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A 12b 100MSps Split Pipeline ADC with Open-Loop Residue Amplification

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12b 100MSps Pipeline ADC with Open-Loop Residue Amplifier

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In partial fulfillment of the requirements for the

Degree of Bachelor of Science

by

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2007-2008 SRC/SIA IC Design Challenge
Worcester Polytechnic Institute
Team 47
February 20, 2008

12b 100MSps Pipeline ADC with Open-Loop Residue Amplifier

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ABSTRACT

The design of a low-power 12-bit 100MSps pipeline analog-to-digital converter (ADC) with open-loop residue amplification using the novel “Split-ADC” architecture is described. The choice of a 12b 100MSps specification targets medical applications such as portable ultrasound. For a representative ADC such as the ADS5270, the figure of merit (FOM) is approximately 1pJ/step and the power dissipation is 113mW. The use of an open-loop residue amplifier resulted in a FOM of 0.571pJ/step and a power dissipation of 11.2mW.
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1 INTRODUCTION

The purpose of this project is to design a 12b 100MSps split pipelined analog-to-digital converter in a 0.18μm BiCMOS Jazz Semiconductor process for a portable ultrasound application.

1.1 Introduction to Analog-to-Digital Converters

An analog-to-digital converter, abbreviated ADC, A/D, or A to D, is an electronic device that converts analog signals to digital signals. An analog signal, shown in Figure 1 on the left, is a piece of real-world information such as force, temperature, or pressure. The reason for converting such a measurement to a discrete digital signal, shown in Figure 1 on the right, is so that a computer can process, transmit, or store that piece of information. Cell phones, cameras, and camcorders, are examples of devices that contain ADCs. An analog signal differs from a digital signal in that an analog signal can take on any $y$-value whereas a digital signal can only take on certain $y$-values.

![Figure 1: Analog v. Digital Signal](image)

Figure 2 shows a block diagram of an ADC. The input to the ADC, $v_{IN}$, is an analog signal. The output of the ADC, $n$, is a digital signal that is proportional to the input signal, $v_{IN}$. The proportionality factor is referred to as the reference voltage and is designated $v_{REF}$ in the block diagram. The analog input cannot be converted to a digital output at every instance in time. Rather, the analog input is converted to a digital output every $T_s$ seconds. The inverse of $T_s$ is referred to as the sampling frequency and it provides an alternative measure for how often an
An analog signal is converted into a digital signal. This transformation is the fundamental procedure behind an analog-to-digital converter; an analog signal is converted into a digital form.

**Sampling Frequency**

An analog signal is converted to a digital signal through sampling. Sampling refers to how often a portion of the input voltage is converted into a digital number. Figure 3 displays the concept of sampling. Each of the green lines represents an instance in time when a sample is taken. When a sample is taken, the value of the input voltage is converted to a digital number. For example, the second green vertical line indicates the second sample. The input voltage has a certain value at the second sample. That value is converted into a digital representation which is designated $n_2$ on the figure. Keep in mind that $n_2$ is proportional to the input voltage but is not necessarily equal to the input voltage. The distance between samples (green lines), is denoted as the sampling time, $T_s$. The sampling time is related to the sampling frequency by the relationship shown in Figure 2. The sampling frequency is a performance specification of all ADC’s and is designated **speed**. How often do you need to sample to obtain a digital signal that accurately represents the original analog signal?
The answer to this question is in the Nyquist-Shannon sampling theorem; the theorem says that you can reconstruct an analog signal completely if you sample the analog signal two times faster than the largest frequency component in the analog signal. Figure 4 displays the fundamental principle behind the Nyquist-Shannon sampling theorem. The figure shows a plot of the frequency components of the input signal. The highest frequency component in the input signal is denoted $f_h$. Therefore, the appropriate sampling frequency, $f_s$, is equal to $2f_h$, as indicated in the figure.

An important result of the Nyquist-Shannon Sampling Theorem is that you can reconstruct an analog signal completely if you retain the value of the input voltage every time you take a sample. This is not possible with an ADC because the output is a digital signal and a digital...
signal can only take on certain \( y \)-values. The fact that a digital signal can only take on certain \( y \)-values leads to a very important error in ADC’s called quantization error.

### 1.2 Quantization Error

The principle of quantization error can be seen in Figure 5. The analog signal being converted to a digital form is shown in blue. The possible output values of the ADC are enumerated on the \( y \)-axis. A sample of the analog signal is taken at some point in time and is denoted by the green line. The value of the input voltage (blue) at the time of the sample lies in between the digital levels \( n_2 \) and \( n_3 \). Which level should be associated with that particular sample? The level that should be associated with the sample is the level that is closest to the original input voltage; in this case the level is \( n_2 \). The original value of the input voltage differs from level \( n_2 \). The exact difference is denoted by the heavy black line in between level \( n_2 \) and the input voltage. This difference is called the quantization error and is an important limitation of all ADC converters. The quantization error is a performance specification of all ADC’s and is designated resolution.

![Figure 5: Quantization Error](image-url)
1.3 Types of ADCs

There are three major categories of ADCs that are showcased in Table 1:

- Low-to-medium speed & high accuracy
- Medium speed & medium accuracy
- High speed & low-to-medium accuracy

<table>
<thead>
<tr>
<th>Low Speed</th>
<th>Medium Speed</th>
<th>High Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Accuracy</td>
<td>Medium Accuracy</td>
<td>Low Accuracy</td>
</tr>
<tr>
<td>Integrating, Oversampling</td>
<td>Successive Approximation</td>
<td>Flash</td>
</tr>
<tr>
<td></td>
<td>Algorithmic</td>
<td>Two-Step</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pipelined</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time-Interleaved</td>
</tr>
</tbody>
</table>

Table 1: Types of ADCs

Each of these categories of ADCs was investigated in order to find a suitable architecture for the application of portable ultrasound. **The 12b 100MSps specifications led to the selection of a pipelined ADC.**

**Low Speed, High Accuracy**

At the low-speed, high-accuracy end of the spectrum are the integrating and sigma-delta oversampling ADCs. The integrating ADC, also known as a single-slope, dual-slope, or multi-slope ADC, uses at least one integrating operational amplifier and one comparator. For a single-slope ADC, the analog input voltage is integrated and then compared to a known reference voltage. The time required for the analog input voltage to exceed the reference voltage is proportional to the input voltage itself. This time is measured by a binary counter that continually counts during the integration.

In general, the integrating ADC requires $2^N$ clock cycles to obtain N bits of resolution. For a 12-bit integrating ADC, $2^{12} = 4096$ clock cycles are required. To obtain higher resolution, more integrating time is required. While such an architecture is useful for certain applications, the prohibitive low speed of the integrating ADC makes it and other slower ADC architectures poor choices for portable ultrasound.
Medium Speed, Medium Accuracy

The next category of ADCs is characterized by medium speed and medium accuracy. One of the ADCs in this category is the successive-approximation register ADC, or the SAR ADC. The operation of this ADC is similar in principle to that of the integrating ADC. In the SAR ADC, instead of a register counting upwards while integration takes place, a special type of register, the successive-approximation register, counts by trying various values of its bits, starting with the MSB and moving down to the LSBs. The digital SAR output is fed through a digital-to-analog (DAC) to convert it to an analog voltage. This voltage is then compared to the input voltage via a comparator. Based upon the result, the counter adjusts its digital output value up or down, depending upon whether the DAC voltage is lower or higher than the input voltage.

The fact that the SAR requires several clock cycles (although not nearly as many as the integrating ADC) to quantize a particular input voltage reduces its maximum throughput to approximately 5MSps. The low speed limit of the SAR makes it unsuitable for our desired specification of 10MSps.

High Speed, Low Accuracy

The high speed, low accuracy converters are the flash, pipeline, two-step and time-interleaved ADCs. The flash ADC is designed with comparators and is the fastest method of converting an analog signal to a digital form. A flash ADC with N bits of resolution requires $2^N$ comparators. Therefore, a 12-bit flash ADC would require $2^{12} = 4096$ comparators. In a flash ADC, complexity increases exponentially with resolution. As a result, high-resolution flash ADCs occupy a large die area and consume large amounts of power. The very high complexity and power consumption of the flash converter renders it unsuitable for a low power application.

The time-interleaved ADC architecture is another ADC architecture that allows very high sampling rates. A time-interleaved ADC utilizes multiple ADCs operating simultaneously on separate clocks with different phase shifts. The outputs of the ADCs are then multiplexed properly to form the output. For example, a time-interleaved ADC might use two different ADCs operating on two clocks 180 degrees out of phase from each other, such that the rising edge of one clock is the falling edge of the other. However, these clocks must be exactly 180 degrees out of phase from one another, or else unwanted frequency components will be
introduced into the multiplexed output. In addition, gain and offset error of the individual ADCs needs to be corrected for, increasing the complexity of the time-interleaved ADC. The increased complexity introduced due to multiple clocks resulted in our group not selecting the time-interleaved ADC.

1.3 ADC Performance Specifications

There are two key performance specifications of ADCs: differential nonlinearity and integral nonlinearity.
**Differential Nonlinearity**

The *differential nonlinearity (DNL)* of an ADC is defined as the difference between the actual step length and the ideal step length. An ideal transfer function for an ADC is displayed in Figure 6. The y-axis displays the digital output values of the ADC and the x-axis displays the analog input voltage values. The transfer characteristic is considered ideal because each of the steps is of equal length. This characteristic is virtually impossible to realize in an ADC.

![Figure 6: Ideal ADC Transfer Function](image)

A desirable characteristic of ADCs is that of ‘no codes lost’. This refers to the ability of an ADC to display all of the possible digital output codes sequentially. In terms of DNL, this means a DNL of less than one LSB at any code. Figure 7 shows two cases: in the first, the DNL error is less than 1 LSB and therefore no codes are lost.

![Figure 7: DNL Error greater and less than 1 LSB](image)
Figure 7 also shows a case with the DNL error is greater than 1 LSB resulting in lost codes. In the figure, it can be seen that the code ‘10’ could be mistaken for code ‘01’ or ‘11’ at certain values of the input voltage. Also, with the analog input voltage at AIN*, the output code could be ‘01’, ‘10’, or ‘11’.

**Integral Nonlinearity**

The integral nonlinearity (INL) of an ADC is the difference between the actual transfer function of an ADC and the ideal transfer function of an ADC. Integral nonlinearity can be thought of as the sum of the differential nonlinearities. A transfer function with no integral nonlinearity is shown in Figure 12. The y-axis displays the digital output values of the ADC and the x-axis displays the analog input voltage values. There is no integral nonlinearity in this figure because a straight line can be fit among the code centers. Any deviations in this straight line fit are captured in integral nonlinearity.

![Figure 8: Integral Nonlinearity [2]](image)

**1.4 Prior Work**

This project represents the continuation of a Major Qualifying Project (MQP) that was completed in 2007 by Abhilash Nair and Sanjeev Goluguri. The goal of their project was to apply the “Split ADC” to a 16-bit 10MHz pipelined analog-to-digital converter in a 0.25µm TSMC CMOS process.
1.5 Research Space and Goals

The ADC designed in this project will be for a multichannel portable ultrasound system. The requirements for such a device are lower power, high sampling rate, and high resolution. The specifications for the ADC will be 100MSps and 12 bits. The project has been accepted into the 2007-2008 SRC/SIA (Semiconductor Research Corporation/Semiconductor Industry Association) Design Challenge. The technology provided by the competition is a 0.18µm BiCMOS Jazz Semiconductor process. The design specifications for the end product are enumerated in Table 2.

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Pipeline ADC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>Split ADC, Digital background</td>
</tr>
<tr>
<td>Resolution</td>
<td>12b</td>
</tr>
<tr>
<td>Speed</td>
<td>100MSps</td>
</tr>
<tr>
<td>Power</td>
<td>~50mW, 0.5pJ/step</td>
</tr>
<tr>
<td>SNR</td>
<td>+70dB</td>
</tr>
<tr>
<td>INL</td>
<td>±0.5LSB</td>
</tr>
<tr>
<td>Process</td>
<td>0.18µ, BiCMOS Jazz Semi</td>
</tr>
</tbody>
</table>

Table 2: Design Specifications

There is a comparable product currently on the market made by Texas Instruments. The ADS5270 is a 12-bit, 40MSps, 8-channel ADC that has 113mW of power dissipation per ADC and approximately 1pJ/step speed-power figure of merit (FOM). The goal of this project is to increase performance with respect to power dissipation (50mW) and speed-power FOM (0.5pJ/step).

1.6 Background

The use of open-loop residue amplification in pipeline ADCs has been the subject of recent investigation due to the power advantages over more precise closed-loop techniques, as well as the general appeal of relaxing accuracy requirements on analog circuitry in deep submicron CMOS. Due to the nonlinearity of the open-loop amplifier, digital calibration is used to restore acceptable linear performance for the overall ADC. Previous work [10] has described statistically based methods for determining the required calibration coefficients, which have the drawback of relatively long adaptation times.
The “Split ADC” concept has been applied to correct linear gain errors in algorithmic and pipeline ADCs [5], [8], and has the advantage of faster calibration convergence. Previous work by an undergraduate project group working in the team leader’s research lab [9] presented an algorithm applying the split ADC approach to digital background correction of errors in pipeline ADCs arising from the nonlinearity of open-loop residue amplifier stages. Compared with [10], the novel contribution of this work is the dramatically reduced time for calibration convergence by adopting the split ADC approach to correct amplifier nonlinearity errors, thereby making the calibration of converters in the 14b to 16b range feasible.

The work completed describes the design of a mixed signal integrated circuit in Jazz Semiconductor’s 180nm SiGe process to provide the analog functionality for a 12b, 100MSps, pipeline ADC using open-loop residue amplifier gain stages. The nonlinearity of the open-loop amplifier will be digitally corrected using a background calibration algorithm to be implemented on an FPGA.

1.7 Purpose of Circuit

The choice of a 12b 100MSps specification targets a performance space associated with medical applications such as portable ultrasound. The 12b 100MSps combination represents a balance between pushing extremes of high performance, while choosing a scope of project reasonable for design and test in the time constraints given.

1.8 High Performance Aspects

*Speed-Power Figure of Merit (FOM)* - Rather than push absolute limits, we have chosen a target in an important application space with moderate speed and resolution; the high performance aspect of the design will be the improvement in the speed-power FOM. For a representative ADC such as the ADS5270 [13], the FOM is approximately 1 pJ/step; in addition, the power dissipation in one ADC is 113mW. Through the use of an open-loop residue amplifier we achieved sufficient power savings to improve the FOM to 0.571pJ/step and the power dissipation in one ADC to 11.2mW. The details of the analysis of the FOM and power dissipation can be found in section 8.1.2 titled Power Consumption.

*Use of bipolar devices in residue amplifier stage* – The output resistance of the residue amplifier proved to be too large to drive the MDAC of the following stage. In order to achieve
further power savings in the residue amplifier stage, we selected a BJT emitter-follower to interface between the residue amplifier and the MDAC. The details of the BJT emitter-follower design are described in the Circuit Design section under Open-Loop Residue Amplifier.
2 PIPELINED ADCs

This section will describe the architecture, operation, and performance considerations of a pipelined ADC.

2.1 Architecture

The pipelined is a popular architecture for modern applications of analog-to-digital converters due to its high sustained sampling rate, low power consumption, and linear scaling of complexity. Figure 9 shows a block diagram of a pipelined ADC. The term “pipelined” refers to the stage-by-stage processing of an input sample $V_{\text{IN}}$.

![Figure 9: Pipelined ADC Block Diagram](image)

In the above diagram, the analog input voltage $V_{\text{IN}}$ enters the ADC. Each subsequent pipeline stage of the ADC resolves a certain $n$ number of bits to be contributed to the final conversion output. The number of bits that each stage is responsible for quantizing is usually on the order of 1 – 5 bits. Simultaneously, after each stage has finished quantizing its input sample to $n$ bits, it outputs an analog residue voltage that serves as the input to the next stage. After $s$ stages of conversion, an $m$-bit ADC resolves the lower bits of the overall ADC digital output. Each stage’s digital decision is then passed to a digital block that properly time-aligns the output bits and corrects for any errors in each stage. The final digital decision is then produced.

2.2 Operation

Each stage displayed in the block diagram shown above can be explored further. A typical pipeline stage is displayed in Figure 10.
Figure 10: Pipelined ADC Stage Block Diagram

The input voltage is sampled and held in the sample-and-hold circuit embedded in each stage. Subsequently, an n-bit flash ADC quantizes the analog voltage and produces a digital decision of n bits. The digital decision is then fed through an n-bit flash DAC to be re-converted into an analog signal. The summation node presented in the above diagram takes the input voltage from the sample-and-hold circuit and subtracts the DAC voltage from it. This difference voltage is then fed through a gain stage with gain G to produce the residue voltage, the output voltage of this stage. In a typical pipelined ADC implementation, the sample-and-hold circuit and flash DAC are typically implemented in a single switched-capacitor circuit called a multiplying DAC, or MDAC. The amplification of the residue usually occurs with a closed-loop operational amplifier, usually consisting of a differential input, gain stage, bias circuitry, and a differential output stage.

In equation form, the output of each pipeline stage can be described as:

\[ V_{RES} \]

The residue voltage, \( V_{RES} \), becomes the input voltage to the next stage. The digital decisions versus input voltage and the residues versus input voltage of a typical pipelined ADC are displayed in Figure 11.
In Figure 11 the input voltage is swept from -2.5V to 2.5V. The resulting digital code that is generated as well as the corresponding residue voltage is displayed above. The residue represents the amplified remainder from the subtraction of the DAC output voltage from the stage input voltage.

The pipelined ADC theory of operation is that each stage is responsible for quantizing a certain set of bits that will eventually become integrated into the final conversion output. For the generalized pipeline ADC described previously, each stage is responsible for quantizing \( n \) bits of the input sample. The final ADC output consists of a weighted sum of each stage’s digital decision. The weightings are determined by the interstage gains, or the gains of the residue amplifiers within each stage. The final output is weighted according to (2):

\[
x = D_1 + \frac{1}{G_1} D_2 + \frac{1}{G_1 G_2} D_3 + \frac{1}{G_1 G_2 G_3} D_4 + \frac{1}{G_1 G_2 G_3 G_4} D_5
\]

(3)
where $D_i$ and $G_i$ represent the digital decision and the residue amplifier gain of each pipeline stage. The above equation suggests that later stages have a smaller weight in the final ADC output. This is indeed the case, as later stages resolve the lower bits of the overall conversion. In the above example, $D_5$ represents the digital decision made by the final flash ADC, responsible for resolving the least significant bits of the output.

As mentioned before, each stage in a generalized pipelined ADC is responsible for resolving $n$ bits of the ADC output, while the final flash ADC is responsible for quantizing the $m$ least significant bits of the ADC output. It is evident from the serialized operation of the pipelined ADC that some sort of time-alignment and error-correction circuitry is required for aligning each stage’s digital decision to produce the final output.

Even though $n$ bits are resolved by each stage and $m$ bits resolved by the final ADC, the maximum resolution of the ADC’s overall output is limited to $s(n-1)+m$ where $s$ is the number of stages. There is a one-bit overlap of the digital decisions between adjacent pipelined stages. For instance, a sample conversion may appear as follows:

$$
\begin{array}{c}
S1: 0 & 0 & 1 \\
S2: 0 & 1 & 0 \\
S3: 0 & 0 & 0 \\
S4: 0 & 1 & 1 \\
S5: 0 & 1 & 1 & 0 \\
\end{array}
$$

Figure 12: Pipelined ADC Redundancy

The output bits of each stage are aligned in such a way that the LSB of one stage overlaps with the MSB of the subsequent stage. The final ADC output is obtained when the columns are added straight down, as shown. This provides a sort of inherent error correction. For example, the output of stage 1 is 001. If the output of stage 1 had encountered an error and produced an output of 000, the residue voltage out of stage 1 and into stage 2 would thus be higher. In Equation 1 $V_{DAC}$ is the digital decision reconverted into an analog voltage via a DAC. If $V_{DAC}$ is lower (000 fed through the DAC instead of a 001), then the resultant $V_{RES}$ will be higher. The interstage gain factor $G$ can be chosen such that the output of the next pipeline stage will reproduce the missing 1 in its MSB. As such, the fact that stage 1 mistakenly produces a digital decision of 000 (instead of 001) is counteracted by the fact that stage 2 will now produce a
digital decision of 110 (instead of 010). The final result of adding the results of these 2 stages’
digital outputs is:

<table>
<thead>
<tr>
<th>S1:</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2:</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

*Figure 13: Pipelined ADC Inherent Error Correction*

Note that the inherent error correction produced by the one-bit overlaps enables the
overall added digital output to remain unchanged.

2.3 Pipelined ADC Performance Characteristics

In general, the pipeline architecture enables the implementation of relatively high-resolution
ADCs without sacrificing processing speed or power draw. Additionally, the linear complexity
scaling inherent to the pipeline architecture makes the implementation of higher-resolution
pipeline ADCs more manageable than with another ADC architecture.

The architecture of the pipelined ADC enables it to have a high throughput rate. This is
evident in that pipelined ADCs can have sampling rates of a few MSps up to 100Msps+. The
reasoning for this is that the sample-and-hold circuit can begin processing the next analog input
voltage sample as soon as the DAC, summation node, and gain amplifier have finished
processing the previous sample. This pipelining action allows a high sustained sampling rate.
Additionally, since each stage is only responsible for quantizing a low number of bits relative to
the overall resolution of the pipeline ADC, each stage processes each sample relatively quickly.

The architecture of the pipelined ADC also allows it to scale linearly as complexity
increases. In the generalized pipeline ADC discussed earlier, each stage has a small flash ADC
that performs the quantization of the input sample. These flash ADCs are comprised of many
comparators that are responsible for quantizing the sample. For an n-bit flash ADC, $2^n$
comparators are needed to perform the conversion. In a pipeline ADC, higher overall resolution
is obtained effectively by adding additional small flash ADCs in the form of having more stages.
A 12-bit pipeline ADC with 4 stages, 3 bits per stage, and a 4-bit LSB flash ADC is implemented
using only $4(2^3)+2^4 = 48$ comparators. This is in stark contrast to a 12-bit pure flash ADC,
which would require $2^{12} = 4096$ comparators in order to quantize the sample! The complexity in
a pipeline ADC scales linearly and not exponentially, as is the case in a flash ADC. It also follows that fewer required comparators translates to much less power dissipation and power draw, another advantage of the pipeline architecture.

Although the pipeline ADC allows for high speed, lower power dissipation, and low complexity, there are still tradeoffs. For instance, the serialized nature of the conversion process means that there is a significant time delay between the sample that enters the first sample-and-hold of the first stage and when the digital alignment circuitry produces the correct output code. Each stage in a pipeline ADC delays the data output by approximately one additional clock cycle. This data latency has to be accounted for when implementing a pipelined ADC.

Even in spite of these tradeoffs, the pipelined ADC architecture enables an ADC to have relatively high resolution, high speed, and low power dissipation, all with very few tradeoffs.
3 BACKGROUND


In his paper, “A 15-b 1-Msample/s Digitally Self-Calibrated Pipeline ADC” published in the IEEE JSSC 12/2003, Karanicolas describes a self-calibration scheme for a pipelined analog-to-digital converter that attempts to compensate for errors such as capacitor mismatch, comparator offset, charge injection, finite op-amp gain, and capacitor nonlinearity. Karanicolas employs a 1-bit per stage, 17-stage design for his 15-bit pipelined ADC, noting that each stage is very simple and fast, whose performance is limited only by the errors listed above.

Functionally, each stage compares its input to a reference voltage with a comparator to obtain the single digital output bit. The residue voltage for each stage is passed through an amplifier to the next stage. The ideal interstage gain in the presented ADC is nominally 2, accomplished by closed-loop operational amplifiers. However, for the first 11 stages of Karanicolas’ ADC, he employs an interstage gain of 1.93. The purpose behind this choice of gain is to ensure that the residue voltage of each stage does not exceed the full scale range of the subsequent stage in a worst case scenario (maximum capacitor mismatch, comparator offset, and charge injection error magnitudes together). This gain reduction ensures that missing decision levels do not result from the output of any stage exceeding the reference voltage. By doing so, the resulting missing codes can be eliminated with digital calibration.

The digital calibration algorithm for a given stage requires the output residue voltage ($V_{IN}$, input to the current stage) and the output bit decision from the previous stage ($D_{IN}$) as well as two calibration constants $S_1$ and $S_2$ determined for that particular stage. Karanicolas cites the eleventh stage for example:
Figure 14: Karanicolas' Digitally Self-Calibrating Scheme

Here, X refers to the uncorrected digital output, and Y refers to the corrected digital output. The following describes the self-calibration algorithm:

\[
Y = \begin{cases} 
X & \text{if } D = 0 \\
X + S_1 - S_2 & \text{if } D = 1 
\end{cases}
\]

S_1 and S_2 can be identified in the residue plot in Figure 14 where \(V_{IN} = 0\) with \(D = 0\) and \(D = 1\), respectively. S_1 is determined when both the input voltage and the input digital decision are both zero. S_2 is determined when the input voltage is set to zero, but the input digital decision is set to one. This transform ensures that the output code is the same when \(V_{IN} = 0\), no matter whether \(D = 0\) or \(D = 1\), eliminating any missing codes resulting from any previously mentioned errors.

From there, stage 11 is calibrated with the determining of constants S_1 and S_2. Stages 11-17 can be used in a similar manner to calibrate stage 10, as shown below:
As such, the rest of the stages, stage one through nine, can be calibrated in a similar fashion. The entire pipeline ADC is calibrated in this fashion. If this converter has a sample rate of 1Msps and 2,048 point averaging, the total calibration time is cited by Karanicolas to be about 70ms.

### 3.2 Murmann: Open-loop residue amplification (2003) [10]

While the ubiquitous pipelined analog-to-digital converter boasts high sampling rates (10-100MSps) and moderate resolution (12-16 bits), the typical pipelined ADC would not be the premier choice in a portable ADC application. The typical pipelined ADC utilizes a precision closed-loop operational amplifier to amplify the residue voltage that serves as the input to the next stage, dominating the power dissipation of the entire ADC. This amplifier must meet rigorous requirements in terms of speed, noise, and gain linearity. Murmann’s 2003 paper explores the prospect of replacing this closed-loop amplifier with a simple open-loop differential pair gain stage in the hope of reducing the power dissipation of a pipelined ADC.

Unfortunately, to replace the complex closed-loop amplifier with a simpler open-loop differential stage results in a nonlinearity in the amplifier gain. Thus, Murmann also proposed a digital background calibration technique to correct for this error in the digital domain, still staying far below the power dissipation of a typical closed-loop amplifier implementation (60% power savings). The general motivation for this approach to residue amplification is the result of
relaxing requirements on analog circuitry and moving that complexity into the digital domain, where it can be more cheaply and efficiently implemented.

Murmann’s pipelined ADC utilized only one open-loop residue amplifier, in the most critical multibit first stage. The nature of the pipelined ADC is such that errors in subsequent stages (after the first stage) become less and less pronounced after each stage iteration. Although subsequent stages still used the typical closed-loop amplifier implementation, Murmann’s work also served as a vehicle to demonstrate that it would be almost trivial to implement this open-loop amplifier and digital correction for every other stage.

By introducing an open-loop residue amplifier into the first stage of his ADC, Murmann needed to correct for the nonlinear gain that would result. He does this by using a digital background calibration technique that utilizes a digital lookup table calculated from measured nonlinearity parameters $p_i$. The parameter $p_i$ that characterizes the nonlinearity in the open-loop amplifier can be determined by introducing a mode-select block into the pipeline stage block diagram, enabling two different residue modes, as shown in the following diagram:

![Figure 16: Mode Select](image)

In Figure 16, the two residue modes are distinctly shown. The residue modes are controlled by a single mode select bit labeled $\text{MODE}$ that essentially shifts over the resulting residue plot when $\text{MODE}$ is enabled. The production of these two redundant residue modes is such that each mode itself is capable of obtaining the conversion result correctly in the case of ideal operation. The nonlinearity parameter $p_i$ can be estimated from the distances between the residues, as shown in Figure 17.
The residue differences $h_1$ and $h_2$ are measured to be the distances between the center and near the edges of the residues, respectively. The difference between $h_1$ and $h_2$ is controlled by nonlinearity parameter $p_2$, where $p_2$ increases with increasing nonlinearity of gain. The distances $h_1$ and $h_2$ are measured by a statistics-based approach whose content is beyond the scope of this project.

### 3.3 Our Approach

In this project, we aim to apply concepts from both Karanicolas’ and Murmann’s work, namely background calibration and open-loop residue amplification, respectively. Digital background calibration shall enable us to utilize open-loop residue amplification in order for us to further reduce power dissipation in our pipeline ADC architecture designed for a portable ultrasound application.
4 SYSTEM LEVEL DESIGN

The work described in the IC Block Diagram section includes a description of the system block diagram and the design methodology used.

4.1 System Block Diagram

The overall system block diagram is shown in Figure 18:

The shaded portion represents the analog integrated circuit comprised of two separate pipeline ADCs in the split ADC architecture. The architecture for each pipeline ADC is shown as utilizing five stages. The first stage has 31 quantization levels and each of the subsequent four stages has 21 quantization levels. The analog IC is responsible for generating the individual digital decisions and transmitting them to the FPGA, enclosed by the dotted box. The FPGA provides for digital correction, background parameter estimation, and calibration as described in [9]. This work is concerned with the design and layout of one of the pipeline ADCs.
**Pipeline Stage**

A block diagram for each pipeline ADC stage is shown in Figure 19. However, it should be noted that the fifth stage will contain only a quantizer.

![Figure 19: Pipeline ADC Stage](image)

The input voltage is sampled and an ADC quantizes the analog voltage and produces a digital decision (Q). Prior to the DAC conversion, the mode select will either (1) pass the digital decision unchanged or (2) add one to the digital decision, similar to the RNG mechanism in [8] and [10]. The DAC processes the digital decision and this decision is subtracted from the input voltage. This differential voltage is then processed by the open-loop residue amplifier to produce the residue voltage, the voltage that will serve as the input voltage to the next stage.

The ADC block will be realized by a flash converter. The S/H, mode select, DAC, and summation node will be realized by an MDAC. Lastly, the gain stage will be realized by an open-loop residue amplifier.

As in [9] and [10], the residue of each pipeline stage can be described as:

$$V_{RES} = G(V_{IN} - V_{DAC}) - \frac{\alpha G(V_{IN} - V_{DAC})^3}{V_{OV}^2}$$  \hfill (4)

The above relation expresses the differential output as the differential input is subjected to a linear gain G and a cubic error term that captures the nonlinearity in the differential pair.

As in [8] and [10], the stage is capable of operating in distinct residue modes as determined by the mode select bit M. Due to the redundancy afforded by the choice of stage gain G and ADC resolution, either residue mode will allow correct operation of the entire ADC. The key to
the split ADC concept is that use of different residue modes allows the two converters to proceed along different decision trajectories.

Two different pipeline stages will be required in the design due to differences in the first and subsequent stages. The first stage will need to process the input voltage from the portable ultrasound machine. Subsequent stage inputs will be the output of the previous stage and therefore will have the same signal swing. Figure 3 and Figure 21 show the critical voltage swings for the first and subsequent stages.

![Figure 20: First Stage Critical Voltages](image1)

![Figure 21: Subsequent Stage Critical Voltages](image2)
4.2 Design Methodology

Behavioral simulations preceded the transistor level design of the pipeline ADC. Using Verilog-A, the entire pipeline ADC was simulated and verified. The transistor-level design followed and verification was conducted by combining the behavioral Verilog-A blocks with circuit-level simulation. Figure 22 shows the analog IC design flow for all phases of the project.

Figure 22: Analog IC Design Flow
5 BEHAVIORAL SIMULATIONS

The work described in the Behavioral Simulations section includes the design of the open-loop residue amplifier, MDAC, and quantizer in Verilog A. The performance of the ADC was verified behaviorally before proceeding with the transistor level design.

5.1 Open-Loop Residue Amplifier & MDAC

The open-loop residue amplifier and MDAC were designed as one behavioral block. The purpose of the multiplying digital-to-analog converter (MDAC) is to subtract the analog representation of the digital decision of the quantizer from the input signal (residue voltage) and the purpose of the open-loop residue amplifier is to amplify the residue voltage. The input to the MDAC and open-loop residue amplifier block is the differential input voltage of the stage, the digital decision of the quantizer, possibly altered by the mode select, and the clock signal. The output of the MDAC and open-loop residue amplifier block is the differential residue voltage. The test bench for the open-loop residue amplifier and MDAC is shown in Figure 23.

![Figure 23: Open-Loop Residue Amplifier & MDAC Test Bench](image)

The block shown in the figure above samples the input, VIP and VIM, on the zero-crossings of the falling edge of the clock (clk). The digital input (D) is converted to the analog voltage levels (VDP and VDM). The analog voltage levels (VDP and VDM) are subtracted from the sampled input (VIP and VIM) to produce the residue voltage. The residue voltage is then amplified by a nonlinear gain, capturing the behavior of the open-loop residue amplifier, and
then outputted as VRP and VRM. The Verilog A code for the open-loop residue amplifier and MDAC block can be found in APPENDIX C: Verilog A Code.

5.2 Quantizer

The purpose of the quantizer is to convert the input signal from an analog signal to a digital signal. The inputs to the quantizer are the differential input voltage (VIP and VIM) and the clock signal. The output of the quantizer is the digital representation of the input voltage Q. The test bench for the quantizer is shown in Figure 24.

![Figure 24: Quantizer Test Bench](image)

The block shown in the figure above samples the input, VIP and VIM, on the zero-crossings of the falling edge of the clock (clk). The input voltage is then correlated to an appropriate value Q which represents an approximation to the input voltage. The input voltage (VIP-VIM) shifts from -1.5V to 1.5V. Appropriately, the quantizer approximates this input voltage on the zero-crossings of the falling edge of the clock to produce the appropriate representations. The Verilog A code for the quantizer can be found in APPENDIX C: Verilog A Code.

5.3 Mode Select

The purpose of the mode select is to ensure that each ADC in the split-ADC architecture takes a different path when converting an analog signal to a digital signal. Different paths are ensured by the mode select. Prior to the DAC conversion, the mode select will either (1) pass the digital decision unchanged or (2) add one to the digital decision. The inputs to the mode select are the quantizer’s output Q and a bit M which specifies whether to pass the digital decision or
add one to the digital decision. The output of the mode select is the digital decision D. The test bench for the mode select is shown in Figure 25.

The mode select is triggered by the bit M. A value of 0 for bit M results in the mode select passing the input to the output unchanged. A value of 1 for bit M results in the mode select adding one to the input Q to produce the output D. The Verilog A code for the mode select can be found in APPENDIX C: Verilog A Code.

Figure 25: Mode Select Test Bench

5.4 Pipeline ADC

The pipeline ADC was constructed by pipelining the stages shown in Figure 26. The residue voltages of the first stage serve as the inputs for the subsequent stage. Four identical stages were pipelined and the final “stage” consisted of a quantizer. The residue voltages (stages 1-4) and output bits (20) for each stage were plotted and verified. The plots can be found in APPENDIX D: Behavioral Simulation Results.
6 DETAILED CIRCUIT DESIGN

The work described in the Circuit Design section includes the design of the open-loop residue amplifier, MDAC, and quantizer, including design equations and simulated results.

6.1 Open-Loop Residue Amplifier

The purpose of the differential amplifier is to provide a gain to the residue in order for the residue to be large enough to fill up the input range of the next stage’s quantizer. Figure 27, a pipelined ADC stage, is repeated below for convenience; the differential amplifier is highlighted.

![Figure 27: Pipelined ADC Stage - Differential Amplifier](image)

The input to the differential amplifier is the difference between the input voltage and the MDAC’s representation of the quantizer’s approximation to the input voltage. The voltage is represented in equation form by: \( V_{IN} - V_{DAC} \). The remaining voltage, the residue, must be amplified in order to be properly quantized by the next stage.

6.1.1 Open-Loop Differential Amplifier

An open-loop differential amplifier will be used over a closed-loop differential amplifier to save power; the nonlinearity introduced by the open-loop differential amplifier will be corrected digitally.

Traditionally, the gain stage has been implemented by a closed-loop operational amplifier. A closed-loop operational amplifier enables a precise linear gain of the residue voltage. However, the major drawback is that the closed-loop operational amplifier consumes a significant amount of power. [10], investigating the use of an open-loop differential amplifier, shows that the power...
reduction of using an open-loop gain stage as opposed to a typical closed-loop amplifier is around 60%, in a pipelined ADC design with a 3V supply. This power figure includes the quantizer, biasing network, gain stage, and the digital post-processor. The major drawback of the open-loop differential amplifier is that a nonlinear gain is applied to the residue voltage.

As in [9] and [10], the nonlinear gain introduced at the output of the open-loop differential amplifier can be approximately expressed in closed form, as in (5). Figure 28 displays the graphical relationship between $v_{id}$ and $v_{od}$, as described in (5).

$$v_{od} = Gv_{id} - \frac{\alpha G v_{id}^3}{V_{ov}^2}$$  \hfill (5)

The overdrive, or effective MOSFET voltage, $V_{OV}$, can be chosen such that third-order nonlinearity dominates in the transfer function. The relation in (5) expresses the differential output as the differential input subjected to a linear gain $G$ and a cubic error term that captures the nonlinearity in the differential amplifier. Although this error needs to be measured and corrected for in order to minimize the ADC output nonlinearity errors, the actual implementation of the digital correction and background calibration circuitry has been described in [9] and is beyond the scope of this project.
6.1.2 Differential Pair with Passive Load

We started the design of the differential amplifier by implementing a passively-loaded differential pair circuit. The inputs to the differential amplifier are two differential signals. It is required that the difference between the two signals be amplified. A differential pair was selected as the appropriate topology for amplifying the difference between two signals.

The design of the pipeline ADC in the 0.18μm process is limited by a 1.8V supply. The low voltage supply leaves little headroom between the supply voltage and ground. Therefore, components that can support a small voltage drop are preferable in the load of the differential pair. A resistive load, or passive load, was selected as the appropriate load for the differential pair. A schematic of the differential pair with a passive load is shown in Figure 29.

![Figure 29: Differential Pair with Passive Load](image)

6.1.3 Cascode

A cascode is inserted into the differential pair to prevent Miller multiplication of $C_{gd}$ in M1 and M2.

There is a gate-drain capacitance that exists due to the geometry of the MOSFET. The Miller effect describes how the gate-drain capacitance of the first stage will be amplified across a gain stage. Note that in Figure 29 the drain node of M1 sees a gain. As a result, a transistor is
inserted between that node and M1 in order to buffer the gate-drain capacitance of the first stage from being Miller multiplied. Similarly, a transistor is inserted between M2 and the resistor on the right side of the differential pair. The resulting schematic is shown in Figure 30. The cascode is represented by MOSFETs M3 and M4. It should be noted that a deep N well was used in M1-M4 to tie the source to the body in order to eliminate the body effect.

![Figure 30: Differential Pair with Passive Load and Cascode](image)

### 6.1.4 Pi-Resistor Network

A pi-resistor network is added to the differential pair in order to provide an extra degree of freedom to control the gain of the amplifier without changing the common-mode as shown in [10].

The pi-resistor network, shown in Figure 31, is inserted above M3 and M4. Additional insight can be gained by analyzing the pi-resistor network using Bartlett’s bisection theorem. Bartlett’s bisection theorem provides two circuit analysis techniques that show which circuit components affect the common-mode voltages and currents and which circuit components affect the differential mode voltages and currents.
To analyze those components that affect the common-mode, we open all leads between points of symmetry. In terms of Figure 31, this means breaking the circuit between the two pi-resistors. No current flows into either of the pi-resistors which leads to the conclusion that the pi-resistors do not affect common-mode voltages and currents. To analyze the components that affect the differential mode, we ground all leads between points of symmetry. In terms of Figure 31, this means inserting a ground between the two pi-resistors. The result is the observation that the pi-resistor influences the differential mode voltages and currents.

Therefore, the pi-resistors provide a “knob” to adjust differential voltages and currents without altering common-mode differential voltages and currents. The network will be particularly useful when determining the gain of the differential amplifier.

![Figure 31: Differential Pair with Passive Load, Cascode, and Pi-Resistor Network](image)

### 6.1.5 Amplifier Biasing

Using the structure in Figure 31, the differential amplifier was designed. The first step in the analysis was to determine roughly how much voltage could be allocated to each of the transistors. Figure 32 shows how we determined the appropriate allocation of voltage from the supply voltage to ground. M3 is the cascode transistor, M1 is the input transistor, and M5 is the
biasing transistor (see Biasing Circuitry Design for more information). The terminals of the NMOS MOSFET are labeled on transistor M5 for convenience.

![Figure 32: Half of Differential Pair Schematic](image)

To begin, we need to allocate portions of the 1.8V supply headroom between each of the components in Figure 32, namely, the drain-source voltages of all MOSFETs, as well as across the load resistor R1. It is desired to have the drain-source voltage large enough so that (1) the output impedance is high and (2) the drain current is primarily altered by the gate-source voltage instead of the drain-source voltage. The drain-source voltage of resistors M3, M1, and M5 were selected to be 0.30V and 0.575V, and 0.175V respectively. The total of these three voltage drops equals 1.05V, which was chosen to be the bottom threshold for the output voltage swing at the drain of M3. The reason that the drain-source voltage for M3 and M5 is lower than M1 is because the common-mode feedback and replica bias will help to increase drain-source voltage of these transistors. More information on these two circuit configurations can be found in their respective sections.
For a symmetrical output voltage swing, the drain of MOSFET M3 needs to have a common-mode voltage of 1.4V and needs to be able to swing up to 1.75V and down to 1.05V. Therefore, the drain of resistor M3 will swing no lower than 1.05V. This voltage defines the available drain-source voltages for transistors M3, M1, and M5.

Knowing the distribution of the 1.05V of drain-source voltage allows for the drain current and transconductance of the differential amplifier to be calculated. The voltage at the drain of transistor M5 is equal to the drain-source voltage of transistor M5 plus the voltage of ground. The voltage at the drain of transistor M5 is set equal to 0.175V, for M5 will not need a significant amount of drain-source voltage to remain properly biased due to the replica biasing techniques used (see Replica Bias Design section). For the common-mode voltage on the gate of M1, we choose a value of 0.9V, halfway between the supply rails. The gate-source voltage can be calculated by subtracting the voltage at the drain of transistor M5 (0.175V) from the common-mode voltage of M1 (900mV). The gate-source voltage of the input transistor M1 is thus 0.725V. Using a fixed W/L = 100 (18u/0.18u), we were able to produce a plot of drain current and transconductance vs. gate-source voltage in Figure 33. Keep in mind that the simulation performed in Figure 33 (drain-source voltage of 1V) served as a sufficiently accurate approximation for the M1 drain-source voltage of 0.575V. From the curve on the left, the threshold voltage was determined to be about 0.5V. Knowing the threshold voltage and gate-source voltage, the effective voltage was calculated to be 0.225V. Using a gate-source voltage of 0.725V, the drain current was determined to be 1.0mA. Finally, using the gate-source voltage of 0.725V, the transconductance was determined to be 6.2mA/V.
Figure 33: 18u/0.18u NMOS Id-Vgs Characteristic

Three values need to be determined next: $R_{LC}$, $R_{LD}$, the series and pi load resistors, and $I_{CM}$, the common-mode feedback “helper” current; the desired values are shown in Figure 34. These three values will help to define the rest of the differential amplifier. The gain of a differential amplifier is generally given by (6), derived from the small-signal model of the MOSFET. The load resistance in the differential amplifier design is given by the parallel combination of the load resistor with the pi-resistor.

$$\text{Gain} = g_m \cdot R_L = g_m \cdot \left( R_{LC} \parallel R_{LD} \right) \quad (6)$$

g_m is already known to be 6.2mA/V. We want to squeeze as much gain as possible out of the amplifier, so we choose the desired gain of the amplifier to be 8 and will reduce the desired gain if it proves unattainable.
A second equation can be derived by analyzing Figure 34. The drain-current \( I_{D3} \) is known to be 1mA. The voltage drop across \( R_{LC} \) is known to be 0.4V for a common-mode voltage of 1.4V. The total current flowing through the source of M1 is \( I_{D3} - I_{CM} \) (KCL). Given the information provided, (7) can be written:

\[
(1\text{mA} - I_{CM}) R_{LC} = 0.4V
\]  

(7)
The third necessary equation can be obtained from “tilting” the differential pair and applying circuit theory. Figure 35 shows the differential amplifier when $V_{D1}$ has moved to its lowest point (1.05V) and $V_{D2}$ has moved to its highest point (1.75V). The tilted currents are determined by taking the single-ended output voltage swing, dividing by the desired gain to obtain the corresponding single-ended input voltage swing, and multiplying by the transconductance of the differential input pair to obtain the current differential:

$$\left(\frac{0.35V}{8}\right) \times \frac{6.2mA}{V} = 271.2\mu A \quad (8)$$

$$1mA + 271.2\mu A = 1.27mA, \quad \text{and} \quad 1mA - 271.2\mu A = 0.73mA \quad (9)$$

Performing a KCL at the node represented by $V_{D1}$ leads to (10):

$$\frac{0.75V}{R_{LC}} + \frac{0.7V}{R_{LD}} = 1.27mA - I_{CM} \quad (10)$$

Solving (6), (7), and (10) leads to values for $R_{LC}$, $R_{LD}$, and $I_{CM}$:

$$R_{LC} = 1.284k\Omega; \quad R_{LD} = 28k\Omega; \quad I_{CM} = 688\mu A$$
6.1.6 Transistor Sizing

Knowing the desired gain, the next step in the design of the differential amplifier was to determine the W/L ratio. In order to determine the optimum ratio, an analysis of varying W/L ratios was performed. The analysis consisted of plotting key voltages and currents in the circuit in order to determine the optimum output resistance and transconductance, to name a few. The plots analyzed, for both NMOS and PMOS transistors, are summarized in Table 3. In addition, logarithmic plots of the dependent variable versus the independent variable were analyzed for all five plots.

<table>
<thead>
<tr>
<th>Plot</th>
<th>Independent Variable</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Drain Current</td>
<td>Gate-Source Voltage</td>
</tr>
<tr>
<td>2</td>
<td>Transconductance</td>
<td>Gate-Source Voltage</td>
</tr>
<tr>
<td>3</td>
<td>Transconductance</td>
<td>Drain Current</td>
</tr>
<tr>
<td>4</td>
<td>Drain Current</td>
<td>Drain-Source Voltage</td>
</tr>
<tr>
<td>5</td>
<td>Drain-Source Resistance</td>
<td>Drain-Source Voltage</td>
</tr>
</tbody>
</table>

Table 3: Transistor Sizing Plots

The W/L ratios plotted were: 18, 50, and 100 with a constant width of 18μm. Using a differential pair, cascode, and resistor network (discussed previously), differential pairs were designed with seven W/L ratios listed above for each transistor. The analysis showed that the gain increases steadily with W/L up to 100. W/L’s greater than 100 lead to diminishing returns, size increases no longer lead to gain increases. The result of this analysis led to a final selection of a W/L of 100. Specifically the width of the transistor is 18μm, and the length of the transistor is .18μm.

6.1.7 CMFB Design

We also chose to include a common-mode feedback circuit into the design of the differential amplifier. The CMFB concept is shown in Figure 36. The common-mode feedback circuitry corrects for differences between the output common-mode voltage of the residue amplifier and the desired common-mode voltage. In our application, the purpose of the CMFB circuit is to ensure that the output common-mode of the differential amplifier stays at a constant 1.4V. By doing so, the drain-source voltage of the cascoding devices can be maintained such that they remain properly biased. This is accomplished via an actively loaded differential pair controlling
a pair of current sources that adjust the differential amplifier currents. Hence, a negative feedback loop is formed.

![Figure 36: Common-mode Feedback Concept](image1)

The inverting input of the CMFB differential pair is tied to the output common-mode voltage, which is present in the middle of the pi-resistor load. Therefore, we choose to split the pi-loaded resistor into two resistors, the point of connection between them containing the output common-mode voltage.

![Figure 37: Common-mode Feedback Circuit](image2)
The non-inverting input of the CMFB differential pair is tied to a resistive divider that produces a reference voltage of 1.4V, the desired output common-mode of the differential amplifier.

If the output common-mode drops below 1.4V, the CMFB differential pair will tilt and cause the gate-source voltage of the CMFB current sources to increase, increasing the current fed into the main differential pair. This will in turn cause the currents in the load resistors to decrease, increasing the output common-mode to compensate. Conversely, the CMFB differential pair will tilt in the opposite direction if the output common-mode increases to above 1.4V. This will decrease the gate-source voltage of the CMFB current sources, decreasing the current fed into the main differential pair. The current in the load resistors will thus increase, decreasing the output common-mode to compensate.

6.1.8 Replica Bias Design

The purpose of the replica biasing technique used in the design of this differential amplifier is to ensure that the current source providing the bias current to the main differential pair remains in the active region by replicating its operating point on another transistor. The replica biasing technique used in the design of this differential amplifier involves a second MOSFET differential pair, whose operating point can be accurately set. The schematic of the replica biasing circuit we used is shown in Figure 38.
Figure 38: Replica Bias

The replica differential pair is biased with a PMOS cascode current mirror, which will be described later in this section. The replica pair’s differential input, gates of M2 and M5, is connected directly to the main differential pair’s differential input, the gates of M1 and M6. Finally, the bias current for the replica pair is fed through an NMOS transistor M3, which in turn mirrors the bias current for the main differential pair onto NMOS transistor M4.

When the differential input voltage swings to its extreme maximum, the inverting input of the differential pair will be at its lowest voltage. Without a replica biasing technique, the drain-source voltage of M4 will decrease, crashing M4 from the active region into the triode region, which is catastrophic for the operation of the main amplifier.

Using the replica amplifier to bias the main amplifier results in a much more stable biasing configuration. The 225uA current from the PMOS biasing circuitry (to be discussed later) will always be biasing the replica amplifier, regardless of what the differential input to the amplifier is. Since the replica amplifier ‘replicates’ the operating point of the main amplifier, there will always be 225uA of current in M3. M4, the current source for the main amplifier, is mirrored off of M3 but wider by a factor of about 8, producing the 2mA required for biasing the main amplifier. Thus, this replica biasing technique provides a stable biasing scheme for the main amplifier, regardless of the magnitude of the differential input voltage to the amplifier.
6.1.9 Biasing Circuitry Design

For the biasing circuitry of this amplifier, we chose to use a series of cascoded current mirrors. The biasing circuitry is shown in Figure 39, while a table of corresponding W/Ls and currents provided by the biasing circuitry is enumerated in Table 4: Biasing Circuitry Currents

<table>
<thead>
<tr>
<th>Bias current for:</th>
<th>Mirror</th>
<th>W/L (um)</th>
<th>Current (uA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main mirror</td>
<td>NMOS</td>
<td>3.24/0.18</td>
<td>200</td>
</tr>
<tr>
<td>CMFB Differential Pair</td>
<td>NMOS</td>
<td>1.62/0.18</td>
<td>85</td>
</tr>
<tr>
<td>NMOS to PMOS Mirroring</td>
<td>--</td>
<td>16.2/0.18</td>
<td>606.0</td>
</tr>
<tr>
<td>Replica bias</td>
<td>PMOS</td>
<td>7.2/0.18</td>
<td>225.0</td>
</tr>
<tr>
<td>2 x CMFB helper currents</td>
<td>PMOS</td>
<td>21.6/0.18</td>
<td>1520.0</td>
</tr>
</tbody>
</table>

Table 4: Biasing Circuitry Currents

![Figure 39: Biasing circuitry](image)

The main 200uA current is set by a 5.5kΩ resistor. NMOS and PMOS current mirrors are then utilized with modified W/L ratios to set the proper bias currents for each section of circuitry, as shown in Figure 39. Recall that our design required a large $I_{CM}$ CMFB “helper” current of around 760uA per half-circuit that would allow the gain of the differential amplifier to
increase while maintaining a relatively high $g_m$ for the differential input pair. This CMFB current is provided by the biasing circuitry, shown in Figure 39.

### 6.1.10 Output Stage Design

As enumerated in the following MDAC design chapter, each MDAC cell uses a 50fF capacitor to perform the subtraction operation required. In the first stage MDAC, there will be 31 MDAC cells per channel, one for each decision level of the first stage quantizer. This represents a total capacitance of $(50\text{fF})(31) = 1550\text{fF}$. Recall that the output resistance $R_O$ of the differential amplifier is typically given by $R_O = R_{LC} \parallel R_{LD}$. This is approximately 1.6kΩ in our design.

We must now ensure that the differential amplifier output is able to drive the MDAC capacitors sufficiently such that we can operate at the desired 100MHz speed. As the MDAC chapter enumerates, the phase $\varphi_2$, where the charge on the capacitor is being ‘evaluated’, lasts 3.0ns, a significant constraint. To ensure proper interfacing of the differential amplifier and the next stage’s input, we must see that the differential amplifier output is able to drive the capacitive load of the next stage’s input within phase $\varphi_2$. In other words, we must ensure that the error at the output of the differential amplifier is within 1 LSB, referred back to the stage input voltage.

For the first stage, this 1 LSB referred back to the stage input voltage is actually $(2V/31)/8 = 8.06\text{mV}$. For all subsequent stages, this value is $(1.4V/21)/8 = 8.33\text{mV}$. The error between the actual voltage and the predicted voltage must be below these 1 LSB-referred error voltages. For simplicity, we will choose the tighter-tolerance value of $8.06\text{mV}$ for all stages. If the first stage output settling is adequate, then the output settling for all subsequent stages will also be acceptable due to lesser capacitance at subsequent stage output nodes (only 21 levels vs. 31 levels in the first stage).

Using the 2.2 times the RC time constant of the charging circuit, we can estimate the length of time required for the output to settle within 1 input-referred LSB. We know that the total capacitance per MDAC cell is 50fF. Next, we select an arbitrary time constant, $RC = 500\text{ps}$, that we assume will be small enough for the desired settling behavior ($2.2RC = 1.1\text{ns} \ll \varphi_2$). We can thus find the maximum series resistance $R$ to achieve this settling behavior. $500\text{ps}/50\text{fF} = 10k\Omega$. 

\[55\]
Figure 40 shows a simplified schematic of the MDAC capacitor charge path. Essentially, the differential amplifier’s output resistance is responsible for charging each 50fF capacitor through two other resistances, an analog transmission gate series resistance, and another series switch resistance. To attain the required 10kΩ series resistance in the charging path, we choose to size the MDAC cell transmission gate and the second switch appropriately in order to obtain approximately a 4.2kΩ $R_{DS(on)}$ for each (this turns out to be 3.6u/0.18u for NMOS and 14.4u/0.18u for PMOS). This should allow the output resistance of the differential amplifier to charge the MDAC cell capacitors and allow the output to settle within the allotted period of $\phi_2$.

However, sizing the MDAC cell switches appropriately does not address the issue of settling in its entirety. As it turns out, the differential amplifier output cannot supply sufficient current to charge all of the MDAC cell capacitors within the allotted period of $\phi_2$. This problem can be solved by implementing a follower amplifier stage at the output of the differential amplifier, serving as an output buffer for driving the next stage. We select an emitter-follower over a source-follower configuration, due to a lower output resistance (higher transconductance) for a given bias current investment. In addition, by choosing an emitter-follower output stage, we take advantage of Jazz Semiconductor’s bipolar devices in the SBC18HX BiCMOS process. The emitter-followers are biased off of 40uA current sources mirrored off of the main differential amplifier biasing circuitry.
After implementing the emitter followers biased with 40uA of current, we run a simulation at 10MHz, 10% of the MDAC’s normal operating speed, to examine the settling behavior. We obtain Figure 42 and see that the settling is insufficient.
The time required to settle to within 1 input-referred LSB is still in the region of 4-5ns, which is unacceptable settling behavior. The MDAC capacitors must charge up to within 1 input-referred LSB well within the length of $\phi_2$, which is 3ns. If we run the same simulation at 100MHz, the problem becomes clearer:

![Figure 43: 100MHz settling simulation, 40uA emitter-followers](image)

It is clear that the capacitor voltages are slewing; that is, the $dV/dt$ is being limited by the amount of bias current driving the capacitors. (Recall that $i_C = C \frac{dV}{dt}$.) Thus, the solution must be to increase the amount of bias current on the emitter followers. To find the maximum (worst-case) amount of current required, we use (11):

$$i_{MAX} = \frac{C \Delta V_{MAX}}{t}$$  \hspace{1cm} (11)

Where $i_{MAX}$ is the maximum emitter-follower current required, $C$ is the capacitor size (in this case, 1550fF, the capacitance of one channel), $\Delta V_{MAX}$ is the maximum voltage across the
capacitor, and $t$ is the desired settling time. $\Delta V_{\text{MAX}}$ is equal to $V_{\text{REFP}} - V_{\text{CM}}$ (see MDAC section) $= 1.4 - 0.9 = 0.5V$. We choose $t$ to be 1ns, and the maximum required follower current is found to be:

$$i_{\text{MAX}} = \frac{C\Delta V_{\text{MAX}}}{t} = \frac{(1550 \, \text{fF})(0.5V)}{1\,\text{ns}} = 775\,\mu\text{A}$$

Note that this is a worst-case scenario. To begin fine-tuning the emitter-follower current, we increase the bias current to 160uA and observe the settling behavior at 100MHz:

The settling time has improved dramatically to around 1.5ns! The settling behavior exhibited by the 160uA emitter-followers is much better, and would work very well in the final design of the differential amplifier. In the interest of low-power consumption, we dial back the emitter-follower bias current to 120uA and run another 100MHz settling simulation:

Figure 44: 100MHz settling simulation, 160uA emitter-followers
Figure 45: 100MHz settling simulation, 120uA emitter-followers

While the settling behavior exhibited in Figure 45 is slightly worse than that of Figure 44 (due to lower emitter-follower bias current), it is still very much acceptable for an output stage solution in the differential amplifier. Finally, we run one last simulation at 10MHz to ensure that the output settles adequately to its final value.
From Figure 46, we see that the settling within merely 1.3ns of $\varphi_2$’s falling edge is within 1 input-referred LSB of the final settling value (8.06mV). Thus, we proceed with this design, and cascade the emitter-follower stages to the outputs of the differential amplifier.

Figure 46: 10MHz settling simulation, 120uA emitter-followers

Figure 47: Final emitter-follower design
6.1.11 Amplifier Schematic

Figure 48: Differential Amplifier Schematic
6.2 MDAC

The purpose of the multiplying digital-to-analog converter (MDAC) is to subtract the analog representation of the digital decision of the quantizer from the input signal. Figure 49, a pipelined ADC stage, is repeated below for convenience; the portions of the stage that the MDAC implements are highlighted.

![Figure 49: Pipelined ADC Stage - MDAC](image)

The input to the MDAC is the differential input voltage of the stage and the digital decision, Q, of the quantizer modified by the state of the mode select. The MDAC calculates the residue voltage, that is, the difference between the analog representation of the quantizer code and the actual input voltage. In this way we can effectively remove the voltage we have already quantized, and send the remaining voltage off to be quantized in further stages. The following section describes the design and simulation of the MDAC.

6.2.1 Theory of Operation

The MDAC uses a switched network of capacitors to implement the subtraction operation of $V_{IN} - V_{DAC}$.

The MDAC uses the principle of charge conservation to implement subtraction. The following example couples the principle of charge conservation and KVL to illustrate the subtraction operation of the MDAC. It is important to note that the MDAC is fully differential; however, the example is done with single-ended signals.

Figure 50 shows a capacitor with a switch attached to the bottom plate and top plate. Utilizing KVL, this capacitor configuration can perform the desired subtraction. There are two phases to the operation of the switched cap network phase one ($\phi1$) and phase two ($\phi2$).
Figure 50: MDAC Theory of Operation

Figure 51 shows the capacitor during the first phase, the charge phase. During phase one the top plate is connected to ground and the bottom plate is connected to a single end of the differential input signal. The charge on the capacitor is then equivalent to the input voltage times the value of the capacitance, as seen in the figure below the capacitor.

\[ Q = CV = CV_{\text{IN}} \]

Figure 51: MDAC Theory of Operation – Phase One

During phase two, as can be seen in Figure 52, the top plate switch is opened in order to freeze the charge on the capacitor. The bottom plate switch is then connected to \( V_{\text{DAC}} \), the output of the digital decision determined by the quantizer, and the top plate is connected to \( V_{\text{OUT}} \).

\[ V_{\text{OUT}} = V_{\text{DAC}} - V_{C} \]

Figure 52: MDAC Theory of Operation – Phase Two
Using KVL, the value of $V_{OUT}$ can be determined. The output voltage is equal to $V_{DAC}$ minus the voltage on the capacitor, or $V_C$, as seen in the equation below the capacitor. Since, ideally, the voltage on a capacitor cannot change during the transition from phase one to phase two, $V_C$ is equal to the voltage on the capacitor during phase one. It should be noted that the actual result of the process is $V_{DAC} - V_{IN}$, however, the desired result of $V_{IN} - V_{DAC}$ can be obtained by inverting the differential outputs. That is, $V_{OUTP}$ becomes $V_{OUTN}$, and $V_{OUTN}$ becomes $V_{OUTP}$. This change occurs only in name, the signals are left unchanged, it is simply their relative naming that is modified. Therefore, the MDAC computes the desired subtraction. This output voltage is then fed off to the open loop differential pair, which is described in detail in section 6.2.2.

6.2.2 Capacitor Sizing

The capacitor size for the MDAC is selected to be 50fF based on our desired signal-to-noise ratio (SNR) of 70dB.

The capacitor is the key component to the MDAC. Capacitors also take up a large amount of die area. Therefore, it is important to keep capacitor sizes as small as possible to reduce die area, yet large enough to reduce noise effects. The ideal signal to noise ratio (SNR) of an n bit ADC is shown in (13). For a 12-bit ADC, the ideal SNR is 74 dB.

$$SNR_{IDEAL}dB = 6.02n + 1.76$$

(13)

Therefore, it seems reasonable to aim for approximately 70dB SNR in our ADC. 70dB SNR correlates to $10^{70/20} = 3162$, that is, 3162 parts signal for every 1 part noise. The largest input voltage swing on the capacitors is 1V peak. This equates to a 0.35V RMS swing. Therefore, the total RMS noise must be $\frac{0.35V}{3162} = 110\mu V$. If we assume that half of the noise comes from the amplifier, and half from thermal noise, we are able to determine the necessary size of the capacitors. "Half" of the total RMS noise of $110\mu V$ is $\frac{110\mu V}{\sqrt{2}} = 77\mu V$ RMS. Given that $\frac{\sqrt{kt}}{\sqrt{C}} = \sigma_V$, we find that the total $C$ should be at least 700fF. For 31 levels, we need 31 individual capacitors of $\frac{700fF}{31} = 22fF$. A capacitor size of 50fF is selected because it is the smallest capacitor that can be used in order to obtain sufficient matching [11].
6.2.3 MDAC Cell

Each of the MDACs is comprised of a collection of individual cells. These cells, as can be seen in Figure 53, are centered around the capacitor, its switches, and the control logic for them. The logic gates can be seen in block form, and the transistor switches can be seen as transistors. This block is simply repeated and connected together at the output to simplify design and layout for stages with different numbers of quantization levels.

![Figure 53: MDAC Cell Schematic](image)

Each individual MDAC cell implements the subtraction mechanism outlined in Figure 50, Figure 51, and Figure 52. The MDAC requires three clocks for its proper operation, which are generated from the ADC master clock with a series of logic gates.

The three clocks required for operation of the MDAC are designated $\Phi_1$, $\Phi_1A$, and $\Phi_2$. $\Phi_1$ and $\Phi_2$ correspond to the two major phases of the MDAC operation, charge and evaluate, respectively. $\Phi_1$ and $\Phi_2$ are non-overlapping, that is, they are never both simultaneously at a
logical high. This allows the connections to the MDAC cell capacitor to “break before make,” so that no shorting of signals occurs. Specifically, the bottom plate of the capacitor is connected to the input voltage during $\Phi_1$ and the digital decision $D$ during $\Phi_2$. It would not be appropriate for the input voltage to be connected digital decision and the cap simultaneously, as the subtraction algorithm would not complete successfully.

The clock logic is designed to implement the following logic equations, as seen in Table 5. The implementation of these equations ensures that the clocks that should not overlap don’t, as they will not be activated until the other clocks have been de-asserted. In this way we can avoid having to create the clocks externally, and have to make predictions as to the length of time required for one clock to transition to another. This method allows for that timing to implement itself, through the use of the additional logic. It should be noted that $\Phi_2$ is creates as per the equation in Table 5, however, when $\Phi_2$ is used to determine other clocks, $\Phi_2$ and $\overline{\Phi_2}$ are actually implemented as $\Phi_{2alt}$ and $\overline{\Phi_{2alt}}$, the combination of the control signals $\Phi_{2P}$ and $\Phi_{2M}$, as seen in (14). This method implements the safest implementation with respect to our need for “break before make” switches.

\[
\begin{align*}
\Phi_1A &= MCLK \cdot \overline{\Phi_2A} \cdot \overline{\Phi_2} \\
\Phi_1 &= \Phi_2A \cdot \overline{\Phi_2} \cdot \Phi_1A \\
\Phi_2A &= MCLK \cdot \Phi_1A \cdot \overline{\Phi_1} \\
\Phi_2 &= \Phi_2A \cdot \Phi_1A \cdot \overline{\Phi_1}
\end{align*}
\]

Table 5: Clock Generator Logic Equations

\[
\Phi_{2alt} = \Phi_{2M} \cdot \overline{\Phi_{2P}}
\]  

(14)

When determining if it is safe to assert the next clock signal, we must know that the gate voltages are in the appropriate range to turn the switch transistors off. We cannot be sure of this condition by sampling the control voltage on up-stream logic gates, due to propagation delays on the “break” side of the clock. The preceding logic may have transitioned to an off state, but the following transistors may not have.

However, it is safe to have propagation delays on the “make” side of the clock, because we can be sure that the previous connections have been broken, if the preceding paragraph has been implemented correctly, as the switching transistors are known to be off.
Figure 54 shows the clocks created by the clock generation block. It can be seen that there is very little, but non-zero, time at which both $\Phi_1$ and $\Phi_2$ are low, which is good. It shows that the “make before break” switching scheme is properly implemented, and also that there is little time spent not charging or discharging the capacitors; time which could be seen as unproductive, as more settling time helps with accuracy.

![Figure 54: MDAC Clocks](image.png)

The required logic based on the above analysis is shown in Figure 55.
6.3 Quantizer

The purpose of the quantizer is to convert the input signal from an analog signal to a digital signal. Figure 56, a pipelined ADC stage, is repeated below for convenience; for our application the quantizer selected is a flash ADC. Two flash ADCs were designed: one for the first stage and one for the subsequent stages. The first stage flash ADC has a 65mV step size and 31 levels. The subsequent stage flash ADC has a 65mV step size and 21 levels. A description of the selection of those parameters can be found in Section 3.1.
The flash ADC is made up of two components: (1) a track and latch comparator and (2) a preamplifier.

### 6.3.1 Number of Stages & Number of Bits per Stage

The pipelined ADC will contain a total of five stages. Four of the five stages will house a differential amplifier. The final fifth stage will consist of a quantizer. The first stage of the pipelined ADC will be characterized by 30 levels (4.9 bits). The four subsequent stages will be characterized by 21 levels (4.4 bits).

The number of bits per stage can be determined by an analysis of the input and output voltages, the quantizer step size, and the parameters of the differential amplifier. The input voltage to the differential amplifier is given by: \( V_{IN} - V_{DAC} \). The input to the differential amplifier is called the residue voltage because it is the leftover of the subtraction shown in the preceding equation. Recall that the input voltage will eventually be converted into (1) n bits and (2) a residue voltage. The residue voltage is the small amount of voltage that could not be quantized by the quantizer because it was too small. Specifically, it was smaller than the quantizer step size \( q \). Therefore, the residue voltage can be expressed as \( \pm q \), where \( q \) is the quantizer step size. Mathematically, the quantizer step size \( q \) is given by (15). In (15), \( q \) is the quantizer step size, \( 2V_r \) is the full-scale voltage range of the input voltage to the stage and not to the differential amplifier (\( V \) is half of the full-scale range), and \( N \) is the number of quantizer levels. Refer to Figure 3 and Figure 21 for a visual of the voltage swings through the system.

\[
q = \frac{2V_r}{N} \tag{15}
\]

Due to offset errors in the quantizer, a safety margin is incorporated into the value of \( q \). For more information on quantizer errors, see the design chapter on the quantizer. The residue voltage will be represented by the value \( \pm q (1+ m) \), where \( m \) is the safety margin. A safety margin of .5 means that the quantizer step size could be 50% larger or smaller than expected.

An equation for the gain of the differential amplifier can now be derived based on the definitions derived above. The gain of the differential amplifier can be expressed as the differential output divided by the differential input. The residue voltage, \( q(1+ m) \), is the differential input swing to the differential amplifier. The differential output swing of the differential amplifier is given in
Table 3 as 700mV. (16) gives the gain of the differential amplifier in terms of the input and output differential voltages. The second expression for the gain, (17), can be found by substituting the value of q (derived in (15)).

\[
G = \frac{0.7}{q(1 + m)} \tag{16}
\]

\[
G = \frac{0.7N}{2V(1 + m)} \tag{17}
\]

The equations make sense from an intuitive point of view. In (16), an increase in the step size (increased q) will need to be compensated for by a lower gain (decreased G). Alternatively, in (17), an increased resolution (increased N) will need to be compensated for by increasing the gain (increased G).

From these equations, the number of bits in the first and subsequent stages was determined.

The number of bits per stage was determined using the analysis provided above. The desired gain for the first stage is eight. In addition, a conservative desired safety factor is of 0.75 (m) is assumed. Furthermore, V is equal to 1V for the first stage (Table 3). Substituting these values into Equation 12 yields a q of approximately 65mV. Substituting these values into (17) yields an N of 30.

The number of bits per stage for the subsequent stage was also determined using the above analysis. Again, the desired gain for the subsequent stage is six. A safety factor of 0.75 was used once again. The only difference is that V is equal to 700 mV for the subsequent stage (Table 4). Substituting these values into (16) yields a q of approximately 65mV. Substituting these values into (17) yields an N of 21.
A summary of the findings of this analysis are provided in Table 6 and Table 7:

<table>
<thead>
<tr>
<th>Number of Bits</th>
<th>4.9 bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantizer Step Size (q)</td>
<td>65mV</td>
</tr>
<tr>
<td>Safety Factor (m)</td>
<td>0.75</td>
</tr>
<tr>
<td>Number of Levels (N)</td>
<td>30 levels</td>
</tr>
<tr>
<td>Gain</td>
<td>8</td>
</tr>
<tr>
<td>Full-Scale Output Range</td>
<td>2V</td>
</tr>
</tbody>
</table>

Table 6: First Stage – Number of Bits per Stage Analysis

<table>
<thead>
<tr>
<th>Number of Bits</th>
<th>4.4 bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantizer Step Size (q)</td>
<td>65mV</td>
</tr>
<tr>
<td>Safety Factor (m)</td>
<td>0.75</td>
</tr>
<tr>
<td>Number of Levels (N)</td>
<td>21 levels</td>
</tr>
<tr>
<td>Gain</td>
<td>8</td>
</tr>
<tr>
<td>Full-Scale Output Range</td>
<td>1.4V</td>
</tr>
</tbody>
</table>

Table 7: Subsequent Stage – Number of Bits per Stage Analysis

6.3.2 Track and Latch Comparator

The operation of the track and latch comparator [3] is as follows: a differential input voltage tips the output voltages to the rails. A larger differential input results in the comparator reaching the rails faster. When the clock is high, the voltages at the input and output nodes reset preventing the previous values from slowing down the comparator or changing the next output. The schematic for the track and latch comparator is shown in Figure 57.

Figure 57: Track and Latch Comparator Schematic
An important design criterion was the size of the MOSFETs. Natural device mismatch due to the size of the MOSFETs can be quite large, in the range of 30% of the width according to [1]. A mismatch in M6 and M9 created a large voltage offset; a 20nm change in the width of M6 resulted in a 30mV offset. Using the Cadence simulation tool, the width necessary for a 5mV offset was determined to be 3μm.

6.3.3 Preamplifier

A preamplifier is inserted in front of the latch in order to prevent kickback into the driving circuitry [3]. Several important considerations must be considered: the effect of the large threshold range on operation, gain and input capacitance. The schematic for the preamplifier is shown in Figure 58.

The preamplifier should have a low transconductance in order to reduce the settling time. In addition, a small amount of input capacitance is required to reduce settling time. The capacitance values given for the gate oxide capacitance-thin oxide at 1.8V for an NMOS device is 8.6 fF/μm². As a result, the 21-level quantizer required a width of 3.07μm, retaining the minimum length and not designing below a capacitor size of 100fF.
6.3.4 Voltage References

The voltage references for the quantizer were established using a resistor ladder and the supplied 1.8V. 1kΩ resistors were used throughout the ladder in order to partition the supply voltage into the required references. The resistor size was chosen due to power and size considerations. Figure 32 and Figure 60 show the configuration of the voltage references for the quantizers.
Figure 59: 31-Level Quantizer Voltage Levels
6.3.5 Binary Encoder Design

Recall that each quantizer has a certain number of digital decision outputs, either 31 (first stage quantizer) or 21 (subsequent stage quantizer). These outputs are generated by separate comparators, which compare the input analog voltage to reference voltages generated on a resistive ladder divider. When the analog input is compared with a lower voltage, the comparator output is high; conversely, when the analog input is compared with a higher voltage, the comparator output is low. These outputs take the form of a “thermometer code”, with all outputs being a logic low above the input voltage, and all outputs being a logic high below the input voltage.
These digital decisions need to be outputted to the external world via the I/O bonding pads on the die. To save space, we chose to binary encode these decisions. Otherwise, outputting five stages’ digital decisions in their original thermometer code would be a waste of I/Os and would require a ridiculous number of I/O pins. A typical solution for binary encoding a thermometer code is shown in Figure 61.

![Typical Thermometer-to-Binary Encoder](image)

**Figure 61: Typical Thermometer-to-Binary Encoder**

A series of XOR gates are used such that the zero-to-one transition (in this case, between D2 and D3) is detected. The activated logic high output is then directed to a ROM, which serves as a lookup table containing the binary encoded values. However, when operating under extremely high frequencies, timing differences between different signal propagation paths or comparator propagation delays can cause what is known as a “bubble”, due to its resemblance to a bubble in the mercury of a thermometer [6]. For example, a logic 1 may be found above a logic 0. This is illustrated in Figure 62.
The presence of a bubble will cause an uncertainty in the true zero-to-one transition point that the XOR gates attempt to find. As a result, two logic lines will be lit up, indicating two separate addresses to the ROM lookup table, which can cause a catastrophic error in conversion. A mechanism must be devised to enable the encoder to be tolerant of bubbles.

Mangelsdorf illustrates such a mechanism in [6]. Essentially, this mechanism can be thought of as a voting process. Each comparator output is compared to the outputs of its adjacent comparators. If the comparator output differs from both neighbors, that comparator’s output is toggled. Mangelsdorf gives the following logic equation for this mechanism: For comparators C1, C2, and C3, the corrected output for comparator C2 is given by C2*:

$$C2* = C1 \times C2 + C1 \times C3 + C2 \times C3$$  \hspace{1cm} (18) $$

This logic function can easily be implemented by a series of AND and OR logic gates. Applied to the predicament in Figure 62, we see that the problem is fixed in Figure 63.
Figure 63: Fixed bubble via correction circuit

[6] further illustrates some fascinating points about this correction scheme. First of all, it makes no difference how far away a lone comparator output ‘1’ is from the transition point. The correction scheme will eliminate a lone ‘1’ output no matter where it may lie. Secondly, he illustrates that either of two patterns, 0011 or 0101, will be recognized as the zero-to-one transition point in the bank of comparators. Finally, he shows a few cases where the bubble-correction scheme will work successfully, and where it will fail:

Figure 64: Examples of thermometer-code bubbles

The correction scheme successfully picks the “best guess” transition point in Examples 1, 2, and 3 of Figure 64. However, in Example 4 of Figure 64, [6] states that the correction scheme does not fix the bubble. He also notes that the error in Example 4 should only ever occur between different blocks of comparators, which in our ADC design, should not be a problem.

With regard to this project, each quantizer output and its two neighboring quantizer’s outputs are fed into a bubble correction block, as shown in Figure 65.
The bubble-correction block effectively implements the logic function in (1). In a logic diagram, this is represented by Figure 66:

![Logic Diagram for Correction Circuit](image)

**Figure 66: Logic Diagram for Correction Circuit**

For the first stage’s quantizer, there will be 31 such blocks. They are connected in a fashion such that each comparator output goes into three adjacent bubble correction blocks, including its own. This is shown in Figure 67.
Figure 67: Quantizer Bubble-Correction Block (detail)

The output of the bubble-correction block will then represent a “fixed” thermometer code, which can then be translated into a ROM address for the lookup table containing the binary encoded quantizer outputs.
7 TOP LEVEL CIRCUIT DESIGN

The work described in the Top Level Design section includes a preliminary layout of the design (floor plan) and a description of the pad ring and I/O.

7.1 Floor Plan

To begin planning how much die area is needed for the IC, we realize that we would like to minimize the die area required for our IC in an effort to increase the density of our design per wafer when the IC is eventually fabricated. There are two main constraints that prevent us from choosing an infinitesimally small die area: the actual die area that our design requires and the number of signals that need to be implemented as I/O pins. Each signal going off-chip will need its own separate bonding pad. In the SBC18HX process, bonding pads are 80µm x 80µm. There is also a required border around each bonding pad of 12µm. This makes each bonding pad effectively require 104µm x 104µm of die area:

![Figure 68: Bonding pad](image)

Bonding pads must go along the edges of the die area, not including corners. Table 8 shows a table of all I/O signals that require at least one bonding pad for each half of the split ADC.
A total of 51 signals need to be routed on/off-chip if we put one ADC per package (to reduce complexity of the chip). We would then use two separate packages, one ADC in each, sending signals to a digital post-processor to generate the output.

We also considered using LVDS (low voltage differential signaling) for the I/Os on this IC. However, that would double the number of required pins for the quantizer digital outputs, as well as for the voltage references and clock signals, requiring a very large package. In the end, we decided to use single-ended CMOS signaling, eliminating the need for copious numbers of I/O pins and large packages.

Using the contest preference for maximum die area of 2.4mm x 2.4mm per chip, we need to first determine the maximum number of bond pads on a 2.4mm x 2.4mm die. We estimate the area required for the pad ring to be approximately double the bond pad width, a border of 208µm around the entire die. By taking this figure into account (in addition to the bond pad size and a safety margin of about 10µm), we see that the maximum number of bond pads on the 2.4mm square die is \( \frac{2400\mu m - 2(104\mu m + 10\mu m)}{104\mu m} = 20.88 \approx 20 \) bond pads per die edge, allowing for a maximum of 80 pins per package. It follows that the dimensions of the usable die area after allocating space for bonding pads, pad ring, and a 10µm safety margin is \( 2400\mu m - 2(104\mu m + 208\mu m + 10\mu m) = 1756\mu m \) per side.

83
We still need to ensure that the usable die area is large enough to accommodate our IC design for each ADC. With a 1756µm square usable die area, the total usable die area is 3.083mm². The estimated die area requirements for each ADC are summarized below in Table 9:

<table>
<thead>
<tr>
<th>Functional Block</th>
<th>Estimated area required (each)</th>
<th>Total area required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantizer latch (x115)</td>
<td>30µm x 10µm = 300µm²</td>
<td>34,500µm²</td>
</tr>
<tr>
<td>Quantizer preamp (x115)</td>
<td>50µm x 10µm = 500µm²</td>
<td>57,500µm²</td>
</tr>
<tr>
<td>MDAC cell (x94)</td>
<td>150µm x 10µm = 1,500µm²</td>
<td>141,000µm²</td>
</tr>
<tr>
<td>Diff. Amp (x4)</td>
<td>100µm x 310µm = 31,000µm²</td>
<td>124,000µm²</td>
</tr>
</tbody>
</table>

Table 9: Required Die Area (per ADC)

Note: for 4 MDACs, there are three 21-level MDACs and one 31-level MDAC. This is equivalent to 94 individual MDAC cells.
Five quantizers are required: four 21-level quantizers and one 31-level quantizer. Each quantizer level requires one latch and one preamplifier, requiring a total of 115 latches and preamps.

From Table 9, we see that the estimated total die area required for one ADC is $357,000\mu\text{m}^2$. Even if we double the estimated total die area as a safety measure to $714,000\mu\text{m}^2$, we are still well within the total usable die area of $3.083\text{mm}^2$ from the I/O pin constraint.

Also as Table 9 shows, a preliminary layout of the quantizer, MDAC, and differential amplifier allows us to estimate how much die area will be required for one ADC. With those numbers, we chose to begin layout of the ADC around a single MDAC cell, which can be laid out to be approximately $150\mu\text{m} \times 10\mu\text{m}$. With one quantizer preamplifier and latch per MDAC cell, we chose to put each MDAC cell adjacent to its corresponding quantizer preamp and latch.

Figure 70 shows a quantizer preamplifier and latch (resized to fit into a $30\mu\text{m}$ square) placed adjacent to an MDAC cell:

![Figure 70: Quantizer and MDAC cell](image)

There will be 31 such cells for the first pipeline stage. When stacked vertically, the result is a rectangular die area $80\mu\text{m}$ wide and $930\mu\text{m}$ tall. The differential amplifier then follows the MDAC. For subsequent pipeline stages, there will only be 21 quantizer and MDAC cells. Both versions are shown in Figure 71.
Note that there will need to be two separate layouts of the differential amplifier to minimize unused area on the die. One layout will be approximately 310µm x 100µm, for use in the first stage, while all subsequent amplifiers’ layout will be approximately 210µm x 150µm.

Using this information, we can layout each ADC in stages, as shown in Figure 72:

Each ADC will require about 1600µm x 310µm of die area for a stage-sequential layout as shown in Figure 72. Figure 73 shows a potential layout of a single ADC on the die.
Figure 73: 80-pad Die with 1 ADC (1/2 split)
Figure 74 shows a diagram of the 80-pad die with potential signals for each of the pads. The signals have been placed for optimal signal routing. Groups of signals for a particular stage have had their pads highlighted in different colors. All stages share the same voltage supplies and clock signals, provided on the left edge of the die.
However, it seems that there is an excessive amount of unutilized die area, as well as a large number of unconnected pads on the die. We therefore seek to reduce the die size further by shrinking down the top and bottom edges of the die, and removing excessive unconnected pads. We also isolate the high-speed digital stage outputs to one side of the die, and move all of the analog pads to the other side, to separate analog and digital power domains, as well as to ease routing of signals on the die. Additionally, we double up on the number of all supply pads. The result is seen in Figure 75.

![Figure 75: Potential Pin Layout and Floor Plan for 60-Pad Die](image)

The die is now considerably smaller at 2400µm x 1268µm, about half the size of the die shown in Figure 74. The high-speed digital output pads are located on the right side of the die and are highlighted in red, while the analog circuitry is to the left and highlighted in blue. In the middle are the digital supplies to each stage’s quantizer and MDAC, as well as the mode select input pads to each stage. These are highlighted in green.
However, we recall that the analog signals will have widely varying voltage swings due to the charging/discharging behavior of the MDAC. To accommodate for this, we allow an extra pad for each of the analog supplies and utilize double-bonded pads for the MDAC analog reference signals. If the die size were not limited to more than 2.4mm per side, we would have allowed an extra pin per MDAC reference voltage. Since this is not the case, using double-bonded pads for each of those signals allows for all analog pads to be routed with two bond wires without increasing the size of the die. Of course, with these extra pins, the die will have to be slightly taller, as in Figure 76.

Figure 76: Potential Pin Layout and Floor Plan for 62-Pad Die
The die is now slightly larger at 2400µm x 1684µm, and there are now 62 pads on the die. Analog signals on the left edge have been given an additional bonding pad, while MDAC reference voltages on the top and bottom edges have double-bonded pads.

Figure 76 represents a feasible floor plan of the die, and is the final revision of our floor plan

7.2 Pad Ring

The pad ring for the ADC consists of ESD-protection circuitry. This includes ESD protection diodes for each I/O to both voltage supply rails (VDD and VSS) as well as voltage-limiting power supply clamp cells. The following figure from [9] illustrates the proper use of ESD diodes and power supply clamp cells in a custom mixed-signal integrated circuit:

![Figure 77: Proper Implementation of ESD Devices in custom Mixed-Signal IC](image)

There are two ESD diodes allocated per bond pad, allowing a path through which excess current can travel when two or more bond pads are subjected to ESD. Additionally, there are large NPN power supply clamp cells regulating voltage supply rails (i.e. VDD and VSS). These ESD protection devices will be located next to the bonding pads on our IC, in a structure known as a pad ring, for which we have allocated a 50µm wide strip encircling the entire die. In schematic form, the pad ring consists of an “ESD cell” for each I/O pad ring, whose schematic is shown in Figure 78:
The ESD cell consists of two ESD diodes, one going from the pad to the positive voltage supply rail, and one going to ground. This ensures an ESD discharge path to either VDD or VSS. The actual pad ring implementation includes one of these ESD cells per I/O, in addition to a set of cross-coupled ESD diodes separating voltage supply grounds and a power supply clamp cell per set of voltage rails. These are shown in Figure 79 and Figure 80.
8 VERIFICATION

The work described in the Verification section includes simulated performance data results of the open-loop residue amplifier, MDAC, and quantizer in addition to system level performance results. Also, the section includes a plan to verify the final layout in Phase Two and a plan to evaluate and test the final fabricated design.

8.1 Open-Loop Residue Amplifier

There are certain criteria that the open-loop differential amplifier must meet or exceed: sufficient differential gain and ample differential output voltage swing.

8.1.1 Differential Gain and Output Swing

To verify the first two criteria of differential gain and differential output swing, we can perform a DC sweep and plot the differential input-output characteristic. Note that the addition of the BJT emitter-follower stage drops the output common-mode of the differential amplifier to 580mV. This is not a problem for the next stage, explained earlier in Section 6.1.10, Output Stage Design.

![Figure 81: DC sweep of Differential Amplifier](image)

Figure 81 shows the DC sweep of the differential amplifier. Both outputs of the differential amplifier are plotted vs. differential input voltage. Taking the two points M2 and M3 in Figure 81, we see that the slope of the transfer function for a single output is approximately -4.02. To
find the differential gain of the entire amplifier, we double this number, giving us a differential gain of 8.04, very close to the gain of 8 that we designed for.

Figure 81 also shows the output range of the differential amplifier in terms of the edges of its roughly-linear range. The output range is seen to be around 692mV, very close to the 700mV output swing we designed for.

### 8.1.2 Power Consumption

In terms of power consumption, we can add up the bias current required for each part of the differential amplifier. This includes the main differential pair, replica biasing, common-mode feedback differential pair, common-mode feedback helper current, current mirroring (current-setting devices and N-to-P devices), and resistive dividers. This is shown in Table 10. The numbers derived in Table 10 come from examining an annotated schematic of the DC bias rail and resistive dividers, as shown in Figure 82.

<table>
<thead>
<tr>
<th>Current Consumption</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Main mirror</td>
<td>200uA</td>
</tr>
<tr>
<td>CMFB Differential Pair</td>
<td>84uA</td>
</tr>
<tr>
<td>N-to-P mirrors</td>
<td>600uA</td>
</tr>
<tr>
<td>Replica Bias</td>
<td>225uA</td>
</tr>
<tr>
<td>CMFB Helper Currents</td>
<td>1520uA</td>
</tr>
<tr>
<td>Output Stage (Emitter Follower)</td>
<td>240uA</td>
</tr>
<tr>
<td>Main Differential Pair</td>
<td>1977uA</td>
</tr>
<tr>
<td>Resistive Dividers</td>
<td></td>
</tr>
<tr>
<td>Resistive Dividers</td>
<td>134uA</td>
</tr>
<tr>
<td></td>
<td>128uA</td>
</tr>
<tr>
<td></td>
<td>86uA</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>5194uA</td>
</tr>
</tbody>
</table>

Table 10: Current consumption
Figure 82: DC Bias Rail and Resistive Dividers

The differential pair draws a total of 5.194mA, at a supply voltage of $V_{DD} = 1.8V$. This is a total of 9.3mW. Using the formula for figure-of-merit in (19),

$$FOM = \frac{\text{Power Consumption} / \text{Speed}}{(\text{Effective Number of Bits})^2}$$

(19)

and assuming that the differential amplifier dominates power consumption in the overall ADC, as well as assuming that differential amplifiers in the 3 subsequent stages can be scaled down such that all 3 amplifiers consume a total current equivalent to that consumed by the first stage amplifier, we see that the figure of merit for our ADC design is roughly 1.29pJ/step, running a 12-bit ADC at 100MHz. This is relatively close to the industry ‘standard’ of about 1pJ/step in efficiency of conversion.

Our target figure of merit for this ADC was 0.5pJ/step. If we re-use the assumptions stated in the previous paragraph, it is possible to tweak the differential amplifier to produce an ADC
with an overall figure of merit of approximately 0.75pJ/step. This would involve lowering the power consumption of the differential amplifier and also decreasing the differential gain of the amplifier. If the gain is lowered to 6, it is estimated that power consumption of the differential amplifier could be reduced by 40%, to a total of 11.2mW for four amplifiers. Also, since we have five 5-bit stages, producing a raw 25-bit output (before overlap, digital post-processing, etc.), it is conceivable that a 14-bit result could be produced rather than a 12-bit result. If these modifications were made, it would be possible to obtain a 14-bit ADC operating at 100MHz with a figure of merit of 0.571pJ/step.

### 8.1.3 Overall Amplifier Specification

From the design of the quantizer, we know that the differential input to the differential amplifier should not exceed 65mV, the quantizer step size. However, we have increased this margin to 87.5mV due to the gain of 8.

The specification table for the amplifier is thus as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage</td>
<td>900mV ± 87.5mV</td>
</tr>
<tr>
<td>Differential Input Swing</td>
<td>± 175mV</td>
</tr>
<tr>
<td>Output Voltage</td>
<td>580mV ± 350mV</td>
</tr>
<tr>
<td>Differential Output Swing</td>
<td>± 700mV</td>
</tr>
<tr>
<td>Differential Gain</td>
<td>8.04</td>
</tr>
<tr>
<td>Voltage Supply</td>
<td>+1.8V</td>
</tr>
<tr>
<td>Current</td>
<td>5.0331mA</td>
</tr>
<tr>
<td>Power Dissipation</td>
<td>9.05mW</td>
</tr>
<tr>
<td>Power FOM</td>
<td>1.25pJ/step</td>
</tr>
<tr>
<td>Input Resistance</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Output Resistance</td>
<td>800Ω</td>
</tr>
</tbody>
</table>

**Table 11: Differential Amplifier Specification**

This specification meets and/or exceeds our design evaluation criteria.

### 8.2 MDAC

As a standalone block, the most important function that the MDAC implement is the subtraction algorithm. As such, the following shows the process used to verify the proper functionality of the implementation of that algorithm, as described in previous sections. Figure 83 shows a portion of a MDAC verification simulation. In this simulation, a 15-level MDAC is
tested. In this test, the input differential voltage was swept, while the digital decisions were incremented correspondingly. This will help to show that the subtraction takes properly, and will help to assess the accuracy of the subtraction. A summary of the values shown in Figure 83 are displayed in Table 12. Shown are the differential input voltages, \( V_{Inn} \) and \( V_{Inp} \). Additionally, the output voltages created by the simulation and the ideal calculated output values are shown. Finally, an error is calculated by examining the difference between the ideal calculated differential output voltage and the simulated differential output.

<table>
<thead>
<tr>
<th>( V_{Inn} )</th>
<th>( V_{Outp Sim} )</th>
<th>( V_{Outp Calc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>535.00</td>
<td>862.00</td>
<td>898.33</td>
</tr>
<tr>
<td>547.00</td>
<td>859.00</td>
<td>886.33</td>
</tr>
<tr>
<td>559.00</td>
<td>847.00</td>
<td>874.33</td>
</tr>
<tr>
<td>572.00</td>
<td>836.00</td>
<td>861.33</td>
</tr>
<tr>
<td>584.00</td>
<td>824.00</td>
<td>849.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( V_{Inp} )</th>
<th>( V_{Outn Sim} )</th>
<th>( V_{Outn Calc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1265.00</td>
<td>871.00</td>
<td>901.67</td>
</tr>
<tr>
<td>1253.00</td>
<td>874.00</td>
<td>913.67</td>
</tr>
<tr>
<td>1240.00</td>
<td>886.00</td>
<td>926.67</td>
</tr>
<tr>
<td>1227.00</td>
<td>898.00</td>
<td>939.67</td>
</tr>
<tr>
<td>1215.00</td>
<td>910.00</td>
<td>951.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( V_{Ind} )</th>
<th>( V_{Outd Sim} )</th>
<th>( V_{Outd Calc} )</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>730.00</td>
<td>9.00</td>
<td>3.33</td>
<td>5.67</td>
</tr>
<tr>
<td>706.00</td>
<td>15.00</td>
<td>27.33</td>
<td>-12.33</td>
</tr>
<tr>
<td>681.00</td>
<td>39.00</td>
<td>52.33</td>
<td>-13.33</td>
</tr>
<tr>
<td>655.00</td>
<td>62.00</td>
<td>78.33</td>
<td>-16.33</td>
</tr>
<tr>
<td>631.00</td>
<td>86.00</td>
<td>102.33</td>
<td>-16.33</td>
</tr>
</tbody>
</table>

Table 12: MDAC Verification Simulation Summary (all voltages in mV)

As can be seen, the errors stay in the range of 15mV. Given a 15-level quantizer and a 1V FSR, the 15mV error is well within \( \frac{1}{2} \) of an LSB, that being 33mV. This shows with reasonable certainty that the MDAC operates properly, at the required frequency.
Figure 83: MDAC Verification Simulation
8.3 Quantizer

Figure 84 verifies the behavior of one decision level of a 21-level quantizer. The levels are numbered n15 (-15) to +15 including 0. Each of the colored lines (quantizer levels) represents the negative differential output subtracted from the positive differential output. The output combination shown in Figure 84 represents an input differential voltage of 950mV, a value in between the threshold for latches +14 and +15. The +15 latch settles at the negative rail because 950mV is lower than the threshold for latch +15. The input voltage of 950mV is higher than the threshold for all latches below +15; as a result, these latches all settle at the positive rail.

![Figure 84: 21-Level Single-Decision Quantizer Verification Simulation](image)

A 31-level quantizer was also constructed, tested, and completely verified. The full-scale input of the quantizer was swept and the digital decisions were plotted. The full data table and plot of the outputs can be found in APPENDIX F: 31-Level Quantizer Verification.
8.4 Phase Two Hardware Plan

We have outlined the path we will take to (1) to verify the final layout and (2) evaluate and test the final fabricated design.

Between the March 15\textsuperscript{th} announcement of the winners of Phase One and the May 15\textsuperscript{th} deadline for Phase Two Initial Submissions, the layout of the IC will be completed and will pass all LVS and DRC requirements. A proposed schedule for the completion of the project is shown below.

The chip will be tested and evaluated using an FPGA coupled with the digital correction algorithm described in [9].

<table>
<thead>
<tr>
<th>ID</th>
<th>Task Name</th>
<th>Start</th>
<th>Finish</th>
<th>Duration</th>
<th>Mar 2008</th>
<th>Apr 2008</th>
<th>May 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Pad Ring Layout</td>
<td>4/14/2008</td>
<td>5/2/2008</td>
<td>15d</td>
<td>4/13</td>
<td>5/4</td>
<td>5/11</td>
</tr>
<tr>
<td>5</td>
<td>Bond Pad Layout</td>
<td>4/14/2008</td>
<td>5/2/2008</td>
<td>15d</td>
<td>4/13</td>
<td>5/4</td>
<td>5/11</td>
</tr>
</tbody>
</table>

Figure 85: Schedule of Layout Completion

A picture of the eventual evaluation of the IC with the FPGA, PC, and data collection is shown in Figure 86.
Figure 86: ADC IC with FPGA, PC, and Data Collection
9 Future Work

This portion of the report details problems that we observed with the operation of the ADC. Due to the obvious time constraints of the MQP, we have been unable to fully solve these problems. What follows is a description of these problems and possible avenues through which a solution may be reached.

9.1 MDAC

A simulation of the voltages of the top and bottom plates of an MDAC cell capacitor is shown in Figure 87. The top plot shows the voltages on the top plate. The discrete waveform shows the actual output top plate voltage, while the continuous waveform shows the ideal output top plate voltage as calculated by \( V_{\text{OUT}} = V_{\text{REFX}} + V_{\text{CM}} - V_{\text{IN}} \). It should be noted that in this simulation, \( V_{\text{REFX}} = V_{\text{REFM}} = 0.4 \text{V} \).

The bottom plot shows the bottom plate voltages; specifically, the bottom plate accurately connects to both \( V_{\text{REFX}} \) and \( V_{\text{IN}} \).

![Figure 87: MDAC Cell Operation - Capacitor Voltages](image)

We are unsure as to the cause of the difference between the actual output voltage and ideal output voltage, given that the other three connections (\( V_{\text{REFX}}, V_{\text{CM}}, \text{and } V_{\text{IN}} \)) are all accurate. We believe that the difference between actual and ideal output voltage could be caused by either parasitic capacitances in the SPICE model of the capacitor, or charge loss during switching of the transistors that connect the capacitor plates to the appropriate voltages.
It should be noted that the voltage discrepancy is proportional to the input voltage. However, we cannot say with any certainty that the input voltage directly causes this error.

In addition, we found that increasing the capacitor size to 250fF (a factor of 5) helped reduce the output voltage error. We are not sure if this is the ideal solution to this problem, but it is a potential one.

Figure 88: MDAC Cell Overall Switching Currents

Figure 88 shows the transient currents in one switching cycle flowing through individual MOSFET terminals that directly connect the capacitor plates to the appropriate voltages.

Figure 89 shows the currents in the transistors switching to reference voltages in a single MDAC cell around the time of $\Phi_1$ rising edge. $\Phi_2$ control the transistors that connect the bottom plate to the reference voltage $V_{REFX}$. When $\Phi_2$ falls, these transistors go into the cutoff operating region, making them effectively open-circuits. Figure 89 shows that there is a flow of current...
through these (and other) transistors. We are unsure as to why this happens; it is our belief that when a transistor “turns off”, there should be no flow of charge.

Additionally, when $\Phi_1A$ and $\Phi_1$ rise in Figure 89, current flows through transistors that are in the cutoff region. The transistors that should be in cutoff are the ones that connect the bottom plate to $V_{REFX}$. Again, we are unsure as to why this happens, but we think that this flow of charge could potentially be causing discrepancies in the subtraction.

Figure 89: Currents during $\Phi_2$ falling edge, $\Phi_1A/\Phi_1$ rising edges

Figure 90 shows similar behavior during the falling edges of $\Phi_1A/\Phi_1$ and the rising edge of $\Phi_2$. During this phase, the transistors that should be in cutoff are the analog transmission gate to $V_{IN}$ and the NMOS switch to $V_{CM}$. However, there is current flowing through these transistors.

Again, we think that this stray current is the charge loss that is causing the voltage discrepancies seen in Figure 87.
9.2 Differential Amplifier

The open-loop differential amplifier is a key component to the appeal of our overall ADC. Murmann [10] showed that the power savings in using an open-loop amplifier stage as opposed to a traditional closed-loop amplifier stage in a pipelined ADC was around 60%. Although the use of an open-loop amplifier has driven the speed-power figure of merit of our ADC to around 0.571\textmu J/step, there are certainly several optimizations to the differential amplifier that could be implemented. However, due to the obvious time constraints of the MQP, this analysis and optimization was overlooked in favor of coarse functionality of the overall ADC.

The most important specification of the differential amplifier (from a system-level point of view) was that it be a low-power solution. It follows that the power draw of the differential amplifier should be optimized, given that it dominates the power dissipation of the entire ADC. However, for a fixed resolution pipelined ADC, there is probably a tradeoff between the attainable differential gain and the overall number of stages required to reach that resolution. In
other words, the differential gain required was determined by the number of quantization levels chosen per stage. Conversely, the number of quantization levels could be determined by the attainable differential gain.

Our design philosophy was to simply choose some design parameters and, based on those decisions, determine the other required parameters. This was done to simplify and hasten the design process. In future work, some analysis could be done to determine a more optimum solution in terms of overall power consumption for a fixed resolution ADC.
10  CONCLUSION

The completed 12b 100MSps pipeline ADC design represents application of the novel “Split ADC” concept to drastically reduce power consumption and improve performance as measured by the pJ/step FOM for ADCs in the important application area of portable ultrasound. Rather than push absolute limits, we have chosen a target in an important application space with moderate speed and resolution; the high performance aspect of the design will be the improvement in the speed-power FOM. For a representative ADC such as the ADS5270 [13], the FOM is approximately 1 pJ/step; in addition, the power dissipation in one ADC is 113mW. Through the use of an open-loop residue amplifier we achieved sufficient power savings to improve the FOM to $0.571\text{pJ/step}$ and the power dissipation in one ADC to $11.2\text{mW}$. 
REFERENCES


APPENDIX A: MATLAB Uncorrelated ADC Simulator

The below code simulates a 12-bit 5-stage uncorrected pipelined ADC resolving 3 bits per stage (except the last stage, resolving 4 bits) utilizing an open-loop differential pair residue amplifier in the first stage. The simulator also implements a mode select feature which effectively shifts the digital output by ±½ LSB.

Additionally, the simulated pipeline ADC operates on supply voltage rails of ±2.5V. The simulator takes an input of an input vector. To obtain residue and decision plots, the input vector can be set as a sweep of the input voltage. The resulting digital decisions from each stage are saved in variables s1d – s5d and the analog residues saved into variables s1r – s5r. The digital decisions from each stage are reconverted to analog voltages and output as the variable ‘result’.

```
function [result] = adcSim(inputVector, mode)
    m = 0;
    if mode
        m = 0.5;
    else
        m = -0.5;
    end
    gain = 4;
    bitsPerStage = 3;
    bitsFinalADC = 4;
    adcLowFSR = -2.5;
    adcHighFSR = 2.5;
    adcFSR = adcHighFSR - adcLowFSR;

    Vstep = adcFSR/(2^bitsPerStage);
    VstepFinalADC = adcFSR/(2^bitsFinalADC);

    s1d = [];
    s1r = [];
    s2d = [];
    s2r = [];
    s3d = [];
    s3r = [];
    s4d = [];
    s4r = [];
    s5d = [];
    Vin = inputVector;

    % stage 1
    s1d = (sign(Vin).*Vstep*round(abs(Vin)/Vstep));
    s1d = s1d + m;
    s1r = Vin-(sign(Vin).*Vstep*round(abs(Vin)/Vstep));
    s1r = diffPairNonLin(s1r, gain);
    %slr = slr.*gain;

    % stage 2
    s2d = (sign(s1r).*Vstep*round(abs(s1r)/Vstep));
    s2d = s2d + m;
```

The function `diffPairNonLin` controls the nonlinearity of the differential pair gain. The differential pair nonlinearity is modeled upon a constant gain (first order term) as well as a cubic nonlinearity term dependent on a nonlinearity coefficient $\alpha$ and a MOSFET overdrive voltage $V_{OV}$. The code for this function is displayed below:

```matlab
function Vod = diffPairNonLin(Vid, G)

% nonlinearity in diff pair
alpha = 0.133/32;
Vov = 0.25;
Vod = G.*Vid - alpha*G/(Vov^2).*Vid.^3;
```

The function `diffPairNonLin` controls the nonlinearity of the differential pair gain.
APPENDIX B: INL/DNL Calculator

The following code will import a dataset as saved in an n-by-1 sized array of digital codes produced from a linear sweep of input voltage and derive from them the corresponding differential and integral nonlinearity in two ways.

Differential nonlinearity is calculated as the ratio of actual hits divided by ideal hits, minus 1. The number if ideal hits per code transition should be the total number of digital decisions divided by $2^N$, where N is the total number of bits, if the input to the ADC is linear.

The first method if INL calculation is simply the cumulative sum of the DNL. The second method of INL calculation is by deviation from ideal transfer function. The slope of the actual data is calculated via the MATLAB function ‘regress’. Knowing this, the slope of the actual data can be corrected. From this gain corrected data the INL can be calculated, given that the input is swept linearly and with an input that is monotonically increasing.

```matlab
%number of bits in adc
nbits = 12;
%load data set
load('\ ... datafile.mat')
%rotate
output = output';
%collect histogram data from input dataset
[counts, bins]=hist(output, min(output):1:max(output));
%ideal 'hits' per code transition based on linear input
datapointsperct = length(output)/2^nbits;
%dnl
inll = (counts./datapointsperct)-1;
%calculate inl as sum of dnl
inl1 = cumsum(dnl);
mainlsum = max(inl1);
%calculate inl as deviation from gain corrected ideal
b = regress(output', Vin');
gaincorrectedoutput = output./b.*(2^nbits/5);
x = min(Vin):max(Vin)/max(bins):max(Vin);
inl2 = Vin./5.*2^12 - gaincorrectedoutput;
maxinlcalc = max(inl2);
%plot results
subplot(3, 1, 1), plot(bins, dnl), title('Differential Nonlinearity'), , xlabel('Digital Code'), ylabel('DNL [LSB]'), grid; v = axis; axis([v(1) 2^nbits v(3) v(4)])
subplot(3, 1, 2), plot(bins, inl1), title('Integral Nonlinearity as sum of DNL'), xlabel('Digital Code'), ylabel('INL [LSB]'), grid; v = axis; axis([v(1) 2^nbits v(3) v(4)])
subplot(3, 1, 3), plot(Vin.*2^12./5, inl2), title('Integral Nonlinearity as Calculated from Ideal'), xlabel('V_In'), ylabel('INL [LSB]'), grid; v = axis; axis([v(1) 2^nbits v(3) v(4)])
```
APPENDIX C: Verilog A Code

MDAC and Open-Loop Residue Amplifier

/*

% This segment of code simulates the behavior of an MDAC.
% 1. MDAC samples input (VIP, VIM) on zero-crossing of falling edge
%    of clock (clk).
% 2. The digital input (D) is converted into analog voltage levels
%    (VDP, VDM).
% 3. The analog voltage levels (VDP, VDM) are subtracted from the
%    sampled input (VIP, VIM). These new values are referred to as
%    the residues (VRP, VRM).
% 4. The residues (VRP, VRM) are amplified by a linear or nonlinear gain.
% 5. The residues (VRP, VRM) are set to 0 when the clock (clk) is high.

% INPUTS: VIP, VIM, clk, D
% OUTPUTS: VRM, VRP

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
*/

`include "disciplines.vams"

`include "constants.vams"

`define bits                 // 4 bit input comes from Mode Select output
`define Vref 2.5

module MDAC (VIP, VIM, clk, D, VRP, VRM); // Defining inputs & outputs of MDAC

parameter real vdd = 2.5;                 // Logic level high

integer i;

input VIP, VIM, clk;

input [0: `bits - 1] D;   // Mode select output D serves as input to MDAC

output VRP, VRM;

// Defining differential residue outputs

voltage VIP, VIM, clk, VRP, VRM;

voltage [0: `bits - 1] D;

real Dprep[0: `bits - 1];

real sampleP, sampleM, sample, sampleNLG, sampleApprox, name, Inter;

real VRPprep, VRMprep, Dint, Dtrue, DVref, G, a, Di, VIMAX, GMAX, N;

genvar k;

analog begin

@(cross (V(clk), -1)) begin

// Falling edge of clock: MDAC samples input and reads output of Mode Select

sampleP = V(VIP);

sampleM = V(VIM);

sample = sampleP - sampleM;

end

endmodule
sampleNLG = sample;
end

for (k = 0; k < `bits; k = k + 1) begin
    Dprep[k] = V(D[k]);
end

@(cross (V(clk), 1)) begin
    // Rising edge of clock: MDAC calculates residue voltage
    Dint = 0;

    for (i = 0; i < `bits; i = i + 1) begin
        // Dint = Integer representation of Mode Select output
        if (Dprep[i] == 2.5) begin
            Dint = Dint + (pow(2,i));
        end
    end

    Di = 5;       // SET NUMBER OF DECISIONS HERE
    N = 15;

    // SET NUMBER OF QUANTIZER LEVELS AND ENSURE BITS DEFINED ABOVE CORRESPONDS!
    name = (Di - 1)/2;
    // name defines number of regions above and below zero region

    sampleApprox = (-(`Vref) + (((2*Dint) - 1) / 2)*((2*`Vref)/N));
    // sampleApprox: approximates the value of the original sample using the
    // quantizer output

    // Although the MDAC has the exact value of sample, the mode select and
    // quantizer combination
    // can cause the MDAC to take a different decision path
    // Equations for algorithm can be found in SN2 pgs. 89-92

    for (i = 1; i <= name; i = i + 1) begin
        // Checks to see if sampleApprox is in one of the positive intervals
        // If it is, DVref and position on nonlinear curve are determined

        if ((sampleApprox > ((`Vref/Di)+((i-1)*((2*`Vref)/Di)))) &&
            (sampleApprox <= ((`Vref/Di)+((i*2*`Vref)/Di)))) begin
            DVref = (`Vref/Di)+((i-1)*((2*`Vref)/Di)) + (`Vref/Di);
            sampleNLG = sampleNLG - DVref;
        end
    end
for (i = 1; i <= name; i = i + 1) begin
// Checks to see if sampleApprox is in one of the negative intervals

if ((sampleApprox < ((-`Vref/Di) - ((i-1)*((2*`Vref)/Di)))) &&
(sampleApprox >= ((-`Vref/Di)-(i*2*`Vref)/Di))))) begin
    DVref = ((-`Vref/Di)-((i*2*`Vref)/Di)) + (`Vref/Di);
sampleNLG = sampleNLG - DVref;
end
end

if ((sampleApprox <= (`Vref/Di)) && (sampleApprox >= (-`Vref/Di))) begin
// Checks to see if sampleApprox is in the zero interval

    DVref = 0;
sampleNLG = sampleNLG - DVref;
end

G = 3; // SET LINEAR GAIN OF DIFFERENTIAL PAIR HERE
GMAX = G * 0.325;
VIMAX = `Vref/Di;
Inter = (GMAX/(3*G));
    a = (0.3333 - Inter)*pow((`Vref/VIMAX), 2));
// alpha parameter of nonlinear gain

// VRPprep = 1.25 + (0.5*G*(sample - DVref));
// These two lines are for linear gain
// VRMprep = 1.25 - (0.5*G*(sample - DVref));
VRPprep = 1.25 + (0.5*G*(sample - DVref)*(1 -a*(pow((sampleNLG/(`Vref)),2))));
// Nonlinear gain
VRMprep = 1.25 - (0.5*G*(sample - DVref)*(1 -a*(pow((sampleNLG/(`Vref)),2))));
end

V(VRP) <+ transition(VRPprep, 0, 1u);
V(VRM) <+ transition(VRMprep, 0, 1u);
end
endmodule
Quantizer

`include "disciplines.vams"
`include "constants.vams"

`define bits 4
// Need constants or constant expressions for genvar for loop, see pg. 40 SN2
`define Vref 2.5

module quantizerN (VIP, VIM, clk, Q);
// Defining quantizer and its respective inputs/outputs

parameter real vdd = 2.5;
// Logic level HIGH for digital output

integer i, N;

input VIP, VIM, clk;
// Differential inputs and clock

output [0: `bits - 1] Q;
// Quantized output value Q

voltage VIP, VIM, clk;
// Defining inputs VIP, VIM, and clock as voltages

voltage [0: `bits - 1] Q;

real Qprep[0: `bits - 1];
// Qprep is the output Q and is assigned as such in the last for loop

real sampleP, sampleM, sample, result, copyresult, name;
// sampleP = VIP, sampleM = VIM, sample = VIP-VIM, result = integer value of Q

genvar k;
// Loop variable used for making final assignment of Q

analog begin

@(cross (V(clk), -1)) begin
// Quantizer activated on zero-crossing of negative edge of clock

sampleP = V(VIP);

sampleM = V(VIM);

sample = sampleP - sampleM;

end

endmodule
$N = 15; \quad \text{// SET NUMBER OF DESIRED LEVELS HERE!}$

$name = (N - 1)/2;$

for (i = 1; i <= name; i = i + 1) begin
  // Checks to see if sample is in "positive range"
  if ($(\text{sample} > ((`Vref/N)+((i-1)*((2*`Vref)/N)))) && (sample <= ((`Vref/N)+((i*2*`Vref)/N))))$ begin
    result = name + i + 1;
  end
end

for (i = 1; i <= name; i = i + 1) begin
  // Checks to see if sample is in "negative range"
  if ($(\text{sample} < ((-`Vref/N)-((i-1)*((2*`Vref)/N)))) && (sample >= ((-`Vref/N)-((i*2*`Vref)/N))))$ begin
    result = name - i + 1;
  end
end

if ($(\text{sample} <= (`Vref/N)) && (\text{sample} >= (-`Vref/N)))$ begin
  result = name + 1;
end

copyresult = result;

for (i = (`bits - 1); i >= 0; i = i - 1) begin
  // Converts integer result into binary number Qprep
  if (result >= (pow(2,i))) begin
    Qprep[i] = vdd;
    result = copyresult - (pow(2,i));
    copyresult = result;
  end else begin
    Qprep[i] = 0;
  end
end

for (k = 0; k < `bits; k = k + 1) begin
  // Assigns Qprep to output Q with a 1 micro-second risetime
  $V(Q[k]) <= \text{transition}(Qprep[k], 0, 1u);$  
end
endmodule
Mode Select

/* Mode Select */

% This segment of code simulates the behavior of a Mode Select.
% 1. M = 0: Mode select passes Q input from Quantizer to output D.
% 2. M = 1: Mode select adds one to input Q to produce output D.
% % INPUTS: Q, M
% % OUTPUTS: D
% */

`include "disciplines.vams"

`include "constants.vams"

`define bits 4    // Number of bits for D and Q
`define Vref 2.5    // Maximum input voltage to any stage

module ModeSelect (Q, M, D);
    parameter real vdd = 2.5;  // Logic level high

    integer i;

    input M;      // M = control signal for Mode Select

    input [0: `bits - 1] Q;   // Q = output of Quantizer

    output [0: `bits - 1] D;        // D = input to MDAC

    voltage M;

    voltage [0: `bits - 1] Q;

    voltage [0: `bits - 1] D;

    real Dprep[0: `bits - 1];    // Will be used to hold intermediate value of D

    real Qprep[0: `bits - 1];   // Will be used to hold intermediate value of Q

    real Mvalue, Qint;
    // Qint = decimal value of Q, Mvalue holds value of M

    genvar k;

    analog begin

        Mvalue = V(M);  // Mvalue obtains value of control signal M

        if (k = 0; k < `bits; k = k + 1) begin
            // Copy value of input to Qprep
            Qprep[k] = V(Q[k]);
        end

    end

endmodule

if (Mvalue == 0) begin  // If control signal = 0V
    for (i = 0; i < `bits; i = i + 1) begin  // Value of Q goes to value of D untouched
        Dprep[i] = Qprep[i];
    end
end

if (Mvalue == 2.5) begin  // If control signal = 2.5V
    Qint = 0;  // Qint will hold decimal value of Q
    for (i = 0; i < `bits; i = i + 1) begin  // Find decimal value of Q
        if (Qprep[i] == 2.5) begin
            Qint = Qint + (pow(2,i));
        end
    end
    Qint = Qint - 1;  // Subtract 1 from decimal value of Q
    for (i = (`bits - 1); i >= 0; i = i - 1) begin  // Convert decimal value of Q back to a binary number
        if (Qint >= (pow(2,i))) begin
            Dprep[i] = vdd;
            Qint = Qint - (pow(2,i));
        end else begin
            Dprep[i] = 0;
        end
    end
end

for (k = 0; k < `bits; k = k + 1) begin  // D equals current value of Dprep with 1 micro-second transition
    V(D[k]) <+ transition(Dprep[k], 0, 1u);
end
end
endmodule
APPENDIX D: Behavioral Simulation Results

Stage 1 Residue Voltage ($V_r$) v. Input Voltage ($V_{in}$)

Stage 2 Residue Voltage ($V_{r2}$) v. Input Voltage ($V_{in}$)
APPENDIX E: Transistor Sizing Plots

<table>
<thead>
<tr>
<th>Plot</th>
<th>Independent Variable</th>
<th>Dependent Variable</th>
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<tbody>
<tr>
<td>1</td>
<td>Drain Current</td>
<td>Gate-Source Voltage</td>
</tr>
<tr>
<td>2</td>
<td>Transconductance</td>
<td>Gate-Source Voltage</td>
</tr>
<tr>
<td>3</td>
<td>Transconductance</td>
<td>Drain Current</td>
</tr>
<tr>
<td>4</td>
<td>Drain Current</td>
<td>Drain-Source Voltage</td>
</tr>
<tr>
<td>5</td>
<td>Drain-Source Resistance</td>
<td>Drain-Source Voltage</td>
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NMOS: \( L = 1\mu m \)
NMOS: \( L = 1\mu m \)

NMOS: \( L = 0.18\mu m \)
NMOS: $L = 0.36 \mu m$
PMOS: $L = 1\mu m$
PMOS 18\mu/m vs. Id

Gm (A/V) vs. Id (A)

Graph showing the relationship between Gm and Id for PMOS transistors.
PMOS: \( L = 0.18 \mu m \)
PMOS: $L = 0.36\mu m$
PMOS 18u/0.36u Gm vs. Id
### Table 13: 31-Level Quantizer Verification

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<th>Time (s)</th>
<th>$V_{INP}$</th>
<th>$V_{INM}$</th>
<th>$V_{DIF}$</th>
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31-Level Quantizer Verification

Figure 91: 31-Level Quantizer Verification