Pulsed Laser Holography in Studies of Bone Motions and Deformations

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Pulsed laser holography in studies of bone motions and deformations

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Abstract. Principles of pulsed laser double-exposure hologram interferometry are discussed. In this presentation, emphasis is placed on biomedical applications of the method. Procedures for quantitative interpretation of images obtained during reconstruction of holograms are presented and illustrated with representative examples. The procedures for quantitative interpretation of holograms allow determination of translations, rotations, and strains of an object, including determination of and compensation for the object's rigid-body motions. A system for automated interpretation of holograms is also discussed. The results shown relate to in vitro and in vivo studies of tooth and bone motions.

Subject terms: holographic interferometry; pulsed lasers; double-exposure hologram interferometry; quantitative interpretation of holograms; translations; rotations; strains; automated interpretation of holograms; biomedical applications; tooth motions; bone motions.


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1. INTRODUCTION

Since its invention, hologram interferometry has found numerous applications in various branches of science and technology, including the field of biomedicine.

In biomedicine, it is often desired to accurately measure three-dimensional changes and/or motions of tissues and bones and to correlate them to corresponding biological changes. In addition to the above, modern biomedicine requires (1) studies of bone growth, (2) accurate determination of the effect of thermal changes on differential expansion and contraction of dissimilar materials, (3) studies of the effect that cavity preparation has on stress distribution in restorative materials, (4) studies of performance characteristics (in three-dimensional space) of artificial joints, (5) detection of micro-fractures, e.g., in cavity preparation or in artificial joints, (6) in vitro and in vivo contouring of surfaces to assure their proper mating, and (7) measurement of surface roughness to evaluate abrasion and wear of materials, just to name a few of the constantly occurring problems.

Because of their inherent limitations, these conventional techniques did not always provide all the information necessary for development of meaningful relationships between the cause and the effect. However, recent advances in the field of laser technology have permitted development of new, laser-based methods. Of the laser-based methods, hologram interferometry seems to be most popular because it allows quantitative, noninvasive, in vivo studies of interest in the field of biomedicine.

Hologram interferometry is particularly suitable for applications in biomedicine because it provides rapid, full field-of-view results in three-dimensional space. Also, the accuracy and precision achievable with this method are not obtainable with any other methods known today. The modern hologram interferometry method allows measurement of displacements to within 0.001 µm and strains to within 0.2 µm/m.1

Out of the variety of existing holographic procedures, the most suitable for biomedical applications is the double-exposure method. In this method, two consecutive positions of an object are recorded in the same recording medium, with the object being displaced and/or deformed between the two exposures. Upon reconstruction of the hologram, two three-dimensional images of the object are formed. Since both images appear in coherent (laser) light and exist in approximately the same location in space, they interfere with each

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other and produce fringes overlaying the reconstructed image. All information about the object’s motion and/or deformation can be determined from this fringe pattern.

Double-exposure holograms can be recorded with continuous-wave (cw) lasers or with pulsed lasers. Although cw holographic systems can be used to record images of resonating objects, they are usually limited, in biomedical applications, to in vitro (static) loadings. On the other hand, pulsed laser holographic systems can be used for biomedical studies in vivo. This paper presents the pulsed laser double-exposure method of hologram interferometry.

2. PULSED LASER DOUBLE-EXPOSURE HOLOGRAM INTERFEROMETRY

The pulsed laser system for recording and reconstruction of double-exposure holograms is shown in Fig. 1. In this setup, the highly coherent and monochromatic light from a pulsed laser is divided into two parts by means of a beam splitter. One of these parts, going directly through the beam splitter, is directed by a mirror and expanded by a lens to illuminate a subject (or scene) to be recorded. This part of the laser output is known as the object beam. The other part of the laser beam, that is, the one reflected from the beam splitter, is expanded and steered toward a high resolution photographic plate with any interference with the “object.” This beam provides a reference beam against which the light modulated by the object is “compared” when they are combined at the photographic plate. The superposition of the object and reference beams results in an interference pattern that is recorded by the high resolution recording medium.

The exposed high resolution recording medium, upon photographic processing, becomes a hologram. The hologram can be used to reconstruct a three-dimensional image of the original object. During this process, the hologram is, most conveniently, illuminated with the original reference beam. The image produced during reconstruction of a hologram is seen covered with a fringe pattern that uniquely relates to motions and/or deformations that the object might have experienced during recording of the hologram. Some of the procedures for quantitative interpretation of holograms are outlined in Sec. 3.

3. QUANTITATIVE INTERPRETATION OF HOLOGRAMS

In biomedical applications of hologram interferometry, it is often desired to investigate objects for which the entire surface has moved and/or deformed. In such applications, it is not possible to determine true fringe orders, and, therefore, conventional procedures of hologram interpretation do not apply because these procedures are based on identification of the zero-order fringe. As a result of this, mathematical procedures for interpretation of fringe patterns where the zero-order fringe is present are simpler than those where the zero-order fringe is not identifiable. This latter case, however, is what usually takes place during biomedical applications of hologram interferometry.

A method for determination of displacements from holograms is presented in Sec. 3.1; Sec. 3.2 deals with determination of strains and rotations, while determination of and compensation for rigid-body motions are discussed in Sec. 3.3.

3.1. Determination of displacements

To obtain quantitative results from fringe patterns produced during reconstruction of holograms, parameters characterizing recording and reconstruction geometry must be determined. This geometry is specified by identifying a point source from which the object is illuminated, point(s) of interest on the object, and direction(s) of observation along which the reconstructed image is viewed (Fig. 2).

Very commonly, these parameters are determined by making only one observation of the holographically reconstructed image, assuming that the motion of the object was in the plane normal to the direction of observation. This approach might generally work in laboratory with inanimate objects where the experimenter can carefully control test conditions. However, even in this in vitro case, the resulting accuracy of the measurements obtained from one observation only is questionable due to errors in determination of various experimental parameters. Also, attempts to measure uniplanar motions from holograms and then to use these motions to determine three-dimensional characteristics of biomedical systems are fallacious because of nonlinear responses of “tissues” to forces acting upon them. On the other hand, use of multiple observations of holographically reconstructed images (Fig. 3) leads to accurate and precise determination of object motions.

In biomedical applications, one is interested in measuring motions, changes under load, growth patterns, etc., which are all three-dimensional in nature. As such, parameters quantifying them can be best described as vectors. Since vectors are completely defined by specifying their three components, then, working with holography, one is usually dealing with three unknowns. These unknowns
are the three components of the vector defining the quantity of interest. Using vector calculus, governing equations of hologram interferometry can be written in such a way that one equation will relate one component of the unknown vector to parameters characterizing fringe patterns as determined from one direction of observation. Thus, ideally, three equations, that is, one for each of the three observations, should be sufficient to quantitatively interpret holograms. In principle the procedure works. However, in practice the results are often subjected to large uncertainties because of experimental errors in the determination of parameters used in the computations. To overcome the above difficulty, a method of interpretation of holograms based on multiple observations was developed, resulting in an overdetermined system of equations, that is, a system having more equations than the number of unknowns. It was also shown that as the number of observations increases so does the accuracy of the corresponding results.

In the method of multiple observations of a hologram, the scalar product of the object's vectorial displacement \( \mathbf{L} \) with the sensitivity vector \( \mathbf{K} \) is related to the fringe order \( n \) by the equation

\[
\mathbf{K} \cdot \mathbf{L} = 2\pi n = \Omega_0 + \Omega,
\]

where \( \Omega \) is the fringe-locus function, constant values of which define fringe loci on the surface of the object, while \( \Omega_0 \) is a constant to be determined from the observed fringe patterns. The sensitivity vector \( \mathbf{K} \), appearing in Eq. (1), is defined as a difference between the observation vector \( \mathbf{K}_2 \) and the illumination vector \( \mathbf{K}_1 \)

\[
\mathbf{K} = \mathbf{K}_2 - \mathbf{K}_1.
\]

In this way, for each observation of the image reconstructed from the hologram, one equation of the type of Eq. (1) can be written relating the observation-illumination geometry and the fringe orders to the unknown displacement vector. Generally, four observations are required in order to determine the vectorial motion of an object and the constant \( \Omega_0 \). However, it is common in holographic analysis to use more than four observations in order to reduce experimental errors. In cases such as this, one solves for the four parameters that yield the least square error, in an attempt to satisfy the overdetermined set of equations that is generated from the excess data.

Using the method of multiple observations, the image reconstructed from the hologram can be observed as shown in Fig. 3. If the first observation is made along the direction defined by the vector \( \mathbf{K}_2 \), then an equation relating the arbitrary constant \( \Omega_0 \) to the scalar product of the unknown object displacement \( \mathbf{L} \) with the sensitivity vector \( \mathbf{K}_1 \)

\[
\mathbf{K}_1 \cdot \mathbf{L} = \Omega_1 = \Omega_0,
\]

where, according to Eq. (2),

\[
\mathbf{K}_1 = \mathbf{K}_2 - \mathbf{K}_1.
\]

Next, the view is changed from observation along the \( \mathbf{K}_2 \) direction to that along the \( \mathbf{K}_3 \) direction. During this change, while continuously observing the object, the number of fringes that pass across the point of interest on the object is determined. In this way, the observed fringe order, more commonly known as the fringe shift \( n_{1,2} \), is determined. This observed fringe order relates to the change \( \Delta \Omega_{1,2} \) in the fringe-locus function; that is,

\[
\Delta \Omega_{1,2} = 2\pi n_{1,2},
\]

thus giving

\[
\mathbf{K}_3 \cdot \mathbf{L} = \Omega^2 = \Omega_0 + \Delta \Omega_{1,2}.
\]

Following the same procedure, the observed fringe orders \( n_{1,m} \) for other positions in observation from \( \mathbf{K}_2 \) to the \( \mathbf{K}_m \) direction can be made, and relationships equivalent to Eqs. (4), (5), and (6) can be written as

\[
\mathbf{K}_m = \mathbf{K}_2^m - \mathbf{K}_1,
\]

\[
\Delta \Omega_{1,m} = 2\pi n_{1,m},
\]

and

\[
\mathbf{K}_m \cdot \mathbf{L} = \Omega^m = \Omega_0 + \Delta \Omega_{1,m}.
\]

respectively, where \( m = 1, 2, \ldots, r \), with \( r \) being the total number of observations. It should be noted that for \( m = 1 \) Eq. (7) is equal to Eq. (4), Eq. (8) gives \( \Delta \Omega_{1,1} = 0 \), and, finally, Eq. (9) yields Eq. (3). In this approach, however, \( r \) has to be equal to or greater than four (i.e., \( r \geq 4 \)) because in addition to the three unknown components of the displacement vector \( \mathbf{L} \) the unknown constant \( \Omega_0 \) must also be determined.

Equation (9) can be rearranged, separating terms involving the unknown parameters from those composed of the known variables, to obtain

\[
\mathbf{K}_m \cdot \mathbf{L} - \Omega_0 = \Delta \Omega_{1,m}, \quad m = 1, 2, \ldots, r.
\]

Since \( \mathbf{L} \) and \( \Omega_0 \) are common to all \( r \) equations, the set of \( r \) equations,
given by Eq. (10), can be combined into a single matrix equation:

\[
\begin{bmatrix} K_1 - I \end{bmatrix} \begin{bmatrix} L \\ \Omega_0 \end{bmatrix} = \Delta \Omega ,
\]

(11)

where \( [K_1 - I] \) is a rectangular \( r \times 4 \) matrix, \( \begin{bmatrix} L \\ \Omega_0 \end{bmatrix} \) is a column \( 4 \times 1 \) matrix, and \( \Delta \Omega \) is a column \( r \times 1 \) matrix. Defining the \( [K_1 - I] \) matrix of sensitivity vectors as \( G \), that is,

\[
[K_1 - I] = G ,
\]

(12)

Eq. (11) can be written as

\[
G \begin{bmatrix} L \\ \Omega_0 \end{bmatrix} = \Delta \Omega .
\]

(13)

Finally, Eq. (13) can be solved to obtain the displacement vector \( L \) and the constant \( \Omega_0 \); i.e.,

\[
\begin{bmatrix} L \\ \Omega_0 \end{bmatrix} = [G^T G]^{-1} (G^T \Delta \Omega) ,
\]

(14)

where \( \Omega_0 \) is equivalent to the fringe order that would have been assigned to the fringe passing the point of interest on the object, while observing it along the direction \( K_1 \), had the zero-order fringe been identifiable.

However, in in vivo biomedical applications of hologram interferometry, the displacements obtained from Eq. (14) may contain rather complex information relating to desired object motions superposed onto undesired motions such as those due to rigid-body motions. To assure that the measured motions are correct, additional computations determining magnitudes and directions of the unwanted motions must be performed, as discussed in Sec. 3.3.

### 3.2. Determination of strains and rotations

Currently, there are a number of methods for strain measurement from holograms, including the optoelectronic fringe interpretation method,\(^4\) the fringe localization method,\(^5\) and the fringe vector method.\(^3\)\(^6\)\(^7\) For the results presented in this paper, the fringe vector method was found to be most applicable.

The fringe vector method recognizes that any combination of homogeneous strain, shear, and rotation of an object yields fringes on its surface that can be described by a single vector. If an object undergoes a homogeneous deformation and/or rotation while recording a hologram, then, during the reconstruction, the object will be seen covered by a pattern of fringes that would appear to be generated along the lines of intersection of the object's surface with a set of parallel, equally spaced planes called fringe-locus planes. The fringe locus planes are uniquely defined by the fringe vector whose magnitude is inversely proportional to the spacing between these planes and whose direction is normal to them. As such, the fringe vector \( K_f \) can be expressed in terms of the matrix \( f \) of strains, shears, and rotations of the object and the first-order variations of the sensitivity vector described by the matrix \( g \) as follows:

\[
K_f = K_f g + L g ,
\]

(15)

where \( g \) is given by

\[
g = \frac{k}{K_0} P_2 - \frac{k}{R_1} P_1 .
\]

(16)

In Eq. (16), \( R_1 \) and \( R_0 \) are radii of curvature of illumination and observation perspectives, respectively, while \( P_1 \) and \( P_2 \) represent corresponding projection matrices.\(^3\)

The projection matrices used in Eq. (16) are based on the directions of illumination and observation, respectively. For example, the projection matrix \( P_2 \) relates to \( K_2 \) as follows:

\[
P_2 = -\mathbf{\hat{K}}_2 \mathbf{\hat{K}}_2 = \mathbf{I} - \mathbf{\hat{K}}_2 \mathbf{\hat{K}}_2 ,
\]

(17)

where \( K_2 \) indicates a \( 3 \times 3 \) antisymmetric matrix of the unit observation vector \( \mathbf{\hat{K}}_2 \), \( I \) is a \( 3 \times 3 \) identity matrix, and \( \mathbf{\hat{K}} \) denotes a matrix product of \( K_2 \) with itself. The operation \( K_2 \mathbf{\hat{K}}_2 \) yields a matrix whose elements are all nine possible products of the three components of \( K_2 \).

What is of interest in Eq. (15) is the matrix \( f \), which can be decomposed into a matrix of strains and shears, \( \epsilon \), and a matrix of rotations, \( \theta \). In order to solve Eq. (15) for the transformation matrix \( f \), multiple observations of the holographically produced image must be made. For each observation the sensitivity vector \( K \) and the fringe vector \( K_f \) must be determined that best fit the data from the entire region examined. Also, multiple views are used to obtain the displacement \( L \) at a point of interest on the object (see Sec. 3.1). For each view, the matrix \( g \) is computed and multiplied by \( L \) to obtain perspective correction to \( K_f \). From multiple views, a set of equations of the type of Eq. (15), with the matrix \( f \) common to all, is generated and solved to obtain

\[
f = [K_f K]^{-1} [K_f f_c] ,
\]

(18)

where \( K_{fc} = K_f - L g \) is the matrix formed by the fringe vectors corrected for perspective. Decomposition of the matrix \( f \), computed from Eq. (18), into the symmetric part \( \epsilon \) and the antisymmetric part \( \theta \), that is,

\[
\epsilon = \frac{1}{2} [f + f^T] , \quad \theta = \frac{1}{2} [f - f^T] ,
\]

(19)

gives strains and shears, and rotations, respectively.

When the object deformations are not homogeneous over the entire body under study, they may, nonetheless, be approximately so over small regions of its surface, and projection matrices are very helpful in formulating the solution to this problem. It can be shown that, in this case, the surface strain-rotation matrix \( f_s \) is

\[
f_s = f_p n ,
\]

(20)

where \( P_n \) is the projection matrix defined as \( P_n = I - \mathbf{\hat{n}} \otimes \mathbf{\hat{n}} \), with \( \mathbf{\hat{n}} \) being surface normal. It should be noted that the derivatives of the observed displacement are not generally equal to surface strains and rotations of an object. They become approximately equal to the extent that the viewing direction can be made parallel to the surface normal.\(^6\) This condition is impossible to achieve on any surface that exhibits three-dimensional contours, and it is difficult to achieve even on a flat surface because of the spherical perspective of most viewing systems. However, the derivatives of observed displacement from two or more viewing directions can be used to extract surface strains and rotations.

### 3.3. Determination of rigid-body motions

Although in in vivo biomedical applications of hologram interferometry pulsed lasers are used, they do not preclude formation of fringes due to rigid-body motions. It is not the exposure time (duration of a pulse), which is usually 15 to 20 ns, but the time interval between the exposures, normally four or more orders of magnitude longer, that plays an important role in the recording of an in vivo hologram. Although subjects used in biomedical studies are usually restricted in specially designed holders, they cannot remain absolutely still even for a fraction of a second, when one is concerned with the "stillness" defined in terms of a wavelength of laser light. Therefore, rigid-body motions will take place naturally. These motions add directly to the motions one wishes to study. Hence, not accounting for these unwanted motions may lead to erroneous results.

To account for the unwanted motions, parameters defining fringe patterns in areas not affected by the applied forces must be investigated (Fig. 4). As a result of analysis of these data, vectors defining all six degrees of freedom, that is, three components of the rigid-body translation vector and three components of the rigid-body rotation...
vector, are determined. These components, together with the vectors defining total motions of the subject, are used to compute relative motions due to the applied force system only, yielding desired force-displacement relationships, as follows.

The displacement \( \mathbf{L}_i \), computed from Eq. (14), represents total motion of the object under investigation. This total motion results from superposition of object motions induced by the applied external force (e.g., tooth motion due to orthodontic forces) and the natural motions (e.g., gross head motion). Therefore, in order to develop meaningful relationships between the applied force system and the resulting biomedical remodeling, the magnitudes and directions of the rigid-body motions must be known. This can be achieved by resolving the motion \( \mathbf{L}_i \) at any point \( i \) on the object into the corresponding bulk body translation \( \mathbf{L}_0 \) and rotation \( \theta \); that is,

\[
\mathbf{L}_i = \mathbf{L}_0 + \mathbf{R}_i \times \theta , \quad i = 1, 2, \ldots, q .
\]

(21)

In Eq. (21), \( q \) denotes the total number of object points considered in calculations, while \( \mathbf{R}_i \) is the position vector from the origin of an arbitrarily chosen coordinate system to the \( i \)th point on the object and is defined as

\[
\mathbf{R}_i = x_i \hat{i} + y_i \hat{j} + z_i \hat{k} , \quad i = 1, 2, \ldots, q ,
\]

(22)

with \( x_i, y_i, \) and \( z_i \) being components of \( \mathbf{R}_i \) in directions of the unit vectors \( \hat{i}, \hat{j}, \) and \( \hat{k} \) of the rectangular coordinate system, respectively.

The set of \( q \) equations given by Eq. (21) can be written in a matrix form as

\[
\begin{bmatrix}
\mathbf{L}_1 \\
\mathbf{L}_2 \\
\vdots \\
\mathbf{L}_q 
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{I} & \mathbf{R}_1 \\
\mathbf{I} & \mathbf{R}_2 \\
\vdots & \vdots \\
\mathbf{I} & \mathbf{R}_q
\end{bmatrix}
\begin{bmatrix}
\mathbf{L}_0 \\
\theta
\end{bmatrix} ,
\]

(23)

and solved to obtain

\[
\begin{bmatrix}
\mathbf{L}_0 \\
\theta
\end{bmatrix} = \left[ \begin{bmatrix}
\mathbf{I} \\
\mathbf{R}_1 \\
\vdots \\
\mathbf{R}_q
\end{bmatrix}^T \begin{bmatrix}
\mathbf{I} \\
\mathbf{R}_1 \\
\vdots \\
\mathbf{R}_q
\end{bmatrix} \right]^{-1} \begin{bmatrix}
\mathbf{I} \\
\mathbf{R}_1 \\
\vdots \\
\mathbf{R}_q
\end{bmatrix}^T \mathbf{L}_i ,
\]

(24)

where \( \mathbf{I} \) is the \( 3 \times 3 \) identity matrix, while \( \mathbf{R}_i \) is the \( 3 \times 3 \) antisymmetric matrix corresponding to the position vector \( \mathbf{R}_i \) defined in Eq. (22).

Using Eq. (24) and considering three or more points on the object, the \( 6 \times 1 \) matrix \( \begin{bmatrix} \mathbf{L}_0 \\ \theta \end{bmatrix} \) can be determined from holograms. If it is also possible to determine total translation \( \mathbf{L}_p \) and total rotation \( \theta_p \) at a point \( P \) on the studied object (see Fig. 4), using, for example, procedures developed in Refs. 6 and 10, then motions \( \mathbf{L}_p \text{rel} \) and \( \theta_p \text{rel} \) at this point, relative to the rest of the object, can be determined. In order to do so, first, the rigid-body translation \( \mathbf{L}_p \text{rig} \) at the point \( P \) on the object must be determined using an equivalent of Eq. (23), that is,

\[
\mathbf{L}_p \text{rig} = [\mathbf{I} \mathbf{R}_p] \begin{bmatrix} \mathbf{L}_0 \\ \theta \end{bmatrix} ,
\]

(25)

where \( \mathbf{L}_0 \) and \( \theta \) have values determined from Eq. (24) and \( \mathbf{R}_p \) is a \( 3 \times 3 \) antisymmetric matrix defining the position vector \( \mathbf{R}_p \). Then,

\[
\mathbf{L}_p \text{rel} = \mathbf{L}_p - \mathbf{L}_p \text{rig} .
\]

(26)

Noting that the rigid-body rotations are independent of the position, \( \theta \) (computed from Eq. (24)) is subtracted directly from total rotation \( \theta_p \) to obtain

\[
\theta_p \text{rel} = \theta_p - \theta .
\]

(27)

Equations (26) and (27) allow determination of translations and rotations at any point on the object relative to the rest of the body.

4. EXPERIMENTAL PROCEDURE

Typically, a tissue acted upon by a force displaces continuously over a certain period of time. Eventually, the tissue reaches a new equilibrium when all external forces are balanced by all internal forces. For practical reasons it is not necessary to record holograms monitoring the tissue over the entire period during which changes take place because a great many holograms would have to be recorded. Therefore, in biomedical applications of hologram interferometry, to reduce the number of holograms that have to be recorded and associated exposures are made at predetermined times (Fig. 5). Then, the results obtained from the corresponding sequence of double-exposure holograms are used to reconstruct the total "monotonic" change experienced by the tissue.

For example, if a hypothetical time-displacement curve for a given point on a tissue were like that shown in Fig. 5, one might choose to record holograms at times of 0 s (i.e., at the instant of application of force), 15 s, 30 s, 45 s, etc., or at some other instants in time covering the entire time period of interest in a given study. Then, knowing the time between the exposures of double-exposure pulsed holograms and the corresponding changes in displacement vectors, instantaneous velocities at the given point can be computed (Fig. 6). In this way, velocity versus time curves, defining temporal characteristics of tissue, can be obtained. Finally, integrating areas under the velocity versus time curves, position (displacement, or motion) at the given point on the tissue can be determined. The resulting vectors give temporal characteristics of shape, size, or other dimensional changes that are all determined in three-dimensional space.

5. COMPUTER-AIDED INTERPRETATION OF HOLOGRAMS

In biomedical applications of hologram interferometry, coordinates of points of interest are known from recording/reconstruction geometry. To use these coordinates effectively in quantitative interpretation of holograms, the points of interest within the studied scene must be "recognized" and identified during the reconstruction of holograms. This process can be performed manually or automatically. In the case in which image analysis is done manually, processing is limited to only a few points because of tediousness of the...
For quantitative interpretation of holograms in biomedical applications, it is natural to use methods of pulsed hologram interferometry while studying its motions. It leads to development of a fully automated system for complete analysis of displacements and deformations at any point on the surface of the studied objects.

6. REPRESENTATIVE APPLICATIONS

To demonstrate the usefulness of double-exposure hologram interferometry, representative applications from the field of dentistry, or more specifically, orthodontics, will be described. The cases used deal with determination of three-dimensional tooth motions and with measurement of displacements of skull bones.

Accurate measurement of tooth motion is a significant factor in evaluation of bone loss in periodontal disease, the tooth response to traumatic occlusion, and tooth response to orthodontic and prosthetic appliances. The biological reaction of the periodontal ligament is determined by stress-strain distributions produced from natural and artificial forces applied at a crown of the tooth. Since tooth response to applied forces takes place in three-dimensional space, it is natural to use methods of pulsed hologram interferometry while studying its motions.12–14

Typical images obtained during reconstruction of double-exposure holograms are shown in Figs. 8 and 9. The fringe patterns are unique for the given object and applied force system. Interpretation of the observed fringe patterns yields, therefore, unique results corresponding to the given experimental situation.

For example, Fig. 10 shows typical results obtained during studies of motions of teeth, which were loaded with forces in the lingual-labial direction normal to the long axis.14 In this study, 300 gram force was applied to the maxillary central incisor. Following application of the force, a sequence of double-exposure pulsed holograms was recorded. Analysis of these holograms resulted in linear and angular velocities. Using the experimentally determined velocity curves, total translations and total rotations were computed and are
Fig. 8. Image reconstructed from a double-exposure hologram of a model of human teeth. The fringe pattern corresponds to a load applied to the maxillary central incisor.

Fig. 9. Image reconstructed from a double-exposure hologram of a human skull. The fringe pattern corresponds to a load applied at the maxilla.

Fig. 10. Typical results obtained during holographic studies of human tooth motions. (a) Translation versus time curves; (b) rotation versus time curves.

Fig. 11. Typical results obtained during holographic studies of human skull bone motions due to forces applied at the maxilla. (a) Displacement versus force curves for a point located on the maxilla; (b) displacement versus force curves for a point located on the zygoma.
shown in Figs. 10(a) and 10(b), respectively.

Figure 10(a) shows that the translations at the end of 45 s after the instant of tooth loading varied from 61 µm to 80 µm in the lingual-labial direction, depending on the subject's root length (the root lengths, as determined radiographically using lateral head films and occlusal films, were 14.5 mm, 11.3 mm, and 15.7 mm, for subjects No. 1, 2, and 3, respectively)—the shorter the root, the greater the translation. Examination of Fig. 10(a) reveals that the teeth were moving at a velocity decreasing nonlinearly with time, and at the time of the last exposure, at 45 s, they still continued to displace. This means that the teeth had not yet reached the state of static equilibrium. Small displacements in the mesial-distal direction, as shown in Fig. 10(a), suggest good alignment of the force in the direction normal to the x-y plane. The displacements along the y-axis, ranging from 10 µm to 17 µm, were associated with the rotations of the corresponding teeth [Fig. 10(b)]. These rotations were also inversely proportional to the length of the root of the tooth and were primarily in the counterclockwise direction with respect to the x-axis. The rotations with respect to the y- and z-axes were small.

In another representative application, motions of skull bones were studied while the skull was loaded at the maxilla. The resulting fringe patterns, Fig. 9, were analyzed using procedures accounting for rigid-body motions, to obtain results shown in Fig. 11.

Displacements of points on the maxilla and on the zygoma are shown in Figs. 11(a) and 11(b), respectively. These results clearly show that for the same load, applied at the maxilla, different points on the skull respond in different manners, depending on their location on the object. However, regardless of the location of the investigated point, the relationships between the force and the resulting displacements were always nonlinear.

Of particular interest in studies of skulls is the effect of sutural expansions on the motion of bones. Methods of hologram interferometry are particularly useful in these studies because they allow observation of the response of the entire skull to a given load. More specifically, use of pulsed laser methods allows studies of viscoelastic effects that tissues have on force-displacement relationships of bones.

7. CONCLUSIONS

The methods of pulsed double-exposure hologram interferometry presented in this paper allow accurate, noninvasive quantification of motions of bones and tissues in three-dimensional space. Procedures that allow mathematical interpretation of the fringe patterns observed during reconstruction of holograms are complex, but solvable with the aid of digital computers. The results obtained yield information on objects' translations, rotations, and deformations. In the analysis of holograms, the necessary data can be obtained directly from the reconstructed images using automated video-digitizer systems. Use of the automated system for interpretation of holograms allows more detailed analysis of images than is possible with methods relying on manual acquisition of data.

8. REFERENCES