**TOGGLE LINKAGE EQUATIONS**

**I. PER MARKS, Pg 859**:

\[ P = \frac{F \cdot s \cdot \cos \alpha}{t} \]

**Simplifying Approximation:**

Take \( F \) as always perpendicular to the toggle \( \phi \):

Then \( s = R \cos \phi \)

And \( t = R \cos [90 - \alpha - \phi] = R \cos [90 - (\alpha + \phi)] \)

\[ t = R \sin (\alpha + \phi) \]

**Substituting:**

\[ P = \frac{F (R \cos \phi) \cos \alpha}{R \sin (\alpha + \phi)} \]

\[ P = \frac{F \cos \beta \cos \alpha}{\sin \alpha \cos \beta + \cos \alpha \sin \beta} \]

**Divide numerator & denominator first by \( \cos \beta \), then by \( \cos \alpha \):**

\[ P = \frac{F}{\tan \alpha + \tan \beta} \] (Approximate)

**II. Full Derivation of the Approximate Solution:**

**Resolving \( F \) into components**

A & B, and looking at \( \alpha \) & \( \beta \)

**Components of A & B:**

\[ F = A_x + B_x = A \sin \alpha + B \sin \beta \]

\[ A_y = B_y; A \cos \alpha = B \cos \beta \]

\[ B = A \ \frac{\cos \alpha}{\cos \beta} \]

\[ F = A \sin \alpha + A \ \cos \alpha \ \tan \beta \]

\[ F = A \sin \alpha + A \ \cos \alpha \ \tan \beta \]

**Now, \( P \) must equal \( A \cos \alpha \), so that \( A = \frac{P}{\cos \alpha} \)**

\[ F = \frac{P}{\cos \alpha} \sin \alpha + P \tan \beta = PTan \alpha + P \tan \beta \]

**\( \therefore \) \[ P = \frac{F}{\tan \alpha + \tan \beta} \] (As Above)**
TOGGLE LINKAGE EQUATIONS

1. PER MARKS, PG 859:

\[ P = \frac{FS \cos \alpha}{t} \]

SIMPLIFYING APPROXIMATION:

TAKE \( F \) AS ALWAYS PERPENDICULAR TO THE TOGGLE \( L \):

THEN \( S = R \cos \beta \)

AND \( t = R \cos [90 - \alpha - \beta] = R \cos [90 - (\alpha + \beta)] \)

\[ t = R \sin (\alpha + \beta) \]

SUBSTITUTING:

\[ P = \frac{F(R \cos \beta) \cos \alpha}{R \sin(\alpha + \beta)} \]

\[ P = \frac{F}{\frac{\cos \beta \cos \alpha}{\sin \alpha \cos \beta + \cos \alpha \sin \beta}} \]

DIVIDE NUMERATOR & DENOMINATOR FIRST BY \( \cos \beta \), THEN BY \( \cos \alpha \):

\[ P = \frac{F}{\tan \alpha + \tan \beta} \] (APPROXIMATE)

II. FULL DERIVATION OF THE APPROXIMATE SOLUTION:

RESOLVING \( F \) INTO COMPONENTS

\( A \& B \), AND LOOKING AT \( x \& y \)

COMPONENTS OF \( A \& B \):

\[ F = A_x + B_x = A \sin \alpha + B \sin \beta \]

\[ A_y = B_y \]

\[ \frac{A \cos \alpha}{\cos \beta} \]

\[ F = A \sin \alpha + A \frac{\cos \alpha \sin \beta}{\cos \beta} \]

\[ F = A \sin \alpha + A \cos \alpha \tan \beta \]

NOW, \( P \) MUST EQUAL \( A \cos \alpha \), SO THAT \( A = \frac{P}{\cos \alpha} \)

\[ F = \frac{P \cos \alpha}{\sin \alpha + P \tan \beta} = P \tan \alpha + P \tan \beta \]

OR \[ P = \frac{F}{\tan \alpha + \tan \beta} \] (AS ABOVE)