measurement is done on a scale with each point equal distance from one another.</td>
<td>When you measure, remember to start at zero.</td>
<td>An image of a finger pointing to the bottom of an object, labeling that point as “0”</td>
</tr>
</tbody>
</table>

During the College Study, participants wrote “Helpful Hints” for another student who might be struggling with the measurement task at hand. These hints by college students were originally intended to be used for the construction of support messages. However, not all participants took this activity seriously and referenced a previously written hint for multiple tasks such as “see the one above”. So instead of relying on hints written by college participants, the proposed hints above (Table 7-1) were constructed based on common misconceptions and lack of procedural skills exhibited by elementary students. The proposed images and support message wording were based on actions and gestures demonstrated by college students during completion of measurement estimation tasks as well as their answer explanations. See Figure 7-1, below, for examples of foundational concepts not yet acquired by elementary students but nicely demonstrated by college students. These gestures by college students used to effectively communicate their understanding of measurement have since been used to create action- and gesture-based images for support messages (Valente, 2019).
<table>
<thead>
<tr>
<th>Foundational Concept of Measure (Lehrer, 2003)</th>
<th>Struggling Elementary Student</th>
<th>Proficient College Student</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proportionality</strong> - The elementary student counted the length of the dowel rather than deducing that if the dowel fits along the cylinder twice, the cylinder must be 24 inches in height. Conversely, the college student on the right demonstrates an understanding of using proportionality for measurement.</td>
<td>“Thirty-seven… [I] Counted. Until I got to the top.”</td>
<td>“This would be the six mark”</td>
</tr>
<tr>
<td><strong>Standardization</strong> - The elementary student did not adhere to the conventional unit for inch. Instead, he counted using segments of roughly four inches.</td>
<td>“Six inches… this was small so I counted big”</td>
<td>“I kinda know how much an inch is with my hand”</td>
</tr>
<tr>
<td><strong>Identical Units</strong> - The elementary student showed her counting process moving up the dowel. She first marked “one” about half an inch above the bottom of the dowel, then marked “two” a little higher, and then moved her finger to the current position to mark both “three” and “four” inches in the same location.</td>
<td>“I like put it right there and then I did one, two, three four.”</td>
<td>“I tried stacking, what I thought was an inch, up”</td>
</tr>
</tbody>
</table>

*Figure 7-1. Select examples of foundational concepts of measurement not yet attained by elementary students but demonstrated through gesture by college students.*
I proposed support messages and image ideas such as the ones above (Table 7-6) to be implemented and tested in *EstimateIT!* as part of an undergraduate Individual Qualifying Project at Worcester Polytechnic Institute (Valente, 2019). From these proposed messages and image ideas, Valente (2019) created and programmed support messages into *EstimateIT!* (Figure 7-2). Some of the messages with supplemental images will be shown as players enter the game, prior to gameplay, while other support messages will be developed and used as hints for specific measurement estimation tasks.

![Figure 7-2. A support message and image in EstimateIT! to reinforce the concept of tiling.](image)

### 7.6 Future Directions

The progress of this project, as well as the massive repository of data collected, suggests next steps for the continuation of this research as well as multiple options to explore related questions. First, the proposed support messages should be iteratively refined and tested for their effectiveness in *EstimateIT!* The aforementioned undergraduate project will implement the first version of support messages then test the effectiveness of this support for elementary-aged children. Specifically, elementary-aged participants will play *EstimateIT!*, with half of the players seeing the original, text-based hints upon request. The other half of players will see support messages with action- and gesture-based images prior to gameplay as well as hints after inputting incorrect answers during the game. The effectiveness of the newly proposed support messages will be assessed by looking at player performance during *EstimateIT!* as well as performance on a pre-
and post-test adapted from prior research on the game, *EstimateIT!* (Rountree, 2015). After this study, support messages with supplemental images should be reassessed and refined as needed.

To study the effectiveness of modeling concepts of measurement for elementary-aged children through action- and gesture-based hints, research should be done to examine differences in learning and performance in *EstimateIT!* between players who receive newly designed support messages with supplemental images and players who receive newly designed support messages that are solely text, without the action- and gesture-based images. This study could begin to investigate the potential benefits of modeling behavior and concepts of measurement over providing text support. Similarly, the behavior observed by children in the Elementary Study suggests that there is a range of knowledge among elementary-aged children for the foundational concepts of measure outlined by Lehrer (2003). To teach and reinforce conceptual knowledge of measurement, *EstimateIT!* may be more effectively used when paired with a short lecture or guided discussion surrounding gameplay. The lecture could focus on the foundations of measure (Lehrer, 2003) and lead students in instructed gesture prior to game play since performing instructed gestures has shown to increase learning in mathematics (Cook et al., 2008). Cook and colleagues (2008) focused on instructed gestures to complete mathematical equations although this work has not been extended to explore how instructed gestures affect performance and learning for measurement estimation. In sum, future work should consider how to effectively implement *EstimateIT!* in classrooms by creating a supplementary activity to be used before or after gameplay to provide teachers with a well-rounded activity for reinforcing measurement estimation skills.

In parallel to studying student behavior and learning with the new supports for *EstimateIT!* the coding guide should also be further refined to be more appropriate for classifying the behavior of elementary students. For instance, “Counting” and “Process of elimination” were not considered unique categories of verbal explanations to measurement tasks, however, a substantial portion of elementary students vocalized those strategies for measurement. Another round of editing and refining the coding guide based on the findings from the College and Elementary Studies will result in a more precise and applicable tool for observing students’ behavior.

A major benefit of this project is the plethora of coded video data that is available to explore related research questions around student behavior and language in the context of measurement estimation tasks. This project has already afforded data to explore differences between the use of static and dynamic gestures among college and elementary students (Smith, Harrison, Ottmar, & Arroyo, 2019) and leaves related questions to explore through the data collected and coded to see how findings on student gestures and behavior in other areas of mathematics, such as explaining geometry proofs or completing mathematical equations, extend to measurement estimation. In sum, this project has provided foundational work to set up the continued exploration of learners’ behavior as it pertains to acquiring measurement understanding and skills.
7.7 Conclusions and Contributions

This chapter presented a comparison of the performance and behavior between college students and elementary-aged children as they completed measurement estimation tasks as well as suggestions for future research and instructional support to help elementary students develop an understanding of measurement. The findings indicated that college students, with years more of experience with measurement estimation in everyday life, performed higher, on average, than did elementary-aged children on measurement estimation tasks. To glean procedural and conceptual differences between age groups, differences in participant actions, language, and gestures were qualitatively explored. Namely, college students tend to exhibit actions and verbal explanations of measurement processes that suggest more proportional reasoning. Elementary students, conversely, displayed behavior more representative of lower-level thinking such as counting without the use of units. By comparing the behaviors of each age group, our team successfully mapped actions, language, and gestures to different concepts of measure (Lehrer, 2003). These conceptual foundations of measurement will be used to create behavior-guiding support for elementary children as they play EstimateIT!, a measurement estimation game, to develop measurement estimation understanding and skills.

This chapter contributes to the larger thesis as a whole. Overall, this body of work has presented contributions to educational research with implications from theory, research, and practice. Namely, Part One strived to define gesture and provide a synthesis of major research surrounding gestures, with a call to consider the development of a common classification system to be used across research teams and projects. To that end, I proposed a coding guide to classify the action, language, and gestures of mathematics learners. While this coding guide was designed for measurement estimation tasks, it could be readily modified for more generalizable use, particularly with the gesture classification system defined by a compilation of previous work on gesture analysis. The second part of this thesis applied the theoretical framework and coding guide to explore the actions, language and gestures of both college and elementary-aged students as they completed measurement estimation tasks in a very exploratory project. Ultimately, this work provided insights about gaps in procedural and conceptual knowledge among elementary students that can be reinforced through theory-driven, behavior-guiding supports in measurement estimation games. Ultimately, the work presented in this thesis suggests that the behaviors of experienced college students can serve as models of procedural strategies to teach younger students through practice-based activities in order to strengthen their conceptual understandings of measurement. Moving forward, this work lays a foundation from which to explore the behavior of learners during measurement estimation tasks with suggestions for continued research and instructional support to apply these findings in practice.
References


Smith, H., Harrison, A., Ottmar, E., & Arroyo, I. (June, 2019). Quantity and quality of gestures are related to performance on an embodied geometric estimation task. Poster to be presented at the 2019 meeting of the Mathematical Cognition and Learning Society in Ottawa, Canada.


# Appendices

## Appendix A. College Estimation Tasks

<table>
<thead>
<tr>
<th>Coding</th>
<th>Task; Verbal Instruction</th>
<th>Shapes to Use</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Estimate the length of the cylinder</td>
<td>Big cylinder</td>
<td>24”</td>
</tr>
<tr>
<td>B</td>
<td>Estimate the diameter of the sphere</td>
<td>Small sphere</td>
<td>7 ¾ “</td>
</tr>
<tr>
<td>C</td>
<td>Pick the cube that seems closest to having a 6” long face</td>
<td>Three cubes</td>
<td>Big cube</td>
</tr>
<tr>
<td>D</td>
<td>Estimate the combined length of the three cubes</td>
<td>Three cubes</td>
<td>14”</td>
</tr>
<tr>
<td>E</td>
<td>Which rectangular prism has a side (with tag) that is 4” long?</td>
<td>Blue, green, grey</td>
<td>Grey</td>
</tr>
<tr>
<td>F</td>
<td>Estimate the length of the longest sides</td>
<td>Grey prism</td>
<td>8”</td>
</tr>
<tr>
<td>G</td>
<td>Estimate the height of the cylinder</td>
<td>Candle cylinder</td>
<td>8”</td>
</tr>
<tr>
<td>H</td>
<td>Estimate the height of the two cylinders, stacked</td>
<td>8” and 24”</td>
<td>32”</td>
</tr>
<tr>
<td>I</td>
<td>Estimate the diameter of the sphere</td>
<td>Big sphere</td>
<td>21”</td>
</tr>
<tr>
<td>J</td>
<td>Pick the cube that seems closest to having a 4” long face</td>
<td>Three cubes</td>
<td>Medium</td>
</tr>
<tr>
<td>K</td>
<td>Estimate the length of the faces on the two other cubes</td>
<td>Small/big cube</td>
<td>Small: 2 ¾ “</td>
</tr>
<tr>
<td>L</td>
<td>Estimate the height of these 3 rectangular prisms with the tags on the side</td>
<td>Three prisms</td>
<td>20 ¾ “</td>
</tr>
<tr>
<td>M</td>
<td>Estimate the height of the short side (with tags) on the green prism</td>
<td>Green prism</td>
<td>3.5”</td>
</tr>
<tr>
<td>N</td>
<td>Estimate the height of this cylinder</td>
<td>Short cylinder</td>
<td>4”</td>
</tr>
<tr>
<td>O</td>
<td>Estimate the diameter of this cylinder</td>
<td>Short cylinder</td>
<td>2 5/8”</td>
</tr>
<tr>
<td>P</td>
<td>Estimate the diameter of the sphere</td>
<td>Basketball</td>
<td>9.5”</td>
</tr>
<tr>
<td>Task Code</td>
<td>Task: Verbal Instruction</td>
<td>Shapes to Use</td>
<td>Answer</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------</td>
<td>---------------</td>
<td>--------</td>
</tr>
<tr>
<td>Q</td>
<td>Estimate the diameter of the sphere</td>
<td>Basketball</td>
<td>9.5”</td>
</tr>
<tr>
<td>O</td>
<td>Estimate the height of this cylinder</td>
<td>Short cylinder</td>
<td>4”</td>
</tr>
<tr>
<td>P</td>
<td>Estimate the diameter of this cylinder</td>
<td>Short cylinder</td>
<td>2 5/8”</td>
</tr>
<tr>
<td>M</td>
<td>Estimate the height of these 3 rectangular prisms with the tags on the side</td>
<td>Three prisms</td>
<td>20 ¾ “</td>
</tr>
<tr>
<td>N</td>
<td>Estimate the height of the short side (with tags) on the green prism</td>
<td>Green prism</td>
<td>3.5”</td>
</tr>
<tr>
<td>J</td>
<td>Pick the cube that seems closest to having a 4” long face</td>
<td>Three cubes</td>
<td>Medium</td>
</tr>
<tr>
<td>K</td>
<td>Estimate the length of the faces on the two other cubes</td>
<td>Small/big cube</td>
<td>Small: 2 ¼ “</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
<td>Big: 6”</td>
</tr>
<tr>
<td>I</td>
<td>Estimate the diameter of the sphere</td>
<td>Big sphere</td>
<td>21”</td>
</tr>
<tr>
<td>G</td>
<td>Estimate the height of the cylinder</td>
<td>Candle cylinder</td>
<td>8”</td>
</tr>
<tr>
<td>H</td>
<td>Estimate the height of the two cylinders, stacked</td>
<td>8” and 24”</td>
<td>32”</td>
</tr>
<tr>
<td>E</td>
<td>Which rectangular prism has a side (with tag) that is 4” high?</td>
<td>Blue, green, grey</td>
<td>Grey</td>
</tr>
<tr>
<td>F</td>
<td>Estimate the length of the longest sides</td>
<td>Grey prism</td>
<td>8”</td>
</tr>
<tr>
<td>C</td>
<td>Pick the cube that seems closest to having a 6” long face</td>
<td>Three cubes</td>
<td>Big cube</td>
</tr>
<tr>
<td>D</td>
<td>Estimate the combined length of the three cubes</td>
<td>Three cubes</td>
<td>14”</td>
</tr>
<tr>
<td>B</td>
<td>Estimate the diameter of the sphere</td>
<td>Small sphere</td>
<td>7 ¾ “</td>
</tr>
<tr>
<td>A</td>
<td>Estimate the length of the cylinder</td>
<td>Big cylinder</td>
<td>24”</td>
</tr>
</tbody>
</table>
Appendix B. Helpful Hints Worksheet

Please write a phrase or short instruction for a student who might be struggling with each task that you complete. If you were helping another student, what would you tell them to do?

1. ____________________________________________________________

2. ____________________________________________________________

3. ____________________________________________________________

4. ____________________________________________________________

5. ____________________________________________________________

6. ____________________________________________________________

7. ____________________________________________________________

8. ____________________________________________________________

9. ____________________________________________________________

10. ____________________________________________________________
Appendix C. Questionnaire for College Study

Please answer the following questions about yourself.

1. Gender: 
   Male     Female     Other: _______________________

2. Age: _______________________

3. Year in school: 
   Freshman     Sophomore     Junior     Senior     Other

4. What is your dominant hand? 
   Left     Right     Both (ambidextrous)

5. Major(s): 
   ___________________________________________ _____________________

6. Minor(s): __________________________

7. Compared to most of your other school subjects, how good are you in math? 
   1 2 3 4 5 6 7 
   Not at all good     Very good

8. How good would you be at learning something new in math? 
   1 2 3 4 5 6 7 
   Not at all good     Very good

9. In general, I find working on math activities… 
   1 2 3 4 5 6 7 
   Very boring     Very interesting

10. How much do you like doing math? 
    1 2 3 4 5 6 7 
    Not at all     Very much

11. In general, how useful is what you learn in math? 
    1 2 3 4 5 6 7 
    Not at all useful     Very useful

12. For me, being good in math is… 
    1 2 3 4 5 6 7 
    Not at all important     Very important
Appendix D. Questionnaire for Elementary Study

Ask the participant to, “Please answer the following questions about yourself by using these smiley faces. Which one suits you the most?”

1. Are you left-handed or right-handed?
   Left       Right       Both (ambidextrous)

2. What is your favorite subject? _____________________________

3. How are you feeling today?
   1  2  3  4  5  6  7
   Not at all good  Very good

4. Compared to most of your other school subjects, how good are you in math?
   1  2  3  4  5  6  7
   Not at all good  Very good

5. How good would you be at learning something new in math?
   1  2  3  4  5  6  7
   Not at all good  Very good

6. In general, I find working on math activities…
   1  2  3  4  5  6  7
   Very boring     Very interesting

7. How much do you like doing math?
   1  2  3  4  5  6  7
   Not at all      Very much

8. In general, how useful is what you learn in math?
   1  2  3  4  5  6  7
   Not at all useful Very useful

9. For me, being good in math is…
   1  2  3  4  5  6  7
   Not at all important Very important
Appendix E. Agreement Scale for Elementary Study
Appendix F. Reference Images for Elementary Study

The **height** is the distance from the bottom to the top.

The **length** is the distance from left to right.

The **diameter** is a straight line through the center of the circle or sphere.
# Appendix G. Elementary Tasks

## Order A (Easiest to Hardest)

<table>
<thead>
<tr>
<th>Task Code</th>
<th>Task; Verbal Instruction</th>
<th>Shapes to Use</th>
<th>Correct Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Can you estimate the <strong>height</strong> of the cylinder?</td>
<td>Big cylinder</td>
<td>24”</td>
</tr>
<tr>
<td>D</td>
<td>Can you estimate the combined <strong>length</strong> of the three cubes?</td>
<td>Three cubes</td>
<td>14”</td>
</tr>
<tr>
<td>C</td>
<td>Can you pick the cube that seems closest to having a <strong>length</strong> of 6 inches?</td>
<td>Three cubes</td>
<td>Big cube</td>
</tr>
<tr>
<td>G</td>
<td>Can you estimate the <strong>height</strong> of the cylinder?</td>
<td>Candle cylinder</td>
<td>8”</td>
</tr>
<tr>
<td>Q</td>
<td>Can you estimate the <strong>diameter</strong> of the sphere?</td>
<td>Basketball</td>
<td>9.5”</td>
</tr>
<tr>
<td>O</td>
<td>Can you estimate the <strong>height</strong> of this cylinder?</td>
<td>Short cylinder</td>
<td>4”</td>
</tr>
<tr>
<td>F</td>
<td>Can you estimate the <strong>length</strong> of the longest sides?</td>
<td>Grey prism</td>
<td>8”</td>
</tr>
<tr>
<td>B</td>
<td>Can you estimate the <strong>diameter</strong> of the sphere?</td>
<td>Small sphere</td>
<td>7 ¾”</td>
</tr>
<tr>
<td>E</td>
<td>Can you estimate which rectangular prism has a <strong>height</strong> of 4”?</td>
<td>Blue, clear, grey</td>
<td>Grey</td>
</tr>
<tr>
<td>P</td>
<td>Can you estimate the <strong>diameter</strong> of this cylinder?</td>
<td>Short cylinder</td>
<td>2 5/8”</td>
</tr>
</tbody>
</table>

## Order B (Hardest to Easiest)

<table>
<thead>
<tr>
<th>Task Code</th>
<th>Task; Verbal Instruction</th>
<th>Shapes to Use</th>
<th>Correct Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Can you estimate the <strong>diameter</strong> of this cylinder?</td>
<td>Short cylinder</td>
<td>2 5/8”</td>
</tr>
<tr>
<td>E</td>
<td>Can you estimate which rectangular prism has a <strong>height</strong> of 4”?</td>
<td>Blue, clear, grey</td>
<td>Grey</td>
</tr>
<tr>
<td>B</td>
<td>Can you estimate the <strong>diameter</strong> of the sphere?</td>
<td>Small sphere</td>
<td>7 ¾”</td>
</tr>
<tr>
<td>F</td>
<td>Can you estimate the <strong>length</strong> of the longest sides?</td>
<td>Grey prism</td>
<td>8”</td>
</tr>
<tr>
<td>O</td>
<td>Can you estimate the <strong>height</strong> of this cylinder?</td>
<td>Short cylinder</td>
<td>4”</td>
</tr>
<tr>
<td>Q</td>
<td>Can you estimate the <strong>diameter</strong> of the sphere?</td>
<td>Basketball</td>
<td>9.5”</td>
</tr>
<tr>
<td>G</td>
<td>Can you estimate the <strong>height</strong> of the cylinder?</td>
<td>Candle cylinder</td>
<td>8”</td>
</tr>
<tr>
<td>C</td>
<td>Can you pick the cube that seems closest to having a <strong>length</strong> of 6 inches?</td>
<td>Three cubes</td>
<td>Big cube</td>
</tr>
<tr>
<td>D</td>
<td>Can you estimate the combined <strong>length</strong> of the three cubes?</td>
<td>Three cubes</td>
<td>14”</td>
</tr>
<tr>
<td>A</td>
<td>Can you estimate the <strong>height</strong> of the cylinder?</td>
<td>Big cylinder</td>
<td>24”</td>
</tr>
</tbody>
</table>