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A Pilot Study of Methods in the Online Socratic Tutoring of Calculus

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A Pilot Study of Methods in the Online Socratic Tutoring of Calculus

A Major Qualifying Report

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Of the

Worcester Polytechnic Institute

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By

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Approved By:

Professor Janice Gobert

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Abstract

We evaluated the potential of the Assisment System to the WPI Community. An empirical study of 27 undergraduate students taking Calculus II showed that the system increased performance in two calculus sub-domains: volumes of rotation and integration by parts. Two learning questionnaires found that students had a positive attitude toward the system and thought it would help them on tests. We concluded that increased use of the Assisment System would benefit the community and made suggestions towards this goal.
Acknowledgements

I first became interested in the learning sciences while taking an undergraduate course in the psychology of learning with Professor Janice Gobert of the Social Sciences Department. I was introduced to Professor Neil Heffernan of the Computer Science Department at this time during a talk he gave to the class on the ACT-R theory and a project he was working on called “Assistments”. By chance, I happened to be living with one of his programmers, Derek Radtke, and we began discussing the possibility of using this technology developed for tutoring middle school algebra to tutor advanced calculus material, which is notoriously difficult, here at WPI. After discussing the future possibility of dynamically assigning material to individual students on demand using a Bayesian Network we decided that I should pursue a major qualifying project working towards this goal. To this end, I contacted Professor Arthur Heinricher of the Mathematics Department who I first met my freshman year during my participation in the Project-Based Learning Center. During that time I had my first encounter with two Online Socratic Tutors called “Mastering Physics” and “WebWork” and I wondered if Professor Heinricher could help me gain a foothold in the Math Department for Assistments. He put me in touch with Professor John Goulet and Professor Gloria Tashjian who were both teaching Calculus II that year. The rest, as they say, is history.

To all of you I give my greatest thanks – without you this project would have never happened!
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1.0 Introduction

This pilot study begins with a literature review of (a) the argument for Online Socratic Tutoring, (b) the role of cognitive modeling in content authoring, (c) the instructional methods used to create a rich tutoring environment through Socratic dialogue, and (d) the methods used to maintain the integrity of that dialogue, before finally setting forth a mission statement and specific objectives to be concluded by the end of the study.

1.1 Argument for Online Socratic Tutoring

1.1.1 Assisting while Assessing

As reviewed by Razzaq et al (2005) in the case of middle school algebra, the value of Online Socratic Tutoring, like that implemented in WPI’s Assistment System (Razzaq et al., 2005), lies in the ability to both assist and assess students during when instructors would typically otherwise have to choose between one or the other because of limited instructional time. The utility of assisting and assessing online has already been established outside of middle schools and namely within the domain of freshman physics at the university level by intelligent tutoring systems (ITS) currently in use at WPI, such as “Mastering Physics”, that have been shown to promote problem-solving transfer in numerous studies (cf., Warnakulasooriya & Pritchard, 2005)\(^1\). There is a large domain overlap between freshman physics and freshman calculus which leads us to believe that the utility of assisting and assessing online will transfer to freshman calculus as well.

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\(^1\) Problem-Solving transfer as a function of time on task, number of hints requested, and number of incorrect responses submitted.
Additionally, the ability to report students’ progress described by Feng and Heffernan (2007; in press) also shows promise for university implementation because the Assistment System provides individual progress reports by student, problem, and concept or “knowledge component”, as well as reports aggregated by class, across classes, and across instructors.

**1.1.2 Compared to other Course Elements**

In addition to being effective at both assisting and assessing, Online Socratic Tutors have also been shown to correlate highly with improvement on final exams when compared to written homework, participation in lectures, and participation in conferences (Morote & Pritchard, 2004b). Furthermore, the level of success shown by these systems is comparable in success to having students do problems collaboratively in groups (Morote & Pritchard, 2004b). While weekly quizzes and final exams in physics have been shown to correlate with background variables detailing prior knowledge in physics, prior knowledge in calculus, the number of math courses being taken simultaneously, and computer access, none of these variables correlated with scores from Online Socratic Tutoring (Morote & Pritchard, 2004a). Thus, Online Socratic Tutoring is valuable, not simply as an “additional” course component, but as one that brings its own unique advantages to a course by supporting students’ learning.

**1.2 Role of Cognitive Modeling in Content Authoring**

**1.2.1 Active Control of Thought Theory**

As outlined by the Adaptive Control of Thought Theory (ACT), intelligent tutoring systems are designed with cognitive models of domain competency and students
are expected to perform within a range of this underlying competency model (Anderson et al., 1995). Anderson distinguishes between two types of knowledge: 1) declarative knowledge of conceptual information that is independent of goals, and 2) procedural knowledge of how to use said information towards some goal. Further, Anderson defines the acquisition of a “cognitive skill” as the conversion of declarative knowledge of concepts into procedural knowledge that can be used in problem-solving. The intelligent tutors described by Anderson can be described as “computational tutors” because they are programmed to have the ability to compute anything the student is asked to compute, i.e., a computational tutor does not simply store a question and correct answer, it has production rules that allow it to take a problem and actually solve it in the same manner that a student would. These tutors differ significantly from the “fixed dialogue tutors” in Razzaq et al (2004) because fixed dialogue tutors do not actually solve problems, but rather store a set of questions and answers. Tutors like Razzaq’s are also designed with a cognitive model, but instead of being able to generate solutions from production rules, they have pre-designed content in the form of hints, scaffolding, worked examples, etc., that is created from “knowledge components” or sub-steps derived from a problem’s annotated solution. The advantage of this is that this drastically reduces the amount of time required to create and implement the cognitive model.

Although Anderson et al (1995) specifies that the primary function of cognitive skill acquisition is the conversion of declarative knowledge into useful procedural knowledge, the importance of having the requisite declarative knowledge cannot be understated. For example, one declarative sub-domain of mathematics is the formal notation used to express relationships. Koedinger and Mitchell (2004) showed that the
main reason for students’ difficulty on story problems is that students lacked declarative knowledge of the formal notation and that these types of problems are actually easier for students than their algebraic counter-parts. One hypothesis explaining this was that story problems gave more context clues, however, Koedinger and Mitchell found that it was not the lack of context clues, but the difficulties associated with declarative knowledge of the formal notation that made algebraic problems more difficult. More recently, Booth et al (2007) showed that: 1) those with less declarative knowledge had less procedural knowledge, and 2) that improved declarative knowledge correlated positively with improved procedural knowledge. In summary, it is important to consider a potential lack of declarative knowledge when attempting to improve the acquisition of a cognitive skill by converting that declarative knowledge to procedural knowledge because not doing so may lead any implementation of the cognitive model to unexpectedly fail to increase performance.

1.2.2 Fine-Grained Performance Models

Recent studies (cf., Pardos et al., 2006; Feng et al., 2006) have shown the value of multidimensional models that contain numerous knowledge components over the value of traditional unidimensional models that only purport to measure one skill (e.g., the math portion of the SAT) as a function of items contained within the assessment. Here they found that while unidimensional models are useful for tests with less items, multidimensional models excel when they have sufficient data points to describe each skill in the model. This supports the idea of having problem sub-steps for problems where there are a sufficiently large number of data points to assess each sub-step.
1.2.3 Automating the Modeling Process

The creation of cognitive models requires skill and can take days in the case of the computational tutors described by Anderson et al (1995) and thus methods have been developed to apply machine learning to the creation of cognitive models using both sample solutions and problems completed by an expert in the domain (cf., Matsuda, 2007; 2006). Similar methods have been developed for Online Socratic Tutors that allow knowledge components to be derived from the text of an encoded problem (Kardian & Heffernan, 2006).

Tools for automatically creating problem sets based on the needs of an individual student are also being developed. For instance, methods developed by Cen et al (2007) used a linear regression to show which knowledge components were understudied and which were over studied, meaning that their method identified which problems required more practice and which ones were sufficiently mastered. Additionally, Radtke (2007) created adaptive scheduling tools that assign items to students based on a variety of different argument models. In summary, the creation of new automation techniques reduces content development time while increasing functionality for presenting that content to students on an individual basis.

1.3 Towards Richer Dialogue during Homework Experiences

1.3.1 Content Building-Blocks within the Assistung System

The Assistung System offers a number of building blocks for crafting the Socratic dialogue around cognitive models (Razzaq et al., 2005). First it allows scheduling through the creation of a “curriculum” or a tree of problems. Problems
themselves can include text and images and store both correct answers and incorrect answers that can be marked with a “buggy message” that gives a student feedback upon submission of that particular incorrect response. Answers can be formatted as a multiple choice, an algebra field, or plain text. Problems also contain a list of sub-problems or “scaffolding problems” that a student is forced to complete if an incorrect answer is submitted. Additionally, all problems that do not contain scaffolding problems (a scaffolding sub-problem can have its own list of scaffolding sub-problems) contain a list of hints made of text and images to be requested on demand. This setup, as seen by the student, is illustrated in Fig 1.1.
Fig. 1.1 An Assiment shown just before a student hits the “Submit” button, showing two scaffolding questions, one buggy message, and a hint message that can occur at different points (Feng et al., 2006).

### 1.3.2 Instructional Methods for Expressing Models as Content

This study explored two methods for expressing a cognitive model comprised of knowledge components as content: 1) scaffolding and hints, and 2) hints-only.
Scaffolding and Hints presents the knowledge components as scaffolds within the Assi stment System with tips for each scaffold expressed as hints. Hints-only presents all knowledge components and tips as one large list of hints. Worked examples show a complete listing of all knowledge components and tips in one large annotated example.

Razzaq and Heffernan (2006) showed this rich combination of hints and scaffolding to be more effective at tutoring eighth grade algebra than hints alone and later they showed that this approach is especially effective with less proficient students in the domain (Razzaq & Heffernan, 2007).\footnote{Razzaq and Heffernan (2006) showed this rich combination of hints and scaffolding to be more effective at tutoring eighth grade algebra than hints alone and later they showed that this approach is especially effective with less proficient students in the domain.} Scaffolding was also shown to be effective over hints alone when implemented through annotated worked examples\footnote{Scaffolding was also shown to be effective over hints alone when implemented through annotated worked examples (Ringenberg et al., 2006) which is in accordance with research by Sweller and Cooper on the same topic (1985; 1987).} (Ringenberg et al., 2006) which is in accordance with research by Sweller and Cooper on the same topic (1985; 1987). A comparison between a self-explanation tutor and a worked examples tutor showed little difference (Salden et al., 2007) and Kalyuga et al (2001) found a negative relationship between domain proficiency and the effectiveness of worked examples as compared to problem-solving. This is similar to the aptitude-treatment interaction by condition observed by Razzaq et al (2007) for scaffolding and hints as compared to hints alone where the scaffolding lost its advantage for students who are more proficient in the domain.

1.3.3 The Importance of Self-Explanation & Note-Taking

One key element of any study of mathematics is showing one’s work. This has been studied within the domain of intelligent tutors in the form of self-explanation and note-taking as a trace of what the student is thinking. Note-taking technologies increase
learning, but simple forms of copy and paste to not provide sufficiently rich processing for deep learning to occur (Bauer & Koedinger, 2007). A technique referred to as self-explanation has been observed to increase learning to such a sufficient degree of richness (Roy & Chi, 2005), however, it has been unclear until recently exactly why this is the case. Two non-mutually exclusive hypotheses exist: 1) the attention hypothesis that explaining a worked example through paraphrase increases on-task attention and so too improves learning, and 2) the generation hypothesis, namely, that explaining examples (with little or no annotation) improves learning through the generation of annotations through inference. Hausman and VanLehn (2007) showed the generation hypothesis to have a greater effect on learning by directly comparing a group that simply paraphrased a solution with expert annotation and one that had to generate explanations of an un-annotated example. In summary, these findings suggest that richer forms of processing like explaining during learning lead to deeper understanding especially when augmented with original inferences on the part of the learner.

1.3.4 The Use of Multimedia & Special Content

Worked examples completed by expert human tutors have also been delivered through video and this instructional method has been compared with live tutoring and intelligent tutoring with mixed results (Craig et al., 2007). Static visuals were also shown to be more effective when integrated together with conceptual text in cognitive tutors in order to promote deep learning (Butcher & Aleven, 2007). The Assisment System currently has the functionality to include videos and other visuals as part of a question, scaffolding question, or hint, although the contribution of this functionality to the current array of instructional methods has not been evaluated empirically.
Since formal mathematic notation is meant to be depicted in two dimensions to make relations more intuitive (i.e., a single line of text input is unintuitive), Anthony et al. (2005; 2006) examined the use of multimodal input including handwriting and voice resulting in increased speed, reduced error, and larger overall gain score.

1.4 Maintaining the Integrity of the Socratic Dialogue

1.4.1 Tracking Progress with Time on Task

In studying the effects of Online Socratic Tutoring it is often difficult to isolate and control the effects of extraneous variables, for example, homework is done at home and outside of the lab and thus one cannot easily control or statistically account for the contribution this has on learning. Warnakulasooriya and Pritchard (2004) devised a method of categorizing students by viewing the percent of students who had completed an assignment as a function of logarithmic time. This plotting produces an s-shaped sigmoid graph. This result suggests that there are three groups of students: 1) students who complete the assignment in less than two minutes and are assumed to already have proficient skill in the domain or are abusing the system; 2) students who complete the assignment using more than two hours, sometimes taking days, are assumed to be somehow receiving outside help, either by seeking it directly or by attending additional lectures, conferences, etc., and 3) students who fall in the middle, completing the assignment between two minutes and two hours. This last group is assumed to be improving based on the Online Socratic Tutoring.
1.4.2 System Exploitation & Gaming

Walonski and Heffernan (2007a) defined three types of student behavior while using an Online Socratic Tutor, (1) on-task behavior, (2) off-task (e.g., web surfing, daydreaming, etc.), and (3) gaming (e.g., rapid-fire guess-and-check, exploiting bottom-out hints, other abuses of tutor behavior, etc.). They then developed methods for preventing this third type of behavior (Walonski and Heffernan, 2007b). Three such methods were created, (1) an algorithm for detecting rapid-fire guess and check which resulted in a feedback message encouraging the student to ask the instructor for help or to pursue other more productive activities, (2) an algorithm for detecting bottom-out hint exploitation which resulted in similar message to the one issued in method 1, and (3) the production of a visual that graphed student behavior and progress as a function of time in an effort to discourage students from engaging in unethical behavior. Additionally, Murray and VanLehn (2005) dissuaded students from over using hints by implementing a ten second delay for hint requests.

1.5 Mission Statement & Objectives

Following the question, “What can our own Assitment System do for us here on campus?”, we aim to develop and run a pilot program for the Assitment System in conjunction with the WPI Department of Mathematics in order to evaluate the system in an undergraduate setting within the domain of freshman calculus. Secondly, we aim to assess additional values of the Assitment System to the WPI Community. To this end, our objectives are:
• To continue the existing dialogue surrounding online undergraduate science and mathematics education and to develop a working relationship between the various faculty members involved, on campus, at WPI – and, more specifically, between those producing research in the learning sciences and those instructing undergraduate mathematics courses.

• To assess potential instructional methods within the Assistment System, more specifically, in two learning conditions, hints and scaffolding, as they are defined in the literature.

• To empirically test the instructional effects of hints and scaffolding in a freshman calculus lab setting and to investigate the potential effects on learning due to attitudinal aspects of learners across the learning conditions.
2.0 Methods

2.1 Participants

A total of 27 (11 female, 16 male) undergraduate students taking the MA1022 Calculus II course at Worcester Polytechnic Institute participated over three regularly scheduled lab periods of two sections each, one at 8am (9 students) and one at 3pm (18 students), and were awarded lab credit for their attendance.

2.2 Design

2.2.1 Testing for Overall Learning

We investigated the effects of the Assitment System treatment on overall learning as in the original study of middle school algebra (Razzaq & Heffernan, 2005). Using this pretest/treatment/posttest design allowed us to evaluate the instructional value of the Assitment System treatment as a pretest/posttest gain, controlling for both previous topic knowledge and any confounding instruction that might have occurred had the posttest been distributed at a later date or time. The pretest, posttest, and treatment used three sets of the same isomorphic problems to insure informational equivalence.4

2.2.2 Testing for Learning by Condition

To investigate any differences in learning between the scaffolding condition and the hints condition, we had the Assitment System randomly assign participants to a set of scaffolding problems or a set of hints problems as in Razzaq and Heffernan (2006).

---

4 If we had a larger sample size, we would have analyzed these data using a repeated measures multivariate analysis of variance (MANOVA).
2.2.3 Exploratory Learning Questionnaires

We created a questionnaire for preliminary use before each of the experiments took place in order to establish: (a) background variables that might influence learning, (b) the participants’ beliefs about the nature of learning, (c) the participants’ learning styles, and (d) the participants’ work habits. The 3 background questions and 14 Likert type items are listed below.

**Background Questions**

a. Have you taken this course before? [Yes, No]

b. Did you take any calculus in high school? [Pre Calculus, AP A/B, AP B/C, No]

c. If you took the AP Exam, what did you get? [1, 2, 3, 4, 5]
Likert Type Items for Nature of Learning, Learning Styles, and Working Habits

We scored items 1 to 5 for “strongly disagree” to “strongly agree” in order to assess our a priori groupings of the items into nature of learning, learning styles, and working habits.

1. I believe that if I work hard at calculus, I can do well.
2. When doing calculus problems, I always use scrap paper instead of doing work in my head.
3. I understand calculus well.
4. I do my homework during calculus lectures.
5. I prefer classes that emphasize facts and data over concepts and ideas.
6. I am creative in the way I do my work.
7. I enjoy calculus class.
8. Breaking a question down into smaller parts helps me to understand how to solve similar problems.
9. I think that some people are just good at math.
10. I am careful about the details of my work.
11. I work hard on calculus.
12. I care a great deal about how I do in calculus.
13. I find it easier to learn concepts rather than facts.
14. I usually seek help when I don’t understand something.
We then created two instances of the same questionnaire to be administered with the posttest of each experiment in order to establish: (a) student strategies participating in the treatment, (b) the level of difficulty experienced by participants during treatment, and (c) participant attitudes toward the Assistment System lab. The 14 Likert type items are listed below.

**Likert Type Items for Completion Strategy, Difficulties During Treatment and Attitudes Toward the Assistment System.**

We scored items 1 to 5 for “strongly disagree” to “strongly agree” in order to assess our a priori groupings for completion strategy, difficulties during treatment, and attitudes toward the Assistment System Lab.

1. I tried to get through difficult problems as quickly as possible.
2. I liked today’s lab using the Assistments System better than doing homework.
3. My goal during today’s lab was to learn new things.
4. When a problem was too hard I would just try to get through so that I could get to easier problems.
5. I liked today’s lab using the Assistments System better than attending lecture.
6. I took today’s lab very seriously and worked hard all the time.
7. I found it hard to stay concentrated all the time.
8. I think the tutoring helped me understand similar problems.
9. I found many of the problems frustrating because they were too hard.
10. I liked today’s lab using the Assistments System better than normal labs.
11. My goal during today’s lab was to get through as many problems as possible.

12. I liked using the web tutoring from the Assistsments System. (Volume Experiment Only)

12. I preferred today’s lab with Assistsments over last week’s lab with Assistsments. (Parts Experiment Only)

13. I think the tutoring will help me on tests in class.

14. I liked today’s lab using the Assistsments System better than doing a test.

2.3 Content Authoring

We created content in two sub-domains of calculus: (a) Volumes of Rotation, and (b) Integration by Parts. The material on Volumes of Rotation was created at the request of Professor Tashjian. The material on Integration by Parts was originally suggested and discussed with Professor Goulet and was later discussed with Professor Tashjian when it was implemented as an alternative to a third sub-domain, Arc-Length, which included integrals not easily solved through the analytic means available to participants.

2.3.1 Volumes of Rotation Material

The volume material consisted of 4 questions based on a cognitive model of a single problem using the “disks method” for calculating the volume of a function revolved around the x-axis.

After deriving knowledge components for this problem by examining an expert\(^5\) annotation of the problem solution, we found that the declarative knowledge required for

\(^5\) We considered the student author of this paper to be a sufficient expert in the domain having taken Calculus II previously and having tutored the topics numerous times since then.
understanding the mathematical concepts behind the procedure was quite complex even though the procedural knowledge required for solving the problem was relatively simple. Recalling student difficulties with conceptual material in algebra (cf., Koedinger & Mitchell, 2004; Booth et al., 2007), we decided that the first of three problems should test progressively more complex conceptual requirements for the fourth and final problem which, only then, tested participants’ knowledge of the volume material. Sample problems taken from the volume pretest illustrate this below. (For a complete description of the material created for the Assistance System treatment, see Appendix A.)

\[
\sum_{i=4}^{5} (i-2)^i
\]

Fig. 2.1 Problem 1: “Evaluate the following summation.”

Problem 1 examines the declarative and procedural knowledge regarding a “summation” or the notation used to sum any number of values for a particular argument between an upper and lower bound – in this example the value of the quantity \( i \) minus 2 raised to the power of \( i \) for values of \( i \) between 4 and 5 inclusive. Conceptual understanding of summations is required for conceptual understanding of integrals which are used for infinite sums.
Fig. 2.2 Problem 2: Riemann Sums

Problem 2 is a multiple choice question, “These three pictures illustrate three different Riemann Sum approximations of the area under the graph of $f(x)$. Of course, neither of them is a perfect approximation and so there is error in all three. If we were to take the limit of the each approximation as the number of boxes increased to infinity and as the width of these boxes decreased to zero, what would happen to their comparative error?” with 4 choices: (a) The error would increase in all the approximations because there were more bars and each bar creates more error, (b) The error would stay the same for each approximation because no matter how many bars there are, the same method is still being used, (c) The error would decrease in all the approximations because as the bars become smaller and greater in number they all approach the true area, and (d) The error for the two approximations using rectangles would stay about the same but the approximation using trapezoids would approach the true area because it more directly mimics the slope of $f(x)$. This question examines only the declarative knowledge regarding an infinite sum of infinitely thin boxes (lines) which are the conceptual product of an integral (i.e., the area under the curve).

![Graph of f(x) and g(x)](image)

Fig. 2.3 Problem 3 & Problem 4
Problem 3 asks participants to, “Find the area bounded between the functions \( f(x) \) and \( g(x) \) on the interval \((0, 3)\).” This problem requires the declarative knowledge that an integral is the area under a curve because only then is it clear that procedurally subtracting the integral of one curve from another is conceptually equivalent to subtracting the area of one curve from another.

Problem 4 asks participants to, “Find the volume of the solid created when the area bounded between the functions \( f(x) \) and \( g(x) \) from Problem 3 is rotated around the x-axis.” This final problem, which only now evaluates knowledge of Volumes of Rotation, is conceptually complex because it requires that the participant connect two pieces of declarative knowledge. First, one must realize that the area of a circle is equal to \( \Pi \) times its radius squared. Second, one must realize that an integral that finds the area under a curve does so by summing an infinite number of infinitely small boxes (lines). Finally one must think to use these lines as the radius of a circle, thereby deriving the procedural knowledge to integrate – or sum – an infinite number, not of lines, but circles, solving, not for an area, but for a volume.

2.3.2. Integration by Parts Material

The parts material consisted of 4 questions based on a hybrid cognitive model created from the similarities between each of the 4 problems. An additional “pre-question” was added to find whether or not participants knew the equation for integration by parts. This equation and the 4 main problems are illustrated below. (For a complete description of the material created for the Assistment System treatment, see Appendix A.)
\[ \int u \, dv = uv - \int v \, du \]

Fig. 2.4 Equation for Integration by Parts

\[ \int x \, e^{5x} \, dx \]

Fig. 2.5 Problem 1

\[ \int x \, \cos(7x) \, dx \]

Fig. 2.6 Problem 2

\[ \int \ln|3x| \, dx \]

Fig. 2.7 Problem 3

\[ \int x^2 \, e^{5x} \, dx \]

Fig. 2.8 Problem 4
After deriving the knowledge components from all 4 of the expert annotated solutions\(^6\), we concluded that the parts model was the opposite of the volume model in that the declarative knowledge required for conceptual understanding was simple while the procedural knowledge for actually implementing the concept was rather difficult because of the large amount of information involved in solving the problems.

We did not originally have a strategy for reducing this strain on working-memory (Anderson, 1995). However, Professor Tashjian introduced us to a method of keeping track of all the numbers. A simple box to hold the values of “\(u\)”, “\(du\)”, “\(v\)”, and “\(dv\)” as they were solved for, we were told, was the key to her instruction. We adapted our scaffolding sub-problems (or hint sub-problems as the hints-only condition dictated) to include graphics depicting how one should sketch the work for an integration by parts. Since there are four pieces of information that must be juggled during the problem, it is intuitively plausible that this visual method of instruction supports student problem solving, especially within the context of studies showing the value of visuals when integrated into the text (cf., Butcher & Aleven, 2007).

### 2.4 Procedure

On the first of three days, we distributed a single packet containing instructions for attaining Assistments logins and passwords along with our preparatory learning questionnaire taking approximately 10 minutes of the regular lab session before it continued as usual. We also introduced ourselves and described the study briefly mentioning that credit would be given for participation.

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\(^6\) We considered the student author of this paper to be a sufficient expert in the domain having taken Calculus II previously and having tutored the topics numerous times since then.
On the second and third days, we distributed instructions in three packets to be completed over the entire 50 minute lab period. We reminded students that credit would be given for participation and handed out the first packet which contained the pencil and paper pretest to be completed within 15 minutes. We collected these pretest packets and distributed a second packet which consisted of a nearly blank sheet of paper containing instructions to log on to the Assignments System and complete the day’s assignment with the instruction to “show your work” in the space provided. We instructed students to stop after 20 minutes and we then collected their tutoring packets while distributing a third packet containing our posttest learning questionnaire and a pencil and paper posttest to be completed over the final 15 minutes of lab after which we dismissed the participants for the day. (For a complete description of the packets distributed on all three days, see Appendix B.)

2.5 Coding & Scoring

2.5.1 Gain Scores Summed and by Problem

Participant responses to individual problems were coded “0” or “1” to represent “incorrect” and “correct” where correctness was defined as having the correct answer on the page. Pretest and Posttest sums were simply the sum of each problem for that experiment. Gain scores were calculated by subtracting the pretest score from the posttest score.

2.5.2 Additional Pretest Correction

We created a second set of measures that removed participants if they correctly responded to a question on the pretest because we hypothesized that these students would
not learn from the treatment because they were already proficient in the sub-domain. This was important because a gain score of zero cannot specify whether the student was correct both times (correct posttest minus correct pretest equals zero) or incorrect both times (incorrect posttest minus incorrect pretest equals zero).  

2.5.3 A Priori Groupings for Questionnaire Items

Individual Likert type items were coded between 1, “strongly disagree” and 5, “strongly agree”. Scales were created to evaluate the a priori concepts mentioned in the content authoring section by taking the average of the items hypothesized to measure that concept. Some questions contributed negatively to a concept and so the reverse of their score was added into the average instead of the original value. The scales for each concept are listed below. Items that contribute negatively to the concept are marked with a negative one.

---

7 This proved to be critical during our analyses of gain score by problem where inflated frequencies for gain score zero either inflated or hid effects and other interactions.
8 E.g., In the case of the two items, “I prefer classes that emphasize facts and data over concepts and ideas”, and “I find it easier to learn concepts rather than facts” we can easily see that the two items are the reverse of each other and that they measure the same concept. Therefore, we would expect a participant who answered “strongly agree” to the first item to then answer “strongly disagree” to the second item and likewise we would expect a participant who neither agreed or disagreed (a score of 3) to answer the same way. To add these two together we would then decide to take one item’s original score (e.g., for a score of 2 we would use 2) and add it with the reverse of the other item’s score (e.g., for a score of 4 we would use 2 because it is the opposite of 4 on the scale). Taking the average keeps the number the same if answers are perfectly consistent (e.g., 2 and 4 turn into 2 and 2 which on the average is 2) and even the score out if answers are inconsistent (e.g., 1 and 1 turn into 1 and 5 which on the average is 3 which represents someone with no opinion one way or the other.)
Preparatory Questionnaire Only

“Nature of Learning”

Item 1 – “I believe that if I work hard at calculus, I can do well.”

Item 9⁻¹ – “I think that some people are just good at math.”

“Learning Styles”

Item 5 – “I prefer classes that emphasize facts and data over concepts and ideas.”

Item 13⁻¹ – “I find it easier to learn concepts rather than facts.”

“Working Habits”

Item 10 – “I am careful about the details of my work.”

Item 11 – “I work hard on calculus.”

Item 14 – “I usually seek help when I don’t understand something.”
Posttest Questionnaires Only

“Treatment Strategies”

Item 1 – “I tried to get through difficult problems as quickly as possible.”

Item 3 – “My goal during today’s lab was to learn new things.”

Item 4 – “When a problem was too hard I would just try to get through so that I could get
to easier problems.”

Item 6 – “I took today’s lab very seriously and worked hard all the time.”

Item 11 – “My goal during today’s lab was to get through as many problems as possible.”

“Difficulty during Treatment”

Item 7 – “I found it hard to stay concentrated all the time.”

Item 9 – “I found many of the problems frustrating because they were too hard.”

“Attitudes towards Assitments System”

Item 2 – “I liked today’s lab using the Assitments System better than doing homework.”

Item 5 – “I liked today’s lab using the Assitments System better than attending lecture.”

Item 8 – “I think the tutoring helped me understand similar problems.”

Item 10 – “I liked today’s lab using the Assitments System better than normal labs.”

Item 13 – “I think the tutoring will help me on tests in class.”

Item 14 – “I liked today’s lab using the Assitments System better than doing a test.”
3.0 Results

We examined the relationship between the Assisment System treatment and: (a) overall learning by gain score, collapsed over condition, (b) learning by condition by comparing scaffolding condition gain score to hint condition gain score, (c) scales and individual items on the learning questionnaire by gain score, and (d) scales and individual items on the posttest learning questionnaire by gain score. Within the Integration by Parts experiment, we also examined the relationship between the Assisment System treatment and: (a) previous Calculus II experience by gain score, and (b) the detail of pencil and paper notes by gain score post hoc. (For a summary of all significant and insignificant findings, see Appendix C.)

3.1 Experiment One – Volumes of Revolution

3.1.1 Overall Learning

In order to assess whether learning occurred due to the Assisment System treatment overall, regardless of tutoring condition, we collapsed the hints and scaffolding groups into a single pool of 23 students who participated in the Volume of Revolutions experiment. Their pretest sums ($M = 1.5 \pm .8$ problems out of a total of 4 problems) and posttest sums ($M = 2.4 \pm .8$ problems out of a total of 4 problems) revealed statistically significant gain scores ($M = .9 \pm .9$ more problems on the posttest out of a total of 4 problems) after a one-tailed, paired-samples t test was conducted, $t(22) = 4.5$, $p < .001$, with $r_{\text{effect size}} = .7$. 
A further examination of gain score by individual item revealed increased performance on all 4 problems, and an analysis using a set of one-tailed, paired-sample t tests confirmed this observation for the first 3 of 4 problems, as illustrated in Table 3.1.

Table 3.1 Paired-Samples Tests for Volume Experiment Pretest and Posttest by Problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
<th>95% Confidence Interval of the Difference</th>
<th>t</th>
<th>df</th>
<th>Sig. (1-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>.348</td>
<td>.487</td>
<td>.102</td>
<td>.137</td>
<td>.558</td>
<td>3.425</td>
<td>.001</td>
</tr>
<tr>
<td>Problem 2</td>
<td>.174</td>
<td>.388</td>
<td>.081</td>
<td>.006</td>
<td>.342</td>
<td>2.152</td>
<td>.022</td>
</tr>
<tr>
<td>Problem 3</td>
<td>.261</td>
<td>.619</td>
<td>.129</td>
<td>-.007</td>
<td>.529</td>
<td>2.021</td>
<td>.028</td>
</tr>
<tr>
<td>Problem 4</td>
<td>.087</td>
<td>.288</td>
<td>.060</td>
<td>-.038</td>
<td>.212</td>
<td>1.447</td>
<td>.081</td>
</tr>
</tbody>
</table>

3.1.2 Learning by Condition

An examination of gain score by condition (\(M = .9 \pm .9\) more problems on the posttest for both the 14 students in the hints condition and the 9 students in the scaffolding condition) found no statistically significant differences after a one-tailed, independent-samples t test analysis was conducted, \(t(21) = -.079, \ p = .469\).

A further examination of gain score by individual item revealed no significant difference between conditions after a set of one-tailed, independent-samples t tests were conducted, as illustrated in Table 3.2.
Table 3.2 Independent-Samples Tests for Volume Experiment Gain Scores by Problem

<table>
<thead>
<tr>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>Equal variances assumed</td>
<td>F 1.529 Sig. .230 t -755 df 21 Sig. (1-tailed) .229 Mean Difference -.15873 Std. Error Difference .21012 Lower -.59570 Upper .27824</td>
</tr>
<tr>
<td>Problem 2</td>
<td>Equal variances not assumed</td>
<td>F .863 Sig. .364 t -471 df 21 Sig. (1-tailed) .322 Mean Difference -.07937 Std. Error Difference .16859 Lower -.42997 Upper .27124</td>
</tr>
<tr>
<td>Problem 3</td>
<td>Equal variances assumed</td>
<td>F .003 Sig. .954 t .235 df 21 Sig. (1-tailed) .409 Mean Difference .06349 Std. Error Difference .27042 Lower -.49887 Upper .62586</td>
</tr>
<tr>
<td>Problem 4</td>
<td>Equal variances not assumed</td>
<td>F 7.889 Sig. .011 t 1.170 df 21 Sig. (1-tailed) .128 Mean Difference .14286 Std. Error Difference .12207 Lower -.11100 Upper .39672</td>
</tr>
</tbody>
</table>

However, by using the corrected gain score, one such statistically significant effect, $t(6) = -2.121$, $p = .039$ (one-tailed), was found for Problem 1, namely that the students in the scaffolding condition scored significantly higher than students in the hints condition on Problem 1 once students who got the problem correct on the pretest were removed.
3.1.3 Preparatory Learning Questionnaire

Of the three scales, 1) nature of learning, 2) learning styles, and 3) work habits, none correlated with gain score. Item 4, “I do my homework during calculus lectures”, was found to have a statistically significant positive correlation to gain score after a two-tailed, Pearson r product-moment correlation coefficient was calculated, $r(21) = .439, p = .047$, meaning that gain score and ratings of how much one does homework during lectures have a positive linear relationship.

3.1.4 Posttest Learning Questionnaire

Of the three scales, 1) treatment strategy, 2) difficulties during treatment, and 3) attitudes toward Assistment System, none correlated with gain score, nor did any of the 14 individual questionnaire items correlate with gain score.

3.2 Experiment Two – Integration by Parts

3.2.1 Overall Learning

In order to assess whether learning occurred due to the Assistment System treatment overall, regardless of tutoring condition, we collapsed the hints and scaffolding groups into a single pool of 22 students who participated in the Integration by Parts experiment. Their pretest sums ($M = .5 \pm .7$ problems out of a total of 5 problems) and posttest sums ($M = 2.2 \pm 1.2$ problems out of a total of 5 problems) revealed statistically

---

9 We cannot really call these scales, per se, due to the few items in each category and the fact that we did not construct a test, in psychometric terms.

10 Pearson’s r product-moment correlation coefficient is calculated by the summation of the product of the z scores for both groups over their sample size. It is expressed as $r(samp\sample) = value, p = significance$. It should not be confused with r effect size correlation or $r^2$ coefficient of determination of variance explained.
significant gain scores \((M = 1.7 \pm 1.0\) more problems on the posttest out of a total of 5 problems) after a one-tailed, paired-samples \(t\) test was conducted, \(t(21) = 7.9, p < .001\), with \(r_{\text{effect size}} = .9\).

A further examination of gain score by individual item revealed a statistically significant increase in performance on the first 4 out of 5 problems after a set of one-tailed, paired-samples \(t\) tests were conducted, as illustrated in Table 3.3.

| Table 3.3 Paired-Samples Tests for Parts Experiment Pretest and Posttest by Question |
|---|---|---|---|---|
| Problem 1 | Posttest-Pretest | Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference |
| | | .500 | .598 | .127 | 95% Lower & Upper |
| | | | | .235 | .765 |
| | | | | 3.924 | 21 | .001 |
| Problem 2 | Posttest-Pretest | .636 | .492 | .105 | 95% Lower & Upper |
| | | | | .418 | .855 |
| | | | | 6.062 | 21 | .000 |
| Problem 3 | Posttest-Pretest | .318 | .477 | .102 | 95% Lower & Upper |
| | | | | .107 | .530 |
| | | | | 3.130 | 21 | .003 |
| Problem 4 | Posttest-Pretest | .227 | .429 | .091 | 95% Lower & Upper |
| | | | | .037 | .417 |
| | | | | 2.485 | 21 | .011 |
| Problem 5 | Posttest-Pretest | .045 | .213 | .045 | 95% Lower & Upper |
| | | | | .049 | .140 |
| | | | | 1.000 | 21 | .165 |

### 3.2.2 Learning by Condition

An examination of gain score by condition \((M = 1.9 \pm 1.2\) more problems on the posttest for the 13 students in the hints condition as compared to \(M = 1.4 \pm .7\) more problems on the posttest for the 9 students in the scaffolding condition out of a total of 5 problems on the posttest) found no statistically significant differences after a one-tailed, independent-samples \(t\) test analysis was conducted, \(t(20) = .885, p = .194\).
A further examination of gain score by individual item revealed no significant differences between conditions after a set of one-tailed, independent-samples t tests were conducted, as illustrated in Table 3.4.

### Table 3.4 Independent-Samples Tests for Parts Experiment Gain Scores by Problem

<table>
<thead>
<tr>
<th>Problem 0</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Problem 2</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Problem 3</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Problem 4</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
</table>
3.2.3 Effects of Prior Coursework

Investigating whether students had ever taken Calculus II before revealed statistically significant differences between pretest sums \((M = .2 \pm .4\) problems for the 11 students who answered “no” compared to \(M = .9 \pm .9\) problems for the 9 students who answered “yes” out of a total of 5 problems) after a two-tailed, independent-samples t test was conducted, \(t(18) = -2.286, p = .035\). Although this means that those who had taken Calculus II before scored significantly higher on the pretest than those who had not, the effect was not found on gain score.

3.2.4 Preparatory Learning Questionnaire

Of the three scales, 1) nature of learning, 2) learning styles, and 3) work habits, none correlated with gain score, nor did any of the individual questionnaire items correlate with gain score.

3.2.5 Posttest Learning Questionnaire

Of the three scales, 1) treatment strategy, 2) difficulties during treatment, and 3) attitudes toward Assistance System, none correlated with gain score. Item 4, “When a problem was too hard I would just try to get through so that I could get to easier problems”, and Item 12, “I preferred today’s lab with Assistance over last week’s lab with Assistance” were found to have a statistically significant positive correlation to gain score after a set of two-tailed, Pearson r product-moment correlation coefficients were calculated, \(r(21) = -.560, p = .008\), and \(r(20) = .501, p = .024\), respectively. This represents a negative linear relationship between moving on when a problem is too hard
and gain score and a positive linear relationship between enjoying the lab more that the previous lab and gain score.

### 3.2.6 Post Hoc Rating of Note Detail

We observed that notes taken during the treatment could easily be coded into categories of detail: 1) Shows Answers Only, 2) Shows Some Knowledge of the Parts Formula, and 3) Sets up the Integral. Comparing the detail of notes taken during the Assisment System treatment (3 students showed answers only, 2 students showed some knowledge of the parts formula, and 17 students set up the integral) with gain score by calculating the two-tailed, Pearson’s r product-moment correlation coefficient revealed a statistically significant positive correlation between the two, $r(22) = .532, p = .011$, meaning that the detail of notes taken during treatment and gain score have a positive linear relationship.
4.0 Discussion

4.1 Summary of Significant Findings

4.1.1 Overall Learning & Learning by Condition

We confirmed our hypotheses about the effectiveness of the Assitement System in a university setting, seeing gains of .9 problems out of 4 in the Volumes of Rotation experiment and gains of 1.7 problems of 5 in the Integration by Parts experiment. This represents a statistically significant total of about 25% improvement in both experiments. Broken down by problem we found statistically significant learning on all but the last problems in both experiments. We also found that the scaffolding condition had a statistically significant advantage over the hint condition on the first problem of the Volumes of Rotation experiment.

In the Integration by Parts experiment, those who had taken Calculus II at WPI before scored 14% higher than those who had not on the Integration by Parts pretest, although this effect was not found for gain score. The level of detail of notes taken during the Assitement System treatment during the Integration by Parts experiment was also shown to have a statistically significant positive linear relationship with gain score.

Our results were consistent with the findings of Razzaq et al (2005) that showed the Assitement System to increase student performance. We had similar issues to Razzaq and Heffernan (2006) in attempting to isolate the difference between hints and scaffolding primarily because of lack of statistical power due to a small sample size in our case. The effects of prior coursework between Calculus II experience and gain score
in the Integration by Parts experiment was shown to be statistically insignificant even though there were significant differences at pretest as consistent with the finding by Morote and Pritchard (2004a) that Online Socratic Tutors were unbiased by background variables.

4.1.2 Learning Questionnaires

None of our a priori learning questionnaire scales correlated with gain score, although we did find three statistically significant correlations with individual learning questionnaire items. In the Volumes of Rotation experiment, doing homework in lecture had a positive linear relationship with gain score. In the Integration by Parts experiment, moving on from a hard problem quickly to get to easier problems had a negative linear relationship with gain score, and enjoying the lab more than the previous lab had a positive linear relationship with gain score.

Two of the three learning questionnaire findings were intuitively plausible, but not particularly interesting. The first of these was a negative linear relationship between moving on quickly when a problem was difficult and gain score. Avoiding a challenge is a plausible reason for being associated with shallower processing on the part of the students, therefore leading to a reduced gain score. The second of these was a positive linear relationship between enjoying a lab more than the previous lab and gain score. Enjoying work when one is doing well is certainly plausible and thus this finding is not interesting.

The third learning questionnaire finding was a statistically significant positive linear relationship between the assertion that one does homework during lecture and gain score. This finding is not initially intuitively plausible as it suggests that those who do
homework in class have higher gain scores, but a look at the lower and upper bounds of student responses shows that the upper bound was 3 so in reality the finding was closer to a positive linear relationship between being ambivalent about doing homework in lecture and gain score which is at least less unintuitive that the alternative.

4.1.3 Effects of Note-Taking

It is possible that there was more an advantage favoring the scaffolding condition over the hints condition that may come from self-explanation and that this advantage would have been nullified by the second fact that we had all participants in both conditions taking notes during the treatment. It would be consistent with other studies on note-taking (Bauer & Koedinger, 2007; Roy & Chi, 2007; Hausman & VanLehn, 2007) for the note-taking to increase students’ processing as much, if not more, than the scaffolds in both the scaffolding condition and the hints condition, thus nullifying the scaffolding condition’s previous advantage. The idea that the notes confounded scaffolding’s advantage over hints is also supported by our finding that the detail of notes taken positively correlated with gain score. Furthermore, the overall success of the Integration by Parts experiment may be due in part to the note-taking aids we provided through images at the suggestion of our domain experts. This is consistent with the visuals literature from Butcher and Aleven (2007).

4.2 Future Research & Development

4.2.1 Comparative Study of Four Conditions

To bring the current Assitement System literature in line with the literature on worked examples (cf., Salden et al., 2007) and video annotated worked examples (Craig
et al., 2007) an experiment should be conducted which matches these two conditions with a scaffolding condition and a hints condition, extending the research presented by this study.

### 4.2.2 Other Exploratory Studies

Other experiments should be conducted to identify and examine the difficulties associated with symbol manipulation in calculus as per the algebra studies put forth by Koedinger and Mitchell (2004) and Booth et al (2007) as well as the exact nature of bottom-out hints and explicitly giving students the correct answer.

The value of reporting to instructors and teaching assistants should also be explored as these experiments are played out, as the current study primarily relied on humans to handle scoring and reporting on paper meaning that half the functionality of the AssiStment System was left un evaluated. For example, it is plausible (in fact the system is designed with this in mind), that professors might change their teaching practices according to the results of the AssiStment data to better align with what strategies and knowledge the students are lacking.

### 4.2.3 Additional System Functionality

Updates to the system should include offline content authoring tools since the current AJAX implementation slows the input of pre-planned content. Namely, there should be tools for batch uploading material to the AssiStments system whether online or offline. There should be functionality for completing problems out of their order and creating problem isomorphs automatically by algebraic formula. The algebra field, itself, should be adapted to handle more advanced arguments allowing for trigonometric and
transcendental functions as well as the notations used in calculus, namely those for summation, differentiation, and integration. Reports should also make use of the time on task findings (cf., Warnakulasooriya & Pritchard, 2004). Finally, the functionality already implemented in the Assistment System by Radtke (2007) should be combined with the functionality described by Cen et al (2007) to allow the scheduling of additional problem isomorphs for knowledge components that are not sufficiently mastered in the domain (e.g., a similar problem is issued at the end of the problem set if scaffolding is required on the first try).

4.3 Conclusions

The pilot successfully established the utility of the Assistments System within the domain of university calculus at an introductory level. A larger sample size will be required to reify our findings. Most importantly the local dialogue surrounding Online Socratic Tutoring at WPI has been shifted toward the Assistment System which will allow the breadth and depth of the content domain to be extended into more advanced topics in calculus and other fields like physics and chemistry in the future. The primary requirements for moving forward with the results of this pilot will be researchers, professors who volunteer their classes, and time.
References


W., (Eds.), *Proceedings of the 8th International Conference on Intelligent Tutoring Systems*, pp. 382-391. Springer-Verlag: Berlin, Germany.


Appendix A
You are currently at: Curriculum Pretty-Print

**Module Name:** Volumes of Revolution Lab

**Questions Only**  **Answer Key**  **Printout for Students**  **Full Details**

Begin '{ChooseOne}' Section

Begin Linear Section

1. "c2 vol problem 1h" (Problem ID: 22406) ALGEBRA_FIELD

| \[ \sum_{i=3}^{5} (i-2)^i \] |

No knowledge components have been assigned

Please evaluate the following Summation.

**Answers:** (Interface Type: ALGEBRA_FIELD)

260

**Hint 1:**

| \[ \sum_{g=0}^{10} g \] |

The Greek capital letter, sigma, denotes a summation, meaning that the function shown with it should be evaluated for every number between and including the number shown below and above the sigma and then that each of these values should be added together. This simple example adds all the numbers up between and including 0 and 10 - please evaluate this summation.

**Hint 2:**

| \[ \sum_{g=0}^{10} g = (1) + (2) + (3) + \ldots + (10) \] |

This is what the summation looks like expanded with each of its values plugged in.

**Hint 3:**

The sum of all of the numbers between and including 0 and 10 is 55.

**Hint 4:**

| \[ \sum_{\alpha=1}^{5} \alpha^2 \] |

This slightly more complicated example squares all the numbers between and including 1 and 5 and then adds them together - please evaluate this summation.

**Hint 5:**
This is what the summation looks like expanded with each of its values plugged in.

**Hint 6:**
The sum of all of the numbers squared between and including 1 and 5 is 55.

**Hint 7:**
\[
\sum_{t=2}^{4} 2^t
\]

This example is slightly different, but just as easy. It sums together every power of 2 between and including 2 and 4 - please evaluate this summation.

**Hint 8:**
\[
\sum_{t=2}^{4} 2^t = (2^2) + (2^3) + (2^4)
\]

This is what the summation looks like expanded with each of its values plugged in.

**Hint 9:**
The sum of all powers of 2 between and including 2 and 4 is 28.

**Hint 10:**
\[
\sum_{i=3}^{5} (i-2)^i
\]

Finally we return to the original problem where our function is more complicated. The work may be more messy, but the idea is the same – plug each value of the summation into the function and then add all the parts together!

**Hint 11:**
\[
\sum_{i=3}^{5} (i-2)^i = (3-2)^3 + (4-2)^4 + (5-2)^5
\]

This is what the summation looks like expanded with each of its values plugged in.

**Hint 12:**
The final answer is 260.
These three pictures illustrate three different Riemann Sum approximations of the area under the graph of \( f(x) \). Of course, neither of them is a perfect approximation and so there is error in all three. If we were to take the limit of the each approximation as the number of boxes increased to infinity and as the width of these boxes decreased to zero, what would happen to their comparative error?

**Answers:** (Interface Type: RADIO BUTTON)

- ✗ The error would increase in all the approximations because there were more bars and each bar creates more error.
- ✗ The error would stay the same for each approximation because no matter how many bars there are, the same method is still being used.
- ✓ The error would decrease in all the approximations because as the bars become smaller and greater in number they all approach the true area.
- ✗ The error for the two approximations using rectangles would stay about the same but the approximation using trapezoids would approach the true area because it more directly mimics the slope of \( f(x) \).

**Hint 1:**

Since the size and number of the boxes are changing, we must think about how this relates to the size of the error. These three pictures illustrate the same rectangle method being used to approximate the area under the graph of \( f(x) \). The only difference between them is the size and number of the rectangles. Which of them has the most error? Which of them has the least error?

Choices:
A. The largest bars have the most error and the smallest bars have the least error.
B. The smallest bars have the most error and the largest bars have the least error.
C. All of the bars have an equal amount of error because the same method is being used.

**Hint 2:**

Compare this graph of the true area under the graph of \( f(x) \) and an approximation and think about what the error is and what causes it.

**Hint 3:**
The largest bars have the most error and the smallest bars have the least error.

3.) "c2 vol problem 3h" (Problem ID: 22408) ALGEBRA_FIELD

No knowledge components have been assigned

Find the area bounded between the functions $f(x)$ and $g(x)$ on the interval $(0, 2)$. 

Answers: (Interface Type: ALGEBRA_FIELD)

8/3

Hint 1:
What is the area bounded between the functions $f(x)$ and $g(x)$ in terms of the area under $f(x)$ and the area under $g(x)$? (Use $F$ to represent the area under $f(x)$ and $G$ to represent the area under $g(x)$.)

Hint 2:
Use this image to help you find the relation between $G$ and $F$.

Hint 3:
The area of the bounded region is equal to the area under $g(x)$ minus the area under $f(x)$ or $G-F$.

Hint 4:
What is the area under $g(x)$ or "G" equal to?

Hint 5:
To find the area under $g(x)$, evaluate the integral of $g(x)$ between 0 and 2.

Hint 6:
The area under $g(x)$ is equal to 16/3.

Hint 7:
What is the area under $f(x)$ or "F" equal to?

Hint 8:
To find the area under $f(x)$, evaluate the integral of $f(x)$ between 0 and 2.

Hint 9:
The area under $f(x)$ is equal to 8/3.

Hint 10:
Now subtract $F$ from $G$ to get the final answer. What is the final answer, the area bounded between $f(x)$ and $g(x)$?
4.) "c2 vol problem 4h"  (Problem ID: 22409) ALGEBRA_FIELD

No knowledge components have been assigned

Find the volume of the solid created when the area bounded between the functions \( f(x) \) and \( g(x) \) is rotated around the x-axis. Use the letter \( p \) for \( \pi \).

**Answers:** (Interface Type: ALGEBRA_FIELD)

\[ \boxed{48p/5} \]

**Hint 1:**
Although we say "rotated around the x-axis" we do not actually have to perform any calculations that involve rotation or polar coordinates. Just as the area between two curves can be approximated by many small rectangles, the volume that is created by rotating that area can be approximated by many small cylinders.

And, just as these rectangles become infinitely thin (lines) when we use an integral (that is to say, a summation where the number of rectangles goes to infinity) these cylinders become infinitely thin (circles) when we use the integral.

So where we normally integrate a function (the length of the line at \( x \)) to find the area - we will integrate \( \pi \) times the function squared (\( \pi r^2 \) is the area of a circle) to find the volume. Knowing this, set up the integral for the volume in terms of \( x \). Use the letter \( p \) for \( \pi \).

**Hint 2:**
Remember from the last problem that we must subtract the volume of \( f(x) \) from the volume of \( g(x) \) to get the volume between them.

**Hint 3:**
To get the volume we must integrate \( p*(8x)-p*x^4 \)

**Hint 4:**
evaluate the integral of \( p*(8x)-p*x^4 \) dx on the interval \( (0, 2) \).

**Hint 5:**
To evaluate a definite integral, subtract the integral evaluated at the lower bound (0) from the integral evaluated at the upper bound (2). Use the letter \( p \) for \( \pi \).

**Hint 6:**
The final answer is \( 48p/5 \).

End Linear Section Begin Linear Section

5.) "c2 vol problem 1s"  (Problem ID: 22382) ALGEBRA_FIELD

No knowledge components have been assigned
Please evaluate the following Summation.

**Answers:** (Interface Type: ALGEBRA_FIELD)

**✓ 260**

(Problem ID: 22383) ALGEBRA_FIELD

No knowledge components have been assigned

\[
\sum_{i=3}^{5} (i-2)^i
\]

The Greek capital letter, sigma, denotes a summation, meaning that the function shown with it should be evaluated for every number between and including the number shown below and above the sigma and then that each of these values should be added together. This simple example adds all the numbers up between and including 0 and 10 - please evaluate this summation.

**Answers:** (Interface Type: ALGEBRA_FIELD)

**✓ 55**

**Hint 1:**

\[
\sum_{g=0}^{10} g = (1)+(2)+(3)+ \ldots +(10)
\]

This is what the summation looks like expanded with each of its values plugged in.

**Hint 2:**

The sum of all of the numbers between and including 0 and 10 is 55.

(Problem ID: 22384) ALGEBRA_FIELD

No knowledge components have been assigned

\[
\sum_{\alpha=1}^{5} \alpha^2
\]

This slightly more complicated example squares all the numbers between and including 1 and 5 and then adds them together - please evaluate this summation.

**Answers:** (Interface Type: ALGEBRA_FIELD)

**✓ 55**

**Hint 1:**
This is what the summation looks like expanded with each of its values plugged in.

**Hint 2:**
The sum of all of the numbers squared between and including 1 and 5 is 55.

(Problem ID: 22385) ALGEBRA_FIELD

No knowledge components have been assigned

This example is slightly different, but just as easy. It sums together every power of 2 between and including 2 and 4 - please evaluate this summation.

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ 28

**Hint 1:**

This is what the summation looks like expanded with each of its values plugged in.

**Hint 2:**
The sum of all powers of 2 between and including 2 and 4 is 28.

(Problem ID: 22386) ALGEBRA_FIELD

No knowledge components have been assigned

Finally we return to the original problem where our function is more complicated. The work may be more messy, but the idea is the same - plug each value of the summation into the function and then add all the parts together!

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ 260

**Hint 1:**
$$\sum_{i=3}^{5} (i-2)^i = (3-2)^3 + (4-2)^4 + (5-2)^5$$

This is what the summation looks like expanded with each of its values plugged in.

**Hint 2:**
The final answer is 260.

6.) "c2 vol problem 2s" (Problem ID: 22155) RADIO BUTTON

No knowledge components have been assigned

These three pictures illustrate three different Riemann Sum approximations of the area under the graph of $f(x)$. Of course, neither of them is a perfect approximation and so there is error in all three. If we were to take the limit of the each approximation as the number of boxes increased to infinity and as the width of these boxes decreased to zero, what would happen to their comparative error?

**Answers:** (Interface Type: RADIO_BUTTON)

✓ The error would decrease in all the approximations because as the bars become smaller and greater in number they all approach the true area.

✗ The error for the two approximations using rectangles would stay about the same but the approximation using trapezoids would approach the true area because it more directly mimics the slope of $f(x)$.

✗ The error would increase in all the approximations because there were more bars and each bar creates more error.

✗ The error would stay the same for each approximation because no matter how many bars there are, the same method is still being used.

(Problem ID: 22156) RADIO BUTTON

No knowledge components have been assigned

Since the size and number of the boxes are changing, we must think about how this relates to the size of the error. These three pictures illustrate the same rectangle method being used to approximate the area under the graph of $f(x)$. The only difference between them is the size and number of the rectangles. Which of them has the most error? Which of them has the least error?

**Answers:** (Interface Type: RADIO_BUTTON)
The largest bars have the most error and the smallest bars have the least error.

All of the bars have an equal amount of error because the same method is being used.

The smallest bars have the most error and the largest bars have the least error.

Hint 1:

Compare this graph of the true area under the graph of f(x) and an approximation and think about what the error is and what causes it.

Hint 2:

The largest bars have the most error and the smallest bars have the least error.

Returning to the original problem, if we were to take the limit of each approximation as the number of boxes increased to infinity and as the width of these boxes decreased to zero, what would happen to their comparative error?

**Answers:** (Interface Type: RADIO_BUTTON)

- The error would increase in all the approximations because there were more bars and each bar creates more error.
- The error for the two approximations using rectangles would stay about the same but the approximation using trapezoids would approach the true area because it more directly mimics the slope of f(x).
- The error would stay the same for each approximation because no matter how many bars there are, the same method is still being used.
- The error would decrease in all the approximations because as the bars become smaller and greater in number they all approach the true area.

7.) "c2 vol problem 3s" (Problem ID: 22158) ALGEBRA_FIELD

No knowledge components have been assigned
Find the area bounded between the functions $f(x)$ and $g(x)$ on the interval $0, 2$.

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ $8/3$

(Problem ID: 22159) ALGEBRA_FIELD

No knowledge components have been assigned

What is the area bounded between the functions $f(x)$ and $g(x)$ in terms of the area under $f(x)$ and the area under $g(x)$? (Use $F$ to represent the area under $f(x)$ and $G$ to represent the area under $g(x)$.)

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ $G-F$

Hint 1:
Use this image to help you find the relation between $G$ and $F$.

Hint 2:
The area of the bounded region is equal to the area under $g(x)$ minus the area under $f(x)$ or $G-F$.

(Problem ID: 22400) ALGEBRA_FIELD

No knowledge components have been assigned

What is the area under $g(x)$ or "$G$" equal to?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ $16/3$

Hint 1:
To find the area under $g(x)$, evaluate the integral of $g(x)$ between 0 and 2.

Hint 2:
The area under $g(x)$ is equal to $16/3$.

(Problem ID: 22401) ALGEBRA_FIELD

No knowledge components have been assigned

What is the area under $f(x)$ or "$F$" equal to?
Answers: (Interface Type: ALGEBRA_FIELD)

8/3

Hint 1:
To find the area under f(x), evaluate the integral of f(x) between 0 and 2.

Hint 2:
The area under f(x) is equal to 8/3.

(Problem ID: 22402) ALGEBRA_FIELD

No knowledge components have been assigned

Now subtract F from G to get the final answer. What is the final answer, the area bounded between f(x) and g(x)?

Answers: (Interface Type: ALGEBRA_FIELD)

8/3

8. "c2 vol problem 4s" (Problem ID: 22403) ALGEBRA_FIELD

No knowledge components have been assigned

Find the volume of the solid created when the area bounded between the functions f(x) and g(x) is rotated around the x-axis. Use the letter p for pi.

Answers: (Interface Type: ALGEBRA_FIELD)

48*p/5

(Problem ID: 22404) ALGEBRA_FIELD

No knowledge components have been assigned

Although we say "rotated around the x-axis" we do not actually have to perform any calculations that involve rotation or polar coordinates. Just as the area between two curves can be approximated by many small rectangles, the volume that is created by rotating that area can be approximated by many small cylinders.

And, just as these rectangles become infinitely thin (lines) when we use an integral (that is to say, a summation where the number of rectangles goes to infinity) these cylinders become infinitely thin (circles) when we use the integral.

So where we normally integrate a function (the length of the line at x) to find the area - we will integrate pi times the function squared (pi r squared is the area of a circle) to find the volume. Knowing this, set up the integral for the volume in terms of x. Use the letter p for pi.

Answers: (Interface Type: ALGEBRA_FIELD)

p*(8*x)-p*x^4
Hint 1:
Remember from the last problem that we must subtract the volume of f(x) from the volume of g(x) to get the volume between them.

Hint 2:
To get the volume we must integrate \( p(8x) - px^4 \)

(Problem ID: 22405) ALGEBRA_FIELD

Now evaluate the integral of \( p(8x) - px^4 \) dx on the interval (0, 2).

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ \( \frac{48p}{5} \)

Hint 1:
To evaluate a definite integral, subtract the integral evaluated at the lower bound (0) from the integral evaluated at the upper bound (2). Use the letter p for pi.

Hint 2:
The final answer is \( \frac{48p}{5} \).

(Problem ID: 22410) RADIO_BUTTON

**Answers:** (Interface Type: RADIO_BUTTON)

End Linear Section
End '{Problem}' Section
Module Worksheet

Module Name: Integration by Parts

Questions Only Answer Key Printout for Students Full Details

Begin '{ChooseOne}' Section
Begin Linear Section

1.) "c2 parts problem 1s" (Problem ID: 21033) ALGEBRA_FIELD

No knowledge components have been assigned

\[ \int x e^{8x} \, dx \]

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero and use $e^x$ for $e$ to the $x$.  

**Answers:** (Interface Type: ALGEBRA_FIELD)  
✓ $(x/8-1/64)*e^{(8*x)}$

(Problem ID: 21127) ALGEBRA_FIELD

No knowledge components have been assigned

\[ \int u \, dv = uv - \int v \, du \]

To complete this problem we'll need…

\[
\begin{align*}
  u = \ ? & & v = \ ? \\
  du = \ ? & & dv = \ ? \\
\end{align*}
\]

Keeping in mind the equation for Integration by Parts, we must first choose values for $u$ and $dv$. What value should be chosen for $u$?  

**Answers:** (Interface Type: ALGEBRA_FIELD)  
✓ $x$

Hint 1:  
The correct answer is 'x'. Please enter/select 'x' (without quotes).

(Problem ID: 21128) ALGEBRA_FIELD

No knowledge components have been assigned
Now that you have chosen $u$, what value should be chosen for $dv$?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ $e^{8x}dx$

- **Hint 1:**
  Make sure to include $dx$ in your answer.
- **Hint 2:**
  The correct answer is $e^{8x}dx$. Please enter/select $e^{8x}dx$ (without quotes).

(Problem ID: 21036) ALGEBRA_FIELD

No knowledge components have been assigned

To complete the equation, we must now solve for $du$ and $v$. What is $du$?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ $dx$

- **Hint 1:**
  Make sure to include $dx$ in your answer.
- **Hint 2:**
  The correct answer is $dx$. Please enter/select $dx$ (without quotes).

(Problem ID: 21037) ALGEBRA_FIELD

No knowledge components have been assigned
Now what is v?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ `e^(8*x)/8`

**Hint 1:**
The correct answer is 'e^(8*x)/8'. Please enter/select 'e^(8*x)/8' (without quotes).

(Problem ID: 21038) ALGEBRA_FIELD

No knowledge components have been assigned

---

Now we can express \(uv\) minus the integral of \(v \, du\) all in terms of \(x\), giving us our final answer. What is the final answer?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ `(x/8-1/64)*e^(8*x)`

**Hint 1:**
The correct answer is '(x/8-1/64)*e^(8*x)'. Please enter/select '(x/8-1/64)*e^(8*x)' (without quotes).
Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero.

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ \( \cos(9x)/81+x\sin(9x)/9 \)

(Problem ID: 21042) ALGEBRA_FIELD

| No knowledge components have been assigned |

\[
\int u \, dv = uv - \int v \, du
\]

To complete this problem we'll need…

\[
\begin{align*}
  u &= ? \\
  v &= ? \\
  du &= ? \\
  dv &= ? \\
\end{align*}
\]

Keeping in mind the equation for Integration by Parts, we must first choose values for \( u \) and \( dv \).

What value should be chosen for \( u \)?

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ \( x \)

Hint 1:
The correct answer is 'x'. Please enter/select 'x' (without quotes).

(Problem ID: 21043) ALGEBRA_FIELD

| No knowledge components have been assigned |

\[
\int u \, dv = uv - \int v \, du
\]

To complete this problem we'll need…

\[
\begin{align*}
  u &= x \\
  v &= ? \\
  du &= ? \\
  dv &= ? \\
\end{align*}
\]

Now that you have chosen \( u \), what value should be chosen for \( dv \)?

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ \( \cos(9x) \, dx \)

Hint 1:
Make sure to include \( dx \) in your answer.

Hint 2:
The correct answer is '\( \cos(9x) \, dx \)'. Please enter/select '\( \cos(9x) \, dx \)' (without quotes).
To complete the equation, we must now solve for du and v. What is du?

**Answers:** (Interface Type: ALGEBRA_FIELD)

- **dx**
  - Hint 1: Make sure to include dx in your answer.
  - Hint 2: The correct answer is 'dx'. Please enter/select 'dx' (without quotes).

Now what is v?

**Answers:** (Interface Type: ALGEBRA_FIELD)

- **sin(9*x)/9**
  - Hint 1: The correct answer is 'sin(9*x)/9'. Please enter/select 'sin(9*x)/9' (without quotes).
Having found all four values, we can reduce the problem to \( uv - \int v \, du \). After evaluating the integral we will have the answer. What is the integral of \( v \, du \) solved in terms of \( x \)?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ \(-\cos(9x)/81\)

<table>
<thead>
<tr>
<th>Hint 1:</th>
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<tbody>
<tr>
<td>The correct answer is (-\cos(9x)/81). Please enter/select (-\cos(9x)/81) (without quotes).</td>
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</tbody>
</table>

(Problem ID: 21047) ALGEBRA_FIELD

No knowledge components have been assigned

Now we can express \( uv - \int v \, du \) all in terms of \( x \), giving us our final answer. What is the final answer?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ \(\cos(9x)/81 + x\sin(9x)/9\)

<table>
<thead>
<tr>
<th>Hint 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The correct answer is (\cos(9x)/81 + x\sin(9x)/9). Please enter/select (\cos(9x)/81 + x\sin(9x)/9) (without quotes).</td>
</tr>
</tbody>
</table>

3. ) "c2 parts problem 3s" (Problem ID: 21048) ALGEBRA_FIELD

No knowledge components have been assigned

\[ \int \ln|7x| \, dx \]

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero.

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ \(x\ln(7x) - x\)

(Problem ID: 21049) ALGEBRA_FIELD

No knowledge components have been assigned
Keeping in mind the equation for Integration by Parts, we must first choose values for \( u \) and \( dv \). What value should be chosen for \( u \)?

**Answers:** (Interface Type: ALGEBRA_FIELD)

- **\( \ln(7x) \)**
  - Hint 1:
    - The correct answer is '\( \ln(7x) \)'. Please enter/select '\( \ln(7x) \)' (without quotes).

(Problem ID: 21050) ALGEBRA_FIELD

No knowledge components have been assigned

Now that you have chosen \( u \), what value should be chosen for \( dv \)?

**Answers:** (Interface Type: ALGEBRA_FIELD)

- **\( dx \)**
  - Hint 1:
    - Make sure to include \( dx \) in your answer.
  - Hint 2:
    - The correct answer is '\( dx \)'. Please enter/select '\( dx \)' (without quotes).

(Problem ID: 21051) ALGEBRA_FIELD

No knowledge components have been assigned
To complete the equation, we must now solve for \( du \) and \( v \). What is \( du \)?

**Answers:** (Interface Type: ALGEBRA_FIELD)

\( \checkmark \) \( \frac{dx}{x} \)

- **Hint 1:**
  Make sure to include \( dx \) in your answer.
- **Hint 2:**
  The correct answer is '\( \frac{dx}{x} \)'. Please enter/select '\( \frac{dx}{x} \)' (without quotes).

(Problem ID: 21052) ALGEBRA_FIELD

| No knowledge components have been assigned |

Now what is \( v \)?

**Answers:** (Interface Type: ALGEBRA_FIELD)

\( \checkmark \) \( x \)

- **Hint 1:**
  The correct answer is '\( x \)'. Please enter/select '\( x \)' (without quotes).

(Problem ID: 21053) ALGEBRA_FIELD

| No knowledge components have been assigned |
Having found all four values, we can reduce the problem to $uv$ minus the integral of $v \, du$. After evaluating the integral we will have the answer. What is the integral of $v \, du$ solved in terms of $x$?

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ $x$

**Hint 1**: The correct answer is 'x'. Please enter/select 'x' (without quotes).

Now we can express $uv$ minus the integral of $v \, du$ all in terms of $x$, giving us our final answer. What is the final answer?

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ $x*\ln(7*x)-x$

**Hint 1**: The correct answer is 'x*ln(7*x)-x'. Please enter/select 'x*ln(7*x)-x' (without quotes).

4.) "c2 parts problem 4s"  (Problem ID: 21055) ALGEBRA_FIELD

No knowledge components have been assigned

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero and use $e^x$ for $e$ to the $x$.

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ $(x^2/8-x/32+1/256)*e^(8*x)$

(Problem ID: 21056) ALGEBRA_FIELD

No knowledge components have been assigned
Keeping in mind the equation for Integration by Parts, we must first choose values for u and dv.

What value should be chosen for u?

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ **x^2**

<table>
<thead>
<tr>
<th align="left">Hint 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td align="left">The correct answer is 'x^2'. Please enter/select 'x^2' (without quotes).</td>
</tr>
</tbody>
</table>

(Problem ID: 21057) ALGEBRA_FIELD

No knowledge components have been assigned

that you have chosen u, what value should be chosen for dv?

**Answers**: (Interface Type: ALGEBRA_FIELD)

✓ **e^(8*x)*dx**

<table>
<thead>
<tr>
<th align="left">Hint 1:</th>
</tr>
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<tbody>
<tr>
<td align="left">Make sure to include dx in your answer.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th align="left">Hint 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td align="left">The correct answer is 'e^(8*x)<em>dx'. Please enter/select 'e^(8</em>x)*dx' (without quotes).</td>
</tr>
</tbody>
</table>

(Problem ID: 21058) ALGEBRA_FIELD

No knowledge components have been assigned
To complete the equation, we must now solve for du and v. What is du?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ 2*x*dx

- **Hint 1:**
  - Make sure to include dx in your answer.
- **Hint 2:**
  - The correct answer is '2*x*dx'. Please enter/select '2*x*dx' (without quotes).

(Problem ID: 21059) ALGEBRA_FIELD

| No knowledge components have been assigned |

Now what is v?

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ e^(8*x)/8

- **Hint 1:**
  - The correct answer is 'e^(8*x)/8'. Please enter/select 'e^(8*x)/8' (without quotes).

(Problem ID: 21060) ALGEBRA_FIELD

| No knowledge components have been assigned |
Having found all four values, we can reduce the problem to \( uv - \int v \, du \). After evaluating the integral we will have the answer. What is the integral of \( v \, du \) solved in terms of \( x \)?

**Answers:** (Interface Type: ALGEBRA_FIELD)

\( x^2e^{8x}/32 - e^{8x}/256 \)

**Hint 1:**

The correct answer is \('x^2e^{8x}/32 - e^{8x}/256'\). Please enter/select \('x^2e^{8x}/32 - e^{8x}/256'\) (without quotes).

(Problem ID: 21061) ALGEBRA_FIELD

Now we can express \( uv - \int v \, du \) all in terms of \( x \), giving us our final answer. What is the final answer?

**Answers:** (Interface Type: ALGEBRA_FIELD)

\( (x^2/8 - x/32 + 1/256)e^{8x} \)

**Hint 1:**

The correct answer is \('(x^2/8 - x/32 + 1/256)e^{8x}'\). Please enter/select \'(x^2/8 - x/32 + 1/256)e^{8x}'\) (without quotes).

---

5.) "c2 parts problem 1h"  (Problem ID: 21005) ALGEBRA_FIELD

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero and use \( e^x \) for \( e \) to the \( x \).

**Answers:** (Interface Type: ALGEBRA_FIELD)

\( x/8 - 1/64 \)

**Hint 1:**
Keeping in mind the equation for Integration by Parts, we must first choose values for u and dv. What value should be chosen for u?

**Hint 2:**
The value for u is x.

**Hint 3:**

Now that you have chosen u, what value should be chosen for dv? Make sure to include dx in your answer.

**Hint 4:**
The value for dv is $e^{8x}dx$.

**Hint 5:**

To complete the equation, we must now solve for du and v. What is du? Make sure to include dx in your answer.

**Hint 6:**
The value for du is dx.

**Hint 7:**
\[ \int u \, dv = uv - \int v \, du \]

To complete this problem we'll need...

\[
\begin{align*}
  u &= x \\
  v &= \, ? \\
  du &= dx \\
  dv &= e^{8x} \, dx
\end{align*}
\]

Now what is \( v \)?

**Hint 8:**
\( e^{(8*x)/8} \)

**Hint 9:**
\[ e^{(8x)/64} \]

Having found all four values, we can reduce the problem to \( uv \) minus the integral of \( v \, du \). After evaluating the integral we will have the answer. What is the integral of \( v \, du \) solved in terms of \( x \)?

**Hint 10:**
\( (x/8-1/64)*e^{(8*x)} \)

**Hint 11:**
Now we can express \( uv \) minus the integral of \( v \, du \) all in terms of \( x \), giving us our final answer. What is the final answer?

**Hint 12:**
The correct answer is '(x/8-1/64)*e^(8*x)'. Please enter/select '(x/8-1/64)*e^(8*x)' (without quotes).

---

6.) "c2 parts problem 2h" (Problem ID: 21062) ALGEBRA_FIELD

No knowledge components have been assigned

\[ \int x \cos(9x) \, dx \]

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero.

**Answers:** (Interface Type: ALGEBRA_FIELD)

✓ \( \cos(9*x)/81+x*\sin(9*x)/9 \)

**Hint 1:**
Keeping in mind the equation for Integration by Parts, we must first choose values for u and dv. What value should be chosen for u?

**Hint 2:**
The value for u is x.

**Hint 3:**

Now that you have chosen u, what value should be chosen for dv? Make sure to include dx in your answer.

**Hint 4:**
The value for dv is cos(9x)*dx.

**Hint 5:**

To complete the equation, we must now solve for du and v. What is du? Make sure to include dx in your answer.

**Hint 6:**
The value for du is dx.

**Hint 7:**
\[ \int u \, dv = uv - \int v \, du \]

To complete this problem we'll need...

\[ \begin{align*}
u &= x & v &= \sin(9x)/9 \\
du &= dx & dv &= \cos(9x) \, dx
\end{align*} \]

Now what is \( v \)?

**Hint 8:**
The value for \( v \) is \( \sin(9x)/9 \)

**Hint 9:**

\[ \int u \, dv = uv - \int v \, du \]

To complete this problem we'll need...

\[ \begin{align*}
u &= x & v &= \frac{\sin(9x)}{9} \\
du &= dx & dv &= \cos(9x) \, dx
\end{align*} \]

Having found all four values, we can reduce the problem to \( uv \) minus the integral of \( v \, du \). After evaluating the integral we will have the answer. What is the integral of \( v \, du \) solved in terms of \( x \)?

**Hint 10:**
- \( \cos(9x)/81 \)

**Hint 11:**
Now we can express \( uv \) minus the integral of \( v \, du \) all in terms of \( x \), giving us our final answer. What is the final answer?

**Hint 12:**
The correct answer is '\( \cos(9x)/81 + x \cdot \sin(9x)/9 \)'. Please enter/select '\( \cos(9x)/81 + x \cdot \sin(9x)/9 \)' (without quotes).

---

7.) "c2 parts problem 3h"  (Problem ID: 21063) **ALGEBRA_FIELD**

No knowledge components have been assigned

\[ \int |x| \ln(7x) \, dx \]

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero.

**Answers:** (Interface Type: ALGEBRA_FIELD)

\[ \checkmark x \ln(7x) - x \]

**Hint 1:**
Keeping in mind the equation for Integration by Parts, we must first choose values for $u$ and $dv$. What value should be chosen for $u$?

**Hint 2:**
The value for $u$ is $\ln(7x)$.

**Hint 3:**

Now that you have chosen $u$, what value should be chosen for $dv$? Make sure to include $dx$ in your answer.

**Hint 4:**
The value for $dv$ is $dx$.

**Hint 5:**

To complete the equation, we must now solve for $du$ and $v$. What is $du$? Make sure to include $dx$ in your answer.

**Hint 6:**
The value for $du$ is $dx/x$.

**Hint 7:**
\[ \int u \, dv = uv - \int v \, du \]

To complete this problem we’ll need...

\[ u = \ln |7x| \quad v = ? \]
\[ du = \frac{dx}{x} \quad dv = dx \]

Now what is \( v \)?

**Hint 8:**

The value for \( v \) is \( x \).

**Hint 9:**

\[ \int u \, dv = uv - \int v \, du \]

To complete this problem we’ll need...

\[ u = \ln |7x| \quad v = x \]
\[ du = \frac{dx}{x} \quad dv = dx \]

Having found all four values, we can reduce the problem to \( uv \) minus the integral of \( v \, du \). After evaluating the integral we will have the answer. What is the integral of \( v \, du \) solved in terms of \( x \)?

**Hint 10:**

\( x \)

**Hint 11:**

Now we can express \( uv \) minus the integral of \( v \, du \) all in terms of \( x \), giving us our final answer. What is the final answer?

**Hint 12:**

The correct answer is '\( x*\ln(7^x)-x \)'. Please enter/select '\( x*\ln(7^x)-x \)' (without quotes).

---

**8.) "c2 parts problem 4h" (Problem ID: 21064) ALGEBRA_FIELD**

No knowledge components have been assigned

\[ \int x^2 e^{8x} \, dx \]

Please evaluate the above integral. Assume the arbitrary constant of integration to equal zero and use \( e^x \) for \( e \) to the \( x \).

**Answers:** (Interface Type: ALGEBRA_FIELD)

\( (x^2/8-x/32+1/256)*e^{(8*x)} \)

**Hint 1:**
Keeping in mind the equation for Integration by Parts, we must first choose values for u and dv. What value should be chosen for u?

Hint 2:
The value for u is $x^2$.

Hint 3:

Now that you have chosen u, what value should be chosen for dv? Make sure to include dx in your answer.

Hint 4:
The value for dv is $e^{8x} * dx$

Hint 5:

To complete the equation, we must now solve for du and v. What is du? Make sure to include dx in your answer.

Hint 6:
The value for $du$ is $2x * dx$.

Hint 7:
Now what is v?

Hint 8:
The value for v is $e^{(8\cdot x)}/8$.

Hint 9:

Having found all four values, we can reduce the problem to $uv$ minus the integral of $v \, du$. After evaluating the integral we will have the answer. What is the integral of $v \, du$ solved in terms of $x$?

Hint 10:
$e^{(8\cdot x)}/32-e^{(8\cdot x)}/256$

Hint 11:
Now we can express $uv$ minus the integral of $v \, du$ all in terms of $x$, giving us our final answer. What is the final answer?

Hint 12:
The correct answer is $(x^2/8-x/32+1/256)e^{(8\cdot x)}$. Please enter/select $(x^2/8-x/32+1/256)e^{(8\cdot x)}$ (without quotes).
Appendix B

We distributed 7 packets in total; 1 for preliminaries, 3 for the Volumes of Rotation Experiment, and 3 for the Integration by Parts Experiment.

Out of three days, 2 students participated only the first day, 1 student participated only the third day, 2 students participated only the first and second days, 1 student participated only the first and third days, 2 students participated only the second and third days, and 19 students participated all 3 days.
### Introduction to Assimilations

#### Section One: Questionnaire

Please answer the following questions regarding your past experiences with calculus.

1. Have you taken this course before? [Yes, No]
2. Did you take any calculus in high school? [Pre Calculus, AP A/B, AP B/C, No]
3. If you took the AP Exam, what did you get? [1, 2, 3, 4, 5]

Please rate how well each of the following statements describes you where 1 refers to a statement that does not describe you at all and 5 refers to a statement that describes you very well.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I believe that if I work hard at calculus, I can do well.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>2. When doing calculus problems, I always use scrap paper instead of doing work in my head.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>3. I understand calculus well.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>4. I do my homework during calculus lectures.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>5. I prefer classes that emphasize facts and data over concepts and ideas.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>6. I am creative in the way I do my work.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>7. I enjoy calculus class.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>8. Breaking a question down into smaller parts helps me to understand how to solve similar problems.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>9. I think that some people are just good at math.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>10. I am careful about the details of my work.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>11. I work hard on calculus.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>12. I care a great deal about how I do in calculus.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>13. I find it easier to learn concepts rather than facts.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>14. I usually seek help when I don’t understand something.</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>
Section 2: Signing onto Assistments

Before you can login to Assistments you must sign up for an account. The following steps are explained to help you create a new account. You should write down your login name and password so you do not forget them!

**Step 1:** Open a web browser and go to [http://www.assistment.org](http://www.assistment.org) and then click on the “Click here for main server” link.

**Step 2:** Click on the “New Student Account” link below the sign-in box.

**Step 3:** Fill in the fields with your first, middle, and last name. Use “WPI” in all capital letters without quotes for your School Code.

**Step 4:** Select a password and your date of birth. You will have to select the exact day again if you change the month or year. Leave the other fields blank.
Step 5: After entering the information, you will be shown your username and password. Write this down so you don’t forget.

Step 6: Now you can sign into the system. Use “WPI” as your School Identifier and the username and password you just created.

Step 7: After logging in you will be asked to join a class. Select “Mr. Krach” and then select “Stratton Hall 306 – Lab”.

Step 8: After joining the class, click the blue hyper link labeled “Stratton Hall 306 – Lab” to continue.

Step 9: Finally, click the link to “start work” that is to the right of the title “sample problems”.


Volumes of Rotation Lab: Pre-Test

Please evaluate the following items related to volumes of rotation and circle whether or not you have had previous experience with similar problems. Please show all your work!

**Problem 1:** __________________________ (I have / have not seen summations before)

Evaluate the following summation.

\[
\sum_{i=4}^{5} (i-2)^i
\]

**Problem 2:** __________________________ (I have / have not seen Riemann Sums before)

These three pictures illustrate three different Riemann Sum approximations of the area under the graph of f(x). Of course, neither of them is a perfect approximation and so there is error in all three. If we were to take the limit of the each approximation as the number of boxes increased to infinity and as the width of these boxes decreased to zero, what would happen to their comparative error?

Choose One:

A. The error would increase in all the approximations because there were more bars and each bar creates more error.
B. The error would stay the same for each approximation because no matter how many bars there are, the same method is still being used.
C. The error would decrease in all the approximations because as the bars become smaller and greater in number they all approach the true area.
D. The error for the two approximations using rectangles would stay about the same but the approximation using trapezoids would approach the true area because it more directly mimics the slope of f(x).
Problem 3: ________________________ (I have / have not seen area under a curve before)

Find the area bounded between the functions \( f(x) \) and \( g(x) \) on the interval \((0, 3)\).

\[
\begin{align*}
f(x) & = x^2 \\
g(x) & = \sqrt{27x}
\end{align*}
\]

Problem 4: ________________________ (I have / have not seen volumes of rotation before)

Find the volume of the solid created when the area bounded between the functions \( f(x) \) and \( g(x) \) from Problem 3 is rotated around the x-axis.
Volumes of Rotation Lab: Assiistments Tutoring

Please login to http://www.assistent.com using the user name and password you created last week (with “WPI” as your school code). Once logged in please do the assigned problems using the following space to show your work.

Problem 1: __________________________

Problem 2: __________________________

Problem 3: __________________________

Problem 4: __________________________
**Volumes of Rotation Lab: Post-Test**

**Part One: Questionnaire**

Please rate how well each of the following statements describes you where 1 refers to a statement that does not describe you at all and 5 refers to a statement that describes you very well.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I tried to get through difficult problems as quickly as possible.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>2. I liked today’s lab using the Ass instments System better than doing homework.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>3. My goal during today’s lab was to learn new things.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>4. When a problem was too hard I would just try to get through so that I could get to easier problems.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>5. I liked today’s lab using the Ass instments System better than attending lecture.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>6. I took today’s lab very seriously and worked hard all the time.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>7. I found it hard to stay concentrated all the time.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>8. I think the tutoring helped me understand similar problems.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>9. I found many of the problems frustrating because they were too hard.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>10. I liked today’s lab using the Ass instments System better than normal labs.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>11. My goal during today’s lab was to get through as many problems as possible.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>12. I liked using the web tutoring from the Ass instments System.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>13. I think the tutoring will help me on tests in class.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>14. I liked today’s lab using the Ass instments System better than doing a test.</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>
Part Two: Problems

Almost done! Please evaluate the following items so we can see if and what you have learned. As usual, please show all your work!

Problem 1: ________________________

Evaluate the following summation.

\[ \sum_{i=3}^{4} (i-2)^i \]

Problem 2: ________________________

These three pictures illustrate three different Riemann Sum approximations of the area under the graph of \( f(x) \). Of course, neither of them is a perfect approximation and so there is error in all three. If we were to take the limit of the each approximation as the number of boxes increased to infinity and as the width of these boxes decreased to zero, what would happen to their comparative error?

Choose One:

E. The error would increase in all the approximations because there were more bars and each bar creates more error.
F. The error would stay the same for each approximation because no matter how many bars there are, the same method is still being used.
G. The error would decrease in all the approximations because as the bars become smaller and greater in number they all approach the true area.
H. The error for the two approximations using rectangles would stay about the same but the approximation using trapezoids would approach the true area because it more directly mimics the slope of \( f(x) \).
Problem 3: __________________________

Find the area bounded between the functions \( f(x) \) and \( g(x) \) on the interval \((0, 4)\).

\[
\begin{align*}
f(x) &= -x^2 \\
g(x) &= \sqrt{64x}
\end{align*}
\]

Problem 4: __________________________

Find the volume of the solid created when the area bounded between the functions \( f(x) \) and \( g(x) \) from Problem 3 is rotated around the \( x \)-axis.

Problem 5: ____________________________ (I have / have not used the “shells” method before)

Find the volume of the solid created when the area bounded between the functions \( f(x) \) and \( g(x) \) from Problem 3 is rotated around the \( y \)-axis. In problem 4, you used horizontal cylinders to slice the solid, making use of the equation: area of a circle equals \( \pi \) radius squared. This time use vertical cylinders to divide the solid into shells, making use of the equation: volume of a cylindrical shell (like a pipe) equals \( 2 \pi \) (average radius) (height of the shell) (thickness of the shell). So where before for a function like \( h(x) \) you used \( \pi h(x) \) squared, now use \( 2 \pi x h(x) \).
Integration by Parts Lab: Pre-Test

Please evaluate the following items related to Integration by Parts. Please show all your work! (If you need more space, use the back of this page.)

(I have / have not seen integration by parts before)

Problem 0: ______________

What is the equation for Integration by Parts?

Problem 1: ______________

\[ \int x e^{5x} \, dx \]

Problem 2: ______________

\[ \int x \cos(7x) \, dx \]

Problem 3: ______________

\[ \int \ln|3x| \, dx \]

Problem 4: ______________

\[ \int x^2 e^{5x} \, dx \]
Name: ________________________ Lab Section: ______

Integration by Parts Lab: Asssitments Tutoring

Please login to http://www.assitment.org using the user name and password you created (with "WPI" as your school code). Once logged in please do the assigned problems using the following space to show your work. (If you need more space, use the back of this page.)

Problem 1: _________________________

Problem 2: _________________________

Problem 3: _________________________

Problem 4: _________________________
# Integration by Parts Lab: Post-Test

## Part One: Questionnaire

Please rate how well each of the following statements describes you where 1 refers to a statement that does not describe you at all and 5 refers to a statement that describes you very well.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I tried to get through difficult problems as quickly as possible.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>2. I liked today's lab using the Assisments System better than doing homework.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>3. My goal during today's lab was to learn new things.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>4. When a problem was too hard I would just try to get through so that I could get to easier problems.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>5. I liked today's lab using the Assisments System better than attending lecture.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>6. I took today's lab very seriously and worked hard all the time.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>7. I found it hard to stay concentrated all the time.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>8. I think the tutoring helped me understand similar problems.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>9. I found many of the problems frustrating because they were too hard.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>10. I liked today's lab using the Assisments System better than normal labs.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>11. My goal during today's lab was to get through as many problems as possible.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>12. I preferred today's lab with Assisments over last week's lab with Assisments.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>13. I think the tutoring will help me on tests in class.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>14. I liked today's lab using the Assisments System better than doing a test.</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>
Part Two: Problems
Almost done! Please evaluate the following items so we can see if and what you have learned. As usual, please show all your work! (If you need more space, use the back of this page.)

Problem 0: ______________________

What is the equation for Integration by Parts?

Problem 1: ______________________

\[ \int x e^{7x} \, dx \]

Problem 2: ______________________

\[ \int x \cos(5x) \, dx \]

Problem 3: ______________________

\[ \int \ln|4x| \, dx \]

Problem 4: ______________________

\[ \int x^2 e^{7x} \, dx \]
### Appendix C

#### C.1 Overall Learning

**Paired Samples Statistics for Overall Learning by Gain Score**

<table>
<thead>
<tr>
<th>Pair</th>
<th>Measure</th>
<th>Mean</th>
<th>N</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vol Pretest Sum</td>
<td>1.52</td>
<td>23</td>
<td>.790</td>
<td>.165</td>
</tr>
<tr>
<td></td>
<td>Vol Posttest Sum</td>
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<td>23</td>
<td>.783</td>
<td>.163</td>
</tr>
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<td>23</td>
<td>.388</td>
<td>.081</td>
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<td>23</td>
<td>.507</td>
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<td>.000</td>
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<td>.060</td>
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<td>.351</td>
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<td>Parts Posttest Question 2</td>
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<td>22</td>
<td>.492</td>
<td>.105</td>
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<td>10</td>
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<td>22</td>
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<td></td>
<td>Parts Posttest Question 3</td>
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<td>22</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Parts Posttest Question 4</td>
<td>.05</td>
<td>22</td>
<td>.213</td>
<td>.045</td>
</tr>
</tbody>
</table>
## Paired Samples Test for Overall Learning by Gain Score

<table>
<thead>
<tr>
<th>Pair</th>
<th>Paired Differences</th>
<th>95% Confidence Interval of the Difference</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Deviation</td>
<td>Std. Error Mean</td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>1 Vol Pretest Sum - Vol Posttest Sum</td>
<td>-.870</td>
<td>.920</td>
<td>.192</td>
<td>-1.267</td>
<td>-.472</td>
</tr>
<tr>
<td>2 Vol Pretest Question 1 - Vol Posttest Question 1</td>
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<td>.487</td>
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<td>3 Vol Pretest Question 2 - Vol Posttest Question 2</td>
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<td>4 Vol Pretest Question 3 - Vol Posttest Question 3</td>
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<td>6 Parts Pretest Sum - Parts Posttest Sum</td>
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<td>1.032</td>
<td>.220</td>
<td>-2.185</td>
<td>-1.270</td>
</tr>
<tr>
<td>7 Parts Pretest Question 0 - Parts Posttest Question 0</td>
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<td>.598</td>
<td>.127</td>
<td>-.765</td>
<td>-.235</td>
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*a* t cannot be computed because the standard deviation is 0.

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## C.2 Learning by Condition

Group Statistics for Volume Experiment by Condition

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a t cannot be computed because the standard deviations of both groups are 0.
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## Independent Samples Test for Parts Experiment by Condition

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C.3 Effects of Prior Coursework

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Independent Samples Test for Effects of Prior Coursework

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### C.4 Preparatory Learning Questionnaire

#### Correlations

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* Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).
## C.5 Posttest Learning Questionnaire

### Correlations for Volume Experiment Posttest Learning Questionnaire and Gain Score

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|                                | Vol Pre/Post Gain |          |          |
| Vol Poor Concentration         | Pearson Correlation | .256     |          |          |
|                                | Sig. (2-tailed)    | .250     |          |          |
|                                | N                  | .22      |          |          |
| Vol Similar Problems           | Pearson Correlation | -.004    |          |          |
|                                | Sig. (2-tailed)    | .988     |          |          |
|                                | N                  | .22      |          |          |
| Vol Frustratingly Hard         | Pearson Correlation | -.180    |          |          |
|                                | Sig. (2-tailed)    | .424     |          |          |
|                                | N                  | .22      |          |          |
| Vol Assistsments Over Lab      | Pearson Correlation | .028     |          |          |
|                                | Sig. (2-tailed)    | .901     |          |          |
|                                | N                  | .22      |          |          |
| Vol As Many As Possible        | Pearson Correlation | -.166    |          |          |
|                                | Sig. (2-tailed)    | .462     |          |          |
|                                | N                  | .22      |          |          |
| Vol Enjoyed Web Tutoring       | Pearson Correlation | -.100    |          |          |
|                                | Sig. (2-tailed)    | .658     |          |          |
|                                | N                  | .22      |          |          |
| Vol Tutoring Helps Tests       | Pearson Correlation | -.116    |          |          |
|                                | Sig. (2-tailed)    | .616     |          |          |
|                                | N                  | .21      |          |          |
| Vol Lab Over Tests             | Pearson Correlation | .038     |          |          |
|                                | Sig. (2-tailed)    | .864     |          |          |
|                                | N                  | .23      |          |          |

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).
## Correlations for Parts Experiment Posttest Learning Questionnaire by Gain Score

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** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).
## C.6 Post Hoc Rating of Note Detail

### Correlations for Note Detail & Parts Gain Score

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*Correlation is significant at the 0.05 level (2-tailed).*