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Stochastic Reserves for Term Life Insurance

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Stochastic Reserves for Term Life Insurance

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Abstract

The team developed a model office consisting of 500 policyholders with varying death benefit amounts, premiums, risk classes and mortality rates. Deterministic reserves were calculated based on interest rates and mortality tables. Next, a simulation was performed consisting of 10,000 trials for each policy to determine an average financial outcome. These outcomes were aggregated to determine stochastic reserves with percentiles and tail value at risk results. Calculations were repeated over a range of interest rates and mortality rates to study the sensitivity of reserves to small changes in these assumptions.

Executive Summary

Our project calculated and analyzed stochastic reserves for a model insurance company based on different factors. These factors include age, policy term length, gender and smoker or non-smoker status. We use a Monte Carlo simulation on a block of 500 fictitious policies. By stressing parameters, we were able to determine reserve sensitivity. By altering mortality rates, interest rates, and other factors we were able to see how reserves for a company would increase or decrease and to what extent.

With the data gathered we computed duration, tail value at risk, averages and percentiles for the simulated reserves. Analysis of this data gave our group a greater understanding of how an insurance company can profit and also how changes in interest rates, mortality rates and other factors affect reserve levels.

Our code is relevant to life insurance companies because it can help actuaries determine reserves (ultimately profitability) and the sensitivity of the reserve estimates.

To aid in our calculations we utilized Excel, VBA and MATLAB.

Contents

1. Background

1.1 Life Insurance Basics

Life Insurance companies can appear to be very complicated, but the basic idea is not so complex. Most people buy a **life insurance policy**, a contract with an insurance company, once they reach a certain age. Life insurance is intended to help prevent any financial distress that could be left behind when a death occurs. The insured person will pay premiums, and, in return, their beneficiary will be provided a payment, or a **death benefit**, upon the insured person's death. A person can buy term life insurance, which means they are only covered by insurance for a defined period. Once this period or term ends, the insured person can either renew the policy, or let the policy expire. This type of policy is in contrast to whole life insurance, which is intended to provide coverage for the entire span of one's life[1]. To pay for the policy, **premium payments,** made by the insured person to the insurance company, are made. Premiums can be a one-time payment, or more commonly an annual payment. Payments for each individual's policy are different. Insurers use risk classes to help determine the premium for each policy. Risk class is determined by a number of different factors including, but not limited to age, overall health, lifestyle choices, family medical history, and tobacco use. For example, someone that uses cigarettes has an increased level of risk, causing their premium payment to be higher than a non-smoker's. However, these risk classes do not affect the amount of coverage provided, or the length of the term[2].

After a person purchases a policy it is important for the company to calculate reserves. Reserves provide for the net future obligations that the insurance company takes on and are considered liabilities on the company's financial statements. Reserves must be evaluated so that

if a claim were to occur, the insurance company would be able to pay the beneficiary. It is also used to calculate profits for insurance companies. An **actuarial reserve** is the expected net present value of each of the future cash flows in the policy.

Formula 1.

Reserve_t = Actuarial Present Value (Death Benefits)$_t$ - Actuarial Present Value (Premiums)$_t$

In other words, reserves are the expected future financial obligations of an insurance company[3].

Before one can understand reserving, you must understand a few other concepts. First of which, the present value of cash flows. Present value, PV, is the current value at time 0, of a future set of cash flows. The cash flows are discounted by the discount factor. The discount factor, v, is

Formula 2.

$$v = \frac{1}{1+i},$$

where i is the defined interest rate. When the interest rate increases, the discount factor decreases, and so the present value of a given set of cash flows will decrease.

Example 1.

Suppose you purchase a five-year annuity with payments of $200 at the end of each year. Let the interest rate be 5%. The timeline below shows the future cash flows.
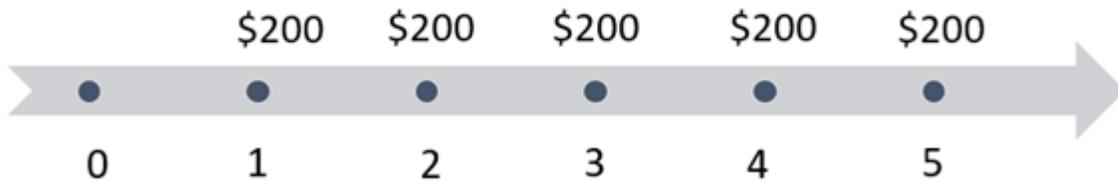
$v = 0.95238$. You must discount each cash flow back to time 0.

$$PV = 200v + 200v^2 + 200v^3 + 200v^4 + 200v^5$$

PV=$865.90

After understanding present value, one must understand actuarial present value, APV. APV is similar to present value, except it takes into account the probability that the payments will actually occur.

Example 2.

Suppose you purchase the same $200, five-year annuity from Example 1. Interest is still 5%. This time in order to receive the payment, the annuitant must be alive. These payments are referred to as contingent payments. The probability of surviving a given year is denoted by p_x, where x is the insured age. The probability of surviving t-years is denoted by $_tp_x$.

Time (t)	Age (x+t)	p_x	$_tp_x$	Benefit
0	x	0.950	1.000	
1	x+1	0.940	0.950	$200
2	x+2	0.930	0.893	$200
3	x+3	0.920	0.830	$200
4	x+4	0.910	0.764	$200
5	x+5	N/A	0.695	$200

$$APV = 200v\ _1p_x + 200v^2\ _2p_x + 200v^3\ _3p_x + 200v^4\ _4p_x + 200v^5\ _5p_x$$

$$APV = \$721.10$$

Actuarial present value is used in the reserves equation from Formula 1. It is important for insurance companies to calculate their reserves so that they can pay benefits. Insurance companies will become insolvent if their prices are too low and they do not have adequate funds from premiums to pay for claims. This can happen if clients are not assessed for risk factors well enough, or if the insurance company does not have enough income saved or reserves to pay the benefits to the beneficiary[4].

Deterministic reserves are calculated using assumptions for mortality and interest rates.

Example 3.

Suppose you purchase a 5-year term life insurance policy with a death benefit of $200 and a premium of $10 paid annually. The discount factor is 5% and 'q' = (1-p).

Time (t)	Age (x+t)	p_{x+t}	q_{x+t}	$_tp_x$
0	x	0.950	0.05	1.000
1	x+1	0.940	0.06	0.950
2	x+2	0.930	0.07	0.893
3	x+3	0.920	0.08	0.830
4	x+4	0.910	0.09	0.764
5	x+5	N/A	N/A	0.695
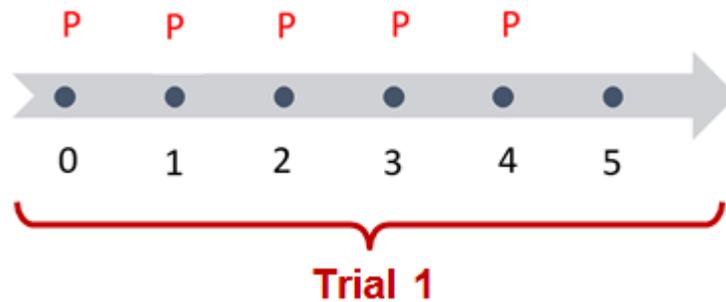
The equation to calculate the deterministic reserve uses the probabilities of a payment occurring in each direction (insured to the company and company to the beneficiary).

$$Deterministic\ Reserve =$$

$$200q_x v + 200\ _1p_x q_{x+1} v^2 + 200\ _2p_x q_{x+2} v^3 + 200\ _3p_x q_{x+3} v^4 + 200\ _4p_x q_{x+4} v^5 - 10$$

$$- 10\ _1p_x v - 10\ _2p_x v^2 - 10\ _3p_x v^3 - 10\ _4p_x v^4$$

$$Deterministic\ Reserve = \$21.74$$

One way to determine the reserves is to simulate the outcomes. Monte Carlo simulation is a simulation technique. It is used to model the probabilities of outcomes. Monte Carlo simulations are used when the outcomes cannot be easily calculated due to changing variables or dimensional complexity. This type of simulation allows one to understand the impact of assumptions on a model. It also makes it possible to run numerous trials and calculations repeatedly and in a relatively quick time period[5]. When using Monte Carlo simulations, it is important to understand how many trials are necessary to receive an accurate answer. A safe answer is anywhere from 100,000 trials to 500,000 trials. A more concrete answer is to find the confidence interval for the population for the agreed upon level of error[6].

Averaging together scenarios to determine the reserve amount means the reserves are calculated stochastically. **Stochastic reserving** is a method insurance companies use to calculate the reserve amount. Using a Monte Carlo simulation, you generate multiple outcomes based on each policy, and average them together to achieve the expected (deterministic) result.

Example 3.

Suppose you run a simulation for a 5-year term life insurance policy. The premium, P, is set at $10.00 and is paid annually at the beginning of each year. The death benefit is $200 paid at the end of the year that a death occurs.
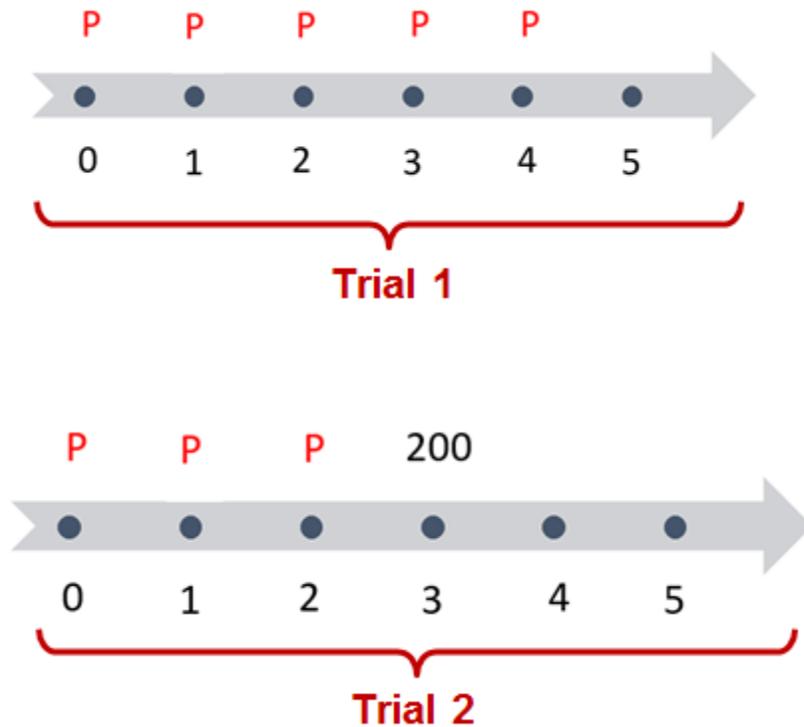
Trial 1

Trial 2

In Trial 1, no deaths occurred, but in trial 2 a death occurred between time 2 and time 3. To calculate the simulated losses for each trial you would have to discount the death benefits and premiums.

Formula 3.

$$Simulated\ Losses = PV(Death\ Benefits) - PV(Premiums)$$

For this example, Trial 1's simulated loss would be

$$0 - 10(v^4 + v^3 + v^2 + v + 1) = -\$45.46.$$

Note that a negative loss translates to a gain for the insurer.

Trial 2's simulated loss would be

$$200v^3 - 10(v^2 + v + 1) = \$144.18.$$

Formula 4.

$$Stochastic\ Reserve = Average\ (Simulated\ losses)$$

The example shows how the reserves can be calculated based on simulated outcomes. In insurance companies this is done on a much larger scale.

1.2 Mortality

Insurance companies use **mortality tables**, death rates by age group, to predict how many people will survive in a certain year. A further refinement that can be used is a mortality adjustment factor. This factor is multiplied by the mortality rate and can either decrease or increase the mortality rate. The multipliers can depend on recent medical exams, region, etc.

Two of the major types of mortality tables include **select mortality tables** and **ultimate mortality tables.** Select mortality tables outline the death rates of recently insured individuals. These death rates are typically lower than other types of mortality tables due to the fact that these are created for individuals who have just recently passed a series of medical exams needed to purchase the insurance. This medical selection process screens out the unhealthier applicants. As time goes on, the selection impact (lower probabilities of dying in early years) wears off, and the policyholder's mortality reverts to that of the general population. After usually 15 or 25 years, select mortality rates are replaced by ultimate mortality rates, which are more conservative.

The SOA website provides many mortality tables. Some are outdated and also some for the most recent years account for the decline in death rates over time as people live longer and longer.

Another step typically taken by insurance companies is to account for the varying levels of health of the policyholders. **Risk multipliers** are a multiplicative factor layered onto the death rates in the tables. These risk multipliers are lower than 1 for healthier individuals and greater than 1 for more unhealthy individuals. These multipliers are just another way insurance companies can refine their mortality assumptions on a policy-level.

1.3 About the "ACTUARIAL" Greeks

The **Greeks** are the major risk measures that should be taken into account when developing predictions. These risk measures are represented by Greek letters (with the exception of vega), hence the name. The financial Greeks, alphabetically, are delta, gamma, rho, theta and vega[7]. The following terminology is used in equity option pricing.

Delta - price sensitivity.

Gamma - second order price sensitivity.

Rho - rate of change between the price of a derivative and a 1% change in the interest rate. It is also known as the sensitivity to the interest rate.

Theta - a measure of the decline rate of an option over time. It is also known as time sensitivity.

Vega - measurement of an option's sensitivity to changes in volatility.

The Greeks are most often used to analyze an option positions' risks and potential rewards. In this code, we created our own "Actuarial Greeks" based off of the financial Greeks to observe the monetary effects on reserves that interest rate changes and mortality changes cause. The change in duration of our portfolio of insureds is used to observe how sensitive the reserves are to changes in the interest rate, as well as changes in mortality rates. We based our duration calculations off of the effective duration formula.

Formulas 5-7

Effective Duration:

$$= \frac{P_{i_-} - P_{i_+}}{P_0(i_+ - i_-)}$$

Interest Rate Duration:

$$= \frac{Reserves_{i_-} - Reserves_{i_+}}{Reserves_0(i_+ - i_-)}$$

Mortality Duration:

$$= \frac{Reserves_{m_-} - Reserves_{m_+}}{Reserves_0(m_+ - m_-)}$$

P_{i_+}	Price using higher interest rate
P_{i_-}	Price using lower interest rate
i_+	Higher interest rate
i_-	Lower interest rate
P_0	Original price using original interest rate
m_+	Higher mortality rate
m_-	Lower mortality rate

The first formula is the standard effective duration formula used for bonds. The second and third formulas are our modified duration formulas to quantify the effects of interest rates and

mortality rates on reserves. In the mortality approximation, m is the mortality rate. These calculations can help insurance companies to quantify the effects of various changes that occur in the real world, and also help these companies make sure that they are reserving enough money to cover all of their insureds even in a relatively unfavorable scenario[8].

1.4 Risk Measurements

Risk measurements that we used in addition to duration to help quantify the impact of interest rates and mortality rates on reserves include percentiles, tail value at risk (TVaR), comparing these to the averages.

The nth percentile is simply the number for which n% of the simulated reserves fall below that number. We calculated various different percentiles, ranging from the 75th percentile up to the 99th percentile, and compared these numbers to the average. Also, we calculated the TVaR, which is the average of the data after the nth percentile, for each of the percentiles spanning from the 75th to the 99th percentile. Insurance companies find the TVaR to be more useful when deciding how much to reserve.

Example 4.

Distribution	X1 ~ uniform [0, 3154]	X2 ~ exponential [θ = 1000]
95th Percentile	2996	2996
TVaR of 95%	3075	3996

In the above example, we can see that these two different distributions have the same 95th percentile, but very different TVaR's. This is why TVaR is used by insurance companies, because they tell the company more information about "how bad is the bad", and insurance companies need to be able to pay off promised benefits even in very negative scenarios. It can be seen more clearly in the below graphs.

Uniform Distribution

$$TVaR = \int\limits_{2996}^{3154} (x)\left(\frac{1}{158}\right)dx = 3075$$

Exponential Distribution

$$TVaR = \int\limits_{2996}^{\infty} (x)\left(\frac{e^{\frac{-(x-2996)}{1000}}}{1000}\right)dx = 3996$$
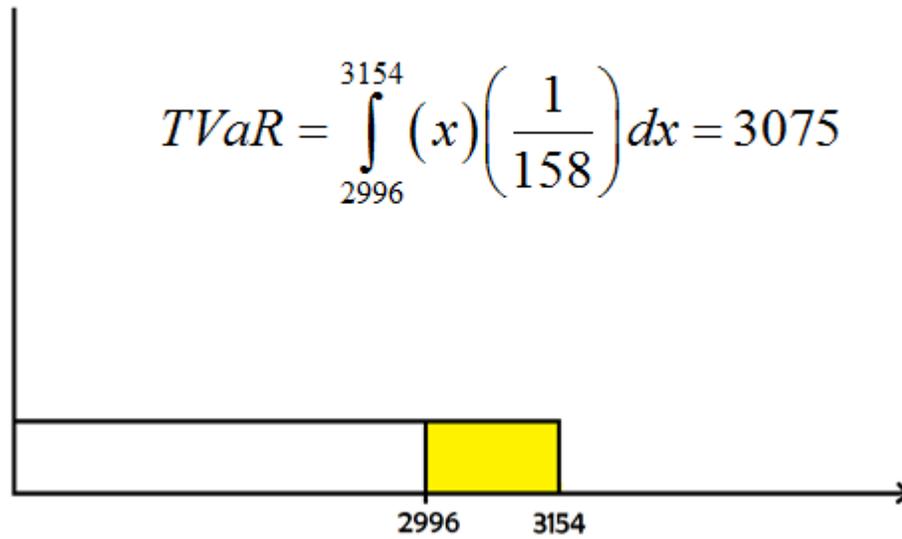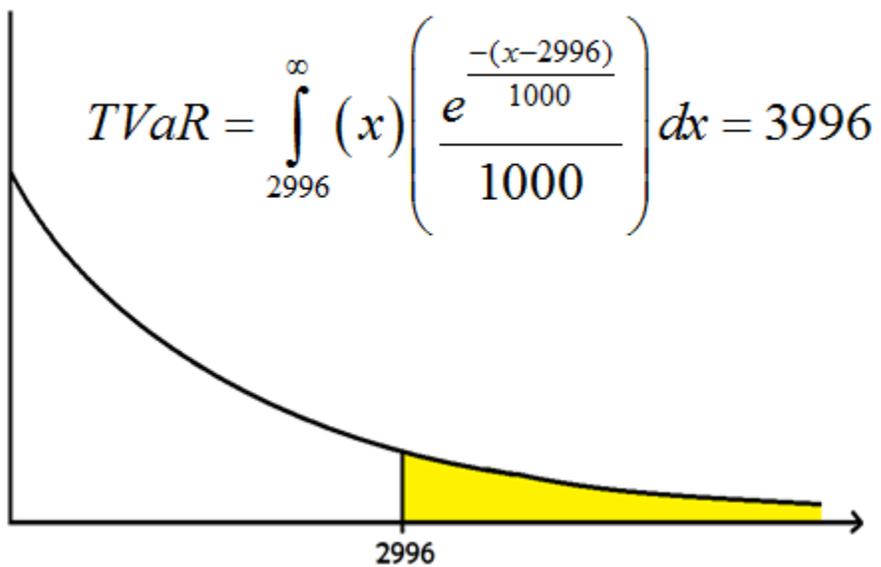
The tail of the exponential distribution goes to infinity. Therefore, the exponential distribution has the possibility of greater losses occurring. Thus, the average of losses above the 95th percentile for this distribution will be greater than the average of losses above the 95th percentile is the previous distribution.

1.5 About the Code

A combination of Excel and MATLAB was used to create the model. MATLAB allows for the importation of Excel documents. We also utilized VBA, Visual Basics for Applications, which is a programming language that is part of Excel. This tool is helpful to run the same calculations repeatedly for varying inputs in Excel. Thus, the data given by a company in an Excel spreadsheet can easily be read by the MATLAB code, making it easy to manipulate data.

2. Methodology

2.1 Monte Carlo and Initial Steps

The initial steps in the project were understanding Monte Carlo simulations and what we needed to build one to simulate actuarial reserves. As previously mentioned, a Monte Carlo simulation is an attempt by mathematicians to model potential outcomes of a situation that is truly deterministic in principle. We started out with a simple Monte Carlo simulation in MATLAB as a base for the reserve calculation. As the simulation progressed, we were able to add more parameters and features to better simulate real world occurrences.

For our simulation, we created a set of random probabilities and a set of starting positions for the probabilities. Specifically, we used Excel's random number generating function to create a set of 251,000 random numbers between zero and one. These are part of the test for death when running the simulation. In order to further randomize the input, we created a set of 1,000,000 random numbers between one and 250,000 also using Excel's random number generating functions. These numbers were used to determine where from the set of 250,000 the simulation would take random numbers for each trial.

The other half of the probabilities used to test for death are probabilities from the Society of Actuaries' mortality tables. We obtained both select and ultimate mortality tables for male and female smokers and non-smokers. These four tables served as the base probabilities of death for each of the policies in the simulation. The select tables provided 25 years of probabilities for ages 18 to 95. This is because in most cases life insurance policies are not sold to people less than 18 years old. The ultimate tables are the tables of probabilities used after the select table is finished. A different table is used because after 25 years all current assumptions about a person are void.

The last major input for the simulation was policy data. For this experiment, a block of 500 policies were created with varying policy parameters. The parameters used for the simulation were risk class, policy year, age at issue, policy term length, risk multiplier, interest rate at issue, premium class, death benefits and premium costs.

The risk class of each policy is which mortality table the simulation should follow for that policy. The risk classes are male smoker (MS), female smoker (FS), male non-smoker (MNS) and female non-smoker (FNS). To simulate an average population, the numbers of males and females were about even with 15% of the population being smokers.

Table 1: Distribution of Risk Classes

Risk Class	Number of Policies	Proportion
FNS	205	0.41
MNS	221	0.442
FS	36	0.072
MS	38	0.076

Policy year refers to how long it has been since the policy was purchased. This combined with the age of the policyholder at issue allows the simulation to determine where in their respective mortality table to take probabilities from.

The policy term length allows the simulation to know after how many years it should stop and go to the next trial.

A risk multiplier was added to simulate the fact that not all people of the same age and risk class would be as healthy as each other. While the majority of policies were held at a

multiplier of 1, some were chosen to be healthier and received a multiplier less than 1 and some were less healthy and received multipliers greater than 1.

Premium class refers to whether the policyholder pays premiums annually or as a single, lump-sum premium. One-tenth of the premiums were chosen to be single premiums and the rest were the annual premiums.

The death benefits are the amount that the insurance company pays out given the death of the insured. In this experiment, death benefits were between $100,000 and $1,000,000.

The interest rate at issue is, as it sounds, the interest rate when the policy was taken out. Since most of the policies have different policy years, the interest rates at issue also vary. The interest rates varied from 3% to 7% in our project.

The final factor that the simulation uses to calculate the actuarial reserve is the premium costs. This value is the amount that the insured pays to the insurance company. It is calculated uniquely for each policy based on that insured's characteristics.

2.2 Excel Premiums

Before the simulation could be run, premiums for each policy had to be calculated in Excel using their policy data and mortality probabilities. Using Excel and VBA, we created a table of values that would update depending on which policy was being indexed. Mortality probabilities, discount vectors and death benefits had to be read into the table in order to calculate the premiums and later the deterministic reserves. With this information we were able to calculate the lump-sum premium, or the premium that the insured would pay if they decided to pay for the entire policy at once. From lump-sum premiums, annual premiums were calculated taking into account the term of the policy, the insured's probabilities of death and a pricing

discount vector. A similar table was made to calculate the expected death benefit. Using the

formula: Present Value (Future Premiums) - Present Value (Death Benefits) = Reserves,

deterministic reserves for both single and annual premiums were calculated. We then used Visual

Basics for Application (VBA) to calculate the reserves and premiums for all 500 policies. This

data became part of the foundation for calculating stochastic reserves.

2.3 MATLAB Simulation

In order to run the simulation, we had to import the random numbers and policy data

from Excel to MATLAB. Vectors and matrices for each of the policy data points were created in

MATLAB to use in the simulation. Before the simulation began, constants for the simulation

were also read in from Excel. These include the number of trials, risk alterations, interest rate,

and the number of policies. Risk alterations were used to vary the mortality rates of each policy

to observe the change in reserves. Similarly, the interest rate was also set in the beginning but

changed to see how changes in interest rates would change reserves. The determined base

interest rate for the experiment was 6%. Once the data was in MATLAB, the first loop of the

simulation begins. The outer loop is where each policy's data is used to build their set of

probabilities of death.

In the first loop of the simulation, each policy is assigned its proper probabilities (policy

q's) for the simulation. MATLAB uses their risk class and age to retrieve their probabilities from

the select and ultimate tables. Any alterations to the select probabilities like risk multipliers from

the policy data are then applied. These probabilities are then combined into one vector and then

shortened to only include the years left in their term.

The next loop of the simulation assigns random numbers from the list to the policy for each trial (Note: The random numbers for the policy change for each trial). To further ensure randomness is introduced into the simulation, the starting point that MATLAB takes random probabilities from is chosen from a different list of over one million numbers. This vector of random probabilities from 0 to 1 and the vector of mortality probabilities are then used in the final loop to simulate death benefits.

The final loop is where all of the prepared data is put to use. In this section, probabilities are compared between the mortality and random probabilities vectors. For each year, the premium calculated in Excel is documented at the beginning of the year. Then, the simulation compares the corresponding probabilities from the two vectors. If the random number is less than the mortality probability; the policyholder dies in that year, the simulation records a death benefit being paid at the end of the year and the trial is ended. However, if the random number is greater than the mortality probability, the policyholder lives through that year and the process repeats until they die or until the end of their term. After MATLAB has completed the specified number of trials for the first policy, it will go back to the first loop and repeat the entire thing for the second policy and so on. (Note: If the policy being simulated is a lump-sum premium policy, no premiums are recorded during their term).

The most important output for the reserve calculation is the matrix of death benefits and the matrix of premiums. These matrices show the per trial death benefit payout and premium intake by the insurance company. Using discount vectors, these are converted into present values to reflect their value now. The difference is then taken to attain the per trial costs to the insurance company. The costs are then averaged over all of the trials to find the stochastic reserve.

2.4 Reserves Accuracy and Duration Calculations

Once reserves could be calculated, there were a series of tests and updates to ensure that the code was functioning properly and giving accurate data. In order to do this, the simulated or stochastic reserve was compared to the deterministic reserve from Excel. One of the major focuses was lining up the years correctly. Starting the simulation, a year too early or late would be difficult to notice in the code but after thousands of trials could have an effect on the simulated reserve. In order to test this, we looked at the quotient of the simulated reserve and the deterministic reserve. Using ten thousand trials, this error was only 11 basis points, or .0011. While it was recommended to use at least one hundred thousand trials, the accuracy of the stochastic reserve after ten thousand trials was within our boundary for error. Additional simulations were conducted to improve its accuracy. After increasing the number of trials to one million, the error was only reduced by 4 basis points. Therefore, due to the accuracy and the speed of the simulation using fewer trials it was determined that ten thousand trials

After it was determined that the simulated reserves were being calculated accurately, alterations on inputs were made to analyze changes in simulated reserves. The initial interest rate of 5% was altered by 5 and 10 basis points in either direction. Similarly, risk multipliers from .95 to 1.05 in .01 increments were applied. The resulting simulated reserves were used to calculate the "Actuarial Greeks". Effective duration calculations allowed us to quantify the effect of changing interest rates and risk multipliers on actuarial reserves.

3. Results

Using our reserve simulation, we were able to record simulated reserves with different risk multipliers and interest rates. As mentioned at the end of the methodology, these reserves were used to create the "Actuarial Greeks". These values tell us how the actuarial reserves change based on changes in the inputs. Durations were calculated using the Effective Duration Equation and are shown in Charts 1 and 2. One trend to notice is the decreasing duration with increasing percentile. This means that the tail of the loss distribution is less sensitive to changes in interest rate than the areas more towards the median. For example, Chart 1 shows a 10-basis point duration for the average reserve of 22.67 and a 99th percentile reserve of 13.16. This means that if the interest rate were to change by 10 basis points, you would expect a 22.67 basis point change in the average reserve and 13.16 basis point change in the 99th percentile reserve. It is interesting to note that while the 99th percentile reserve would change less in either a 5-basis point or 10 basis point change, the amount of change was more consistent in the average. The same trend can be seen in the duration for changes in risk multiplier. While the 99th percentile reserve had a lower duration, the lower percentiles and the average had less variance in their durations.

Another interesting point to note is how low the durations for risk multipliers are compared to those for interest rates. This can be accounted for due to the rigidity of the simulated outcomes based on risk multipliers as opposed to interest rates. Altering the interest rate effects, the reserves after they've already been simulated. Once premiums and death benefits are calculated they are then discounted using the interest rate. This means that all of the data is being affected by the change. However, changes in risk multiplier could have different effects on

different trials. In order to change the result of a trial, the risk multiplier must change the probability of death in a year enough for it to cross over the random number for that year. Since the random numbers are fixed prior to the simulation, the probabilities compared in corresponding simulations will be the same. This means that in many cases, a small change in risk multiplier may not have an effect on the losses for a given trial. The difference is that when the risk multiplier does affect the outcome of a trial, it has a significant effect. When an extra person dies due to increasing the risk multiplier or the opposite, it will change the losses for that trial significantly.

Chart 1: Duration Interest Rate

TVaR	Effective Duration by Changes in Rates	
	10BP Change	5BP Change
75	-16.35	-16.35
80	-16.01	-16.01
85	-15.54	-15.54
90	-15.05	-15.05
95	-14.38	-14.38
99	-13.46	-13.46
Deterministic	-22.67	-22.67

Chart 2: Duration Risk Multiplier

	First Order Reserve Approximation				
	Risk Multiplier Factors				
TVaR	1.05	1.04	1.03	1.02	1.01
75	1.67	1.66	1.65	1.7	1.62
80	1.61	1.6	1.59	1.64	1.55
85	1.54	1.53	1.52	1.56	1.48
90	1.45	1.44	1.44	1.47	1.37
95	1.34	1.34	1.35	1.36	1.27
99	1.16	1.2	1.21	1.21	1.27
Deterministic	2.72	2.71	2.7	2.71	2.64

Another way to interpret duration is to visualize it. Duration is the slope of the line where the endpoints used are a given distance apart. For example, if you wanted the 10-basis point duration and you're centered around 5% interest, you would use 4.9% and 5.1% as the endpoints for the slope calculation.

The following graphs show the TVaR reserves for varying interest rates and risk multipliers. Duration calculations were only done for the first set of graphs as duration is only a useful measure in a close environment of the original point. This is because duration, being a straight-line estimator, becomes a worse approximation as you go further from the original point.

Graph 1:
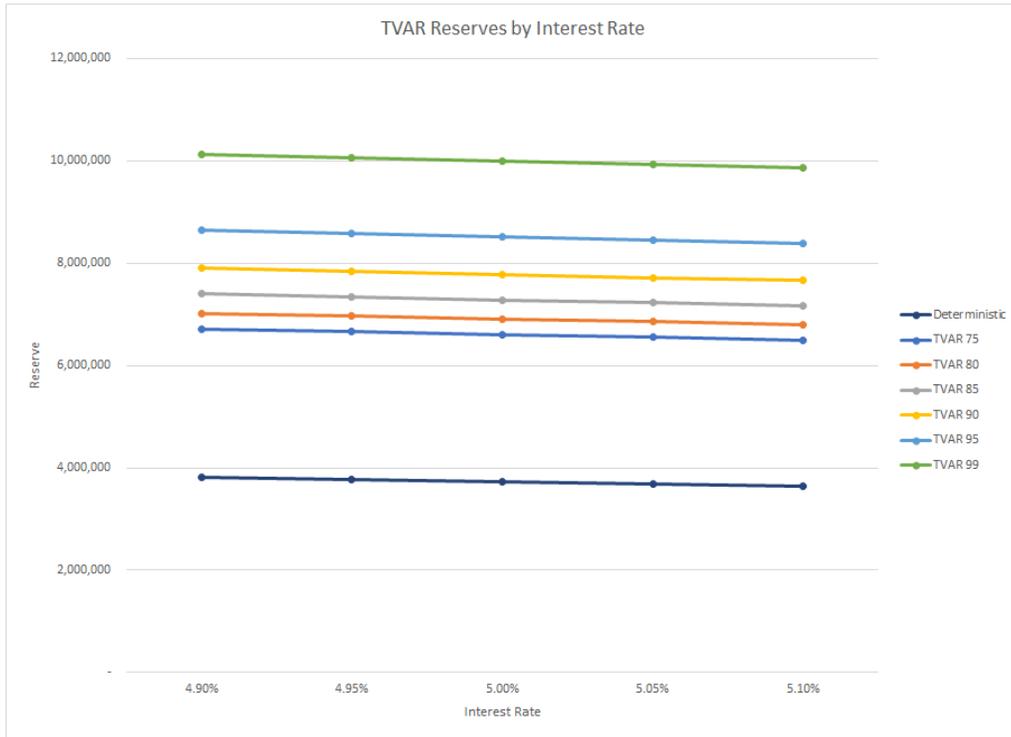
Graph 2:
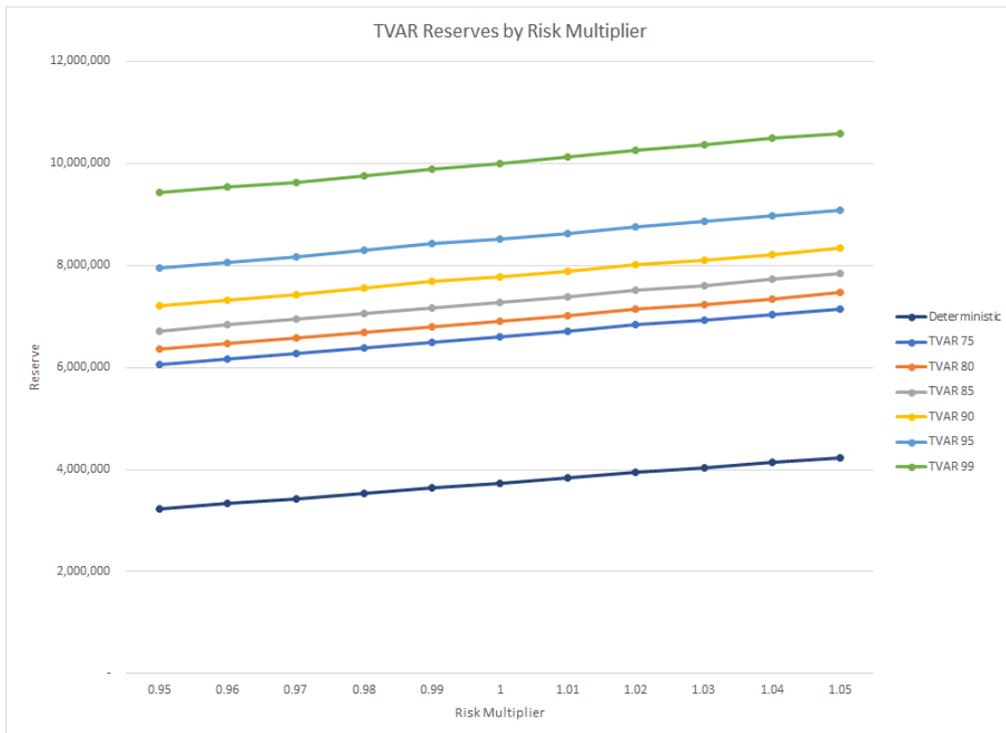
The next set of graphs show the same TVaR reserves but simulated using larger steps away from the base run. In this way, you can better see the trend of actuarial reserves with changing interest and mortality rates. As mortality rates increase, the actuarial reserves will increase. This is intuitive since more people dying means more money that the insurance company has to pay out. The opposite is true for interest because if interest rates increase, money held now will be worth more in the future. Therefore, the insurance company would have to hold less to cover the same losses.

Graph 3:
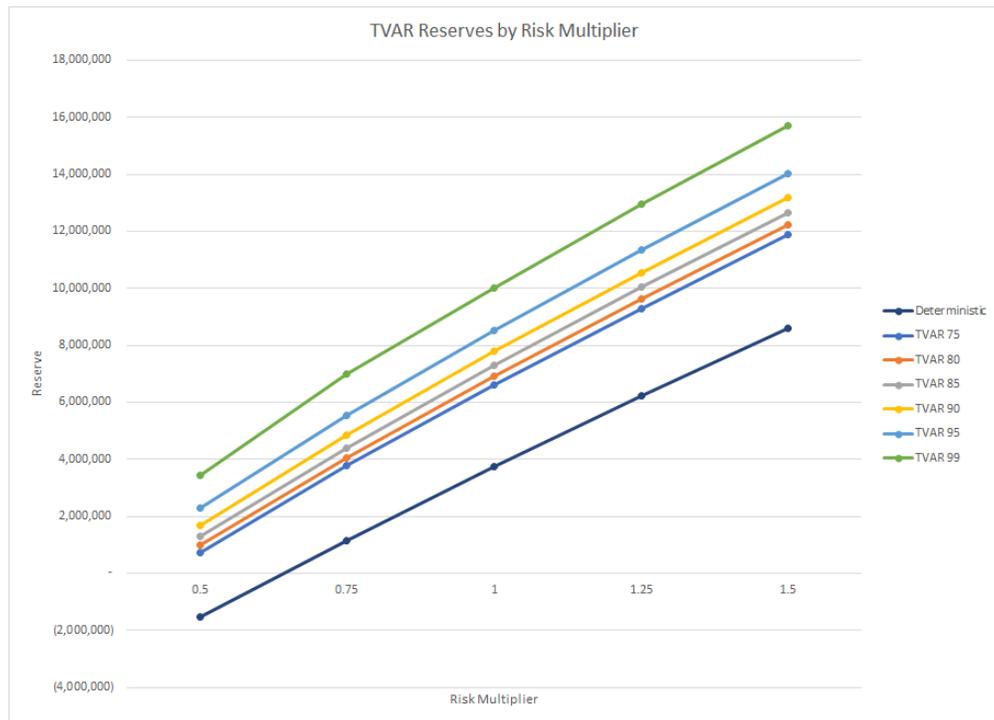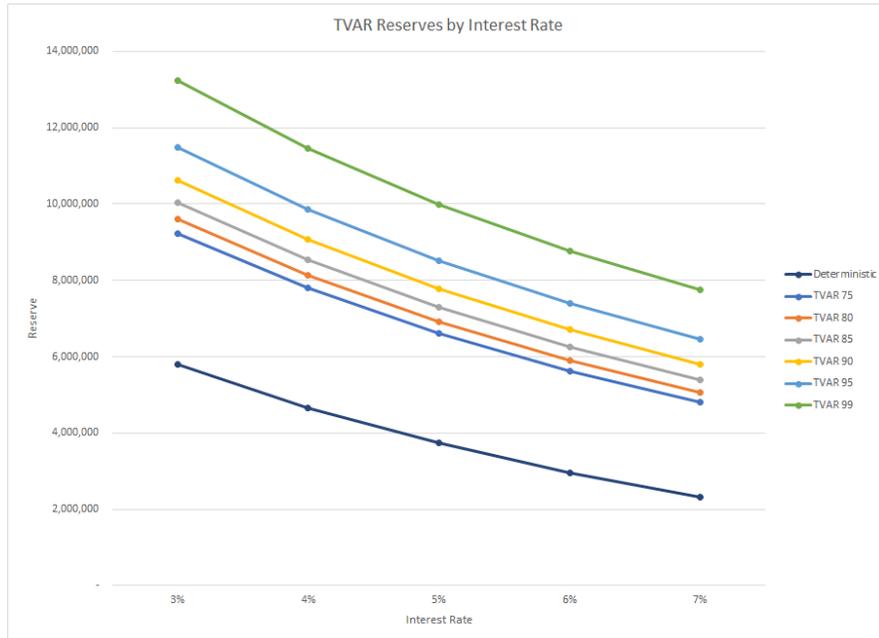
The final two graphs show the difference between the average, 95th percentile and TVaR$_{95}$ reserve for changes in both interest rates and risk multipliers. Again, regulators require that insurance companies hold the TVaR reserve instead of the percentile reserve since the TVaR gives a better idea of how bad losses are past the given percentile.

Also, without the use of a Monte Carlo simulation only the average reserve would be able to be calculated. The deterministic reserve calculated in Excel does not include any randomness and relies on the probabilities and policy data. Therefore, the only information it gives is what should happen based on the probabilities. Through the introduction of randomness with a Monte Carlo simulation, we were able to more accurately determine what the insurance company should reserve for account for their future obligations.

Graph 5:
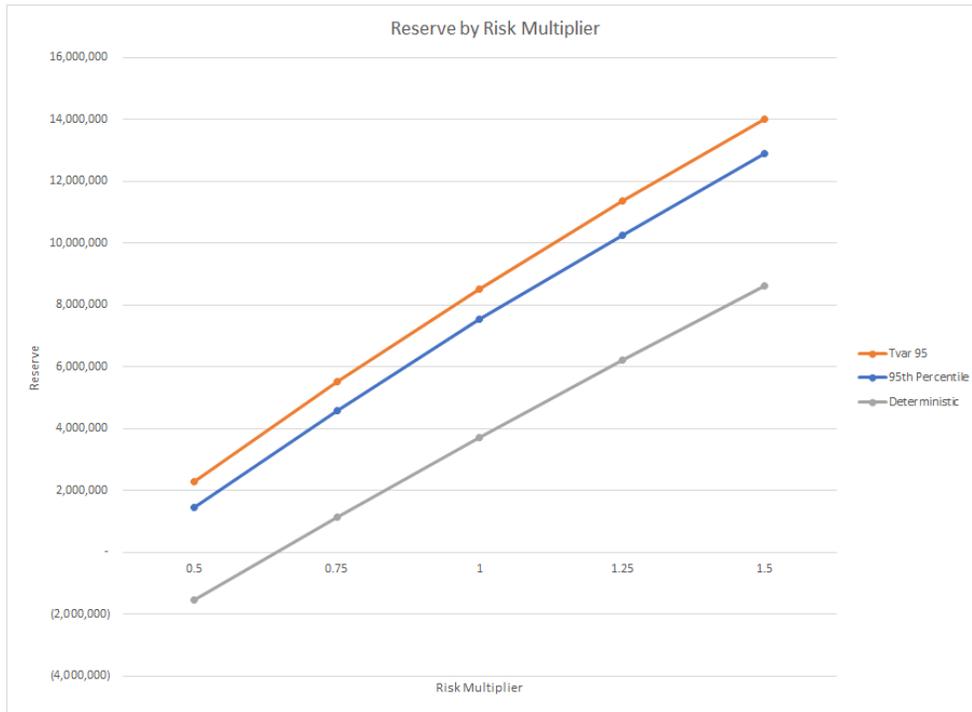
Graph 6:
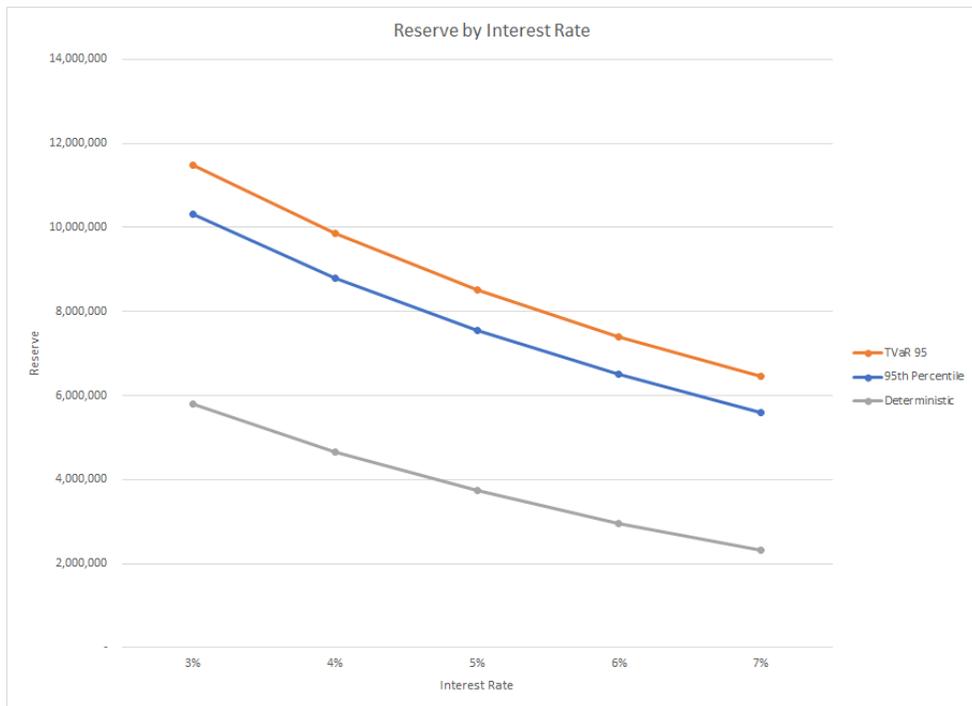
4.Recommendations

In the future, if this project were to continue, we have a few recommendations.

Recommendation 1: Build the code so that it can do whole life insurance.

The code can currently only do term life insurance. It would be interesting to see how the reserves change if whole life was included.

Recommendation 2: Add more mortality tables to the code or adjust risk multipliers to make them more realistic.

We simply picked certain people to have high and low risk multipliers. We recommend that future groups research the probabilities of varying risk multipliers. It would make the runs more realistic if the risk multipliers accounted for this.

Recommendation 3: Adjust the code so that the error of the deterministic and the stochastic reserves is less.

There is a small error when comparing the deterministic reserves to the stochastic reserves. We believe that more runs and more random numbers could improve this error. We urge future groups to attempt to bring this error down.

5. The Code

```
%% MQP Reserve Model

%% Building matricies and reading in data from excel files
format short
Sim_Death_Benefit=zeros(500,120);
Deaths_Per_Year=zeros(500,120);
Matrix_Policy_qs = zeros(500,120);
Years_Survived=zeros(500,120);
Per_Trial_Reserve=zeros(500,10000);
Aggregate_Premium=zeros(500,120);
Discount_Vector_Premium=zeros(120,1);
Discount_Vector_DB=zeros(120,1);
Rand_Num = xlsread('OldRands','sheet1','A1:A251000');
Trial_Start_Num = xlsread('OldRands','sheet1','B1:B1000000');
Policies = xlsread('RealPremiumCalculation','Premiums','A5:M504');


%% Reading in Risk Factor Mortality Tables
Mort_FNS_Select = xlsread('RealPremiumCalculation','FNS','b2:z97');
Mort_MNS_Select = xlsread('RealPremiumCalculation','MNS','b2:z97');
Mort_MS_Select = xlsread('RealPremiumCalculation','MS','b2:z97');
Mort_FS_Select = xlsread('RealPremiumCalculation','FS','b2:z97');

Mort_FNS_Ultimate = xlsread('RealPremiumCalculation','FNS Ult','a1:a121');
Mort_MNS_Ultimate = xlsread('RealPremiumCalculation','MNS Ult','a1:a121');
Mort_FS_Ultimate = xlsread('RealPremiumCalculation','FS Ult','a1:a121');
Mort_MS_Ultimate = xlsread('RealPremiumCalculation','MS Ult','a1:a121');

%% Reading in constants
Num_Trials = xlsread('RealPremiumCalculation','Run Parameters', 'a2'); %Reads in Number of
trials from Policy Listings
Risk_Alteration = xlsread('RealPremiumCalculation','Run Parameters', 'e2');
Interest_Rate = xlsread('RealPremiumCalculation','Run Parameters', 'c2');
Num_Policies = length(Policies(:,1));
select_period = 25; %assume select 25 before ultimate
Random_Number_Position = 0;
Discount_Vector_Premium(1)=1;
Discount_Vector_DB(1)=1;

for i = 1:119
    Discount_Vector_Premium(i+1)=Discount_Vector_Premium(i)/(1+Interest_Rate);
end
Discount_Vector_DB(1)=1/(1+Interest_Rate);
```

```
for i=1:119
    Discount_Vector_DB(i+1)=Discount_Vector_DB(i)/(1+Interest_Rate);
end



%% Start of Simulation Loops

for index_policies=1:Num_Policies
        %% Reading Policy variables
        policy_year_policy = Policies(index_policies,1);
        risk_class_policy = Policies(index_policies,2);
        premium_class_policy = Policies(index_policies,3);
        risk_multiplier_policy = Policies(index_policies,4);
        issue_age_policy = Policies(index_policies,5);
        term_policy = Policies(index_policies,6);
        death_benefit_policy = Policies(index_policies,7);
        if premium_class_policy == 1
            policy_premium = Policies(index_policies,11);
        else
            policy_premium = 0;
        end




    %% Risk Class Assignment
    % Assigns the correct select and ultimate tables for each policy
        if risk_class_policy == 1
            Mort_Table_Policy_Select = Mort_FNS_Select;
            Mort_Table_Policy_Ultimate = Mort_FNS_Ultimate;
        elseif risk_class_policy == 2
            Mort_Table_Policy_Select = Mort_MNS_Select;
            Mort_Table_Policy_Ultimate = Mort_MNS_Ultimate;
        elseif risk_class_policy == 3
            Mort_Table_Policy_Select = Mort_FS_Select;
            Mort_Table_Policy_Ultimate = Mort_FS_Ultimate;
        elseif risk_class_policy == 4
            Mort_Table_Policy_Select = Mort_MS_Select;
            Mort_Table_Policy_Ultimate = Mort_MS_Ultimate;
        end
    %% Policy q's Assignment and Risk multiplications
    % Set policy level q since issue
        Select_qs = Mort_Table_Policy_Select(issue_age_policy + 1,:); %Add Ability to read
in risk factor to go to different table
        for i=1:10 %Multiplies first 10 Select_qs by the risk multplier for that policy
```

```matlab
            Select_qs(i)=Select_qs(i)*risk_multiplier_policy;
        end
        for i=1:length(Select_qs) %Multiplies first 10 Select_qs by the risk multplier for that
policy
            Select_qs(i)=Select_qs(i)*Risk_Alteration;
            %Ensure that risk alterations don't bring q's above
            %.975
            if Select_qs(i)>0.975
                Select_qs(i)=0.975;
            end


        end
%% Policy q's assignment
Ultimate_qs = Mort_Table_Policy_Ultimate(issue_age_policy + select_period + 1:end);
Policy_qs = [transpose(Select_qs);Ultimate_qs];
        % Chop qs until policy_year_policy
Simulation_Policy_qs = Policy_qs(policy_year_policy:term_policy);


 %% Trials # Assignment
    %Trials on innter loop
    for index_trials = 1:Num_Trials
        %% Random Number Assignment
        Random_Number_Position = Random_Number_Position + 1;
        if Random_Number_Position >=length(Trial_Start_Num)
            Random_Number_Position = 1;
        end
        random_numbers_policy =
Rand_Num(Trial_Start_Num(Random_Number_Position):Trial_Start_Num(Random_Number_
Position)+149);

        %% Death Check and Premium/DB assignments
        %First premium is always paid
        continue_flag = 1;
        Aggregate_Premium(index_policies,1) = Aggregate_Premium(index_policies,1) +
policy_premium;
            for year_index=1:length(Simulation_Policy_qs) %If whole life, make term 121,
determine if policy_year_policy needs +1
                if (random_numbers_policy(year_index)<= Simulation_Policy_qs(year_index)) &&
(continue_flag == 1)

Sim_Death_Benefit(index_policies,year_index)=Sim_Death_Benefit(index_policies,year_index)
+death_benefit_policy;

Deaths_Per_Year(index_policies,year_index)=Deaths_Per_Year(index_policies,year_index)+1;
                continue_flag = 0;
```

```matlab
        elseif (random_numbers_policy(year_index)> Simulation_Policy_qs(year_index)) &&
(continue_flag == 1)
            Aggregate_Premium(index_policies,year_index + 1) =
Aggregate_Premium(index_policies,year_index + 1) + policy_premium;
            Years_Survived(index_policies,year_index) =
Years_Survived(index_policies,year_index)+1;
            Aggregate_Premium(index_policies,term_policy-policy_year_policy+2)=0;

            %Calculate Premium per person to get reserves
            %Premium 1 is always paid

        end %if random_numbers_policy(year_index)<= Policy_qs(year_index)
      end %for year_index=policy_year_policy:term_policy
      %% Code that allows to track per trial reserves
      PV_of_Premiums=Aggregate_Premium*Discount_Vector_Premium;
      PV_of_Death_Benefit = Sim_Death_Benefit*Discount_Vector_DB;
      Per_Trial_Reserve(index_policies,index_trials)=PV_of_Death_Benefit(index_policies)-
PV_of_Premiums(index_policies);
      Aggregate_Premium=zeros(500,120);
      Sim_Death_Benefit=zeros(500,120);
   end %for index_trials=1:Num_Trials
end %for index_policies=1:Num_Policies

%% Post Simulation Calculations
%Sim_Death_Benefit;
%Deaths_Per_Year;
%Aggregate_Premium;
%Aggregate_Premium(:,1)=0;   Only use if first premium is not used for
%reserve cals
%PV_of_Premiums=(Aggregate_Premium*Discount_Vector_Premium)/Num_Trials;
%PV_of_Death_Benefit = (Sim_Death_Benefit*Discount_Vector_DB)/Num_Trials;
%AVG_Sim_Reserve=(PV_of_Death_Benefit - PV_of_Premiums);
%AVG_Sim_Reserve_Sum = sum(AVG_Sim_Reserve);
%Percentiles = [prctile(AVG_Sim_Reserve,75),...
%prctile(AVG_Sim_Reserve,80),...
%prctile(AVG_Sim_Reserve,85),...
%prctile(AVG_Sim_Reserve,90),...
%prctile(AVG_Sim_Reserve,95),...
%prctile(AVG_Sim_Reserve,99)];
Sum_Per_Trial_Reserve = sum(Per_Trial_Reserve);
```

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