# Statistical Models for NCAA Indoor Track and Field Data

A Major Qualifying Project Report, completed in partial fulfillment
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## Abstract

The goal of this project was to continue the research of finding conversion standards for NCAA Indoor Track and Field. Conversion standards quantify the advantages of certain types of indoor tracks over others. Data from the 2010, 2011, and 2012 indoor seasons was fit to a series of statistical models, and these models were compared with one another. Final conversion standards computed from the models are provided. Based on the results of this project, suggestions and recommendations were made.

## Acknowledgements

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## Executive Summary

The collegiate sport of track and field is comprised of two seasons: indoor track and field and outdoor track and field. There is a standard size and shape for an outdoor track, and most college tracks adhere to this standard. However, indoor tracks tend to vary in size and shape due to space availability and cost. Indoor tracks can be grouped into four major types: flat, oversized, undersized, and banked. It is well known that certain types of tracks have advantages over others.

When ranking athletes for the NCAA Championships, the advantages (or disadvantages) of running on a particular track type should be taken into account. The NCAA adjusts race times to account for these advantages (disadvantages) using conversion standards. A conversion standard takes a race time from one track type and converts it to an equivalent race time on another track type. After converting all race times to one track type, the athletes can be properly ranked.

Last year, an NCAA committee calculated new conversion standards for indoor track and field. The committee wanted to update the old standards, which were derived from limited data and personal experience. In addition, the committee wanted to unify the three NCAA divisions under one set of standards, as opposed to a separate set for each division. The new conversion standards, which went into effect for the 2013 indoor season, were based on data from the 2010 and 2011 regular indoor seasons, and then validated using the 2012 season's data.

This Major Qualifying Project continued the research of finding conversion standards that accurately quantify the advantages of certain track types over others. The data used in the project consisted of all three (2010, 2011, and 2012) indoor track seasons. Each event and season was analyzed separately, as it was determined that the seasons were independent of each other (due to the large time gap between seasons). The statistical software program SAS 9.2 was utilized to model and analyze the data.

The indoor track data was fit to a series of statistical models. First, to explore the data, a basic linear regression model was fit to each athlete's sequence of race times and the collection of fitted parameters studied. Based on this analysis, a linear mixed model was proposed and fit to the data. The linear mixed model incorporates both fixed (track type) and random (athlete-to-athlete) effects, along with a repeated-measures covariance structure. The response variable of the data (race time) did not follow a normal distribution, and thus a generalized linear mixed model (GLMM) was next used to represent the data. GLMMs are similar to linear mixed models, but do not assume that the response and errors follow a normal distribution. Finally, a generalized linear repeated measures model was also fit to the data.

The goodness-of-fit of each model was analyzed and models were compared with one another (through residual diagnostics and information criteria, for example). Conversion standards were computed from the models through the differences between least squares means. The conversion standards from the models were compared with one another, as well as with the NCAA committee's standards.

The GLMM appeared to be the best-fitting model, based on the significance of the parameter estimates, residual analysis, and information criteria. However, the conversion standards calculated from this model were extremely low compared with the NCAA's conversion standards. The only major similarity between the NCAA and GLMM conversion standards was the banked to oversized track conversion, which says there is no difference between these two track types.

To assess the accuracy of the conversions standards, the NCAA, linear mixed model, and GLMM conversion standards were each applied to the data. Analyses showed that the NCAA and GLMM conversions did not eliminate the difference between track types. However, application of the linear mixed model's standards to the data did result in the elimination of track type effects from both the linear mixed model and the GLMM. This showed that the linear mixed model standards were the most accurate of the conversion standards studied here. The linear mixed model's conversion standards are larger than the NCAA's, and unlike those standards, suggest that there is a difference between banked and oversized tracks.

More research is highly suggested, and recommendations for future work are provided.

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# Chapter 1: Introduction

The National Collegiate Athletic Association (NCAA) sport of track and field is comprised of two seasons: indoor track and field and outdoor track and field. There is a standard size and shape for an outdoor track, and most college tracks adhere to this standard (Pederson, Larson, Jones, Podkaminer, 2012). The “circumference” is exactly 400 meters (400m) and the track typically has 8 or 9 lanes. Here, circumference refers to the distance of exactly one lap on the track in the innermost lane (lane one). Indoor tracks, unfortunately, do not follow one set standard. Several factors, such as space availability and cost, affect the size and type of indoor track (Pederson et al., 2012).

The different types of indoor track can be grouped into four categories: flat, undersized, oversized, and banked. Flat tracks are 200m in circumference and are completely flat all around. Undersized tracks are less than 200m in circumference and are flat all around. Oversized tracks are greater than 200m in circumference and flat. Banked tracks are 200m in circumference, but unlike the other three types, the turns of the track are sloped upwards, to create a bowl-like shape. The steepness of the banked turns can differ between facilities.

Like many other NCAA sports, track and field has its own NCAA Championships competition. The selection process for choosing who can attend this championship meet varies among the three divisions of the NCAA. In Division III, for example, the top 15 female athletes and top 13 male athletes in each individual event, as well as the top 10 relay teams, qualify for the meet. To determine the top athletes, the NCAA maintains a performance list for each track and field event that will occur at the championship meet (Pederson et al., 2012). An event performance list contains the best mark from every athlete who has competed in that specific event during the current season. The best marks are then ranked from fastest to slowest, longest to shortest, or highest to lowest (depending on the event).

However, regardless of the championship selection process, issues arise because of the different track types. Certain track types have advantages over others, which influence a runner’s race time. For example, it is well known that athletes run faster on banked tracks as opposed to flat tracks. The banked turns help reduce loss of speed when running along the curve, which clearly aids the athlete during a race. If two athletes were to run the same race time in a 400m dash, with Runner A on a banked track and Runner B on a flat track, Runner B would be considered to have the better time. On the banked track, Runner A would have had help from sloped turns, unlike Runner B. If one were to compare the two races, Runner A should, in a sense, be penalized for the help received from the banked curves.

In order for the NCAA to properly choose the fastest athletes for the NCAA championships, the advantages (or disadvantages) of running on a particular track type should be taken into account (Pederson et al., 2012). In order to do this, there must be a way to compare race times coming from different track types. The NCAA uses "conversion standards" to allow for this comparison. The conversion standards take a race time from one track type, and convert it to an equivalent time on another track type. By adjusting all the race times to reflect track type, the NCAA can then properly choose athletes for the championships.

Up through the 2012 indoor track season, the conversion from one track type to another simply consisted of adding (or subtracting) a certain amount to a runner's race time. In Division III, for example, to covert a women's 400m banked track time to a flat track time, one would add 0.4 seconds onto the banked track time. The amounts to add on (or subtract off) were based on limited data and personal experience, but updated yearly if necessary (Pederson et al., 2012). Each Division had its own separate set of standards as well.

In 2010, it became mandatory for all meet results to be reported online, and thus, because of much more readily available data, better analyses could be performed (Pederson et al., 2012). From September 2011 to the spring of 2012, an NCAA committee conducted a study to compute new conversion standards. The study used much more data than previous studies had used. The committee came up with a set of multiplicative standards, as opposed to the previous additive conversion standards. The new standards went into effect for the 2013 indoor season, and were used by all three NCAA Divisions.

In certain cases, the new standards affect race times much more than the old conversion standards. In 2011, as stated above, one added 0.4 seconds onto a Division III woman's banked track 400m race time to find the equivalent flat track race time. Multiplying by the new conversion standard causes a much more drastic change. For a 58.0 second 400m runner, multiplying by the new standard (to convert a banked track time to a flat track time) is approximately the same as adding 0.78 seconds to the banked race time. In track and field, the difference between 0.4 and 0.78 seconds is a large amount in a 400m dash, and could sometimes mean an athlete is ranked 20$^{th}$ as opposed to 10$^{th}$. In addition, since the new standards are multiplicative, larger race times are more affected than lower times. As said before, for a 58.0 second 400m, the conversion standard adds 0.78 seconds onto a banked track time. For a 65 second 400m, the conversion standard adds 0.87 seconds onto a banked track time.

The effect of the new conversion standards sparked my curiosity. As a track and field athlete who has attended several NCAA championships, these standards directly applied to me, and I thought it important to get them right. I had also heard negative comments about the new standards from other runners and coaches, who were not confident in the techniques used to compute them. Because of these things, I decided to look further into the NCAA indoor track data.

This Major Qualifying Project continued the research of finding NCAA conversion standards for track and field. I focused on the following goals:

- Model NCAA Indoor Track and Field data
- Determine the quantitative advantages/disadvantages of running on a particular track type
- Compute standards that properly rank athletes for NCAA championships

With the completion of these goals, I hope to aid the NCAA with properly choosing the correct athletes for the Indoor Track and Field Championships. I also hope to offer insight into the nature and behavior of track and field data, specifically data coming from collegiate runners.

# Chapter 2: Background

## 2.1 The NCAA and the Indoor Track and Field Championships

> *"The NCAA is made up of three membership classifications that are known as Divisions I, II and III. Each division creates its own rules governing personnel, amateurism, recruiting, eligibility, benefits, financial aid, and playing and practice seasons – consistent with the overall governing principles of the Association. Every program must affiliate its core program with one of the three divisions."*
>
> - www.ncaa.org, 2012

The most well-known way to distinguish between the three NCAA divisions regards financial aid for athletic participation. Division I schools provide the most athletic financial aid to student athletes, while Division II schools give limited financial aid. Division III schools provide no financial aid for athletic participation to its student athletes (www.ncaa.org, 2012).

Each NCAA division holds its own Indoor Track and Field Championships. The following events occur at the indoor championship meets:

- Sprints/Hurdles: 60m, 60m Hurdles, 200m*, 400m
- Distance: 800m, Mile, 3000m**, 5000m
- Relays: 4x400m, Distance Medley Relay (DMR, 1200m-400m-800m-1600m)
- Field: Long Jump, Triple Jump, High Jump, Pole Vault, Shot Put, Weight Throw
- Multi: Pentathlon (women), Heptathlon (men)

* 200m does not currently occur at the DIII meet

** 3000m does not currently occur at the DIII meet

Conversion standards apply to the events contested on the track that involve running on a curve (as opposed events contested on the track straight-aways, the infield, etc.). These events are the 200m, 400m, 800m, 1000m (a heptathlon event), Mile, 3000m, 5000m, 4x400m, and the DMR.

## 2.2 NCAA Track and Field Committee's Study

To start my project, I first needed the report that discussed the new conversion standards calculated by the NCAA committee. I contacted Bob Podkaminer, the NCAA Track and Field Secretary and Rules Editor, who provided me with the report. He also directed me towards Scott Jones, a coach at the University of Akron and one of the members of the NCAA committee. The two were able to clarify certain aspects of the report for me, and answered questions about the methodologies the NCAA committee employed.

The NCAA committee obtained its data from the TFFRS system, a website where track and field meet results are submitted (Pederson et al., 2012). The committee collected all performances from athletes who competed for NCAA member institutions during the 2010 and 2011 indoor seasons (Pederson et al., 2012). The data was then reduced: athletes who only competed in an event once or who only competed on the same track type during a season were removed. Using the reduced data, the committee then paired consecutive race performances in order to see if there was a relationship between different track types. Races from different years were not paired. They found it "unmistakably clear" that there was a track effect, or in other words, that track type played a role in an individual's performance (Pederson et al., 2012). They also determined that each Division's current conversion standards were not sufficient, and thus, new, universal ones were needed.

The NCAA committee proceeded to refine the data more, in order to begin the process of determining new conversion standards (Pederson et al., 2012). For example, only consecutive races within a set number of days were considered in the analysis. Consecutive races whose time lapse (number of days between races) was outside the set limit were excluded. Athletes who competed on only one track type were temporarily added back into the data. These performances would help calculate the training effect: an athlete's progression from race-to-race (Pederson et al., 2012). Without accounting for an athlete's improvement over the season, the conversion standards would be larger than in actuality (this is under the assumption that an athlete improves from one race to the next). The conversion standards would encompass both training effect the track effect (Pederson et al., 2012). By removing the training effect, the quantitative track effect could then be calculated.

Two separate approaches were used to calculate the conversion ratios. The first approach dealt with the race time difference, in seconds, between consecutive races. The second used the percent difference between consecutive races (Pederson et al., 2012). For more details, see *Indoor Facility Indexing For NCAA Running Event Performances* by Pederson et al. (2012). "The two approaches achieved very similar results," and the committee used the 2012 season's data (when it became available) to validate the results (Pederson et al., 2012). The final conversion standards are shown below in Figures 1 and 2. A comparison between the new conversion standards and the old Division III conversion standards is provided in Figure 3.

***Figure 1: New NCAA Indoor Track and Field Conversion Standards- Women***

Women

| Event | Undersized to Flat | Banked to Flat | Oversized to Flat |
|---|---|---|---|
| **200** | 0.9900 | 1.0155 | 1.0155 |
| **400** | 0.9929 | 1.0133 | 1.0133 |
| **800** | 0.9951 | 1.0115 | 1.0115 |
| **1000** | 0.9958 | -- | -- |
| **Mile** | 0.9969 | 1.0099 | 1.0099 |
| **3000** | 0.9981 | 1.0086 | 1.0086 |
| **5000** | 0.9989 | 1.0077 | 1.0077 |
| **4x4** | 0.9929 | 1.0133 | 1.0133 |
| **DMR** | 0.9959 | 1.0107 | 1.0107 |

***Figure 2: New NCAA Indoor Track and Field Conversion Standards- Men***

Men

| Event | Undersized to Flat | Banked to Flat | Oversized to Flat |
|---|---|---|---|
| **200** | 0.9872 | 1.0179 | 1.0179 |
| **400** | 0.9901 | 1.0160 | 1.0160 |
| **800** | 0.9923 | 1.0143 | 1.0143 |
| **1000** | 0.9929 | 1.0138 | 1.0138 |
| **Mile** | 0.9941 | 1.0128 | 1.0128 |
| **3000** | 0.9953 | 1.0116 | 1.0116 |
| **5000** | 0.9961 | 1.0107 | 1.0107 |
| **4x4** | 0.9901 | 1.0160 | 1.0160 |
| **DMR** | 0.9931 | 1.0136 | 1.0136 |

*Figure 3: Conversion Standards Comparison (Division III)*

Women

| Event | Old Conversion Standards (used for the 2012 season) | New Conversion Standard* (used for the 2013 season) |
|---|---|---|
| **400m** | 0.4 seconds | 0.78 seconds (0:58.0) |
| **800m** | 0.5 seconds | 1.56 seconds (2:15.0) |
| **Mile** | 0.9 seconds | 2.97 seconds (5:00.0) |
| **5000m** | 3.3 seconds | 8.00 seconds (17:18.0) |
| **4x400m** | 1.6 seconds | 3.14 seconds (3:56.0) |
| **DMR** | 2.5 seconds | 7.75 seconds (12:04.0) |

Men

| Event | Old Conversion Standards (used for the 2012 season) | New Conversion Standard* (used for the 2013 season) |
|---|---|---|
| **400m** | 0.5 seconds | 0.79 seconds (0:49.0) |
| **800m** | 0.6 seconds | 1.63 seconds (1:54.0) |
| **Mile** | 1.0 seconds | 3.24 seconds (4:13.0) |
| **5000m** | 3.6 seconds | 9.12 seconds (14:40.0) |
| **4x400m** | 2.0 seconds | 3.17 seconds (3:18.0) |
| **DMR** | 3.0 seconds | 8.20 seconds (10:03.0) |

* The conversion standards used for the 2013 season are multiplicative standards as opposed to additive standards. The above "New Conversion Standards" are the equivalent additive standards for the race times in the parentheses. If given a different race time, the additive standard would change. The purpose of this table is to show the effect of the new conversion standards on DIII race times.

*To convert from a banked track time to a flat track time, add the appropriate conversion standard above to the banked track time. For example: 400m banked track time- 58.0 seconds; flat track time- 58.4 (using old standard), 58.78 (using new standard)*

## 2.3 NCAA Track and Field Data

I obtained some of the men's and women's data that was used in the NCAA committee's study (prior to any refinement). The data was comprised of race times from the 2010, 2011, and 2012 regular track and field seasons. The regular season begins in mid-January and ends in early March. Very early meets, such as a few in December and early January, as well as the NCAA Championship meets were excluded from the data. Each gender had its own excel file and each event had its own spreadsheet tab. Race times from Divisions I, II, and III were combined together into the one spreadsheet tab. Data was given for the following events:

- Women’s Events: 200m, 400m, 800m, Mile, 3000m
- Men’s Events: 200m, 400m, 800m, Mile, 3000m, 5000m

An observation in the data consisted of an ID (the individual/runner), time (race time), the dates of the meet, and the use date. The use dates are the days the NCAA committee included in their calculations. Specifically, the committee found the race time difference between consecutive races, and thus dates were important. Issues arose for the committee, however, because some meets last longer than one day, so the runner could have competed on any one of the days. To solve this problem, the NCAA committee did the following: If a meet was only held for one day, the use date was the same as the meet date. If a meet was two days or more, the use date was listed as the end date of the meet.

I uploaded all of the data into SAS 9.2, software product that allows one to “perform a wide range of statistical and analytical tasks” (www.support.sas.com, 2013). In SAS, I could conduct involved statistical analyses of the data, such as more advanced regressions that Excel could not provide.

Once the data was in SAS, I separated an event’s data by year. I wasn’t sure what type of analyses I would be doing at that point, but I did not think I should combine data from different years in my analyses. The time gap between each indoor season is large and I thought that each season was essentially independent of one another. In other words, each season could be seen as a “new start” for returning athletes. Returning athletes refers to those who ran in more than one indoor season from 2010, 2011, or 2012.

## Chapter 3: Modeling

As stated before, the NCAA's study essentially focused on two tasks: calculating the "training effect" and then calculating the "track effect" through conversion ratios. The training effect is an athlete's race-to-race progression. An athlete typically improves as the season progresses, due to the fact that the athlete trains daily. The athlete also learns and practices race strategies, helping him or her improve even more. In order to find the true effect of different types of track, one has to account for this training effect. Thus, both the track type and the date of a race are the two important factors to include in the modeling.

### 3.1 Basic Regression Model

To begin with, I wanted to familiarize myself with the data, as well as learn how to code procedures in SAS. Because of this, I decided to keep my initial modeling simple. I wanted to see if there was any particular trend or relationship between a race time and the date of the race. I decided to hold off on incorporating a track effect into any models. Using the REG procedure in SAS, I fit a regression line for each runner, and then found an overall regression line. For each individual, I found that a linear regression line was a good fit for the data (as opposed to a non-linear fit). The response was the race time, while the date of the race was a predictor. In other words:

$$Y_{ij} = a_i + b_i X_{ij} + \varepsilon_{ij} \quad individual\ i, race\ j, \quad \varepsilon_{ij} \sim N(0, \sigma^2)$$

$$Race\ Time = Intercept + (Slope)(Date\ of\ the\ Race) + Error$$

In SAS, dates are represented as the number of days after January 1, 1960. If the day occurs before 1960, the date will be a negative number. Thus, for 2010, 2011, and 2012, the use dates were extremely large (above 19000). As mentioned before, use date was a variable created by the NCAA committee to deal with meets that lasted longer than one day. When I fit the regression lines with the use date, the regression lines were hard to make sense of. For example, the intercepts did not reflect an actual race time, and thus it was hard to see if the model was a good fit. Because of this, I created a new variable called fixed date.

Fixed date is essentially the same as use date, just shifted so the regression lines would make more sense. An individual's first race in an event would have a fixed date of 0. By doing this, the intercept of each regression line would measure an athlete's starting point for the season in that particular race. Then, for a subsequent race, fixed date would be the number of days after the first race. For example, if an individual first competed in the 400m on January 13, the variable fixed date had a value of 0. Then, a second 400m race on January 20 would have a fixed date of 7.

The slope of each individual's regression line roughly reflects that athlete's improvement as time passes, or in other words, the training effect. The estimate was rough because track effect was not accounted for in the model. Since the variable fixed date was used, and since fixed date is in terms of days, the slope actually shows an athlete's daily improvement. To find the improvement between races, simply multiply the slope by the number of days between races.

An overall regression line could then be used to find an estimate of the average training effect. To find this overall regression, I essentially used the STS method discussed by Henry Feldman in his article, *Families of Lines: Random effects in linear regression analysis* (1987). The STS method reduces each individual's data to a slope and intercept, and then treats the slopes and intercepts as "raw data for further analysis" (Feldman, 1987). After finding a regression line for each individual, I found the mean slope and intercept, which gave estimates for an overall slope and intercept. I also looked at the median slope and intercept, because the median is less affected by outliers than the mean.

In addition, I examined the weighted mean and the weighted median of the slope and intercepts. For these weighted statistics, I used the number of races that the individual competed in as the weight. Thus, individuals with more races would have more effect on the overall regression. It would be difficult to find the true training effect for an individual who only ran a couple of races, as opposed to someone who ran four or more times over the course of the season. Similarly, when the regression line was fit for each individual, standard errors were calculated for the slope and intercept. The reciprocal of these standard errors could be used as a weight as well. The individual regression lines with better fits (those with smaller standard errors) would have more weight in the overall mean.

When calculating the four different types of overall regression (mean, median, weighted mean, weighted median), I looked at different scenarios/cases. For example, I looked at the situation when individuals who only ran a race once were removed from the calculations for the overall regression.  One cannot find a training effect from one data point.  I also investigated other scenarios, such as when intercept (slope) outliers were removed from the overall regression calculation.  These outliers came from individuals who had observations that were out of the norm.  For example, for the women's 400m dash, if an individual runs a couple of races around 57 seconds, and then a third race in 67 seconds, this would be considered an outlier.  Clearly, something abnormal happened during the third race, quite possibly an injury.

While calculating the regression lines for different scenarios, a noteworthy point came up.  For an individual runner, a slope equal to zero could mean two things: a.) the runner had only one observation or b.) the runner had multiple observations with the same race time.  Both cases resulted in a horizontal line as the regression line.  When looking at the training effect, I had wanted to remove all individuals with a slope equal to zero, as a way to remove individuals with only one observation.  However, this was not a good approach, since I was also removing individuals with multiple observations, who did in fact have a training effect (equal to zero).

## 3.2 Linear Mixed Model

Following the basic regression, I decided to build a linear mixed model.  Linear mixed models contain both fixed and random effects (support.sas.com, 2013).  Fixed effects are factors that have multiple levels, and all of the levels are represented in the data (Smith, 2013).  In my case, track type is a fixed effect because all of the levels (the four track types) are represented. Random effects are factors where the levels represented in the data are only a random sample of all possible levels (a population).  The athletes in the NCAA data are only a random sample of all possible runners, and thus, this is where the randomness in the model comes from.

The track and field data also falls under the category of repeated measures data.  For a particular race, an individual runner can have multiple observations (races).  These observations are likely to be correlated, since they come from the same individual.  The model can account for this phenomenon, which I will discuss later.

### 3.2.1 The Model

All of the formulas and model theory below follows the SAS/STAT® 9.2 User's Guide, Section Edition. Specifically, all information was from "The MIXED Procedure" section of the guide. A more detailed and theoretical treatment can be found in McCulloch, Searle and Neuhaus (2008).

A linear mixed model takes the following form:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{Z\gamma} + \boldsymbol{\varepsilon}$$

where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}, \text{vector of observed data}$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_p \end{pmatrix}, \text{unknown vector of p } fixed \text{ effects parameters}$$

$$\boldsymbol{X} = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix}, \text{known design matrix for } fixed \text{ effects}$$

$$\boldsymbol{\gamma} = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_q \end{pmatrix}, \text{unknown vector of q } random \text{ effects parameters}$$

$$\boldsymbol{Z} = \begin{pmatrix} z_{11} & \cdots & z_{1q} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nq} \end{pmatrix}, \text{known design matrix for } random \text{ effects}$$

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{pmatrix}, \text{unknown random error vector}$$

The random effects $(\boldsymbol{\gamma})$ and errors $(\boldsymbol{\varepsilon})$ are assumed to be normally distributed, with $\boldsymbol{\gamma} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{G})$ and $\boldsymbol{\varepsilon} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{R})$. The observed data $(\boldsymbol{y})$ are also assumed to come from a normal distribution, with a variance equal to $\boldsymbol{V} = \boldsymbol{ZGZ}' + \boldsymbol{R}$, where $\boldsymbol{Z}'$ is the transpose of matrix $\boldsymbol{Z}$.

For a linear model, a model with only fixed effects and no random effects, the errors are assumed to be independent and homogeneous. This means that the errors associated with one particular individual are not correlated, and that each individual's errors come from the same normal distribution (same mean and variance). For a linear mixed model, however, the errors do not need to be independent and homogeneous. This is due to the randomness in the model.

### 3.2.2 Covariance Matrix

As stated before, because the track and field data is repeated measures data, the observations are likely to be correlated. This correlation can be modeled through ***R***, the variance matrix of the errors. The matrix ***R*** has a certain structure, known as a covariance structure, which provides both the variance of the errors, as well as the correlation between different observations for an individual. When modeling the NCAA data, I used the spatial power structure, SP(POW)(c). Here, c refers to the variable that defines the "coordinates" of a data observation (support.sas.com, 2013). In the case of my model, c was the fixed date variable. The SP(POW) structure has the following form (Note: this is for an individual with 4 observations. The size will change with more or less observations, but will stay a square):

$$\sigma^2 \begin{pmatrix} 1 & \rho^{d_{12}} & \rho^{d_{13}} & \rho^{d_{14}} \\ \rho^{d_{21}} & 1 & \rho^{d_{23}} & \rho^{d_{24}} \\ \rho^{d_{31}} & \rho^{d_{32}} & 1 & \rho^{d_{34}} \\ \rho^{d_{41}} & \rho^{d_{42}} & \rho^{d_{43}} & 1 \end{pmatrix}$$

Here, $\sigma^2$ is the variance, and the $\rho$'s are the correlations. The $d_{ij}$'s are the distances between the $i^{th}$ and $j^{th}$ observations, or in my case, the time lapse between different races. The SP(POW) structure was a good fit for the NCAA data because it allows for unequally spaced observations, unlike other covariance structures that are designed to model observations that are equally spaced apart (support.sas.com, 2013). The SP(POW) structure also follows the layout that as observations become further and further apart (in time), they become less correlated. This seemed to fit well with the track and field data: a race will affect a subsequent race the following week more than it will a race three weeks later.

Minor adjustments to the data were necessary in order to use the SP(POW) structure. This structure assumes that there is only one observation at a specific time unit (support.sas.com, 2013). In the track data, some individuals ran preliminary and final races at the same meet. However, these races were listed as occurring on the same day. This was due to the NCAA's creation of use date, the variable the committee utilized in its analyses. However, preliminaries and finals typically occur on different days. To fix the problem, I changed the day of any preliminary races to one day prior to the final race. For example, if two races occurred on January 28, I changed the day of the preliminary race to January 27.

### 3.2.3 The MIXED Procedure

I used SAS's MIXED procedure to create the linear mixed model for the NCAA data. The code I used is given below:

```
PROC MIXED data = 'Name of Input Dataset'    method=reml;          (1)
CLASS id track_type;                                                (2)
MODEL time = fixed_date track_type / s residual                     (3)
      outp= 'Name of Output Dataset';
RANDOM int fixed_date / subject=ID;                                 (4)
REPEATED / subject = ID    type = SP(POW)(fixed_date);              (5)
LSMEANS track_type / diff;                                          (6)
RUN;
QUIT;
```

The first line of the code gives the name of the dataset and the method used to estimate the fixed and random effects. As shown above, I used the REML method. REML is an iterative method that "performs residual (restricted) maximum likelihood, and it is the default method" (support.sas.com, 2013). First, REML zeroes out the design matrix for the fixed effects. Then, the random effects and their variances are estimated. The fixed effects are estimated, under the assumption that the variances of the random effects are the true variances the model. These variances are then used as weights for the original data, and the process repeats itself, until optimal values are found that satisfy certain criteria.

The second line of the code defines the ID and track type variables as class variables. Class variables take on nominal values as opposed to numerical values. The third line provides the layout of the model: to the left of equal sign is the response variable, and to the right are the predictor variables. This line also tells SAS to output the solutions (for the effects) in the results window, and output the residuals in a separate dataset.

The random statement line (4), "defines the random effects constituting the $\boldsymbol{\gamma}$ vector" (support.sas.com, 2013). Here, one can see that the intercepts and estimates for fixed date are different among each individual. In line (5), "the REPEATED statement is used to specify the $\boldsymbol{R}$ matrix" and provides the correlation structure that is used (support.sas.com, 2013).

Line (6) calculates the least squares means (of race time) for each of the track types, and calculates the difference between two different track types. This part of the code is crucial: it essentially provides the conversion standards based on the model.

### 3.2.4 Conversion Standard Calculations

When I first fit the linear mixed model, I used (race) time as the response variable with track type and fixed date as the predictor variables. In this case, the difference between least squares means gave an additive conversion standard (CS) between the two track types. In other words:

$$Time_{track\ type\ 1} = Time_{track\ type\ 2} + CS$$

However, the conversion standards calculated by the NCAA were multiplicative, not additive. In order to more easily compare standards, I wanted to also compute multiplicative standards. This was easily done by taking the natural logarithm of all race times, and refitting the model with log(time) as the response variable, with track type and fixed date as the predictors. The differences between least squares means gave the following:

$$\log(Time_{track\ type\ 1}) = \log(Time_{track\ type\ 2}) + CS$$

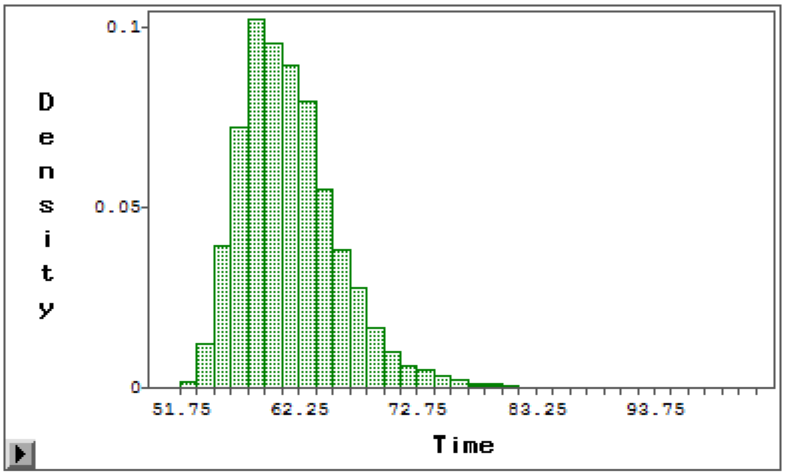
$$\Leftrightarrow e^{\log(Time_{track\ type\ 1})} = e^{\log(Time_{track\ type\ 2} + CS)}$$

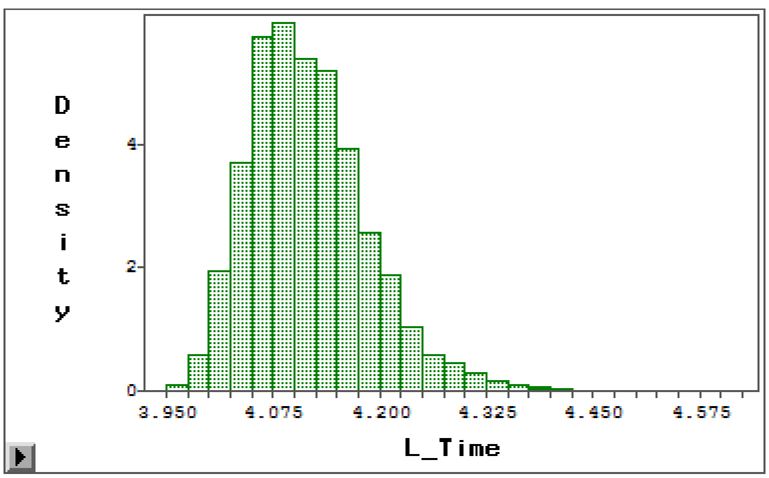
$$\Leftrightarrow e^{\log(Time_{track\ type\ 1})} = \left(e^{\log(Time_{track\ type\ 2})}\right)(e^{CS})$$

$$\Leftrightarrow Time_{track\ type\ 1} = (Time_{track\ type\ 2})(\underbrace{e^{CS}}_{\text{Multiplicative Conersion Standard}})$$

Another benefit to using the natural logarithm dealt with the distribution of the race times. As mentioned before, the linear mixed model assumes that the response variable follows a normal distribution. However, the distribution of the race times was not normal: it was heavily skewed to the right, (whereas the normal distribution is symmetric). Taking the natural logarithm of the (race) times reduced the skewness of the distribution, although it did not eliminate it.

***Figure 4: Distribution of Race Times (Women 400m 2012)***




*The first graph shows the distribution of race time. The second graph shows the distribution of the natural log of the race time.*

The following figure is an example of a portion of the outputted results for the Women's 400m (2012), with log(time) as the response variable. The example includes the regression parameter estimates, as well as the least squares means and their differences.

*Figure 5: Sample Output for Women's 400m 2012*

Solution for Fixed Effects

| Effect | Track Type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | | 4.1308 | 0.001302 | 3374 | 3172.21 | <.0001 |
| fixed_date | | -0.00059 | 0.000016 | 2168 | -36.38 | <.0001 |
| Track_Type | banked | -0.00029 | 0.000708 | 2934 | -0.41 | 0.6843 |
| Track_Type | flat | 0.01446 | 0.000705 | 2934 | 20.52 | <.0001 |
| Track_Type | oversized | 0 | . | . | . | . |

Least Squares Means

| Effect | Track Type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Track_Type | banked | 4.1230 | 0.001258 | 2934 | 3278.52 | <.0001 |
| Track_Type | flat | 4.1378 | 0.001219 | 2934 | 3393.57 | <.0001 |
| Track_Type | oversized | 4.1233 | 0.001297 | 2934 | 3180.17 | <.0001 |

Differences of Least Squares Means

| Effect | Track Type | Track Type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|---|
| Track_Type | banked | flat | -0.01475 | 0.000600 | 2934 | -24.59 | <.0001 |
| Track_Type | banked | oversized | -0.00029 | 0.000708 | 2934 | -0.41 | 0.6843 |
| Track_Type | flat | oversized | 0.01446 | 0.000705 | 2934 | 20.52 | <.0001 |

From this, it is easy to calculate the conversion standards, using the differences between least squares mean. The estimate gives the difference (of least squares means) between the second track type and the first track type. For example, the estimate of -0.01475 is the flat – banked difference, and thus, 0.01475 is the banked – flat difference. Then, since the log(race time) was used in this model, the conversion standard for banked to flat would be $e^{0.01475} = 1.01486$.

The output also provides the results from a t-test, used to determine if the difference between the least squares means is significant. A test with a p-value (Pr > |t|) less than 0.05 says that the difference is considered significant. In the example above, the difference between banked and flat is significant, while the difference between banked and oversized is not.

To find the overall estimate for the conversion standard, I took the mean ($\bar{X}$) of the estimates from 2010, 2011, and 2012. The output gave the standard error (SE) for each individual estimate, so the standard error for the mean estimate could be calculated as well. Since I assumed that each year was independent from one another, the following formula applies:

$$SE_{\bar{X}} = \frac{\sqrt{(SE_{2010})^2 + (SE_{2011})^2 + (SE_{2012})^2}}{3}$$

Then, a 100(1 - 0.5) = 95% confidence interval for the overall estimate could be calculated:

$$(\bar{X} - 1.96 * SE_{\bar{X}}, \bar{X} + 1.96 * SE_{\bar{X}})$$

Bonferroni-adjusted confidence intervals could also be calculated. The Bonferroni method adjusts the confidence level based on the total number of comparisons being considered. In the case of the NCAA data, a total of 36 comparisons (banked-flat, banked-oversized, etc. for all events) are considered. The Bonferroni method gives a 100(1 – 0.5/36) = 99.58 % confidence interval with formula:

$$(\bar{X} - 3.18 * SE_{\bar{X}}, \bar{X} + 3.18 * SE_{\bar{X}})$$

### 3.2.5 Distribution of the Response

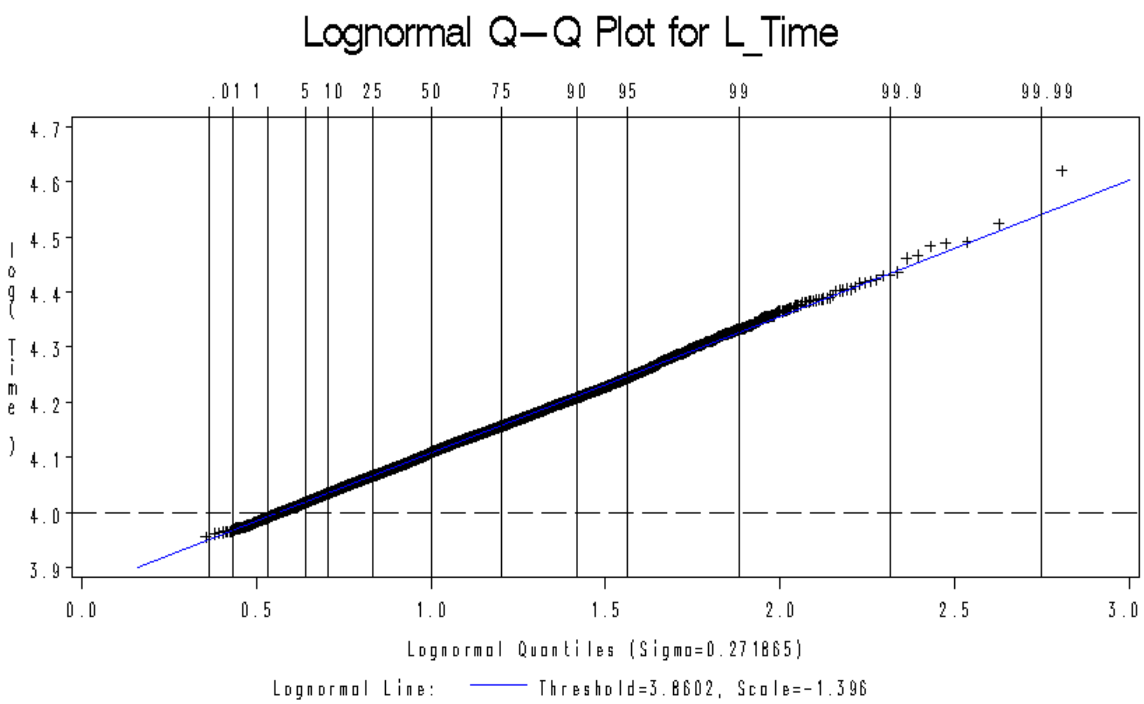
As mentioned previously, the response variable (race time), did not follow a normal distribution. Taking the natural logarithm of the race times helped make the distribution less skewed, however, even after this action, the response variable was still skewed. In addition, the residuals of the models I fit did not follow a normal distribution. A major assumption for linear mixed models is that the response variable and the residuals each follow a normal distribution. Since my models did not uphold this assumption, the fit of the models became questionable. In order to try and fix this problem, I began transforming the response variable. My hope was that one transformation would make the response variable normally distributed.

I discovered that the log(race time) followed a three parameter lognormal distribution. The three parameters are $\theta$ (threshold), $\mu$ (scale), and $\sigma$ (shape). Figure 6 shows the estimated parameters for the Women's 400m (2012). If I subtracted $\theta$ from the log(race time), the new variable, log(race time) - $\theta$, followed a two parameter lognormal distribution. It is known that if variable X follows a two parameter lognormal distribution, then log(X) follows a normal distribution.  So in the case of the NCAA data:

$$\log\left(\log(race\ time) - \theta\right) \sim Normal(\mu, \sigma)$$

However, the above transformation made the calculations of the conversion standards much more difficult than those using the log(race time).  In addition, after fitting a linear mixed model with the transformed variable, the residuals still did not follow a normal distribution.  I looked at the outliers from the residuals, and removed the individuals where these outliers came from.  Even after removing the outliers, the results did not change substantially.  As a result, I decided to try a different type of model: a generalized linear mixed model.

***Figure 6: Distribution of log(race time) for Women's 400m 2012***




*The picture above is a Quantile-Quantile plot.  Q-Q plots are used to compare probability distributions.  The blue line is a lognormal distribution and the black dots are all the log(race time) data points.  This shows that the data points follow the lognormal distribution.*

## 3.3 Generalized Linear Mixed Model

A generalized linear mixed model (GLMM) is closely related to a linear mixed model. The GLMM has both fixed and random effects. However, the observed data and the errors are not assumed to have a normal distribution, unlike those of the linear mixed model (support.sas.com). In addition, the GLMM utilizes a link function, to be discussed later.

The following exposition follows the SAS/STAT® 9.2 User's Guide, Section Edition. GLMM's are discussed under "The GLIMMIX Procedure" section.

### 3.3.1 The Model

As with the linear mixed model, $\boldsymbol{y}$ is a vector of observed data, $\boldsymbol{\beta}$ is an unknown vector of fixed effects parameters, $\boldsymbol{X}$ is the known design matrix for the fixed effects, $\boldsymbol{\gamma}$ is an unknown vector of random effects parameters, $\boldsymbol{Z}$ is the known design matrix for the random effects, and $\boldsymbol{\varepsilon}$ is the unknown vector of random errors. The observed data and errors do not necessarily follow a normal distribution, however the errors still have a variance matrix $\boldsymbol{R}$, with a specific covariance structure. In addition, $\boldsymbol{\gamma} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{G})$ like with the linear mixed model.

A GLMM assumes that the observed data have mean

$$E(\boldsymbol{y}|\boldsymbol{\gamma}) = g^{-1}(\boldsymbol{\eta}) = \boldsymbol{\mu}$$

where

$$\boldsymbol{\eta} = \boldsymbol{X\beta} + \boldsymbol{Z\gamma}$$

and variance

$$Var(\boldsymbol{y}|\boldsymbol{\gamma}) = \sqrt{\boldsymbol{A}}\boldsymbol{R}\sqrt{\boldsymbol{A}}$$

Here, $\boldsymbol{\eta}$ is called the linear predictor and $g^{-1}$ is the inverse of the function $g$, known as the link function. The link function is both monotonic and differentiable, and when it equals the identity function, the model reduces to a linear model. "The matrix $\boldsymbol{A}$ is a diagonal matrix and contains the variance functions of the model. The variance function expresses the variance of a response as a function of the mean." (support.sas.com).

### 3.3.2 Distribution, Link Function, and Covariance Structure of NCAA data

When I first fit the GLMM to the NCAA data, I considered different distributions for the response. As with the linear mixed model, I used log(race time) as the response, in order to obtain multiplicative conversion standards. It turned out that that among the distributions available in the GLIMMIX procedure, the gamma distribution was the best fit for the data, because (1) The gamma distribution is right skewed and takes on only positive values, and (2) The -2 Res Log Pseudo-Likelihood function, a measure of fit, was minimized with the gamma distribution (compared with the final function values for other distributions). I will discuss this function shortly.

The default link function for the gamma distribution is the log link function.
In other words:

$$g(\mu) = \log(\mu)$$

$$log(\mu) = log(E(\boldsymbol{y}|\boldsymbol{\gamma})) = \boldsymbol{\eta}$$

For the $\boldsymbol{R}$ matrix, I again used the SP(POW) covariance structure, for the same reasons as given before in the linear mixed model section.

### 3.3.3 The GLIMMIX Procedure

I used SAS's GLIMMIX procedure to create the GLMM, and the code I used is given below:

```
PROC GLIMMIX data = 'Name of Input Dataset'
CLASS id track_type;
MODEL l_time = fixed_date track_type / DIST=gamma s;
NLOPTIONS TECHNIQUE=NRRIDG;                                         (1)
RANDOM int fixed_date / subject = id;
RANDOM _residual_ / subject = id type = SP(POW)(fixed_date);        (2)
LSMEANS track_type / diff;
OUTPUT out = 'Name of Output Dataset'
      pred = p student = student_resid residmu(noblup) = resid_mu;
RUN;
QUIT;
```

Line (1) specifies the nonlinear optimization method used to fit the model. In many cases, "models fit with the GLIMMIX procedure have one or more nonlinear parameters, and estimation requires nonlinear optimization methods." (support.sas.com). NRRIDG is the

Newton-Raphson Ridge technique, which utilizes the gradient vector $\mathbf{g}(\boldsymbol{\theta}^{(k)})$ and the Hessian matrix $\mathbf{H}(\boldsymbol{\theta}^{(k)})$ to optimize the objective function. Here, $\boldsymbol{\theta}$ is the parameter(s) being estimated. I used this method simply because it is "akin to the optimization method in the MIXED procedure." (support.sas.com). For more information about NRRIDG, see "Optimization Algorithms" under the Details section of the NLMIXED procedure in the SAS/STAT® 9.2 User's Guide.

The objective function just mentioned is the -2 restricted log pseudo-likelihood function (-2 Res Log Psuedo-Likelihood). The GLIMMIX procedure aims to minimize this function when estimating model parameters. The estimation is based on linearization, and proceeds as follows:

1. Find the first-order Taylor series of $\boldsymbol{\mu}$ about $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$

$$\boldsymbol{\mu} = g^{-1}(\boldsymbol{\eta}) = g^{-1}(\widehat{\boldsymbol{\eta}}) + \tilde{\Delta}\boldsymbol{X}\left(\boldsymbol{\beta} - \widetilde{\boldsymbol{\beta}}\right) + \tilde{\Delta}\boldsymbol{Z}(\boldsymbol{\gamma} - \widetilde{\boldsymbol{\gamma}})$$

where

$$\tilde{\Delta} = \left(\frac{\partial g^{-1}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}}\right)_{\widetilde{\boldsymbol{\beta}},\widetilde{\boldsymbol{\gamma}}}$$

2. Rearrange the first-order Taylor series to obtain:

$$\tilde{\Delta}^{-\mathbf{1}}\left(\boldsymbol{\mu} - g^{-1}(\widetilde{\boldsymbol{\eta}})\right) + \boldsymbol{X}\widetilde{\boldsymbol{\beta}} + \boldsymbol{Z}\widetilde{\boldsymbol{\gamma}} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{Z}\boldsymbol{\gamma}$$

And thus

$$\tilde{\Delta}^{-\mathbf{1}}\left(\boldsymbol{y} - g^{-1}(\widetilde{\boldsymbol{\eta}})\right) + \boldsymbol{X}\widetilde{\boldsymbol{\beta}} + \boldsymbol{Z}\widetilde{\boldsymbol{\gamma}} = \boldsymbol{P}$$

where $\boldsymbol{P}$ is called the pseudo-response.

3. Define the function:

$$l_R(\boldsymbol{\theta}, \boldsymbol{p}) = -\frac{1}{2}\log|\boldsymbol{V}(\boldsymbol{\theta})| - \frac{1}{2}\boldsymbol{r}'\boldsymbol{V}(\boldsymbol{\theta})^{-\mathbf{1}}\boldsymbol{r} - \frac{1}{2}\log\left|\boldsymbol{X}'\boldsymbol{V}(\boldsymbol{\theta})^{-\mathbf{1}}\boldsymbol{X}\right| - \frac{f-k}{2}\log\{2\pi\}$$

where

$$\boldsymbol{V}(\boldsymbol{\theta}) = \boldsymbol{ZGZ}' + \tilde{\Delta}^{-1}\sqrt{\boldsymbol{A}}\boldsymbol{R}\sqrt{\boldsymbol{A}}\tilde{\Delta}^{-1}$$

$$\boldsymbol{r} = \boldsymbol{p} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X})^{-}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{p}$$

$\boldsymbol{\theta}$ is the vector containing all unknowns in $\boldsymbol{G}$ and $\boldsymbol{R}$, $f$ is the sum of frequencies used in the analysis, and $k$ is the rank of $\boldsymbol{X}$

4. Minimize $-2l_R(\boldsymbol{\theta}, \boldsymbol{p})$ and then estimate the fixed and random effects:

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{V}(\boldsymbol{\theta})^{-1}\boldsymbol{X})^{-}\boldsymbol{X}'\boldsymbol{V}(\boldsymbol{\theta})^{-1}\boldsymbol{p}$$

$$\widehat{\boldsymbol{\gamma}} = \widehat{\boldsymbol{G}}\boldsymbol{Z}'\boldsymbol{V}(\widehat{\boldsymbol{\theta}})^{-1}\widehat{\boldsymbol{r}}$$

5. Re-compute the pseudo-response ($\boldsymbol{P}$) and error weights of the linearized model.

6. Repeat steps 4 and 5 “until the relative change between parameter estimates at two successive (outer) iterations is sufficiently small” (support.sas.com)

For more detailed information, please see the “Pseudo-likelihood Estimation Based on Linearization” section under the Generalized Linear Mixed Models Theory of the GLIMMIX Procedure in the SAS/STAT® 9.2 User’s Guide.

***

The GLIMMIX procedure does not use a REPEATED statement. Instead, it utilizes a RANDOM _residual_ statement. In the MIXED procedure, I had used the line of code:

```
REPEATED / subject = ID    type = SP(POW)(fixed_date);
```

The equivalent statement in GLIMMIX is found in line (2) from the code above.

## 3.4 Generalized Linear Model

After fitting the GLMM, I decided to also fit a generalized linear model (GLM). A GLM, like a GLMM, does not assume the normality of the response and errors. A GLM also utilizes a link function $g$. However, the GLM only incorporates fixed effects, and no random effects. In other words:

$$g^{-1}(\boldsymbol{\eta}) = \boldsymbol{\mu}$$

where

$$\boldsymbol{\eta} = \boldsymbol{X\beta}$$

As with the other models, $\boldsymbol{\beta}$ is the unknown vector of fixed effects and $\boldsymbol{X}$ is the design matrix for the fixed effects (support.sas.com).

I fit a generalized linear model to the NCAA data because I wanted to see if conversion standard estimates were similar to those computed from the GLMM. Track effect is a fixed effect, so the results from both models should be similar.

For the GLM, I used the log link function and the SAS procedure GENMOD. The GENMOD procedure utilizes a REPEATED statement (like the MIXED procedure) to deal with the repeated measures. However, the SP(POW) structure was not available. Instead, I chose the Autoregressive(1) structure, AR(1), which has the following form:

$$Corr(Y_{ij}, Y_{ik}) = \begin{cases} 1 & j = k \\ \rho^{|j-k|} & j \neq k \end{cases} \quad individual\ i, observations\ j\ and\ k$$

This structure was not perfect, since the time gap (in days) between races was not equal for all observations and individuals (this structure assumes observations are equally spaced). However, it was the best one available.

The code for the GENMOD procedure can be found in Appendix A.

# Chapter 4: Results and Discussion

## 4.1 Basic Regression Model

The basic regression models showed that there is clearly a linear relationship between race time and the day the race occurred. The correlation between the two variables was high (close to a value of 1). Also, the p-values (testing for significance of the model estimates), indicated that the estimates were significant. In other words, it proved that there is a training effect. The training effect estimates computed by the models were negative: as time (in days) passes, an athlete's race time decreases.

In addition, the slopes and intercepts of each event regression line seemed like reasonable estimates. The intercepts reflected actual race times, and the slopes (training effect) were comparable to the estimates provided by the linear mixed model (I will discuss these later). Training effect estimates were not provided in the NCAA report, so I could only base my judgments on personal experience and the significance of the estimates.

I noted earlier that one could not find a training effect from one observation (one race). When examining the overall regression lines, I looked at two cases: when all the individuals were considered, and when the individuals with only one observation were removed. The slopes of the overall regression line differed greatly between the two cases. The slope was much smaller, and very close to zero, when all individuals were considered. Thus, removing the individuals with only one observation was an important step.

## 4.2 GLMM Conversion Standards

The following two tables display the conversion standards estimated from the generalized linear mixed model (GLMM). The tables also show an approximate 95% confidence interval for the GLMM estimates, as well as a Bonferroni-adjusted interval. Finally, the tables include the conversion standards computed by the NCAA committee.

### *Table 1: Conversion Standards (GLMM) - Women*

| Event | Conv. Type | NCAA | GLMM | GLMM 95% C.I. | Bonferroni C.I. |
|---|---|---|---|---|---|
| 200 | B – F | 1.0155 | 1.0048 | (1.0045, 1.0050) | (1.0044, 1.0052) |
| | B – O | 1.0000 | 1.0006 | (1.0003, 1.0009) | (1.0001, 1.0010) |
| | O – F | 1.0155 | 1.0042 | (1.0040, 1.0044) | (1.0038, 1.0046) |
| | U – F | 0.9900 | -- | -- | -- |
| 400 | B – F | 1.0133 | 1.0038 | (1.0036, 1.0040) | (1.0035, 1.0041) |
| | B – O | 1.0000 | 1.0006 | (1.0004, 1.0008) | (1.0002, 1.0009) |
| | O – F | 1.0133 | 1.0032 | (1.0030, 1.0034) | (1.0029, 1.0035) |
| | U – F | 0.9929 | -- | -- | -- |
| 800 | B – F | 1.0115 | 1.0028 | (1.0026, 1.0030) | (1.0025, 1.0031) |
| | B – O | 1.0000 | 1.0000 | (0.9998, 1.0003)* | (0.9997, 1.0004)# |
| | O – F | 1.0115 | 1.0028 | (1.0026, 1.0030) | (1.0024, 1.0031) |
| | U – F | 0.9951 | -- | -- | -- |
| Mile | B – F | 1.0099 | 1.0022 | (1.0020, 1.0023) | (1.0019, 1.0024) |
| | B – O | 1.0000 | 1.0004 | (1.0002, 1.0006) | (1.0000, 1.0007)# |
| | O – F | 1.0099 | 1.0018 | (1.0016, 1.0020) | (1.0015, 1.0021) |
| | U – F | 0.9969 | -- | -- | -- |
| 3000 | B – F | 1.0086 | 1.0019 | (1.0017, 1.0020) | (1.0016, 1.0021) |
| | B – O | 1.0000 | 1.0005 | (1.0003, 1.0007) | (1.0002, 1.0008) |
| | O – F | 1.0086 | 1.0014 | (1.0012, 1.0016) | (1.0011, 1.0017) |
| | U – F | 0.9981 | 0.9990 | (0.9986, 0.9995) | (0.9983, 0.9997) |

*Notes: (1) For the 200m, 400m, 800m, and Mile, there was no data on undersized tracks. Thus, an Undersized-to-Flat conversion standard could not be calculated.*
*(2) B-F = Banked to Flat, B-O = Banked to Oversized, O-F = Oversized to Flat, and U-F = Undersized to Flat*

### *Table 2: Conversion Standards (GLMM) - Men*

| Event | Conv. Type | NCAA's | GLMM | GLMM 95% C.I. | Bonferroni C.I. |
|---|---|---|---|---|---|
| 200 | B – F | 1.0179 | 1.0063 | (1.0060, 1.0065) | (1.0059, 1.0067) |
| | B – O | 1.0000 | 1.0006 | (1.0003, 1.0009) | (1.0001, 1.0010) |
| | O – F | 1.0179 | 1.0057 | (1.0055, 1.0060) | (1.0053, 1.0061) |
| | U – F | 0.9872 | -- | -- | -- |
| 400 | B – F | 1.0160 | 1.0051 | (1.0048, 1.0053) | (1.0046, 1.0055) |
| | B – O | 1.0000 | 0.9997 | (0.9994, 1.0000)* | (0.9992, 1.0002)# |
| | O – F | 1.0160 | 1.0054 | (1.0051, 1.0057) | (1.0049, 1.0058) |
| | U – F | 0.9901 | -- | -- | -- |
| 800 | B – F | 1.0143 | 1.0041 | (1.0039, 1.0043) | (1.0037, 1.0045) |
| | B – O | 1.0000 | 0.9998 | (0.9995, 1.0001)* | (0.9993, 1.0003)# |
| | O – F | 1.0143 | 1.0043 | (1.0040, 1.0045) | (1.0038, 1.0047) |
| | U – F | 0.9923 | -- | -- | -- |
| Mile | B – F | 1.0128 | 1.0055 | (1.0048, 1.0061) | (1.0044, 1.0065) |
| | B – O | 1.0000 | 1.0006 | (0.9997, 1.0014)* | (0.9992, 1.0019)# |
| | O – F | 1.0128 | 1.0049 | (1.0042, 1.0056) | (1.0037, 1.0061) |
| | U – F | 0.9941 | -- | -- | -- |
| 3000 | B – F | 1.0116 | 1.0026 | (1.0024, 1.0028) | (1.0023, 1.0030) |
| | B – O | 1.0000 | 1.0006 | (1.0003, 1.0008) | (1.0002, 1.0009) |
| | O – F | 1.0116 | 1.0021 | (1.0019, 1.0023) | (1.0017, 1.0024) |
| | U – F | 0.9953 | -- | -- | -- |
| 5000 | B – F | 1.0107 | 1.0022 | (1.0020, 1.0024) | (1.0019, 1.0026) |
| | B – O | 1.0000 | 0.9998 | (0.9996, 1.0001)* | (0.9995, 1.0002)# |
| | O – F | 1.0107 | 1.0024 | (1.0022, 1.0026) | (1.0021, 1.0027) |
| | U – F | 0.9961 | 1.0000 | (0.9990, 1.0010) | (0.9984, 1.0016) |

*Notes:(1) For the 200m, 400m, 800m, Mile, and 3000m, there was no data on undersized tracks. Thus, an Undersized-to-Flat conversion standard could not be calculated.*
*(2) B-F = Banked to Flat, B-O = Banked to Oversized, O-F = Oversized to Flat, and U-F = Undersized to Flat*

In essentially all cases, the GLMM estimates for the conversion standards are much lower than the ones computed by the NCAA. In other words, the GLMM calculated less of a track effect than the NCAA. The NCAA standards fell within the 95% confidence interval (for the linear mixed model estimates) in only 5 of 35 cases (denoted by *). The NCAA standards were included in 6 of 35 Bonferroni intervals (denoted by #). Because of this, there is a significant difference between the two sets of standards. Figure 7 also shows that the GLMM conversion standards (for a banked to flat conversion) are even lower than the old conversion standards for Division III.

One exception to the statement above is the undersized to flat conversion for the women's 3000m. The conversion standard estimated by the GLMM is fairly close to the one estimated by the NCAA. The GLMM conversion standard confidence interval almost contains the NCAA's conversion standard, which further supports the idea that the two are not that different.

The major similarity between the NCAA's and the GLMM's conversion standards regards the banked to oversized conversion. In many cases, the GLMM showed no significant difference between banked and oversized tracks, a discovery also made by the NCAA. The GLMM estimates for the banked to oversized conversion were all extremely close or equal to one. Even if the estimate did equal one itself, the confidence interval for the estimate did contain one, which shows no difference between the two track types.

Earlier on, I noted that the NCAA committee's conversion standards affect slower times more than faster times. As race times become larger, more time is added on (or subtracted off) when converting from one track type to another. This is due to the fact that the conversion standards are multiplicative. The GLMM conversion standards also have this same effect. Again, this is because the standards are multiplicative. However, the effect from the GLMM conversion standards is not nearly as much as that from the NCAA's conversion standards. As race time increases, the amount of time added on (or subtracted off) changes very little. Figure 8 provides an example of this using 400m dash times (converting from a banked track time to a flat track time).

*Figure 7: Conversion Standards Comparison (Division III), Revisited*

Women

| Event | Old Conversion Standards (used for the 2012 season) | NCAA Standards (Approx) * | GLMM Standards (Approx) * |
|---|---|---|---|
| **400m** | 0.4 seconds | 0.78 seconds (0:58.0) | 0.22 seconds |
| **800m** | 0.5 seconds | 1.56 seconds (2:15.0) | 0.38 seconds |
| **Mile** | 0.9 seconds | 2.97 seconds (5:00.0) | 0.66 seconds |
| **5000m** | 3.3 seconds | 8.00 seconds (17:18.0) | -- |
| **4x400m** | 1.6 seconds | 3.14 seconds (3:56.0) | -- |
| **DMR** | 2.5 seconds | 7.75 seconds (12:04.0) | -- |

Men

| Event | Old Conversion Standards (used for the 2012 season) | NCAA Standards (Approx) * | GLMM Standards (Approx) * |
|---|---|---|---|
| **400m** | 0.5 seconds | 0.79 seconds (0:49.0) | 0.25 seconds |
| **800m** | 0.6 seconds | 1.63 seconds (1:54.0) | 0.47 seconds |
| **Mile** | 1.0 seconds | 3.24 seconds (4:13.0) | 1.40 seconds |
| **5000m** | 3.6 seconds | 9.12 seconds (14:40.0) | 1.94 seconds |
| **4x400m** | 2.0 seconds | 3.17 seconds (3:18.0) | -- |
| **DMR** | 3.0 seconds | 8.20 seconds (10:03.0) | -- |

* The NCAA and GLMM conversion standards are multiplicative standards as opposed to additive standards. The above approximate conversions are the equivalent additive standards for the race times in the parentheses. If given a different race time, the additive standard would change.

*To convert from a banked track time to a flat track time, add the appropriate conversion standard above to the banked track time.*

*Figure 8: Banked-to-Flat Conversion Comparison*

| Time (400m) | NCAA | GLMM |
|---|---|---|
| 54.0 seconds | 0.72 seconds | 0.21 seconds |
| 58.0 seconds | 0.78 seconds | 0.23 seconds |
| 61.0 seconds | 0.82 seconds | 0.24 seconds |
| 65.0 seconds | 0.87 seconds | 0.25 seconds |

Apart from the banked to oversized conversion standard, the results from the GLMM are not what I expected at all. I had anticipated that the model would have produced results much closer to the results from the NCAA's study.

The GLMM I used appeared to be a well-fit model. The estimates for the fixed effects were all significant, indicated by very low p-values (<0.0001 in essentially all cases). The restricted log pseudo-likelihood function was the smallest out of all the models I tried as well (Recall: the GLIMMIX procedure minimizes this function, so the smaller, the better). The residuals followed a gamma distribution, like the response, which also suggested a good fit.

As anticipated, the GLM estimates were very similar to the GLMM estimates. Figure 9 shows both the GLM and GLMM differences between least squares means (for each track type) for the women. Recall, log(race time) was used as the response, so to find the multiplicative conversion standard, simply raise *e* to the powers in the below.

***Figure 9: Estimate Comparison- GLMM vs. GLM (Women)***

| **Event** | **Conv.** | **Estimate (GLMM)** | **Estimate (GLM)** |
|---|---|---|---|
| 200 | B-F | 0.00477 | 0.00455 |
| | B-O | 0.00058 | 0.00068 |
| | O-F | 0.00419 | 0.00387 |
| 400 | B-F | 0.00380 | 0.00393 |
| | B-O | 0.00059 | 0.00062 |
| | O-F | 0.00322 | 0.00331 |
| 800 | B-F | 0.00283 | 0.00267 |
| | B-O | 0.00005 | 0.00006 |
| | O-F | 0.00278 | 0.00261 |
| Mile | B-F | 0.00216 | 0.00174 |
| | B-O | 0.00037 | 0.00025 |
| | O-F | 0.00179 | 0.00149 |
| 3000 | B-F | 0.00185 | 0.00326 |
| | B-O | 0.00048 | 0.00077 |
| | O-F | 0.00138 | 0.00249 |
| | U-F | -0.00097 | -0.00127 |
| | U-B | -0.00282 | -0.00453 |
| | U-O | -0.00235 | -0.00377 |

## 4.3 Assessment of GLMM and NCAA Conversion Standards

After computing the conversion standards from the GLMM, I wanted to assess the accuracy of these standards. I applied the GLMM standards to the data in order to delete the track effect. Then, I re-fit this adjusted data to both the linear mixed model and the GLMM. If the track effect was truly deleted, then the difference of least squares means should not be significant. However, after applying the GLMM conversion standards, the difference between least squares means was still significant. I determined this by the extremely small p-values. Figure 10 shows the results for the Women's 400m (2012). Thus, these conversion standards were not accurate.

***Figure 10: Accuracy Results: GLMM Conversion Standards (Women 400m 2012)***

Differences of Track_Type Least Squares Means

| Track Type | Track Type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| banked | flat | -0.01102 | 0.000606 | 2934 | -18.19 | <.0001 |
| banked | oversized | 0.000307 | 0.000715 | 2934 | 0.43 | 0.6673 |
| flat | oversized | 0.01133 | 0.000711 | 2934 | 15.92 | <.0001 |

I then applied the NCAA conversion standards to the data and fit both a linear mixed model and a GLMM. Again, the difference between least squares means was significant, suggesting the NCAA standards were not accurate as well (see Figure 11). The p-values for the NCAA conversion standards were not as low as the ones from the GLMM. This suggests that the NCAA conversion standards are closer to being correct relative to the GLMM standards.

***Figure 11: Accuracy Results: NCAA Conversion Standards (Women 400m 2012)***

Differences of Track_Type Least Squares Means

| Track Type | Track Type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| banked | flat | -0.00160 | 0.000606 | 2934 | -2.64 | 0.0083 |
| banked | oversized | -0.00029 | 0.000715 | 2934 | -0.41 | 0.6846 |
| flat | oversized | 0.001308 | 0.000711 | 2934 | 1.84 | 0.0660 |

## 4.4 Linear Mixed Model Conversion Standards

After seeing these results, I took a look at the conversion standards computed by the linear mixed model. The linear mixed model conversion standards were larger than the NCAA standards. The NCAA standards fell within the 95% confidence interval (for the linear mixed model estimates) in only 12 of 36 cases (denoted by *). The NCAA standards were included in 14 of 36 Bonferroni intervals (denoted by #). The linear mixed model estimates, however, were more similar to the NCAA standards than the GLMM estimates. Recall, the GLMM estimates were much lower than the NCAA standards. The linear mixed model conversion standards are shown below in Figures 12 and 13.

*Figure 12: Conversion Standards (Linear Mixed Model) - Women*

| Event | Conv. | NCAA | MIXED | 95% C.I. | Bonferroni C.I. |
|---|---|---|---|---|---|
| 200 | B – F | 1.0155 | 1.0162 | (1.0156, 1.0169) | (1.0152, 1.0173)# |
| | B - O | 1.0000 | 1.0022 | (1.0015, 1.0030) | (1.0010, 1.0035) |
| | O – F | 1.0155 | 1.0139 | (1.0132, 1.0146) | (1.0128, 1.0150) |
| | U - F | 0.9900 | -- | -- | -- |
| 400 | B – F | 1.0133 | 1.0157 | (1.0150, 1.0165) | (1.0145, 1.0170) |
| | B - O | 1.0000 | 1.0024 | (1.0015, 1.0033) | (1.0010, 1.0039) |
| | O – F | 1.0133 | 1.0133 | (1.0124, 1.0141)* | (1.0119, 1.0146)# |
| | U - F | 0.9929 | -- | -- | -- |
| 800 | B – F | 1.0115 | 1.0142 | (1.0131, 1.0152) | (1.0125, 1.0159) |
| | B - O | 1.0000 | 1.0003 | (0.9991, 1.0014)* | (0.9984, 1.0022)# |
| | O – F | 1.0115 | 1.0139 | (1.0129, 1.0149) | (1.0122, 1.0156) |
| | U - F | 0.9951 | -- | -- | -- |
| Mile | B – F | 1.0099 | 1.0126 | (1.0116, 1.0136) | (1.0109, 1.0142) |
| | B - O | 1.0000 | 1.0022 | (1.0010, 1.0034) | (1.0002, 1.0042) |
| | O – F | 1.0099 | 1.0104 | (1.0040, 1.0167)* | (1.0001, 1.0207)# |
| | U - F | 0.9969 | -- | -- | -- |
| 3000 | B – F | 1.0086 | 1.0121 | (1.0110, 1.0132) | (1.0103, 1.0138) |
| | B - O | 1.0000 | 1.0031 | (1.0018, 1.0044) | (1.0010, 1.0052) |
| | O – F | 1.0086 | 1.0089 | (1.0078, 1.0100)* | (1.0071, 1.0108)# |
| | U - F | 0.9981 | 0.9936 | (0.9908, 0.9965) | (0.9890, 0.9983)# |

*Figure 13: Conversion Standards (Linear Mixed Model) - Men*

| Event | Conv. | NCAA | MIXED | 95% C.I. | Bonferroni C.I. |
|---|---|---|---|---|---|
| 200 | B – F | 1.0179 | 1.0201 | (1.0157, 1.0245)* | (1.0130, 1.0273)# |
| | B - O | 1.0000 | 1.0018 | (1.0008, 1.0027) | (1.0002, 1.0033) |
| | O – F | 1.0179 | 1.0183 | (1.0174, 1.0192)* | (1.0169, 1.0197)# |
| | U - F | 0.9872 | -- | -- | -- |
| 400 | B – F | 1.0160 | 1.0205 | (1.0197, 1.0214) | (1.0192, 1.0219) |
| | B - O | 1.0000 | 0.9995 | (0.9985, 1.0006)* | (0.9978, 1.0012)# |
| | O – F | 1.0160 | 1.0210 | (1.0200, 1.0220) | (1.0194, 1.00226) |
| | U - F | 0.9901 | -- | -- | -- |
| 800 | B – F | 1.0143 | 1.0199 | (1.0189, 1.0209) | (1.0183, 1.0215) |
| | B - O | 1.0000 | 0.9993 | (0.9981, 1.0005)* | (0.9973, 1.0012)# |
| | O – F | 1.0143 | 1.0206 | (1.0195, 1.0217) | (1.0189, 1.0224) |
| | U - F | 0.9923 | 0.9911 | (0.9881, 0.9941)* | (0.9862, 0.9960)# |
| Mile | B – F | 1.0128 | 1.0169 | (1.0157, 1.0181) | (1.0150, 1.0188) |
| | B - O | 1.0000 | 1.0015 | (1.0000, 1.0029)* | (0.9991, 1.0038)# |
| | O – F | 1.0128 | 1.0154 | (1.0141, 1.0167) | (1.0133, 1.0175) |
| | U - F | 0.9941 | -- | -- | -- |
| 3000 | B – F | 1.0116 | 1.0164 | (1.0153, 1.0174) | (1.0147, 1.0180) |
| | B - O | 1.0000 | 1.0033 | (1.0020, 1.0045) | (1.0012, 1.0053) |
| | O – F | 1.0116 | 1.0131 | (1.0120, 1.0142) | (1.0113, 1.0149) |
| | U - F | 0.9953 | -- | -- | -- |
| 5000 | B – F | 1.0107 | 1.0153 | (1.0139, 1.0167) | (1.0130, 1.0176) |
| | B - O | 1.0000 | 0.9989 | (0.9974, 1.0004)* | (0.9964, 1.0015)# |
| | O – F | 1.0107 | 1.0164 | (1.0150, 1.0177) | (1.0142, 1.0186) |
| | U - F | 0.9961 | 1.0001 | (0.9958, 1.0039)* | (0.9866, 1.0087)# |

I applied the linear mixed model conversion standards to the data, and fit both the linear mixed model and GLMM to the adjusted data. In both cases, the difference between least squares means was not significant (see Figure 14). Because of this, the linear mixed model estimates for the conversion standards appeared more accurate than the GLMM and NCAA standards. I found this surprising, since the linear mixed model was not the best-fitting model out of the ones I tried.

***Figure 14: Accuracy Results: Linear Mixed Model Standards (Women 400m 2012)***

Differences of Least Squares Means

| Effect | Track Type | Track Type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|---|
| Track_Type | banked | flat | 0.04778 | 0.03840 | 2934 | 1.24 | 0.2135 |
| Track_Type | banked | oversized | 0.1121 | 0.04535 | 2934 | 2.47 | 0.0135 |
| Track_Type | flat | oversized | 0.06435 | 0.04512 | 2934 | 1.43 | 0.1539 |

As mentioned before, the linear mixed model conversion standards are larger than the NCAA estimates. Figure 15 compares the NCAA and linear mixed model standards. The linear mixed model standards also suggest that there may be a difference between banked and oversized tracks (see Figure 16). The difference between least squares means between banked and oversized tracks was significant for most events and years, with the exception of a few, such as the Women's 400m in 2012. For all women's events, the final linear mixed model banked-oversized standards are greater than one, implying that banked tracks have advantages over oversized tracks. However, for the men, only three out of the six banked-oversized conversion ratios were greater than one.

### *Figure 15: Conversion Standards Comparison (Division III), Revisited (Again)*

Women

| **Event** | **Old Conversion Standards** (used for the 2012 season) | **NCAA Standards (Approx) *** | **MIXED Standards (Approx) *** |
|---|---|---|---|
| **400m** | 0.4 seconds | 0.78 seconds (0:58.0) | 0.91 seconds |
| **800m** | 0.5 seconds | 1.56 seconds (2:15.0) | 1.92 seconds |
| **Mile** | 0.9 seconds | 2.97 seconds (5:00.0) | 3.78 seconds |
| **5000m** | 3.3 seconds | 8.00 seconds (17:18.0) | -- |
| **4x400m** | 1.6 seconds | 3.14 seconds (3:56.0) | -- |
| **DMR** | 2.5 seconds | 7.75 seconds (12:04.0) | -- |

Men

| **Event** | **Old Conversion Standards** (used for the 2012 season) | **NCAA Standards (Approx) *** | **MIXED Standards (Approx) *** |
|---|---|---|---|
| **400m** | 0.5 seconds | 0.79 seconds (0:49.0) | 1.01 seconds |
| **800m** | 0.6 seconds | 1.63 seconds (1:54.0) | 2.27 seconds |
| **Mile** | 1.0 seconds | 3.24 seconds (4:13.0) | 4.28 seconds |
| **5000m** | 3.6 seconds | 9.12 seconds (14:40.0) | 13.47 seconds |
| **4x400m** | 2.0 seconds | 3.17 seconds (3:18.0) | -- |
| **DMR** | 3.0 seconds | 8.20 seconds (10:03.0) | -- |

### *Figure 16: Banked – Oversized Significance Women's 200m 2012*

Differences of Least Squares Means

| Effect | track_ type | track_ type | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|---|
| track_type | banked | flat | -0.01584 | 0.000588 | 3653 | -26.93 | <.0001 |
| track_type | banked | oversized | -0.00247 | 0.000701 | 3653 | -3.52 | 0.0004 |
| track_type | flat | oversized | 0.01337 | 0.000585 | 3653 | 22.85 | <.0001 |

*The low p-value of 0.0004 indicates that there is a significant difference between banked and oversized tracks.*

Another noteworthy aspect of the results regards the training effect. As mentioned before, the training effect is in terms of days. In order to find how much an athlete improved between two races, one multiplies the training effect by the number of days between the two races.

However because I used the natural log of the race times, the training effect from the models is actually multiplicative (like the conversion standards). Assuming two consecutive races were contested on the same track type:

$$e^{((estimate)(\#days\ between\ races))} * Race\ Time1 = Race\ Time2$$

Here, estimate refers to the estimate provided by the models. Approximate additive training effects (between race time 1 and race time 2) could then be calculated:

$$Training\ Effect\ (Additive) \approx Race\ Time1 - Race\ Time2$$

Unfortunately, the NCAA committee did not provide their estimates for the training effect, so I was unable to compare my results with theirs. I could only assess the training effect based on personal experience.

For the Women's 400m, the estimate directly from the linear mixed model was -0.00059. Thus, for two races that were one week apart, the training effect would equal to $e^{(-0.00059*7)}$ = 0.9959. If an athlete ran a 58.0 second 400m week one, the second week, it's predicted that the athlete would run 58.0 * 0.0059 = 57.76 seconds. A weekly drop of 0.24 seconds in the 400m seems reasonable. For the Women's 200m dash, an athlete who ran a race in 26.5 seconds would drop about 0.11 seconds by the next week. This training affect also seems reasonable.

# Chapter 5: Recommendations and Conclusions

## 5.1 Recommendations

Based on the results of my project, I have a few recommendations for future research related to the calculation of indoor track conversion standards:

- Take into consideration the banking angle. Certain banked tracks are more sloped than others, which I believe may affect the conversion standards. Tracks with a larger banking angle will result in a higher conversion standard. Potentially, more than one conversion standard may be necessary to ensure fairness and accuracy.
- Likewise, take into consideration the size of an oversized track. Oversized tracks range from 215m to 352m, a rather large range. Again, potentially more than one conversion standard (based on size) may be necessary.
- Consider modeling the data from each indoor season all together, instead of analyzing each season separately (as I did). This could be done by creating a new variable in the model(s) that indicate the year a race occurred.
- Consider fitting the NCAA data to other statistical models, such as a nonlinear mixed model. Compare the results from this model with the results of this project.
- Apply the linear mixed model conversion standards to the recently available 2013 indoor track season data. Then, model the data and see if there if the difference between means (for each track type) is still significant.
- Fit the 2013 indoor track season data to the statistical models in this project. See what conversion standards originate from this data.
- As more data becomes available (2014 season and on), re-evaluate the conversion standards. The more years that are analyzed, the more accurate the standards will be.

## 5.2 Conclusions

Based on my statistical models, I was able to determine that the new NCAA standards (which went into effect for the 2013 indoor season) do not entirely eliminate the track effect. Even after the new standards are applied to the data, there is still a significant difference between the least squares means of race time (for each track type). If the track effect had been removed, the least squares differences would not be significant.

I was able to fit various statistical models to the NCAA data and come up with my own conversion standards for indoor track. These standards, which come from a linear mixed model, appear to eliminate the differences in track type (based on statistical analysis). The linear mixed model estimates are slightly different than the NCAA standards. The linear mixed model standards are larger, so more time is added on (or subjected off) when converting from one track type to another. In other words, the linear mixed model estimates more of a track effect than the NCAA. For my proposed conversion standards, please see Appendix B.

The results of the statistical analysis also indicate that there may be a difference between banked and oversized tracks, and thus different conversion standards are needed. Further analysis of conversion standards is also suggested.

# Appendix

## Appendix A

### A.1 Basic Regression Model Code

```
PROC REG data = 'Name of Input Dataset'   outset = 'Name of Output Dataset';
      BY id;
      MODEL time = fixed_date;
RUN;
QUIT;
```

### A.2 Linear Mixed Model Code

```
PROC MIXED data = 'Name of Input Dataset'    method=reml;
CLASS id track_type;
MODEL time = fixed_date track_type / s residual
      outp= 'Name of Output Dataset';
RANDOM int fixed_date / subject=ID;
REPEATED / subject = ID    type = SP(POW)(fixed_date);
LSMEANS track_type / diff;
RUN;
QUIT;


/* the following code shows how I determined the distribution for the
response variable */
/* Note: l_time stands for log(time) */
/* L_Time_M_Z = log(Time) - Theta */
/* L_Time_M_Z follows 2-parameter lognormal */

ODS GRAPHICS ON;
PROC CAPABILITY data = 'Name of Input Dataset' noprint;
   QQPLOT l_time / lognormal(sigma=est theta=est slope=est)
                     pctlaxis(grid)
                     vref  = 4 5 6 ;
RUN;
ODS GRAPHICS OFF;


ODS GRAPHICS ON;
PROC CAPABILITY data = 'Name of Input Dataset' noprint;
   QQPLOT l_time_m_z / lognormal(sigma=est slope=est)
                     pctlaxis(grid)
                     vref  = 4 5 6 ;
RUN;
ODS GRAPHICS OFF;
```

```
/* L_L_Time_M_Z = log(log(time) – theta)), follows normal */

PROC CAPABILITY data = 'Name of Input Dataset' noprint;
      QQPLOT L_L_Time_M_Z / normal(mu=est sigma=est);
RUN;
```

### A.3 Generalized Linear Mixed Model (GLMM) Code

```
PROC GLIMMIX data = 'Name of Input Dataset'
CLASS id track_type;
MODEL l_time = fixed_date track_type / DIST=gamma s;
NLOPTIONS TECHNIQUE=NRRIDG;
RANDOM int fixed_date / subject = id;
RANDOM _residual_ / subject = id type = SP(POW)(fixed_date);
LSMEANS track_type / diff;
OUTPUT out = 'Name of Output Dataset'
      pred = p student = student_resid residmu(noblup) = resid_mu;
RUN;
QUIT;
```

### A.4 Generalized Linear Model (GLM) Code

```
proc genmod data = sasuser.w_400_2012;
class ID track_type;
model l_time = fixed_date track_type / dist=gamma link=log;
REPEATED subject = id / type = EXCH COVB;
lsmeans track_type / diff;
run;
```

## Appendix B

Below are the final conversion standards that I calculated, using a linear mixed model.

| Event | Conv. | WOMEN | MEN |
|---|---|---|---|
| 200 | Banked- Flat | 1.0162 | 1.0201 |
| | Banked- Oversized | 1.0022 | 1.0018 |
| | Oversized- Flat | 1.0139 | 1.0183 |
| | Undersized- Flat | -- | -- |
| 400 | Banked- Flat | 1.0157 | 1.0205 |
| | Banked- Oversized | 1.0024 | 0.9995 |
| | Oversized- Flat | 1.0133 | 1.0210 |
| | Undersized- Flat | -- | -- |
| 800 | Banked- Flat | 1.0142 | 1.0199 |
| | Banked- Oversized | 1.0003 | 0.9993 |
| | Oversized- Flat | 1.0139 | 1.0206 |
| | Undersized- Flat | -- | 0.9911 |
| Mile | Banked- Flat | 1.0126 | 1.0169 |
| | Banked- Oversized | 1.0022 | 1.0015 |
| | Oversized- Flat | 1.0104 | 1.0154 |
| | Undersized- Flat | -- | -- |
| 3000 | Banked- Flat | 1.0121 | 1.0164 |
| | Banked- Oversized | 1.0031 | 1.0033 |
| | Oversized- Flat | 1.0089 | 1.0131 |
| | Undersized- Flat | 0.9936 | -- |
| 5000 | Banked- Flat | -- | 1.0153 |
| | Banked- Oversized | -- | 0.9989 |
| | Oversized- Flat | -- | 1.0164 |
| | Undersized- Flat | -- | 1.0001 |

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